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USE OF LOGARITHMIC DECREMENT OF OSCILLATION DAMPING FOR PREDICTION OF THE AVIATION STRUCTURES SERVICE LIFE

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Problem of predicting the residual service life of airplanes and helicopters is highly relevant for flight safety. In this paper, on the basis of the conducted research on the change of mechanical characteristics during materials fatigue accumulation, it is proposed to control the service life by changing the dissipative characteristics. In case of fatigue damage, the accumulative logarithmic decrement of oscillation damping δ *increases to the limit maximum value* δ_m, *which corresponds to the critical length of the main fatigue crack, which leads to failure. The limit value* δ_m *can be set depending on the amount of energy spent on the development of the main fatigue crack, taking into account the dangerous part of the consumed energy. With the accumulation of fatigue damage, the growth of logarithmic decrement occurs at the expense of energy expenditure for the growth of fatigue cracks and internal friction. This is taken into account by the coefficient* $\boldsymbol{\alpha}$, *which allows to allocate a dangerous part of the energy that goes into the development of a main fatigue crack. The problem of durability prediction consists of two stages. At first, it is needed to determine* δ_m *for the critical crack length. Then, based on the two values of logarithmic decrement at the corresponding load cycles, the number of cycles to failure – to the critical length of the crack – is predicted by the Peris formula.*

Keywords: *service life, logarithmic decrement, energy expended to destruction, "dangerous" part of the dissipated energy, critical crack length.*

Introduction

The problem of predicting the residual life of airplanes and helicopters is very relevant for flight safety because it is necessary to know when to stop flights due to insufficient strength of the structure. At the National Aerospace University "KhAI", studies on the change of mechanical characteristics of materials and structures during the accumulation of fatigue damage were conducted. Changes in the modulus of elasticity E, Poisson's ratio μ, natural oscillation frequency ω, and the logarithmic decrement (LD) of oscillation damping δ were studied. All of them change with the fatigue accumulation, but the changes occur to different degrees. The modulus of elasticity, Poisson's ratio, natural oscillation frequency change by 5–10%. The LD δ increases by 60–80%. Equipment that allows to measure this value quickly and simply, without any sensors, has been developed during research. Therefore, based on the results of our research, it is suggested to control the service life by changing the dissipative characteristics of LD.

Determination of fatigue energy parameters and critical crack length

In case of fatigue damage, accumulative LD δ increases to the limit value δ_m. The limit value δ_m corresponds to the critical length of the main fatigue crack. The limit value δ_m is set depending on the amount of energy spent on the development of the main fatigue crack, taking into account the harmless part of the consumed energy.

With the accumulation of fatigue damage, the growth of LD occurs at the expense of energy expenditure for the growth of fatigue cracks and internal friction. This is taken into account by the coefficient α [1], which allows to allocate a dangerous part of the energy that goes to the main fatigue crack development.

The problem of durability prediction consists of two stages. The first stage is the determination of the critical crack length. Then, based on the two values of LD at the corresponding operating load cycles, the number of cycles to failure – to the critical length of the crack – is predicted by the Peris formula.

In [1] it is shown that for metals, regardless of the number of cycles to destruction, the share of energy spent on destruction W_d, is constant

$$W_d = N_f \left[W_{tc} - W_{-1} \left(\frac{W_{tc}}{W_{-1}} \right) \right], \tag{1}$$

where N_f is the number of cycles to failure; W_{tc} is a total energy that is irreversibly dissipated during load cycles; W_{-1} is the energy dissipated per cycle irreversibly with a stress equal to the fatigue limit (for the base $N_f=10^7$ symmetrical load cycles); α is the fatigue failure parameter.

Energy W_d is called the "dangerous" part of the dissipated energy from deformation cycles leading to fatigue failure.

LD δ due to the energy dissipated in the sample, the structure and the total energy of elastic oscillations is determined as follows:

$$\delta = \frac{\Delta W_c}{2W_c}, \tag{2}$$

where W_c is the energy of elastic deformation of the body during the load cycle; ΔW_c is the total dissipated irreversible energy per load cycle.

At destruction, the limit value δ_m is equal to the sum of the initial value of LD δ_0 to fatigue load, increase due to dangerous side scattered energy δ_d and safe part of dissipated energy δ_n

$$\delta_m = \delta_0 + \delta_d + \delta_n .$$

The amount of energy that is spent on the fatigue crack development during one load cycle ΔW_d, can be found as [2]

$$\Delta W_d = \Delta G_1 \cdot \Delta F , \tag{3}$$

where $\Delta F=\Delta h \cdot \Delta l$ is the increase in fatigue crack area; Δh, Δl are accordingly, an increase in the fatigue crack depth and length; ΔG_1 is the energy required for the development of a crack surface unit.

The wing, fuselage, and strut panels operate in a plane stress state for which this energy ΔG_1 is defined as follows [2]:

$$\Delta G_1 = \frac{(\Delta K_1)^2}{E},$$

where $\Delta K_1=K_{1\,\max}-K_{1\,\min}$ is the amplitude of the stress intensity factor; E is the modulus of elasticity.

Formula (1) considers a symmetrical load cycle, for which it is set that $K_{1\,\min}=0$ [2], then $\Delta K_1=K_{1\,\max}$.

The stress intensity factor is defined as [2]

$$K_1 = \sigma \sqrt{\pi l_{cr}} F , \tag{4}$$

where σ is the operating stress; l_{cr} is the crack length; $F = f(\bar{l})$ is the relative crack length; $\bar{l}=l_{cr}/b$ is the plate width; b is the correction factor taking into account the crack length.

By substituting these values in (3), it is possible to determine the increase in dangerous energy

$$ddW = \frac{\sigma^2 \pi l_{cr} F^2}{E} dl\, dh . \tag{5}$$

After integration, (5) can be obtained in general form

$$W_d = \frac{\sigma^2 \pi}{E} \varphi(l_0, l_{cl})(h_{cl} - h_0) ,$$

where l_0 is the initial crack length; l_{cl} is the critical fatigue crack length; h_{cl} is the critical fatigue crack depth; h_0 is the initial crack depth; function $\varphi(l_0, l_{cl}) = (h_{cl} - h_0) \int_{l_0}^{l_{cl}} l_{cr} F^2 dl = t \int_{l_0}^{l_{cl}} l_{cr} F^2 dl$.

In the first approximation, it can be assumed that $h_0=0$, $h_{cl}=t$, where t is the cladding thickness.

The limit value of the "dangerous" energy and, accordingly, the LD is determined by the fatigue crack critical length. Critical crack length l_{cl} can be found by the formula [2]

$$l_{cl}=\frac{1}{\pi F^2}\left(\frac{K_{1c}}{\sigma_{02}}\right)^2,$$

where K_{1c} is the critical coefficient of stress intensity; σ_{02} is the yield strength.

Only part of the energy is spent on destruction, the other part of the energy is spent on internal friction. Full growth of LD $\Delta\delta_t$ consists of two parts:

$$\Delta\delta_t=\delta_d+\delta_n,$$

Full LD δ, caused by crack growth, is obtained according to (2).

The ratio of "dangerous" energy per load cycle to all absorbed energy per cycle can be obtained from relation (1)

$$\frac{W_{dc}}{W_{tc}}=1-\left(\frac{W_{-1}}{W_{tc}}\right)^{\beta}, \tag{6}$$

where W_{dc} is the "dangerous" energy absorbed per load cycle; W_{tc} is the total energy absorbed per load cycle; $\beta=1-\alpha$.

On the basis of (2) and (6), the total increase of LD $\Delta\delta_t$ can be determined

$$\Delta\delta_t=\frac{\delta_d}{1-\left(\frac{W_{-1}}{W_{tc}}\right)^{\beta}}. \tag{7}$$

The unknown fatigue failure parameters of the material W_{-1} and β can be determined from experiments during fatigue tests [3, 4, 5] taking into account (7) at different stress levels

$$\frac{\delta_d}{\delta_{c1}}=1-\left(\frac{W_{-1}}{W_{tc1}}\right)^{\beta};\ \frac{\delta_d}{\delta_{c2}}=1-\left(\frac{W_{-1}}{W_{tc2}}\right)^{\beta},$$

where δ_{c1}, δ_{c2} are just like LD, are measured in fatigue experiments at different stress levels; W_{tc1}, W_{tc2} is the total energy absorbed per load cycle at various voltage levels corresponding to LD.

Prediction of total durability

The durability prediction is based on the fatigue crack growth equation [2]

$$\frac{dl}{dN}=C(\Delta K)^m,$$

where C and m are constants determined experimentally; ΔK is the stress intensity factor amplitude.

As mentioned earlier, $\Delta K_1=K_{1\max}$ is taken for a symmetrical cycle

$$\frac{dl}{dN}=C(K_{1\max})^m. \tag{8}$$

In this expression we have two unknown parameters C and m.

The value $K_{1\max}$ is taken from (4) and substituted into (8)

$$\frac{dl}{dN}=C\cdot\sigma_a^m\cdot\pi^{0.5m}\cdot l_{cr}^{0.5m}\cdot F^m.$$

Separating the variables and integrating gives

$$\int_{l_0}^{l_{cl}}\frac{dl}{l_{cr}^{0.5m}\cdot F^m}=C\cdot\sigma_a^m\cdot\pi^{0.5m}\int_{N_0}^{N_f}dN,$$

where l_0 is the length of the inchoate crack; N_0 is the number of cycles before the appearance of an inchoate fatigue crack; N_f is the number of cycles to failure; σ_a is the stress amplitude.

This equation can be transformed into the following form

$$\frac{\Psi(l_{cl},m)}{C\cdot\pi^{0.5m}}=\sigma_a^m\cdot N_f\,, \tag{9}$$

where $\Psi(l_{cl},m)=\int\limits_{l_0}^{l_{cl}}\frac{dl}{l_{cr}^{0.5m}\cdot F^m}$.

On the basis of (9) N_f can be defined as follows:

$$N_f=\frac{\Psi(l_{cl},m)}{C\cdot\sigma_a^m\cdot\pi^{0.5m}}\,. \tag{10}$$

To find the values C and m, at least two LD measurements must be taken for different operating hours. Value C can be obtained from (10)

$$C=\frac{\Psi(l_{cr},m)}{\sigma_a^m\cdot\pi^{0.5m}\cdot N_i}\,, \tag{11}$$

where l_{cr} is the current crack length; N_i is the current number of load cycles.

From equation (11), C for two measurements should be

$$\frac{\Psi_1(l_{cr1},m)}{\Psi_2(l_{cr2},m)}=\frac{N_1}{N_2}\,,$$

where Ψ_1, Ψ_2 are values $\Psi(l_{cr}, m)$ for two numbers of load cycles; l_{cr1}, l_{cr2} are crack lengths for two numbers of cycles.

Accordingly, the value m can be determined from the equation

$$1-\frac{N_1}{N_2}=\left(\frac{\delta_0}{\delta_1}\right)^{0.5(0.5m-1)}-\frac{N_1}{N_2}\left(\frac{\delta_0}{\delta_2}\right)^{0.5(0.5m-1)}.$$

It should be emphasized that the value m is not a constant material characteristic, but a constant value for a specific part or structural element.

This allows to find the number of cycles to failure N_f from the following equation:

$$N_f=N_i\frac{1-\left(\frac{\delta_0}{\delta_d}\right)^{0.5(0.5m-1)}}{1-\left(\frac{\delta_0}{\delta_i}\right)^{0.5(0.5m-1)}}\,, \tag{12}$$

where N_i is the number of cycles at the i-th testing stage; δ_i is LD on the i-th testing stage; δ_d is the value LD, which corresponds to "dangerous" energy that leads to destruction and is found according to formula (7).

According to the values of the full service life (12) and the current value of the fatigue load, the residual resource N_{res} is found

$$N_{res}=N_f-N_i\,,$$

where N can be measured in load cycles for samples, in load programs during tests of aircraft components, aggregates and structures, in flight hours during the operation of aircraft and helicopters.

Conclusion

A method of prediction of the aircraft structures residual resource by changing the logarithmic decrement of the natural oscillations damping has been developed. This technique was verified when testing samples from alloys 2024, Ti-6A1-4B, steel 449A. Experimentally, according to the methodology developed at KhAI; α is the parameters of fatigue failure were determined. Experiments on the durability prediction were carried out at the KhAI on samples that simulated the work on the wing and feathers bending. For this purpose, the samples were also tested for bending. The results of the tests and predictions are shown in the table.

The experiments analysis shows that at the early stages of testing, the prediction overestimates durability by 10–15%, the error gradually decreases, and at the last stage of testing, durability is underestimated by a margin of 4–5%. It is necessary to conduct a sufficient number of experiments on full-scale structures to verify this technique and implement it in practice.

Table. Durability prediction on alloy 2024 samples

Oscillations amplitude, mm	Number of load cycles $N_c \cdot 10^4$	δ_i	Durability $N_c \cdot 10^4$	
			Calculated	Experimental
19	0	0.0184	–	–
	2	0.0202	11.80	–
	4	0.0225	11.54	–
	6	0.0258	11.45	–
	8	0.0311	10.30	–
	10	0.0388	10.00	10.44
25	0	0.0207	–	–
	1	0.0225	5.84	–
	2	0.0247	5.76	–
	3	0.0275	5.64	–
	4	0.0316	5.45	–
	5	0.0385	5.00	5.22

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Використання логарифмічного декременту згасання коливань для прогнозування ресурсу авіаційних конструкцій

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Проблема прогнозування залишкового ресурсу літаків і вертольотів є дуже актуальною з точки зору безпеки польотів. У даній роботі на базі проведених досліджень зміни механічних характеристик при накопиченні втоми матеріалів пропонується контролювати строк служби по зміні дисипативних характеристик.

При втомному пошкодженні накопичувальний логарифмічний декремент згасання коливань δ зростає до граничного максимального значення δ_m, що відповідає критичній довжині магістральної втомної тріщини, яка призводить до руйнування. Граничне значення δ_m може встановлюватися залежно від кількості енергії, витраченої на розвиток магістральної втомної тріщини з урахуванням нешкідливої частини спожитої енергії. Із накопиченням втомних пошкоджень логарифмічний декремент зростає за рахунок витрат енергії на збільшення втомних тріщин і внутрішнього тертя. Це враховується коефіцієнтом α, який дозволяє виокремити небезпечну частину енергії, що йде на розвиток магістральної втомної тріщини. Задача прогнозування довговічності складається з двох етапів. Спочатку визначається δ_m для критичної довжини тріщини. Після цього за двома значеннями логарифмічного декременту при відповідних циклах навантаження за формулою Періса прогнозують кількість циклів до руйнування – до критичної довжини тріщини.

Ключові слова: *ресурс, логарифмічний декремент, енергія, витрачена на руйнування, «небезпечна» частина розсіяної енергії, критична довжина тріщини.*

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