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# NONLINEAR DEFORMATION OF CYLINDERS FROM MATERIALS WITH DIFFERENT BEHAVIOR IN TENSION AND COMPRESSION

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*A new numerical-analytical method for solving physically nonlinear deformation problems of axisymmetrically loaded cylinders made of materials with different behavior in tension and compression has been developed. To linearize the problem, the uninterrupted parameter continuation method was used. For the variational formulation of the linearized problem, a functional in the Lagrange form, defined on the kinematically possible displacement rates, is constructed. To find the main unknowns of the problem of physically nonlinear cylinder deformation, the Cauchy problem for the system of ordinary differential equations is formulated. The Cauchy problem was solved by the Runge-Kutta-Merson method with automatic step selection. The initial conditions were established by solving the problem of linear elastic deformation. The right-hand sides of the differential equations at fixed values of the load parameter corresponding to the Runge-Kutta-Merson's scheme are found from the solution of the variational problem for the functional in the Lagrange form. Variational problems are solved using the Ritz method. The test problem for the nonlinear elastic deformation of a thin cylindrical shell is solved. Coincidence of the spatial solution with the shell solution was obtained. Physically nonlinear deformation of a thick-walled cylinder was studied. It is shown that failure to take into account the different behavior of the material under tension and compression leads to significant errors in the calculations of stress-strain state parameters.*

***Keywords***: *thick-walled cylinder, different behavior in tension and compression, physically nonlinear deformation, uninterrupted parameter continuation method.*

## Introduction

Axisymmetrically loaded cylinders are widely used in modern technology, for example, as pressure vessels (hydraulic cylinders, gun barrels, nozzles, boilers, fuel tanks), battery housings, cylinders for the aerospace industry, nuclear reactor pipelines, etc.

Many materials (light alloys, superalloys, gray cast iron, polymers, composites, etc.) are characterized by different resistance to tension and compression beyond linear elasticity [1–3]. The problem of deformation of bodies made of such materials becomes physically nonlinear. However, when studying the physically nonlinear deformation of cylinders, scientists face certain mathematical difficulties associated with modeling the nonlinear behavior of the material, developing methods of linearization, and solving nonlinear boundary value problems.

The study of physically nonlinear deformation (nonlinear-elastic, elastic-plastic problems, creep problems) of cylinders and cylindrical shells is studied in, for example, papers [4–19]. Only a few papers [11–19] study the nonlinear deformation of cylinders and shells made from materials with different behavior in tension and compression. Thus, in paper [11], the solution of the problem of nonlinear elastic bending of a cylindrical shell made from material with different behavior in tension and compression was obtained by integrating the Cauchy problem by the Runge-Kutta-Merson's method with a simultaneous five-fold solution of the boundary problems at each step for the original equations by the method of discrete orthogonalization. In the monograph [12, P. 1], the problems of elastic-plastic and nonlinear-elastic deformation of thick-walled cylinders were reduced to the solution of initial boundary value problems. The methods of discrete orthogonalization of S. K. Godunov and movement along the load in combination with iterative refinement of the solution were used to integrate the equilibrium equations. In the paper [13], a comparative analysis of the spatial and shell solutions of the axisymmetric problem of creep and damage of a cylinder under the action of external pressure is performed. Both in the spatial and in the shell formulation, the problem was reduced to an initial boundary problem. Integration by time was

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performed using the Runge-Kutta-Merson's method, and boundary value problems at each step were solved using the R-function method and the S. K. Godunov discrete orthogonalization method.

The theory and methods of calculating the nonlinear deformation of cylinders made of non-traditional materials are currently being developed. The aim of the paper is the development of a numerical and analytical method for solving the problems of physically nonlinear deformation of cylinders from materials with different behavior in tension and compression, based on the use of the uninterrupted parameter continuation, Ritz and Runge-Kutta-Merson's methods.

**Formulation of the problem. Solution method**

An axisymmetrically loaded isotropic hollow cylinder of finite length in a cylindrical coordinate system $Or\varphi z$ is considered. Axis $Oz$ coincides with the axis of symmetry.

To formulate and linearize the problem of physically nonlinear deformation of cylinders, we will use the uninterrupted parameter solution continuation method [20]. Let's introduce a parameter $t \in [t_0, t_*]$, which is related to the external load acting on the cylinder. In this case $t_0$ is the value of the parameter at which the deformation problem is linear, and $t_*$ corresponds to the specified loading level. Let's mark the derivative with respect to the parameter with a dot above the symbol $t$. Further on derivatives by parameter $t$ will be called rates.

Let's assume that the components $\dot{\varepsilon}_{kl}$ of strain rate tensor consist of linear $\dot{e}_{kl}$, which are subject to Hooke's law, and non-linear $\dot{\eta}_{kl}$ components, i.e

$$\dot{\varepsilon}_{rr} = \dot{e}_{rr} + \dot{\eta}_{rr}, \quad \dot{\varepsilon}_{zz} = \dot{e}_{zz} + \dot{\eta}_{zz}, \quad \dot{\varepsilon}_{\varphi\varphi} = \dot{e}_{\varphi\varphi} + \dot{p}_{\varphi\varphi}, \quad \dot{\varepsilon}_{rz} = \dot{e}_{rz} + \dot{\eta}_{rz}. \tag{1}$$

After differentiating the Cauchy dependencies in cylindrical coordinates by the load parameter $t$ [21], we obtain the relation between the strain rates and the displacement rates

$$\dot{\varepsilon}_{rr} = \dot{u}_{r,r}, \quad \dot{\varepsilon}_{\varphi\varphi} = \dot{u}_r / r, \quad \dot{\varepsilon}_{zz} = \dot{u}_{z,z}, \quad \dot{\gamma}_{rz} = 2\dot{\varepsilon}_{rz} = \dot{u}_{r,z} + \dot{u}_{z,r}, \tag{2}$$

where $\dot{u}_r, \dot{u}_z$ are displacement rates along the axes $Or$ and $Oz$.

Similarly, by differentiating Hooke's law [21] with respect to $t$ and taking into account (1), we obtain the relation between stress rates and strain rates

$$\dot{\sigma}_{rr} = \lambda\left(\dot{\varepsilon}_{zz} + \dot{\varepsilon}_{\varphi\varphi} - \dot{\eta}_{zz} - \dot{\eta}_{\varphi\varphi}\right) + \lambda_1\left(\dot{\varepsilon}_{rr} - \dot{\eta}_{rr}\right),$$
$$\dot{\sigma}_{zz} = \lambda\left(\dot{\varepsilon}_{rr} + \dot{\varepsilon}_{\varphi\varphi} - \dot{\eta}_{rr} - \dot{\eta}_{\varphi\varphi}\right) + \lambda_1\left(\dot{\varepsilon}_{zz} - \dot{\eta}_{zz}\right),$$
$$\dot{\sigma}_{\varphi\varphi} = \lambda\left(\dot{\varepsilon}_{rr} + \dot{\varepsilon}_{zz} - \dot{\eta}_{rr} - \dot{\eta}_{zz}\right) + \lambda_1\left(\dot{\varepsilon}_{\varphi\varphi} - \dot{\eta}_{\varphi\varphi}\right),$$
$$\dot{\sigma}_{rz} = G\left(\dot{\gamma}_{rz} - 2\dot{\eta}_{rz}\right), \tag{3}$$

where $\lambda = \dfrac{E\nu}{(1-2\nu)(1+\nu)}$, $\lambda_1 = \lambda + 2G$; $E$, $G$, $\nu$ are Young's modulus, shear modulus and Poisson's ratio of the material.

To describe the nonlinear behavior of the material, we use tensor-linear constitutive equations that describe the different behavior of the material under tension and compression [22]

$$\dot{\eta}_{ij} = n\sigma_e^{n-1}\dot{\sigma}_e\left(\frac{C\sigma_{ij} + AI_1\delta_{ij}}{\sigma_{e2}} + B\delta_{ij}\right), \ (i, j = 1,2,3), \tag{4}$$

where $\sigma_e = \sigma_{e2} + \sigma_{e1}$ is the equivalent stress; $\sigma_{e1} = BI_1$, $\sigma_{e2} = \sqrt{AI_1^2 + CI_2}$; $I_1 = \delta_{ij}\sigma_{ij}$, $I_2 = \sigma_{ij}\sigma_{ij}$ are linear and quadratic invariants of the stress tensor; $A$, $B$, $C$ – material parameters determined from experimental data.

If elastic-plastic deformation is studied, then relations (4) should be supplemented by the condition of plasticity.

Consider the method of determining the parameters of the material $A$, $B$, $C$ in relations (4). For this, it is necessary to have experimental data for material samples, for example, under uniaxial stress state and under pure torsion.

Let's assume that as a result of experiments on uniaxial tension ($\sigma_{11}>0$) it is established that in the direction of the applied load is

$$\eta_{11} = K_{+}\sigma_{11}^{n}, \tag{5}$$

and on uniaxial compression ($\sigma_{11}$<0) it is

$$\eta_{11} = -K_{-}\left|\sigma_{11}\right|^{n}. \tag{6}$$

Similarly for pure torsion ($\sigma_{12}\neq 0$) it is

$$2\eta_{12} = K_{0}\sigma_{12}^{n}, \tag{7}$$

where $K_{+}, K_{-}, K_{0}, n$ are material constants.

In the case of a simple load, equations (4) can be integrated and written in the form

$$\eta_{ij} = \sigma_{e}^{n}\left(\frac{C\sigma_{ij} + AI_{1}\delta_{ij}}{\sigma_{e2}} + B\delta_{ij}\right). \tag{8}$$

From relations (8) for uniaxial tension, we will have

$$\eta_{11} = \sigma_{11}^{n}\left(\sqrt{A+C} + B\right)^{n+1}. \tag{9}$$

In case of compression

$$\eta_{11} = -\left|\sigma_{11}\right|^{n}\left(\sqrt{A+C} - B\right)^{n+1}. \tag{10}$$

For pure torsion

$$2\eta_{12} = \sigma_{12}^{n}\left(\sqrt{2C}\right)^{n+1}. \tag{11}$$

Next, matching (5) and (9), (6) and (10), (7) and (11) in pairs, we get the following system of equations

$$\left(\sqrt{A+C} + B\right)^{n+1} = K_{+}, \quad \left(\sqrt{A+C} - B\right)^{n+1} = K_{-}, \quad \left(\sqrt{2C}\right)^{n+1} = K_{0},$$

from which it is easy to find the material parameters

$$A = 0{,}25\left(K_{+}^{\frac{1}{n+1}} + K_{-}^{\frac{1}{n+1}}\right)^{2} - C, \quad B = 0{,}5\left(K_{+}^{\frac{1}{n+1}} - K_{-}^{\frac{1}{n+1}}\right), \quad C = 0{,}5K_{0}^{\frac{2}{n+1}}.$$

For the variational statement of the problem, we will use a functional in the Lagrange form, given on the kinematically possible displacement rates, which for a rotation body has the form [12]

$$\Lambda(\dot{u}_r, \dot{u}_z) = 0.5\iint_{\Omega}\int_{0}^{2\pi}\left[\dot{\sigma}_{rr}\left(\dot{\varepsilon}_{rr} - \dot{\eta}_{rr}\right) + \dot{\sigma}_{zz}\left(\dot{\varepsilon}_{zz} - \dot{\eta}_{zz}\right) + \dot{\sigma}_{\varphi\varphi}\left(\dot{\varepsilon}_{\varphi\varphi} - \dot{\eta}_{\varphi\varphi}\right) + \right.$$

$$\left. + \dot{\sigma}_{rz}\left(\dot{\gamma}_{rz} - 2\dot{\eta}_{rz}\right)\right] r\,dr\,dz\,d\varphi \; - \int_{\partial\Omega_p}\int_{0}^{2\pi}\left(\dot{P}_{n}^{0}\dot{u}_{n} + \dot{P}_{\tau}^{0}\dot{u}_{\tau}\right)d\partial\Omega\,d\varphi, \tag{12}$$

where $\partial\Omega_p$ is the part of the border $\partial\Omega$, to which surface loads are applied; $P_n^0$, $P_\tau^0$ are normal and tangent components of external loads; $n$, $\tau$ are external normal and tangent to the contour $\partial\Omega_p$; $\dot{u}_n = \dot{u}_r n_r + \dot{u}_z n_z$, $\dot{u}_\tau = \dot{u}_z n_r - \dot{u}_r n_z$; $n_r$, $n_z$ are direction cosines of the normal $n$.

In formula (12) the rates of the nonlinear components $\dot{\eta}_{rr}, \dot{\eta}_{zz}, \dot{\eta}_{\varphi\varphi}, \dot{\eta}_{rz}$ are considered given for each fixed value of the parameter $t$ and do not vary.

Substituting (2), (3) into (12) and integrating along the angular coordinate, we obtain the following functional:

$$\Lambda = 0.5\iint_{\Omega}\left[\lambda_1\left(\dot{u}_{r,r}^{2} + \dot{u}_{z,z}^{2} + \frac{\dot{u}_r^{2}}{r^{2}}\right) + G\left(\dot{u}_{r,z} + \dot{u}_{z,r}\right)^{2} + 2\lambda\left(\dot{u}_{r,r}\dot{u}_{z,z} + \frac{\dot{u}_r\left(\dot{u}_{r,r} + \dot{u}_{z,z}\right)}{r}\right)\right] rdrdz -$$

$$-\iint_{\Omega}\left[\dot{u}_{r,r}\dot{N}_{r}^{f} + \dot{u}_{z,z}\dot{N}_{z}^{f} + \frac{\dot{u}_r\dot{N}_{\varphi}^{f}}{r} + \dot{N}_{rz}^{f}\left(\dot{u}_{r,z} + \dot{u}_{z,r}\right)\right] rdrdz - \int_{\partial\Omega_p}\left(\dot{P}_{n}^{0}\dot{u}_{n} + \dot{P}_{\tau}^{0}\dot{u}_{\tau}\right)d\partial\Omega\;, \tag{13}$$



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where $\dot{N}_r^f = \left(\lambda_1\dot{\eta}_{rr} + \lambda\left(\dot{\eta}_{zz} + \dot{\eta}_{\varphi\varphi}\right)\right)$, $\dot{N}_z^f = \left(\lambda_1\dot{\eta}_{zz} + \lambda\left(\dot{\eta}_{rr} + \dot{\eta}_{\varphi\varphi}\right)\right)$, $\dot{N}_\varphi^f = \left(\lambda_1\dot{\eta}_{\varphi\varphi} + \lambda\left(\dot{\eta}_{rr} + \dot{\eta}_{zz}\right)\right)$, $\dot{N}_{rz}^f = 2G\dot{\eta}_{rz}$ are "fictitious" forces caused by nonlinear components of deformation.

Solution of the variational equation $\delta\Lambda=0$ gives the distribution of the displacement rate fields for fixed values of the parameter $t>t_0$, at any point of the cylinder. The main unknown problems of nonlinear deformation can be found by integrating the corresponding rate fields from the solution of the Cauchy problem with the parameter $t$ for a system of ordinary differential equations of the form

$$\frac{du_r}{dt} = \dot{u}_r, \quad \frac{du_z}{dt} = \dot{u}_z,$$

$$\frac{d\varepsilon_{rr}}{dt} = \dot{u}_{r,r}, \quad \frac{d\varepsilon_{zz}}{dt} = \dot{u}_{z,z}, \quad \frac{d\varepsilon_{\varphi\varphi}}{dt} = \frac{\dot{u}_r}{r}, \quad \frac{d\gamma_{rz}}{dt} = 2\frac{d\varepsilon_{rz}}{dt} = \dot{u}_{r,z} + \dot{u}_{z,r},$$

$$\frac{d\sigma_{rr}}{dt} = \lambda_1\left(\dot{\varepsilon}_{rr} - \dot{\eta}_{rr}\right) + \lambda\left(\dot{\varepsilon}_{zz} + \dot{\varepsilon}_{\varphi\varphi} - \dot{\eta}_{zz} - \dot{\eta}_{\varphi\varphi}\right),$$

$$\frac{d\sigma_{zz}}{dt} = \lambda_1\left(\dot{\varepsilon}_{zz} - \dot{\eta}_{zz}\right) + \lambda\left(\dot{\varepsilon}_{rr} + \dot{\varepsilon}_{\varphi\varphi} - \dot{\eta}_{rr} - \dot{\eta}_{\varphi\varphi}\right),$$

$$\frac{d\sigma_{\varphi\varphi}}{dt} = \lambda_1\left(\dot{\varepsilon}_{\varphi\varphi} - \dot{\eta}_{\varphi\varphi}\right) + \lambda\left(\dot{\varepsilon}_{rr} + \dot{\varepsilon}_{zz} - \dot{\eta}_{rr} - \dot{\eta}_{zz}\right),$$

$$\frac{d\sigma_{rz}}{dt} = G\left(\dot{\gamma}_{rz} - 2\dot{\eta}_{rz}\right),$$

$$\frac{d\eta_{rr}}{dt} = \dot{\eta}_{rr}, \quad \frac{dp_{zz}}{dt} = \dot{\eta}_{zz}, \quad \frac{dp_{\varphi\varphi}}{dt} = \dot{\eta}_{\varphi\varphi}, \quad \frac{dp_{rz}}{dt} = \dot{\eta}_{rz}. \tag{14}$$

The nonlinearity of the system (14) is due to the nonlinearity of the constitutive equations (4). The initial conditions for the sought functions are found from the solution of the problem of linear elastic deformation. For this purpose, we can use a functional of the form (13), in which we replace the rates of the functions with the functions themselves and assume that the "fictitious" forces are $\dot{N}_r^f = \dot{N}_z^f = \dot{N}_\varphi^f = \dot{N}_{rz}^f = 0$.

We will solve the initial problem for the system of equations (14) using the Runge-Kutta-Merson's method with automatic step selection [23]. To calculate the right-hand sides of equations (14) at fixed values $t>t_0$, corresponding to the Runge-Kutta-Merson's scheme, it is necessary to solve the variational problems for the functional (13) five times at each step. Variational problems were solved using the Ritz method.

**Numerical results**

As a test example, the non-linear elastic deformation of a thin cylindrical shell made of gray cast iron СЧ 15-32, which is loaded with an internal pressure of $P_{inn}$=4 MPa, is considered. The shell is rigidly fixed on one edge and free from fastening and effort on the other. The geometric dimensions are as follows: length $l$=0.2 m, the radius of the inner surface $r_1$=0.195 m, the radius of the outer surface $r_2$=0.205 м.

For gray cast iron, the equality of the modulus of elasticity under tension and compression on the initial linear sections of the deformation diagrams was experimentally established. At a higher load, the nonlinear character of the deformation, in which the diagrams of deformation under tension and compression differ significantly, is manifested [1].

Young's modulus and Poisson's ratio of the material are: $E=1.07\times10^5$ MPa, $\nu$=0.22. Material constants for nonlinear component deformations [12, P.1] are: $K_+=1.53\times10^{-12,4}$ MPa$^{-n}$, $K_-=8.1\times10^{-14,4}$ MPa$^{-n}$, $K_0=9{,}07\times10^{-12,4}$ MPa$^{-n}$, $n$=4,4.

A linear law for the load is assumed as

$$P_{inn}\left(t\right) = P_1 + tP_2, \tag{15}$$

where $t \in \left[0, t_*\right]$.

The origin of the coordinates is placed on the fixed edge. Then the kinematic boundary conditions will have the form

$$\dot{u}_r = \dot{u}_z = 0 \text{ for } z=0. \tag{16}$$

Sequences of coordinate functions, which satisfy the conditions (16), can be written as:

$$\dot{u}_r = z\Phi_1,\ \dot{u}_z = z\Phi_2,$$

where $\Phi_1(r,z,t)=\sum_{n=1}^{N_1} C_n^{(1)}(t) f_n^{(1)}(r,z)$, $\Phi_2(r,z,t)=\sum_{n=1}^{N_2} C_n^{(2)}(t) f_n^{(2)}(r,z)$; $C_n^{(1)}$, $C_n^{(2)}$ are indefinite coefficients, which are found at each step by the Ritz method; $t$ – some fixed value of the load parameter; $\{f_n^{(1)}\}$, $\{f_n^{(2)}\}$ are systems of linearly independent functions. In this paper, bicubic Schoenberg splines were used as $\{f_n^{(1)}\}$, $\{f_n^{(2)}\}$. Spline systems were built on a regular grid $N_r \times N_z$, where $N_r$, $N_z$ is the number of discretization segments along the axes $Or$ and $Oz$, respectively.

Rates of equivalent stresses $\dot{\sigma}_e$ in the constitutive equations were assumed to be constant at each step by $t$ and were calculated from the stress values and stress rates in the previous step.

Figs. 1, 2 show graphs of changes of radial $w=u_r(r_0, z)$ and axial $u_{z0}=u_z(r_0, z)$ displacements of the middle surface of the shell $r = r_0 = \frac{r_2 - r_1}{2}$ along the axis $Oz$.

Dashed lines show the results obtained within the framework of the theory of shells [11], using relations (4), and solid lines show the results obtained using the method developed in the paper. When using it for the loading in formula (15), the following parameters were accepted: $P_1$=0.02 MPa, $P_2$=$10^{-1}$ MPa, and the initial step and the given calculation error in the Runge-Kutta-Merson's method were equal to: $\Delta t_0=10^{-2}$, $\varepsilon=10^{-5}$.

From Figs. 1, 2, it can be seen that the results of the shell displacements calculation, obtained by the shell and spatial models, almost completely coincided.

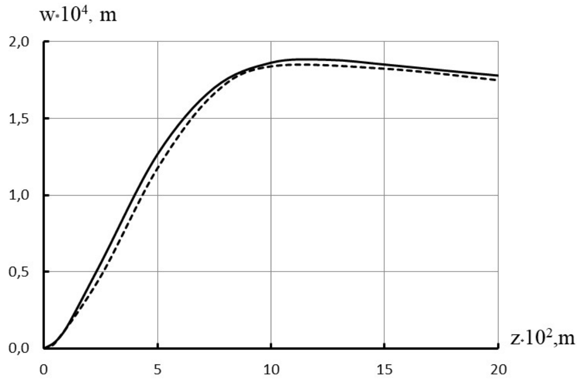


***Fig. 1. Radial displacements along the shell axis***

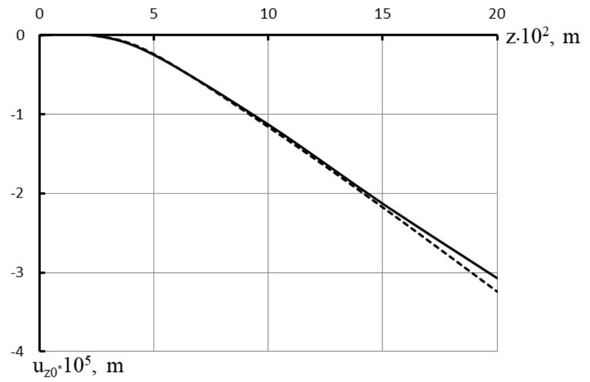


***Fig. 2. Axial displacements along the shell axis***

Next, the nonlinear deformation of a thick-walled hollow cylinder made of the material СЧ 15-32, loaded with internal pressure of $P_{inn}$=22.0 MPa was considered. Geometric dimensions were: $l$=0.2 m, $r_1$=0.18 m, $r_2$=0.22 m.

The ends of the cylinder are free of loading and fixed in such a way that the radial displacements are equal to zero. The origin of the coordinates is placed in the center of the cylinder. Then the kinematic boundary condition will be written as

$$\dot{u}_r = 0 \text{ for } z = \pm l/2. \tag{17}$$

In this case, sequences of coordinate functions have the form

$$\dot{u}_r = \left((l/2)^2 - z^2\right)\Phi_1, \quad \dot{u}_z = z\Phi_2. \tag{18}$$

In formulas (18), the second expression is not related to the satisfaction of boundary conditions, but only ensures the fulfillment of symmetry conditions for axial displacements.

Fig. 3 shows graphs of changes along the axis of the cylinder of the radial displacements of the middle surface, and Fig. 4 – graphs of the distribution of axial stresses $\sigma_{zz}=\sigma_{zz}(r_1, z)$ on the inner surface of the cylinder. The solid lines show the results obtained on the basis of the constitutive equations (4), and the dashed lines show the results based on a simplified model built only on experimental data obtained during uniaxial tension [12].

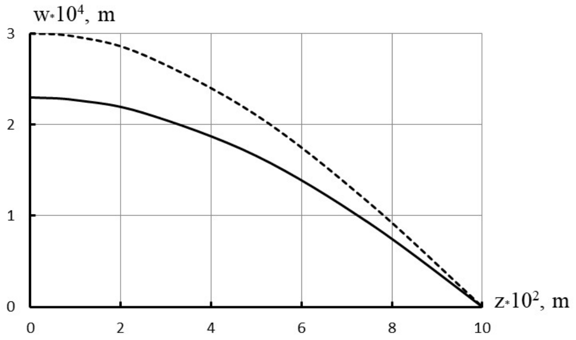

*Fig. 3. Radial displacements along the cylinder axis*

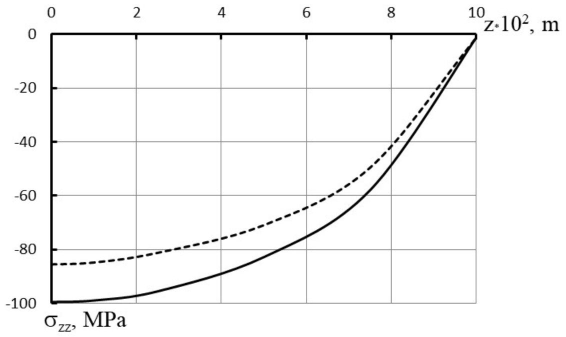

*Fig. 4. Axial stresses along the cylinder axis*

For loading, the following parameters were taken in formula (15): $P_1$=1.0 MPa, $P_2$=0.1 MPa, and the initial step and the given calculation error in the Runge-Kutta-Merson's method were equal to: $\Delta t_0=10^{-2}$, $\varepsilon=10^{-5}$.

The graphs given in Figs. 3, 4 show that failure to take into account the different behavior of the material under tension and compression leads to significant errors in determining the components of the stress-strain state.

Spline systems in both solved problems were built on a regular dimensional grid $N_r \times N_z=5\times10$. At the same time, the total number of coordinate functions was equal to 208.

## Conclusions

A numerical-analytical method for solving the problems of physically nonlinear deformation of cylinders from materials with different behavior in tension and compression has been developed. The method is based on the joint use of uninterrupted parameter continuation, Ritz and Runge-Kutta-Merson's methods. The test problem of the deformation of the cylindrical shell was solved, and a coincidence with the shell solution was obtained. The calculation of nonlinear elastic deformation of a thick-walled hollow cylinder under the action of internal pressure was performed. It is shown that failure to take into account the different behavior of the material under tension and compression leads to significant errors in the results of the stress-strain state parameters calculation.

## References


1. Zhukov, A. M. (1986). *Soprotivleniye nekotorykh materialov chistomu rastyazheniyu i szhatiyu* [Resistance of some materials to pure tension and compression]. *Izvestiya AN SSSR. Mekhanika tverdogo tela – Mechanics of Solids*, no. 4, pp. 197–202 (in Russian).
2. Miklyayev, P. G. & Fridman, Ya. B. (1986). *Anizotropiya mekhanicheskikh svoystv metallov* [Anisotropy of mechanical properties of metals]. Moscow: Metallurgiya, 224 p. (in Russian).
3. Sarrak, V. I. & Filippov, G. A. (1977). *Effekt raznogo soprotivleniya deformatsii pri rastyazhenii i szhatii martensita zakalennoy stali* [The effect of different resistance to deformation in tension and compression of hardened steel martensite]. *Fizika metallov i metallovedeniye – Physics of Metals and Metallography*, vol. 44, iss. 4, pp. 858–863 (in Russian).
4. Shevchenko, Yu. N., Babeshko, M. Ye., Piskun, V. V., & Savchenko, V. G. (1980). *Prostranstvennyye zadachi termoplastichnosti* [Spatial problems of thermoplasticity]. Kyiv: Naukova dumka, 264 p. (in Russian).
5. Podgornyy, A. N., Bortovoy, V. V., Gontarovskiy, P. P., Kolomak, V. D., Lvov, G. I., Matyukhin, Yu. I., & Morachkovskiy, O. K. (1984). *Polzuchest elementov mashinostroitelnykh konstruktsiy* [Creep of elements of machine-building structures]. Kyiv: Naukova dumka, 264 p. (in Russian).
6. Shevchenko, Y. N., Piskun, V. V., & Kovalenko, V. A. (1992). Elastoplastic state of axisymmetrically loaded laminated bodies of revolution made of isotropic and orthotropic materials. *International Applied Mechanics*, vol. 28, iss. 1, pp. 25–32. https://doi.org/10.1007/BF00847326.
7. Rvachev, V. L. & Sinkop, N. S. (1990). *Metod R-funktsiy v zadachakh teorii uprugosti i plastichnosti* [R-functions method in problems of the theory of elasticity and plasticity]. Kiyev: Naukova dumka, 216 p. (in Russian).
8. Breslavs'kyi, D. V., Korytko, Y. M., & Morachkovs'kyi, O. K. (2011). Cyclic thermal creep model for the bodies of revolution. *Strength of Materials*, vol. 43, iss. 2, pp. 134–143. https://doi.org/10.1007/s11223-011-9279-8.
9. Braam, H. & Haverkate, B. R. W. (1994). Creep lifetime prediction of pressurized tubes with continuum damage mechanics. *Structural Integrity: Experiments – Models – Applications*: Proceedings of the 10th Biennial Euro-

pean Conference on Fracture – ECF 10 (20–23 September 1994, Berlin, Federal Republic of Germany). Auspices of the European Structural Integrity Society, pp. 279–284.
10. Galishin, A. Z., Zolochevsky, A. A., & Sklepus, S. M. (2018). Study of creep and damage for a hollow cylinder on the basis of space and refined shell models. *Journal of Mathematical Sciences*, vol. 231, iss. 5, pp. 629–640. https://doi.org/10.1007/s10958-018-3840-y.
11. Zolochevskiy, A. A. & Damasevich, S. V. (1990). *Metodika rascheta nelineyno-uprugogo deformirovaniya obolochek iz materialov, raznosoprotivlyayushchikhsya rastyazheniyu i szhatiyu* [Method for calculating nonlinear elastic deformation of shells made of materials differently resistant to tension and compression]. *Izvestiya vuzov. Mashinostroyeniye – Bauman Moscow State Technical University Journal of Mechanical Engineering*, no. 5, pp. 30–34 (in Russian).
12. Zolochevskiy, A. A., Sklepus, A. N., & Sklepus, S. N. (2011). *Nelineynaya mekhanika deformiruyemogo tverdogo tela* [Nonlinear mechanics of a deformable solid body]. Kharkiv: «Biznes Investor Grupp», 720 p. (in Russian).
13. Zolochevsky, A., Sklepus, S., Galishin, A., Kühhorn, A., & Kober, M. (2014). A comparison between the 3D and the Kirchhoff-Love solutions for cylinders under creep-damage conditions. *Technische Mechanik – European Journal of Engineering Mechanics*, vol. 34, no. 2, pp. 104–113. https://doi.org/10.24352/UB.OVGU-2017-056.
14. Podgornyy, A. N., Gontarovskiy, P. P., Kirkach, B. N., Matyukhin, Yu. I., & Khavin, G. L. (1989). *Zadachi kontaktnogo vzaimodeystviya elementov konstruktsiy* [Problems of contact interaction of structural elements]. Kyiv: Naukova dumka, 232 p. (in Russian).
15. Voronin, V. Ye. & Shirshov, A. A. (1989). *Neustanovivshayasya polzuchest' tolstostennoy truby iz materiala s razlichnymi kharakteristikami pri rastyazhenii i szhati* [Unsteady creep of a thick-walled pipe made of a material with different characteristics under tension and compression]. *Izvestiya vuzov. Mashinostroyeniye – Bauman Moscow State Technical University Journal of Mechanical Engineering*, no. 8, pp. 7–9 (in Russian).
16. Gorev, B. V. & Tsvelodub, I. Yu. (1974). *Primeneniye energeticheskikh uravneniy polzuchesti k raschetu tolstostennoy tsilindricheskoy truby* [Application of energy creep equations to the calculation of a thick-walled cylindrical pipe]. *Dinamika sploshnoy sredy – Continuum dynamics*, iss. 17, pp. 99–105 (in Russian).
17. Gao, J., Huang, P., & Yao, W. (2017). Analytical and numerical study of temperature stress in the bi-modulus thick cylinder. *Structural Engineering and Mechanics*, vol. 64, no. 1, pp. 81–92. https://doi.org/10.12989/sem.2017.64.1.081.
18. He, X.-T., Wang, X.-G., & Sun, J.-Y. (2023). Application of the variational method to the large deformation problem of thin cylindrical shells with different moduli in tension and compression. *Materials*, vol. 16 (4), article 1686. https://doi.org/10.3390/ma16041686.
19. He, X.-T., Chang, H., & Sun, J.-Y. (2023). Axisymmetric large deformation of thin shallow shells with different moduli in tension and compression. *Thin-Walled Structures*, vol. 182, part B, article 120297. https://doi.org/10.1016/j.tws.2022.110297.
20. Grigolyuk, E. I. & Shalashilin, V. I. (1988). *Problemy nelineynogo deformirovaniya: metod prodolzheniya resheniya po parametru v nelineynykh zadachakh mekhaniki tverdogo deformiruyemogo tela* [Problems of nonlinear deformation: method of continuing the solution with a parameter in nonlinear problems of mechanics of a solid deformable body]. Moscow: Nauka, 232 p. (in Russian).
21. Demidov S. P. (1979). *Teoriya uprugosti* [Theory of elasticity]. Moscow: Vysshaya shkola, 432 p. (in Russian).
22. Zolochevskii, A. A. (1990). Determining equations of nonlinear deformation with three stress-state invariants. *Soviet Applied Mechanics*, vol. 26, iss. 3, pp. 277–282. https://doi.org/10.1007/BF00937216.
23. Krylov, V. I, Bobkov, V. V., & Monastyrnyy, P. I. (1977). *Vychislitelnyye metody* [Computational methods]. Moscow: Nauka, 399 p. (in Russian).

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## Нелінійне деформування циліндрів із матеріалів, що неоднаково опираються розтягу і стиску

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*Розроблено новий чисельно-аналітичний метод розв'язування фізично нелінійних задач деформування осесиметрично навантажених циліндрів із матеріалів, що неоднаково опираються розтягу і стиску. Для лінеаризації задачі використано метод неперервного продовження за параметром. Для варіаційної постановки лінеаризованої* *задачі побудовано функціонал у формі Лагранжа, заданий на кінематично можливих швидкостях переміщень. Для знаходження основних невідомих задачі фізично нелінійного деформування циліндра сформульовано задачу Коші для системи звичайних диференціальних рівнянь. Задачу Коші розв'язано методом Рунґе-Кутта-Мерсона з автоматичним вибором кроку. Початкові умови встановлювалися шляхом розв'язання задачі лінійно-пружного деформування. Праві частини диференціальних рівнянь при фіксованих значеннях параметра навантаження, що відповідають схемі Рунґе-Кутта-Мерсона, знайдено із розв'язку варіаційної задачі для функціонала у формі Лагранжа. Варіаційні задачі розв'язано методом Рітца. Розв'язано тестову задачу для нелінійно-пружного деформування тонкої циліндричної оболонки. Отримано збіг просторового розв'язку з оболонковим. Досліджено фізично нелінійне деформування товстостінного циліндра. Показано, що неврахування різної поведінки матеріалу за розтягу і стиску призводить до значних похибок у результатах розрахунку параметрів напружено-деформованого стану.*

***Ключові слова**: товстостінний циліндр, різноопірність розтягу і стиску, фізично нелінійне деформування, метод неперервного продовження за параметром.*

## Література

1. Жуков А. М. Сопротивление некоторых материалов чистому растяжению и сжатию. *Известия АН СССР. Механика твердого тела*. 1986. № 4. С. 197–202.
2. Микляев П. Г., Фридман Я. Б. Анизотропия механических свойств металлов. Москва: Металлургия, 1986. 224 с.
3. Саррак В. И., Филиппов Г. А. Эффект разного сопротивления деформации при растяжении и сжатии мартенсита закаленной стали. *Физика металлов и металловедение*. 1977. Т. 44. № 4. С. 858–863.
4. Шевченко Ю. Н., Бабешко М. Е., Пискун В. В., Савченко В. Г. Пространственные задачи термопластичности. Киев: Наукова думка, 1980. 264 с.
5. Подгорный А. Н., Бортовой В. В., Гонтаровский П. П., Коломак В. Д., Львов Г. И., Матюхин Ю. И., Морачковский О. К. Ползучесть элементов машиностроительных конструкций. Киев: Наукова думка, 1984. 264 с.
6. Шевченко Ю. Н., Пискун В. В., Коваленко В. А. Упругопластическое состояние осесимметрично нагруженных слоистых тел вращения из изотропных и ортотропных материалов. *Прикладная механика*. 1992. Т. 28. № 1. С. 31–39.
7. Рвачев В. Л., Синкоп Н. С. Метод R-функций в задачах теории упругости и пластичности. Киев: Наукова думка, 1990. 216 с.
8. Бреславський Д. В., Коритко Ю. М., Морачковський О. К. Модель циклічної термоповзучості для тіл обертання. *Проблеми міцності*. 2011. № 2. С. 33–46.
9. Braam H., Haverkate B. R. W. Creep lifetime prediction of pressurized tubes with continuum damage mechanics. *Structural Integrity: Experiments – Models – Applications*: Proceedings of the 10th Biennial European Conference on Fracture – ECF 10 (20–23 September 1994, Berlin, Federal Republic of Germany). Auspices of the European Structural Integrity Society, 1994. P. 279–284.
10. Галішин О. З., Золочевський О. О., Склепус С. М. Дослідження повзучості та пошкоджуваності порожнистого циліндра на основі просторової та уточненої оболонкової моделей. *Математичні методи та фізико-механічні поля*. 2016. Т. 59. № 2. С. 116–124.
11. Золочевский А. А., Дамасевич С. В. Методика расчета нелинейно-упругого деформирования оболочек из материалов, разносопротивляющихся растяжению и сжатию. *Известия вузов. Машиностроение*. 1990. № 5. С. 30–34.
12. Золочевский А. А., Склепус А. Н., Склепус С. Н. Нелинейная механика деформируемого твердого тела. Харьков: «Бизнес Инвестор Групп», 2011. 720 с.
13. Zolochevsky A., Sklepus S., Galishin A., Kühhorn A., Kober M. A comparison between the 3D and the Kirchhoff-Love solutions for cylinders under creep-damage conditions. *Technische Mechanik*. 2014. Vol. 34. No. 2. P. 104–113. https://doi.org/10.24352/UB.OVGU-2017-056.
14. Подгорный А. Н., Гонтаровский П. П., Киркач Б. Н., Матюхин Ю. И., Хавин Г. Л. Задачи контактного взаимодействия элементов конструкцій. Киев: Наукова думка, 1989. 232 с.
15. Воронин В. Е., Ширшов А. А. Неустановившаяся ползучесть толстостенной трубы из материала с различными характеристиками при растяжении и сжатии. *Известия вузов. Машиностроение*. 1989. № 8. С. 7–9.
16. Горев Б. В., Цвелодуб И. Ю. Применение энергетических уравнений ползучести к расчету толстостенной цилиндрической трубы. *Динамика сплошной среды*. 1974. Вып. 17. С. 99–105.
17. Gao J., Huang P., Yao W. Analytical and numerical study of temperature stress in the bi-modulus thick cylinder. *Structural Engineering and Mechanics*. 2017. Vol. 64. No. 1. P. 81–92. https://doi.org/10.12989/sem.2017.64.1.081.
18. He X.-T., Wang X.-G., Sun J.-Y. Application of the variational method to the large deformation problem of thin cylindrical shells with different moduli in tension and compression. *Materials*. 2023. Vol. 16 (4). Article 1686. https://doi.org/10.3390/ma16041686.

19. He X.-T., Chang H., Sun J.-Y. Axisymmetric large deformation of thin shallow shells with different moduli in tension and compression. *Thin-Walled Structures.* 2023. Vol. 182. Part B. Article 120297. https://doi.org/10.1016/j.tws.2022.110297.
20. Григолюк Э. И., Шалашилин В. И. Проблемы нелинейного деформирования: метод продолжения решения по параметру в нелинейных задачах механики твердого деформируемого тела. Москва: Наука, 1988. 232 с.
21. Демидов С. П. Теория упругости. Москва: Высшая школа, 1979. 432 с.
22. Золочевский А. А. Определяющие уравнения нелинейного деформирования с тремя инвариантами напряженного состояния. *Прикладная механика.* 1990. Т. 26. № 3. С. 74–80.
23. Крылов В. И, Бобков В. В., Монастырный П. И. Вычислительные методы. Москва: Наука, 1977. 399 с.