DOI: https://doi.org/10.15407/pmach2023.04.050

UDC 539.3

STUDY OF THE STABILITY OF THE MATHEMATICAL MODEL OF THE COUPLED PENDULUMS MOTION

Yuliia E. Surhanova
surganova.julia@gmail.com
ORCID: 0000-0002-6540-3025

Yurii V. Mikhlin
yuri.mikhlin@gmail.com
ORCID: 0000-0002-1780-9346

National Technical University "Kharkiv Polytechnic Institute"
2, Kyrpychova str., Kharkiv, 61002, Ukraine

The paper presents a study of the dynamics of the oscillatory dissipative system of two elastically connected pendulums in a magnetic field. Nonlinear normal vibration modes of the pendulum system are studied in the paper taking into account the resistance of the medium, and the damping moment created by the elastic element. A system with two degrees of freedom is considered. The masses of the pendulums in that system differ significantly, which leads to the possibility of localization of oscillations. In the following analysis, the mass ratio was chosen as a small parameter. For approximate calculations of magnetic forces, the Padé approximation, which satisfies the experimental data the most, is used. This approximation provides a very accurate description of magnetic excitation. The presence of external influences in the form of magnetic forces and various types of loads that exist in many engineering systems leads to a significant complication in the analysis of vibration modes of nonlinear systems. A study of nonlinear normal vibration modes in this system was carried out, one of the modes is a connected mode, and the second one is localized. Vibration modes are constructed by the multiples scales method. Both regular and complex behavior is studied when changing system parameters. The influence of these parameters is studied for small and significant initial angles of the pendulum inclination. An analytical solution, which is based on the fourth-order Runge-Kutta method, is compared to numerical simulation results. The initial conditions for calculating the vibration modes were determined by the analytical solution. Numerical simulation, which consists of constructing phase diagrams, trajectories in the configuration space and spectra, allows to estimate the dynamics of the system, which can be both regular and complex. The stability of vibration modes is studied using numerical analysis tests, which are an implementation of the Lyapunov stability criterion. The stability of vibration modes is determined by the estimation of orthogonal deviations of corresponding trajectories of vibration modes in configuration space.

Keywords: *coupled pendulums, magnetic forces, nonlinear normal vibration modes, the multiples scales method, stability.*

Introduction

Pendulum models are often used in nonlinear dynamics. One of the most important stages of studying the dynamics of nonlinear systems with several independent variables is the study of nonlinear normal modes of oscillations (NNMs). It is of great importance for engineering applications to establish the possibility of localization of oscillations, which sometimes harms the normal functioning of machines and devices. In addition, it is important and difficult to study the oscillations of systems under the influence of magnetic forces.

In recent papers [1–3], a theoretical and experimental study of the dynamics of two coupled pendulums in a magnetic field was conducted. In addition, the NNMs of oscillations in such a system without taking into account the influence of dissipative forces for the case when the masses of these coupled pendulums differ significantly is considered in [4]. It should be noted that from now on, various aspects of the theory of NNMs and various cases of its application are presented in many papers. The main elements of this theory and references to papers can be found, in particular, in the reviews [5, 6] and the book [7]. It should also be noted that the problem of localization of oscillations is very important both for theory and for engineering practice. In view of this, it has been studied in recent decades in numerous papers, among which it is worth to highlight the [8–10] ones.

Based on the fact that there is a significant nonlinearity in the system, it is necessary to use asymptotic methods to analyze the forms of oscillations. Considering what has been said, the multiples scales method, which can be successfully applied specifically to dissipative systems, as well as numerical simulation, is used. In addition to the construction of vibration modes, their stability was studied, and conclusions about the influence of changes in system parameters on its dynamics were drawn.

Main body

The studied mathematical model of the coupled pendulums movement is shown in Fig. 1, where m_1=μm_2=μm; μ is the ratio of masses of two pendulums; ε is the conditional small parameter; ε=1; m is the mass of the larger pendulum; $k_l^* = \frac{k_l}{I}$; $I = 4ms^2$ is the stiffness of the associated elastic element; $k_l^*(\varphi_1 - \varphi_2)$ is the moment of torsional deformation of an elastic element; γ is the magnetic excitation intensity; $M^*_{mag_{1,2}} = \frac{M_{mag_{1,2}}}{I}$; $M_{mag_{1,2}}$ is the moment of magnetic influence; $C^*_{1,2} = \frac{C_{1,2}}{I}$; $C_{1,2}$ is the coefficient of resistance of viscous air forces (drag forces); $C_e^* = \frac{C_e}{I}$; C_e is the coefficient of the damping moment created by the elastic element; $r^* \sin\varphi$ is the moment of return of gravity; $r^* = \frac{r}{I}$; s is the distance between the center of mass of the pendulum and the rotation axis. The system is described by a system of differential equations (1).

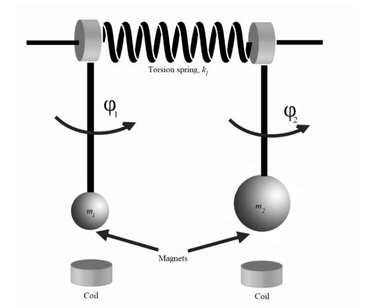


Fig. 1. The coupled pendulums system

$$\begin{cases} \varepsilon\mu\ddot{\varphi}_1 = \varepsilon\gamma M^*_{mag_1} - \varepsilon C_1^*\dot{\varphi}_1 - \varepsilon C_e^*(\dot{\varphi}_1 - \dot{\varphi}_2) - \varepsilon\mu r^* \sin\varphi_1 - k_l^*(\varphi_1 - \varphi_2), \\ \ddot{\varphi}_2 = \varepsilon\gamma M^*_{mag_2} - \varepsilon C_2^*\dot{\varphi}_2 - \varepsilon C_e^*(\dot{\varphi}_2 - \dot{\varphi}_1) - r^* \sin\varphi_2 - k_l^*(\varphi_2 - \varphi_1). \end{cases} \tag{1}$$

It should be noted that the measurement units of the parameters are as follows: m is measured in kg, s – in m, r – in N·m, I – in kg·m^2, k_l – in N·m/rad, φ – in rad (their values in degrees will also be given in brackets, ε and γ – dimensionless quantities, and g=9.81 m/s^2). In numerical calculations, the initial velocities are zero: $\dot{\varphi}_1(0) = \dot{\varphi}_2(0) = 0$.

Considering the fact that we are studying the behavior of the system when the angles of rotation of the pendulums are not very significant, we will use the expansion of sine in the Maclaurin series. In the expansion, we will use only terms not higher than the third order.

We will use the Padé approximation of the magnetic effect in the form (2).

$$M_{mag}(\varphi) = \left(a_0 + \frac{a_1\varphi + a_2\varphi^3}{1 + b_1\varphi^2 + b_2\varphi^4} \right) \mathrm{sign}(\varphi), \tag{2}$$

where a_0, a_1, a_2, b_1, b_2 are coefficients of the model obtained using the least squares method in order to satisfy the experimental data the most [1–3]. A comparison of this approximation with experimental data is shown in Fig. 2.

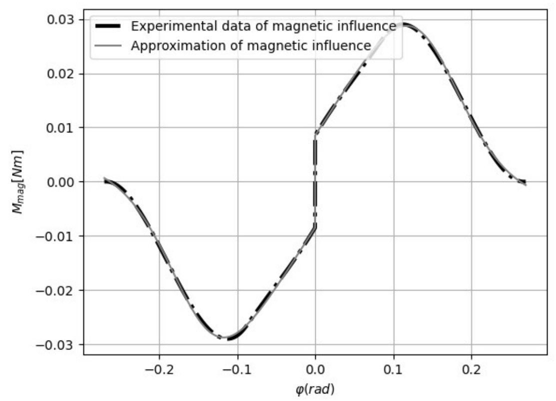


Fig. 2. Experimental data of the magnetic moment in comparison with a numerically consistent model

The solution (1) is given in the form of an expansion by a small parameter

$$\varphi_1 = \varphi_{10} + \varepsilon\varphi_{11} + O(\varepsilon^2); \ \varphi_2 = \varphi_{20} + \varepsilon\varphi_{21} + O(\varepsilon^2), \tag{3}$$

where φ_{10}, φ_{20} is the solution of the generating linear system; φ_{11}, φ_{21} is the solution of the first approximation for a small parameter ε.

The multiples scales method is used [10]. In accordance with it, the following time scales are introduced, namely:

$$T_0 = \tau\,;\; T_1 = \varepsilon \cdot \tau\,;\; \tau = \omega_0 \cdot t\,, \tag{4}$$

where T_0 – is the fast time; T_1 is the slow time.

When performing the standard transformations of this method, the systems of equations (5) and (6) are obtained. These systems correspond to the first two approximations with a small parameter ε:

$$\varepsilon^0: \begin{cases} -k_l^*(\varphi_{10} - \varphi_{20}) = 0, \\ \omega_0^2 \dfrac{\partial^2 \varphi_{20}}{\partial T_0^2} = -r^* \varphi_{20} - k_l^*(\varphi_{20} - \varphi_{10}). \end{cases} \tag{5}$$

$$\varepsilon^1: \begin{cases} \mu\omega_0^2 \dfrac{\partial^2 \varphi_{20}}{\partial T_0^2} = \gamma M^*_{mag_1} - C_1^* \dfrac{\partial \varphi_{10}}{\partial T_0} - \mu r^* \varphi_{10} - k_l^*(\varphi_{11} - \varphi_{21}), \\ \omega_0^2\left(2\dfrac{\partial^2 \varphi_{20}}{\partial T_0 \partial T_1} + \dfrac{\partial^2 \varphi_{21}}{\partial T_0^2}\right) = \gamma M^*_{mag_2} - C_2^* \dfrac{\partial \varphi_{20}}{\partial T_0} - r^*(\varphi_{21} - \dfrac{1}{6}\varphi_{20}^3) - k_l^*(\varphi_{21} - \varphi_{11}), \end{cases} \tag{6}$$

$\varphi_{10} = \varphi_{20} = A_1(T_1)\cos(T_0 + \nu)$ is the solution for (5), which corresponds to *coupled (in-phase) vibration mode*. The magnetic moment acting on the first pendulum is represented by the Fourier series according to relation (7) (for the magnetic effect on the second pendulum we will use the coefficients $h_i, i = \overline{(0,6)}$).

$$M^*_{mag_1} \approx \frac{1}{I}\left(\frac{g_0}{2} + \sum_{i=1}^{6} g_i \cos i(T_0 + \nu)\right), \tag{7}$$

where $g_i = \dfrac{2}{\pi}\displaystyle\int_0^{\pi/2} \operatorname{sign}(\varphi_{10})\left(a_0 + \dfrac{a_1\varphi_{10} + a_2\varphi_{10}^3}{1 + b_1\varphi_{10}^2 + b_2\varphi_{10}^4}\right)\cos(i(T_0 + \nu))dT_0,\, i = \overline{(0,6)}$.

To prevent the appearance of secular terms in the solution of the system of equations (6), we exclude terms containing functions $\cos(T_0 + \nu)$ and $\sin(T_0 + \nu)$ in the right-hand side of these equations, and as a result we obtain the equations (8) and (9).

$$\cos(T_0 + \nu):\;\; 2\omega_0^2 A_1 \frac{\partial \nu}{\partial T_1} + \frac{\gamma}{I}(g_1 + h_1) + \frac{r^* A_1}{8} = 0\,; \tag{8}$$

$$\sin(T_0 + \nu):\;\; 2\omega_0^2 \frac{\partial A_1}{\partial T_1} + A_1(C_1^* + C_2^*) = 0\,. \tag{9}$$

It follows that $A_1 = e^{A_3 - \frac{(C_1^* + C_2^*)T_1}{2\omega_0^2}}$; $\nu = \dfrac{-\gamma(g_1 + h_1)}{I(C_1^* + C_2^*)} e^{\frac{(C_1^* + C_2^*)T_1}{2\omega_0^2} - A_3} + \dfrac{\omega_0^2}{16(C_1^* + C_2^*)} e^{2A_3 - \frac{(C_1^* + C_2^*)T_1}{\omega_0^2}}$; where A_3 is the arbitrary constant determined by the initial deflection of the pendulum. A comparison of the analytical solution with the numerical one, which is applied to the base system (1) using the Runge–Kutta method of the 4th order, is carried out for the initial values of the variables that are set from the analytical solution. Such a comparison shows the good accuracy of the analytical approximation at sufficiently small values of the μ parameter, and for such values of the initial angles of the pendulums, which do not exceed approximately 60º.

To study the influence of the parameters and initial conditions of the system on the stability of the in-phase mode, the parameters are fixed as follows: γ=0.1; k_l=1; m=1; s=1.5; C_1=3.1×10^{-5}; C_2=7.2×10^{-5}; C_e=13.736×10^{-5}, and parameter A_3, which is responsible for the initial angle of the pendulum, will be set as an array of elements located at equal intervals in the range from -2 to 2. In addition, the value of the ratio of the masses of the two pendulums will change in the range from 0.01 to 0.25. The number of elements in the given arrays is 50 and 25, respectively. Based on this, a study of the growth of the number of instability nodes in relation to the duration of the simulation time of the system behavior with the specified parameters was carried out. The analysis showed that the number of instability nodes stops increasing after the duration of the system simulation of 1000 seconds. Both with a simulation duration of 1000 seconds and 6000 seconds, the number of unique angle values does not exceed 28. Therefore, studying the stability of the model, we will limit ourselves to a simulation time of 1000 seconds.

The stability of the associated vibration mode depending on the parameters A_3, μ is analyzed by numerical implementation of the Lyapunov stability criterion, which was proposed and described in [11]. The stability of the vibration mode is determined by orthogonal deviations from its trajectory in the configuration space, and the initial conditions for deviations from the trajectory are determined by the initial values of the angles of the two pendulums on the oscillation form as $\widetilde{\varphi}_{1,2}(0)=1{,}01\cdot\varphi_{1,2}(0)$. After that, the specified deviations when changing time are calculated. The instability of the vibration mode is detected when the deviations according to the modulus $\left|\widetilde{\varphi}_{1,2}(t)\right|$ exceed the values $\rho\left|\varphi_{1,2}(0)\right|$. As shown in [11], the values ρ can be chosen in a sufficiently wide range of numbers exceeding 1. In this paper, it is set that ρ=1.1. Taking into account that we study the stability of vibration modes depending on the parameter values A_3, μ, then on the corresponding plane, a grid of values in a rectangle is selected as $A_3\in[-2, 2]$, $\mu\in[0.01, 0.25]$. The results of the calculations are shown in Fig. 3, where the regions of instability are highlighted.

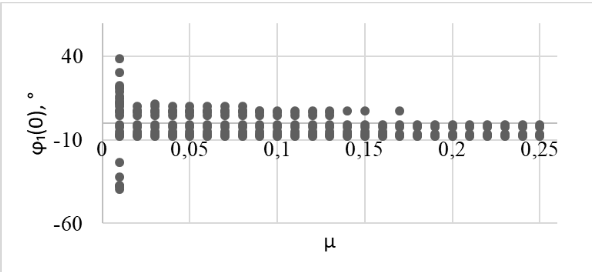


Fig. 3. Instability regions of the coupled mode on the plane of parameters $\varphi_1(0)$ and μ depending on the system simulation time

The examples of stable and instability grid nodes from Fig. 3 are considered. It is set that $\varphi_1(0)$=30.14º, $\varphi_2(0)$=30.454º and the corresponding values are μ={0,01, 0,1}. The result of such simulation is shown in Fig. 4.

The results of the calculations demonstrate that the coupled mode is unstable at small values of the parameter μ, if the initial values of the angles are small. This follows from the fact that at small initial angles, the influence of magnetic forces significantly exceeds the influence of elastic forces in the system. In other words, an increase in the value of the mass proportionality factor leads to a decrease in the trajectories wandering in the configuration space near the mode.

The next step is to study the influence of the distance from the center of mass of the pendulums to the rotation axis on the mode stability. To do this, it should be noted that the parameter $s\in[0.1, 4]$. 25 equidistant points from a given range are considered. The result is shown in Fig. 5.

The values of μ are set in Fig. 5 legend. The three-dimensional Figure 6, which depicts the regions of instability of the studied vibration modes in the space of parameters $\varphi_2(0)$, μ, s is considered as well.

The in-phase mode is more pronounced at a greater distance between the center of mass and the rotation axis with a considerable initial deviation of the pendulums, since the influence of the magnetic moment is smaller. This mode is observed when both the distance and the mass of the smaller pendulum increase. However, with a significant increase in the ratio of the pendulums, regions of instability appear even at significant distances, as we can see in Figs. 6 and 7. We will give an example of instability in Figs. 8 and 9, respectively.

In Fig. 7, $\varphi_1(0)$= –52,8515º; $\varphi_2(0)$= –52,88º, s=2,2125; μ=0,25. At the same time, in Fig. 7, one can see the occurrence of beating, which is the result of the addition of natural and forced oscillations near the resonance, at the same initial energies. The amplitude of the oscillations changes from a minimum value equal to the difference of the initial amplitudes to a maximum, which is equal to the sum of the amplitudes of the initial oscillations, and again to a minimum value. The beating period is the repetition time of this process.

The influence of the coupling coefficient $k_l\in[0{,}01, 1]$ is studied. From the specified range, 25 equidistant points were considered. Regions of instability in the space of parameters $\varphi_2(0)$, μ, k_l is constructed in Fig. 8.

In Fig. 9, we will demonstrate several examples of the region of instability on the plane at different values of parameters $\varphi_2(0)$ and μ and at fixed values k_l (values are given in the legend to the diagram).

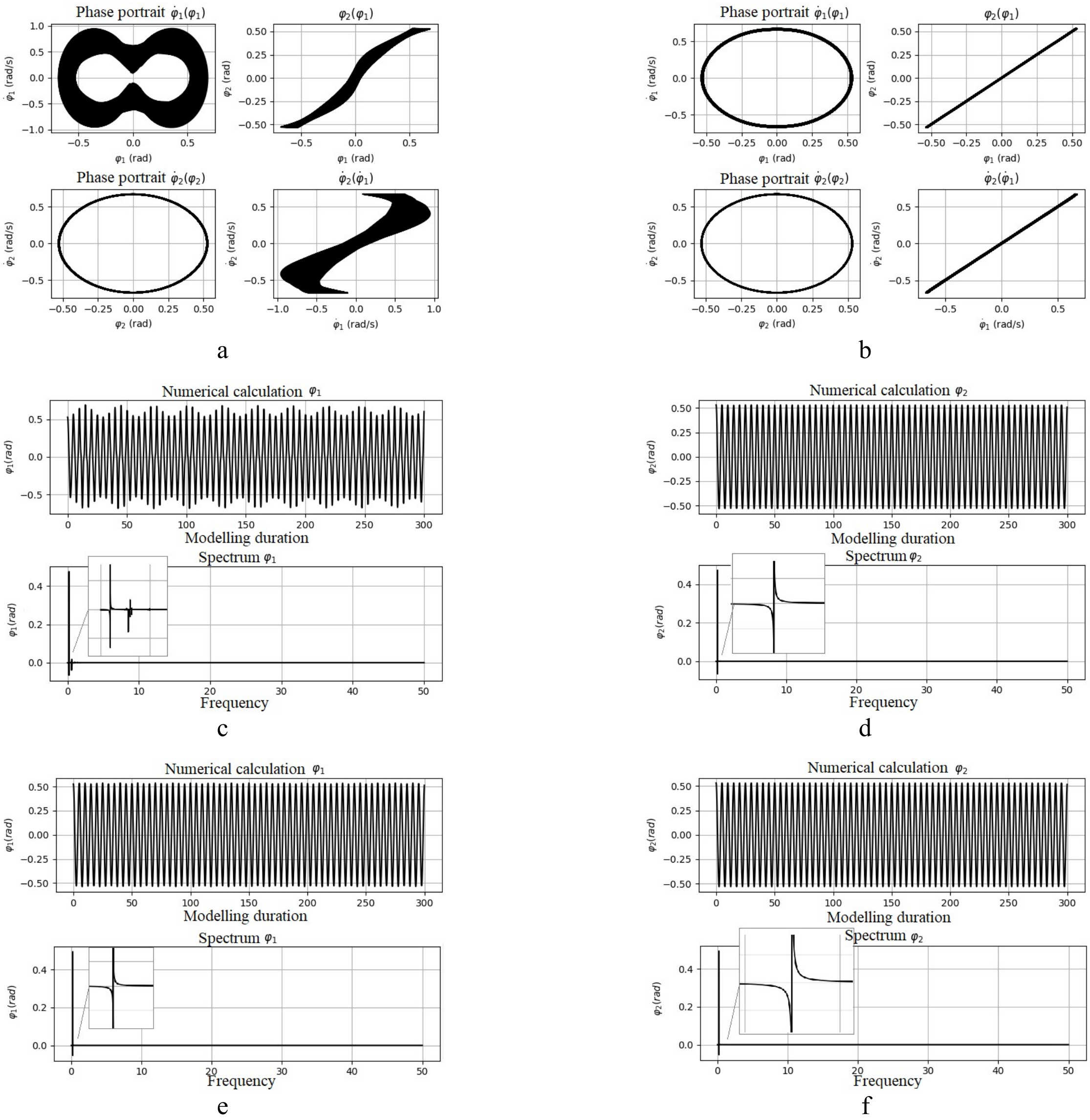


Fig. 4. Spectra of the coupled vibration mode

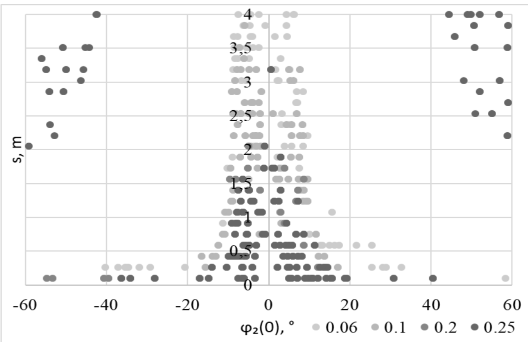


Fig. 5. Instability regions of the in-phase mode on the plane of parameters $\varphi_2(0)$ and s depending on the simulation time of the system

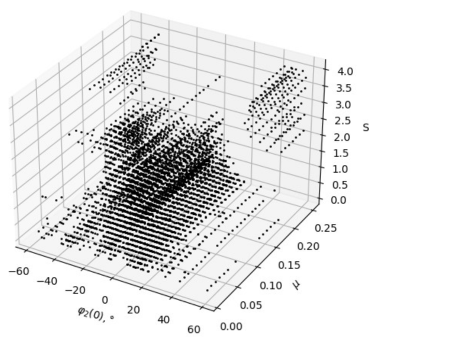
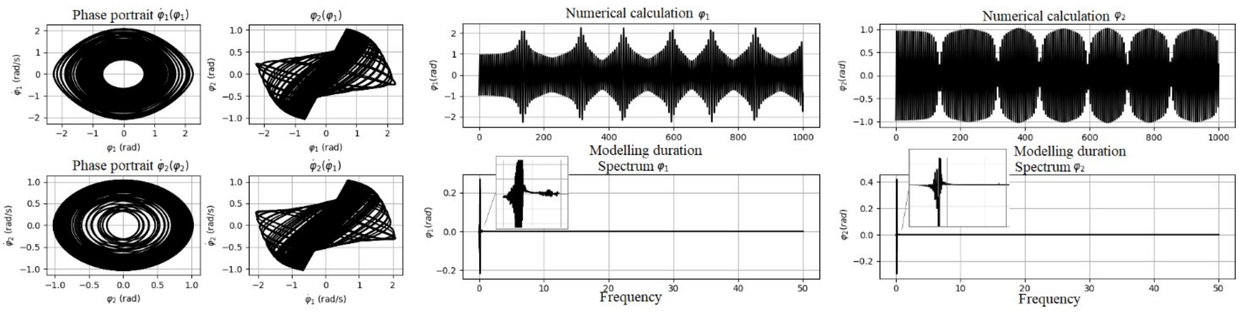


Fig. 6. Instability regions of the in-phase mode on the plane of parameters $\varphi_2(0)$, μ and s depending on the simulation time of the system



a b c

Fig. 7. Spectra of the coupled vibration mode

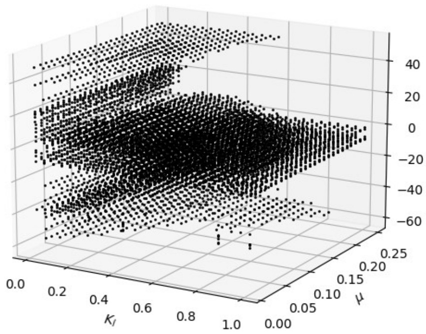


Fig. 8. Instability regions of the in-phase mode in the space of parameters $\varphi_2(0)$, μ and k_l depending on the simulation time of the system

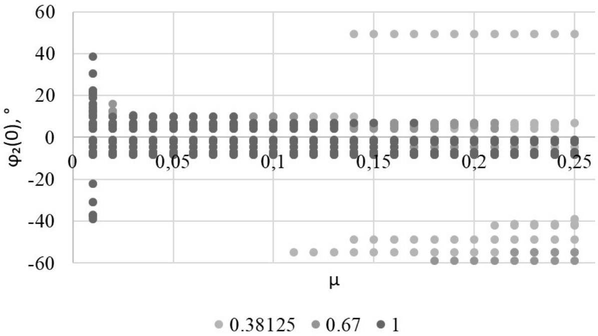


Fig. 9. Instability regions of the in-phase mode in the space of parameters $\varphi_2(0)$ and μ depending on the simulation time of the system

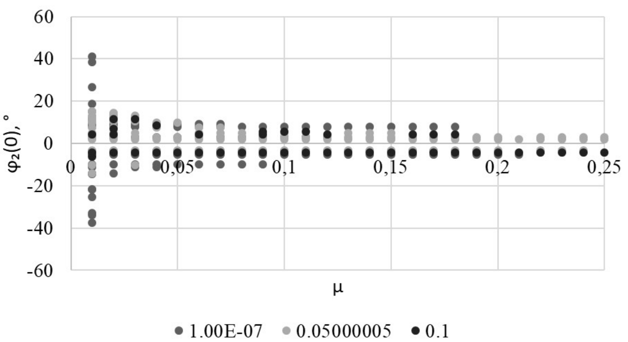


Fig. 10. Instability regions of the in-phase mode in the space of parameters $\varphi_2(0)$ and μ depending on the simulation time of the system

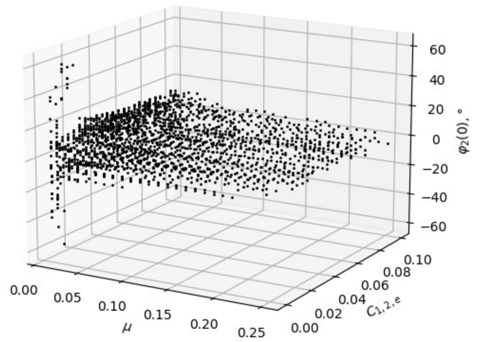


Fig. 11. Instability regions of the in-phase mode in the space of parameters $\varphi_2(0)$, μ and $C_{1, 2, e}$ depending on the simulation time of the system

The influence of the dissipation coefficients on the stability of the associated form of oscillations should also be studied. The values of all friction coefficients from the range of $C_{1, 2, e} \in [10^{-7}, 10^{-1}]$ (25 points from the specified interval were considered) are provided. The regions of instability will be derived on the plane of parameters $\varphi_2(0)$, μ and in the space of parameters $\varphi_2(0)$, μ, $C_{1, 2, e}$. The results are shown in Figs. 10–11.

Fig. 10 depicts the decrease in the number of instability nodes when increasing values $C_{1, 2, e}$.

Now, a study of the *localized vibration modes* will be conducted. It can be analytically represented after introducing the time transformation $t = \sqrt{\varepsilon}\tau$. Then system (1) takes the form (10).

$$\begin{cases} \mu\ddot{\varphi}_1 = \varepsilon\gamma M^*_{mag_1} - \varepsilon C_1^*\dot{\varphi}_1 - \varepsilon C_e^*(\dot{\varphi}_1 - \dot{\varphi}_2) - \varepsilon\mu r^* \sin\varphi_1 - k_l^*(\varphi_1 - \varphi_2), \\ \ddot{\varphi}_2 = \varepsilon^2\gamma M^*_{mag_2} - \varepsilon^2 C_2^*\dot{\varphi}_2 - \varepsilon^2 C_e^*(\dot{\varphi}_2 - \dot{\varphi}_1) - \varepsilon r^* \sin\varphi_2 - \varepsilon k_l^*(\varphi_2 - \varphi_1). \end{cases} \tag{10}$$

Similarly, just as in the case of the in-phase mode, fast and slow time scales will be introduced according to the multiples scales method. The found functions will be expanded by a small parameter ε, similar to formulas (3) and (4). Two systems corresponding to two approximations with a small parameter are written as follows:

$$\varepsilon^0 : \begin{cases} \mu\omega_0^2 \dfrac{\partial^2\varphi_{10}}{\partial T_0^2} = -k_l^*(\varphi_{10} - \varphi_{20}), \\ \omega_0^2 \dfrac{\partial^2\varphi_{20}}{\partial T_0^2} = 0. \end{cases} \tag{11}$$

$$\varepsilon^1 : \begin{cases} \mu\omega_0^2\left(2\dfrac{\partial^2\varphi_{20}}{\partial T_0\partial T_1} + \dfrac{\partial^2\varphi_{11}}{\partial T_0^2}\right) = \gamma M^*_{mag_1} - C_1^*\dfrac{\partial\varphi_{10}}{\partial T_0} - C_e^*\left(\dfrac{\partial\varphi_{10}}{\partial T_0} - \dfrac{\partial\varphi_{20}}{\partial T_0}\right) - \mu r^*\varphi_{10} - k_l^*(\varphi_{11} - \varphi_{21}), \\ \omega_0^2\left(2\dfrac{\partial^2\varphi_{20}}{\partial T_0\partial T_1} + \dfrac{\partial^2\varphi_{21}}{\partial T_0^2}\right) = -r^*\varphi_{20} - k_l^*(\varphi_{20} - \varphi_{10}). \end{cases} \tag{12}$$

$\varphi_{20}=0$, $\varphi_{10} = A_1(T_1)\cos(T_0+\nu)$, $\omega_0^2 = \dfrac{k_l^*}{\mu}$ is the solution of (11). The magnetic moment is given in the form (7). Again, terms containing $\cos(T_0+\nu)$ and $\sin(T_0+\nu)$ are excluded, therefore

$$\cos(T_0+\nu):\quad 2\mu\omega_0^2 A_1\frac{\partial\nu}{\partial T_1} + \frac{\gamma}{I}g_1 - \mu A_1(r^* + k_l^*) = 0\,; \tag{13}$$

$$\sin(T_0+\nu):\quad 2\mu\omega_0^2\frac{\partial A_1}{\partial T_1} + A_1(C_1^* + C_e^*) = 0\,. \tag{14}$$

It follows that $A_1 = e^{A_3 - \frac{(C_1^*+C_e^*)T_1}{2k_l^*}}$; $\nu = \dfrac{-\gamma g_1}{C_1^* + C_e^*}e^{\frac{(C_1^*+C_e^*)T_1}{2k_l^*} - A_3} + \dfrac{(r^* + k_l^*)T_1}{2\omega_0^2}$.

When studying the influence of parameters A_3, which is determined by the initial angles of the pendulums, and μ for the localized mode, we came to the same conclusions as when studying the in-phase mode. As the value of the pendulum masses ratio increases, the deviations near the mode decrease and the localized mode becomes more defined. It is clear that at small initial angles the localized mode does not exist, since the influence of the magnetic moment is very significant.

Conclusions

A stable in-phase (coupled) vibration mode does not exist over the entire range of initial conditions. It is unstable at small initial values of the deflection angles of the pendulums, if the masses of the pendulums differ significantly. An increase in the pendulum masses ratio leads to a decrease in the trajectories wandering near the mode. The in-phase mode is more pronounced at a greater distance between the center of mass and the rotation axis, since the influence of the magnetic moment is smaller. This mode is observed when both the mass of the smaller pendulum and the distance increase. In most of the considered cases, at large values of the distance from the rotation axis to the center of mass of the pendulum and the mass ratio coefficient, an increase in the value of the coupling coefficient leads to stabilization of the in-phase mode and a decrease in the trajectories wandering near such a mode. An increase in dissipation does not always force the trajectories into an vibration modes.

Like the in-phase mode, the localized mode does not exist over the entire range of the initial deflections of the pendulums. As the value of the pendulum masses ratio increases, the deviations near the mode decrease and the mode becomes more defined. It turned out, as for the related mode, that localization manifests itself when the connection and the distance between the center of mass and the rotation axis of the pendulums increase, or when both the connection and the proportionality factor of the pendulum masses increase. Large coefficients of masses proportionality along with large values of bounding and distance with increasing friction coefficients reduce the trajectories wandering near the localized mode, or pull the trajectories to this mode.

References

1. Polczyński, K., Wijata, A., Awrejcewicz, J., & Wasilewski, G. (2019). Numerical and experimental study of dynamics of two pendulums under a magnetic field. *Proceedings of the Institution of Mechanical Engineers. Part I: Journal of Systems and Control Engineering*, vol. 233, iss. 4, pp. 441–453. https://doi.org/10.1177/0959651819828878.
2. Wijata, A., Polczyński, K., & Awrejcewicz, J. (2021). Theoretical and numerical analysis of regular one-side oscillations in a single pendulum system driven by a magnetic field. *Mechanical Systems and Signal Processing*, vol. 150, 107229. https://doi.org/10.1016/j.ymssp.2020.107229.
3. Polczyński, K., Skurativskyi, S., Bednarek, M., & Awrejcewicz, J. (2021). Nonlinear oscillations of coupled pendulums subjected to an external magnetic stimulus. *Mechanical Systems and Signal Processing*, vol. 154, 107560. https://doi.org/10.1016/j.ymssp.2020.107560.
4. Surganova, Yu. E. & Mikhlin, Yu. V. (2022). Localized and non-localized nonlinear normal modes in a system of two coupled pendulums under a magnetic field. *International Journal of Non-Linear Mechanics*, vol. 147, 104182. https://doi.org/10.1016/j.ijnonlinmec.2022.104182.
5. Mikhlin, Yu. V. & Avramov, K. V. (2010). Nonlinears normal modes for vibrating mechanical systems. review of theoretical developments. *Applied Mechanics Reviews*, vol. 63, iss. 6, 060802. https://doi.org/10.1115/1.4003825.
6. Avramov, K. V. & Mikhlin, Yu. V. (2013). Review of applications of nonlinear normal modes for vibrating mechanical systems. *Applied Mechanics Reviews*, vol. 65, iss. 2, 020801. https://doi.org/10.1115/1.4023533.
7. Kerschen, G. (ed.) (2014). Modal Analysis of Nonlinear Mechanical Systems. Vienna: Springer. https://link.springer.com/book/10.1007/978-3-7091-1791-0.
8. Manevitch, L. I. & Smirnov, V. V. (2010). Limiting phase trajectories and the origin of energy localization in nonlinear oscillatory chains. *Physical Review E*, vol. 82, iss. 3, 036602. https://doi.org/10.1103/physreve.82.036602.
9. Vakakis, A. F., Gendelman, O. V., Bergman, L., McFarland, D. M., Kerschen, G., & Lee, Y. S. (2008). Nonlinear targeted energy transfer in mechanical and structural systems I. In: Solid Mechanics and its Applications. Vol. 156. Springer, pp. 1–1033.
10. Nayfeh, A. H. & Mook, D. T. (1995). Nonlinear oscillations. John Wiley & Sons, 720 p. https://doi.org/10.1002/9783527617586.
11. Mikhlin, Yu. V., Shmatko, T. V., & Manucharyan, G. V. (2004). Lyapunov definition and stability of regular or chaotic vibration modes in systems with several equilibrium positions. *Computers & Structures*, vol. 82, iss. 31–32, pp. 2733–2742. https://doi.org/10.1016/j.compstruc.2004.03.082.

Received 16 October 2023

Дослідження стійкості математичної моделі руху пов’язаних маятників

Ю. Е. Сурганова, Ю. В. Міхлін

Національний технічний університет «Харківський політехнічний інститут»
61002, Україна, м. Харків, вул. Кирпичова, 2

У статті представлено дослідження динаміки коливальної дисипативної системи двох пружно пов’язаних маятників у магнітному полі. Досліджено нелінійні нормальні моди коливань маятникової системи з урахуванням опору середовища, моменту демпфування, створеного пружним елементом. Розглянуто систему з двома ступенями свободи, в якій маси маятників суттєво розрізняються, що приводить до можливості появи локалізації коливань. У наступному дослідженні співвідношення мас обрано як малий параметр. Для наближених розрахунків магнітних сил використовується апроксимація Паде, яка найбільше *задовольняє експериментальним даним. Це наближення забезпечує дуже точний опис магнітного збудження. Наявність зовнішніх впливів у вигляді магнітних сил і різного типу навантажень, які існують в багатьох інженерних системах, значно ускладнює аналіз мод коливань нелінійних систем. Проведено дослідження нелінійних нормальних мод коливань у даній системі, причому одна з мод є пов'язаним режимом, а друга – локалізованою. Моди коливань побудовано методом багатьох масштабів. Вивчено як регулярну, так і складну поведінку при зміні параметрів системи. Вплив цих параметрів досліджено для малих і значних початкових кутів нахилу маятника. Аналітичний розв'язок, який базується на методі Рунге-Кутти четвертого порядку, порівняно з результатами чисельного моделювання. Початкові умови для розрахунку мод коливань визначалися аналітичним розв'язком. Чисельне моделювання, яке складається з побудови фазових діаграм, траєкторій у конфігураційному просторі й амплітудно-частотних характеристик, дозволяє оцінити динаміку системи, що може бути як регулярною, так і складною. Стійкість режимів коливань досліджено за допомогою тестів чисельного аналізу, які є реалізацією критерію стійкості Ляпунова. При цьому стійкість режимів коливань визначається шляхом оцінки ортогональних відхилень відповідних траєкторій режимів коливань у конфігураційному просторі.*

***Ключові слова**: пов'язані маятники, магнітні сили, нелінійні нормальні моди коливань, метод багатьох масштабів, стійкість.*

Література

1. Polczyński K., Wijata A., Awrejcewicz J., Wasilewski G. Numerical and experimental study of dynamics of two pendulums under a magnetic field. *Proceedings of the Institution of Mechanical Engineers. Part I: Journal of Systems and Control Engineering*. 2019. Vol. 233. Iss. 4. P. 441–453. https://doi.org/10.1177/0959651819828878.
2. Wijata A., Polczyński K., Awrejcewicz J. Theoretical and numerical analysis of regular one-side oscillations in a single pendulum system driven by a magnetic field. *Mechanical Systems and Signal Processing*. 2021. Vol. 150. 107229. https://doi.org/10.1016/j.ymssp.2020.107229.
3. Polczyński K., Skurativskyi S., Bednarek M., Awrejcewicz J. Nonlinear oscillations of coupled pendulums subjected to an external magnetic stimulus. *Mechanical Systems and Signal Processing*. 2021. Vol. 154. 107560. https://doi.org/10.1016/j.ymssp.2020.107560.
4. Surganova Yu. E., Mikhlin Yu. V. Localized and non-localized nonlinear normal modes in a system of two coupled pendulums under a magnetic field. *International Journal of Non-Linear Mechanics*. 2022. Vol. 147. 104182. https://doi.org/10.1016/j.ijnonlinmec.2022.104182.
5. Mikhlin Yu. V., Avramov K. V. Nonlinears normal modes for vibrating mechanical systems. review of theoretical developments. *Applied Mechanics Reviews*. 2010. Vol. 63. Iss. 6. 060802. https://doi.org/10.1115/1.4003825.
6. Avramov K. V., Mikhlin Yu. V. Review of applications of nonlinear normal modes for vibrating mechanical systems. *Applied Mechanics Reviews*. 2013. Vol. 65. Iss. 2. 020801. https://doi.org/10.1115/1.4023533.
7. Kerschen G. (ed.) Modal Analysis of Nonlinear Mechanical Systems. Vienna: Springer, 2014. https://link.springer.com/book/10.1007/978-3-7091-1791-0.
8. Manevitch L. I., Smirnov V. V. Limiting phase trajectories and the origin of energy localization in nonlinear oscillatory chains. *Physical Review E*. 2010. Vol. 82. Iss. 3. 036602. https://doi.org/10.1103/physreve.82.036602.
9. Vakakis A. F., Gendelman O. V., Bergman L., McFarland D. M., Kerschen G., Lee Y. S. Nonlinear targeted energy transfer in mechanical and structural systems I. In: Solid Mechanics and its Applications. Vol. 156. Springer, 2008. P. 1–1033.
10. Nayfeh A. H., Mook D. T. Nonlinear oscillations. John Wiley & Sons, 1995. 720 p. https://doi.org/10.1002/9783527617586.
11. Mikhlin Yu. V., Shmatko T. V., Manucharyan G. V. Lyapunov definition and stability of regular or chaotic vibration modes in systems with several equilibrium positions. *Computers & Structures*. 2004. Vol. 82. Iss. 31–32. P. 2733–2742. https://doi.org/10.1016/j.compstruc.2004.03.082.