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DISCRETIZATION OF THIN-WALLED SECTIONS WITH VARIABLE WALL THICKNESS

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At the stage of designing thin-walled aircraft structures, to simplify calculations, their cross sections are idealized. To do this, the section with the skin and the longitudinal elements reinforcing it is replaced (when determining normal stresses) by a discrete one, consisting of concentrated areas at characteristic points. In this case, the equality of the moments of inertia of the initial and discrete sections is preserved. Such idealization is used in the calculation of thin-walled rods for normal and shear stresses (Wagner model). For sections consisting of a system of rectangular strips of constant thickness, discretization allows to set approximate values of normal and shear stresses and accurately find the locations of the singular points of the bending center (in an open contour) and the center of rigidity (in a closed one). The discrete model of a strip consists of three lumped areas: two at the edges and one in the center. The paper proposes to extend the discrete model to sections in which the skin thickness changes according to a linear law. In addition to the rectangular strip, it is possible to use elongated triangles and trapezoids, replaced by three and four concentrated areas, respectively. The possibility of using a discrete model for calculating some thin-walled sections of open and closed contours is considered. The section of an open contour is studied - the problem of transverse bending without torsion of a channel having flanges with a linearly variable thickness. Differences in the flows of tangential forces calculated from the exact and discrete models are shown. The coincidence of the results in determining the position of the bending center according to two models was established. When studying the application of a discrete model to a closed contour, its simplified option is proposed. The problem of transverse bending without torsion and finding the center of rigidity in a section with a contour line in the form of a trapezoid with front and rear walls of constant thickness and upper and lower skins and a similar section with a contour in the form of a rectangle was considered. Differences in the flows of tangential forces calculated by exact and discrete models are established. For a closed section in the form of a rectangle, the decrease in the moment of inertia for torsion due to the redistribution of material in the cross section was studied separately. It has been established that when finding the position of the center of rigidity, the discrepancy between the results of the exact and discrete models in sections with geometric parameters close to real ones was less than 1% for a rectangular contour, and 4% for a trapezoidal contour. The results indicate the possibility of extending the application of the discrete model of thin-walled cross section to the design calculations of thin-walled rods with variable skin thickness, representing practical structures.

***Keywords**: discretization of thin-walled sections, variable wall thickness of a cross section, Wagner model, stiffness center, bending center.*

Introduction

To simplify the calculations of thin-walled rods, their cross sections are idealized [1], replacing the actual cross section with a skin, and the cross section is reinforced with longitudinal elements when determining the normal stresses by discrete elements, in which there are only concentrated elements at characteristic points of the cross section. At the same time, it is necessary to ensure the exact or very close equality of the axial and centrifugal moments of inertia of the cross section.

A similar idealization is used in the calculation of thin-walled rods for normal and tangential stresses. At the same time, Wagner's model is used with a shell that does not work for normal stresses [1, 2]. If the thin-walled section consists only of rectilinear strips of constant thickness, a technique called discretization [3, 4] is used, in which each strip is replaced by three concentrated areas (at the edges and in the middle) while maintaining the exact correspondence of the axial and centrifugal moments of inertia in the original transverse sections. The presence of reinforcing elements on the walls of the section is taken into account separately.

The use of discretization in the calculation of tangential stresses naturally leads to errors in their determination, but it ensures the accurate establishment of the most important parameters of the cross section: the position of the bending center (open cross section) and the center of stiffness (closed section), simplifying the search for their location [4–6].

It is proposed to expand the use of the method of discretization of a thin-walled cross section consisting of rectilinear strips by considering strips in the form of not only elongated rectangles, but also elongated trapezoids and triangles.

Discretization of strips

An elongated triangular strip, as well as a rectangular one, fulfilling the requirement for the equivalence of the three indicated moments of inertia, will be represented by three concentrated areas: two at the edges (f_b at the base of the triangle and f_a at its apex) and one at the center of gravity of the triangle f_c.

By combining a triangular strip with a rectangular one, we get a discrete model for the strip in the form of an elongated trapezoid (Fig. 1).

In practical calculations, the discrete model of a rectangular strip for sections of a thin-walled rod consisting of such strips finds sufficient application [4–6].

The proposed discrete models of strips with linearly varying width along the length can be useful in the development of new types of profile sections for reinforcing panels in aircraft structures. These are cross sections with an open contour. For the cross sections of a thin-walled rod with a closed profile, these models appear to be in demand when developing new structural and technological solutions for the main spar of a helicopter, the tip of a wing of a light aircraft, and caisson parts of aircraft structures.

The application of the proposed discrete models of strips with a width that varies linearly along the length is considered. The error introduced by discretization in the calculations of thin-walled sections is estimated.

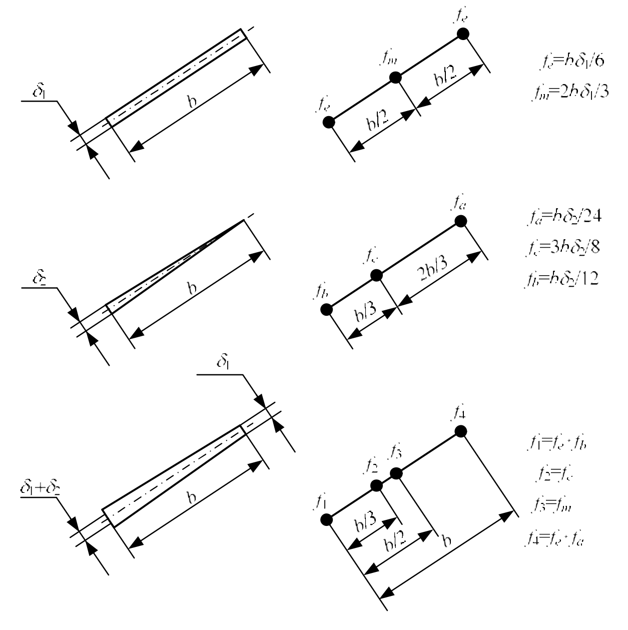

Fig. 1. Discrete models for stripes

Open contour research

When studying a thin-walled section of an open contour, we will consider the problem of transverse bending (without torsion) of a channel, the thickness of the shelves of which varies linearly. Fig. 2 shows three options of the cross section of the channel: the left and middle sections with shelves of variable thickness, and the third one, shown for comparison, has a constant thickness of shelves. The section is symmetrical about the x axis and has general data h=40 cm, b=20 cm and δ=1 cm. The purpose of the calculations was to determine the flows of tangential forces q_p in shelves and find the position of the bending center. The moment of inertia about the x axis for all sections was I_x=29333.3 sm^4.

For an accurate analytical solution, expressions for the current static moment at the starting point of the contour coordinate t bypass were obtained. The said coordinate was located at the upper free point of the channel: for the left section $S_x(t)=\frac{h}{2}\left(\delta t+\frac{\delta}{2b}t^2\right)$, for the middle one – $S_x(t)=\frac{h}{2}\left(2\delta t-\frac{\delta}{2b}t^2\right)$ and for the right one $-S_x(t)=\frac{h\delta}{2}t$. The graphs are shown above the images of sections and determine the type of graphs of the flows of tangential forces q_p in the shelves.

Parts of a discrete section model including the top shelf for three sections are shown at the top of Fig. 2. The concentrated areas marked there will be respectively:

$$F_{11}=\frac{b\delta}{6}+\frac{b\delta}{24}=4.167\ \text{sm}^2;$$

$$F_{12}=\frac{2b\delta}{3}=13.33\ \text{sm}^2;$$

$$F_{13}=\frac{3b\delta}{8}=7.5\ \text{sm}^2;$$

$$F_{14}=\frac{b\delta}{6}+\frac{b\delta}{12}+\frac{h\delta}{6}=11.67\ \text{sm}^2;$$

$$F_{21}=\frac{b\delta}{6}+\frac{b\delta}{12}=5.0\ \text{sm}^2;$$

$$F_{22}=\frac{3b\delta}{8}=7.5\ \text{sm}^2;$$

$$F_{23}=\frac{2b\delta}{3}=13.33\ \text{sm}^2;$$

$$F_{24}=\frac{b\delta}{6}+\frac{b\delta}{24}+\frac{h\delta}{6}=10.83\ \text{sm}^2;$$

$$F_{31}=\frac{b\delta}{6}=3.667\ \text{sm}^2;$$

$$F_{32}=\frac{2b\delta}{3}=13.33\ \text{sm}^2;$$

$$F_{33}=\frac{b\delta}{6}+\frac{h\delta}{6}=10.0\ \text{sm}^2.$$

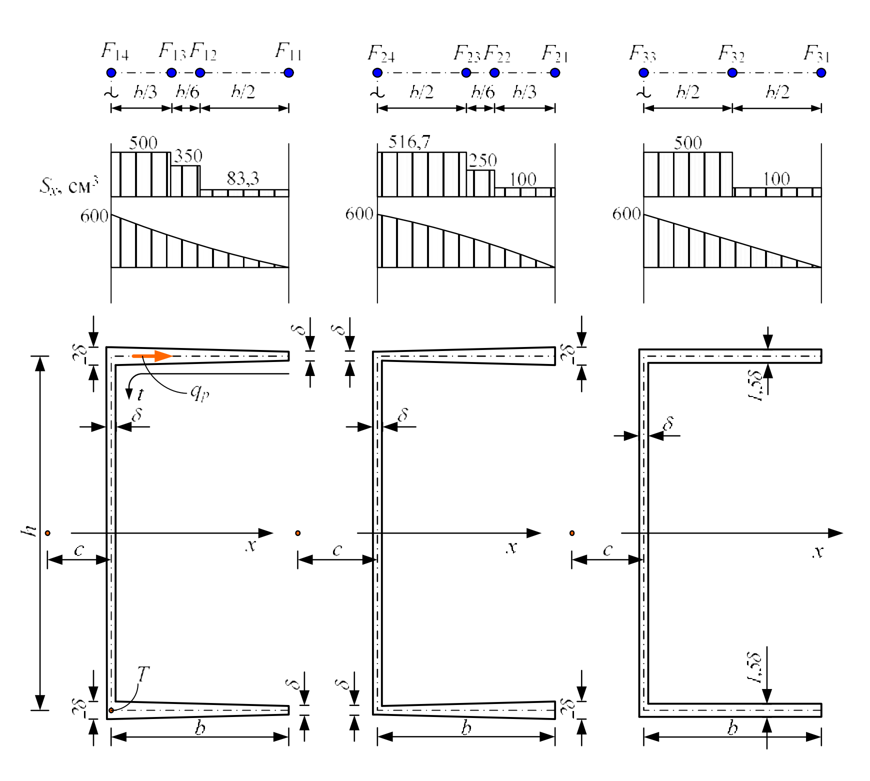

Fig. 2. Three options for the channel section

Based on the obtained concentrated areas for the three models, the values of the current static moment $S_x(t)$ were obtained in the areas of the upper shelf. Graphs $S_x(t)$ are shown in Fig. 2 over the corresponding graphs of the exact solution. As can be seen, the differences in the results in the graphs for the exact and discrete models are significant, but acceptable for the design calculation. There will be the same discrepancy in the magnitudes of the flows of tangential forces q_p. It is clear that the discretization for strips with linearly varying thickness did not affect the difference in the results for q_p. Further, of course, the discretization for strips with linearly varying thickness has little effect on the graphs $S_x(t)$ and on the difference in results for q_p.

However, the discretization turns out to be useful in finding the position of the bending center of a section of a thin-walled rod of an open profile. Here, as it was shown in [4–6], calculations of exact and discrete sections coincide for cross sections made of strips of constant thickness. For the sections shown in Fig. 2, when using the fictitious force method (at the same time, the lower point of the channel wall was taken as momentary), the position of the bending center, which is indicated in the same figure (on the x axis and at a distance c from the contour line of the wall), is determined. The results of the two solution methods coincided and were compiled for the left section c=7,273 sm, for the average one – c=9,091 sm and for the right one – c=8,182 sm.

Therefore, the proposed discretization can be useful in the calculation of the cross sections of a thin-walled rod of an open contour, its application is especially beneficial when finding the position of the bending center of an open contour.

Research of a closed loop

Moving on to the use of the proposed discretization model of strips with a linearly variable width in the cross sections of a thin-walled rod with a closed contour, we will indicate some difficulties that arise in this case.

First of all, we emphasize that when calculating the flows of tangential forces, there were no problems with the use of this discrete model. The results of calculations based on the exact and approximate discrete models are quite similar (this will be shown below), and the approximate calculation can be used to estimate the value of tangential force flows q.

Difficulties are associated with finding the position of the center of rigidity in a closed thin-walled section. The problem is that it is necessary to calculate the torsional stiffness of the section and a series of integrals over sections [3, 4]. This may affect the error when calculating using a discrete model.

The torsional stiffness of a single-locked section of a thin-walled rod is calculated as follows:

$$GI_T = \frac{\Omega^2}{\oint \frac{dt}{G\delta}}, \tag{1}$$

where Ω is the doubled area of the closed loop; t is the contour coordinate; G is the material shear modulus; $\delta=\delta(t)$ is the skin thickness, in the considered model it is a linear function.

Integrals used in calculations have the form

$$\oint \frac{q(t)dt}{G\delta}. \tag{2}$$

It was set that G=const (this is true for most problems). Then the calculation of torsional stiffness GI_T can be carried out using a discrete model according to the given formula, which does not pose any difficulties with a linear change in thickness.

However, when obtaining the given integrals using an exact calculation method, certain computational inconveniences arise ($q(t)$ – power function), especially when conducting parametric studies at the initial stages of designing structures.

A discrete model is useful to overcome these inconveniences.

In order to study the issue of the possibility of applying a discrete model, the problem of determining the position of the center of rigidity of a closed cross section of a thin-walled rod was considered.

Fig. 3 shows a section, the contour line of which has the shape of a rectangle, and has front and back walls of constant thickness, as well as upper and lower skins with variable thickness, which changes linearly and coincides with the thickness of the walls at the extreme points.

The calculation of the section (Fig. 3) was carried out using the exact method and the basic (Fig. 4) and simplified (Fig. 5) options of the discrete model.

Both shown models (Figs. 4 and 5) have the same values of concentrated areas

$$F_{p1} = \frac{H\delta_2}{6} + \frac{B\delta_2}{6} + \frac{B(\delta_1-\delta_2)}{24};\ F_{p2} = \frac{2B\delta_2}{3};$$

$$F_{p3} = \frac{3B\delta_2}{8};\ F_{p4} = \frac{H\delta_1}{6} + \frac{B\delta_2}{6} + \frac{B(\delta_1-\delta_2)}{12}. \tag{3}$$

In the model (Fig. 4), the thickness of the upper and lower skins varies linearly along the length of the contour line, and integrals of the type $\oint \frac{dt}{\delta(t)}$. The simplified model (Fig. 5) additionally includes the constancy of the thickness on the contour sections to eliminate the need to calculate the specified integrals. For example, thicknesses δ_{p1}, δ_{p2}, δ_{p3} represent the average thickness of the corresponding part of the contour; their values are used at "calculation" of the specified integrals.

The performed calculations showed that the use of two discrete models leads to approximately the same results.

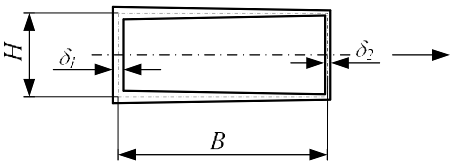

Fig. 3. Rectangular section

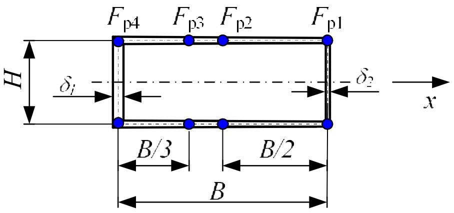

Fig. 4. Discrete model

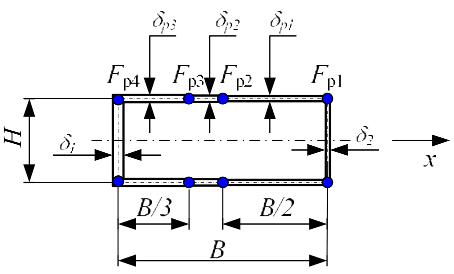

Fig. 5. Simplified discrete model

Fig. 6 shows the graphs of tangential force flows in the section (Fig. 3 with the following parameters B/H=2.5 and δ_1/δ_2=2) in transverse bending of a thin-walled rod without torsion, when the transverse force Q_y is applied in the center of rigidity. At the same time, it turned out that the ratio of the maximum values of the flows of tangential forces q on the front and rear walls of the section was 1.74 with an exact solution, and 1.71 with the use of a discrete model.

The error of the discrete model in determining the position of the center of rigidity for a rectangular section was also compared (Fig. 3) $\bar{x}_{c.s} = \frac{x_{c.s}}{B}$, where $x_{c.s}$ is the distance from the front wall to the center of rigidity. Value $x_{c.s}$ was calculated with the help of two methods of fictitious force and fictitious moment (extraction of torsion), while the coincidence of the results was observed. Sections with different lengths λ=B/H and the ratio of the thicknesses of the back and front walls δ_2/δ_1 were considered. The error increases as the ratio decreases. The results of calculations with an indication of the border where the error in the calculation $\bar{x}_{c.s}$ (relative distance to the center of stiffness) is absorbed by 1%, are shown in Fig. 7.

The results (Fig. 7) demonstrate the possibility of using a discrete model regarding the position of the center of rigidity of a rectangular cross section (Fig. 3) with parameters characteristic of the section geometry of real thin-walled structures.

The possibility of reducing the moment of inertia during rotation I_T is of some interest for sections with variable thickness of the upper and lower skins (Fig. 3). For a section of constant area, the ratio δ_1/δ_2 changed and a relative decrease in value I_T in relation to the value obtained with the same thickness of the skin along the entire contour $\delta_1/\delta_2=1$ was established. Two sections with different lengths λ=B/H, equal to 2 and 5, were considered. The results are shown in Fig. 8.

As can be seen (Fig. 8), the redistribution of material along the contour of the section is in a practically interesting range of changes in λ and δ_1/δ_2 does not lead to a significant fall of I_T, and therefore, the torsional stiffness of the cross section.

The application of a discrete model was also studied regarding the position of the center of rigidity in the cross section, the contour line of which is a trapezoid. Fig. 9 shows a section, the contour line of which has the form of a trapezoid (symmetric about the *x* axis) with front and back walls of constant thickness, upper and lower skins with linearly variable thickness, which coincides at the extreme points with the thickness of the walls.

In the discrete model (Fig. 10), the values of concentrated areas were calculated

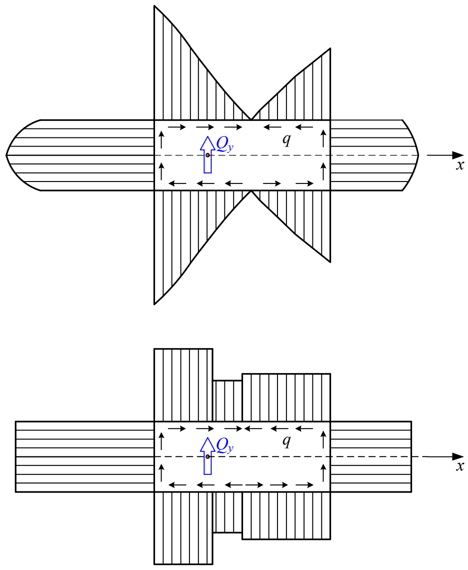

Fig. 6. Distribution of values of the flows of tangential forces q

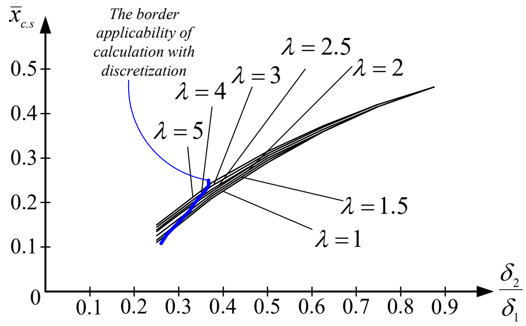

Fig. 7. The position of the center of rigidity in a rectangular section

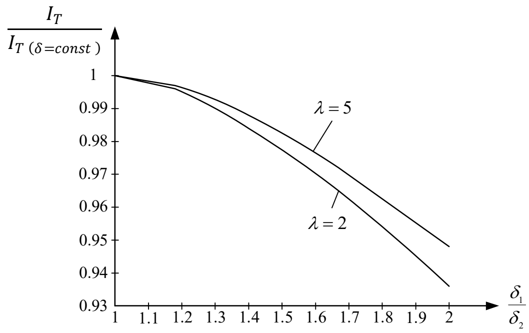

Fig. 8. Moment of inertia during torsion

$$F_{ptr1} = \frac{H_2\delta_2}{6} + \frac{L\delta_2}{6} + \frac{L(\delta_1-\delta_2)}{24};\; F_{ptr2} = \frac{2L\delta_2}{3};\; F_{ptr3} = \frac{3L\delta_2}{8};\; F_{ptr4} = \frac{H_1\delta_1}{6} + \frac{L\delta_2}{6} + \frac{L(\delta_1-\delta_2)}{12}, \quad (4)$$

where L is the the length of the contour line of the upper and lower skins.

A simplified discrete model was also used during the calculations and gave approximately the same results.

Fig. 11 shows the diagrams of the flows of tangential forces in the section (with the following parameters B/H_1=2.5; H_1/H_2=2 and δ_1/δ_2=2) in transverse bending of a thin-walled rod without torsion, when the transverse force Q_y is applied in the center of rigidity.

At the same time, it turned out that the ratio of the maximum values of the tangential flows of forces q for the front and back walls of the section was 2.10 with an exact solution, and 1.80 with the use of a discrete model. The specified difference decreases with a decrease in the ratio H_1/H_2.

In calculations carried out for symmetrical sections in the form of a trapezoid with front and back walls of constant thickness, upper and lower skins with linearly variable thickness, which coincides at the extreme points with the thickness of the walls, the error Δ in determining the position of the center of rigidity of the discrete model was studied. Cross sections with various geometric parameters, close to the parameters of real thin-walled sections of aircraft structures, were considered. The results of calculations for parametric studies are shown in Fig. 12.

Error Δ in determining the position of the center of rigidity when using a discrete model in all considered tasks did not exceed 4%. An increase in the error is observed with an increase in the ratio B/H_1. For sections with fixed B/H_1 the error increased with decreasing ratios H_2/H_1 and δ_2/δ_1.

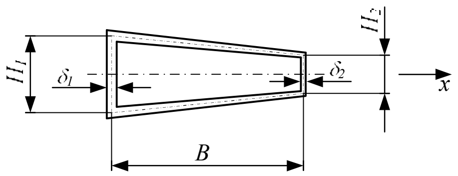

Fig. 9. Cross section

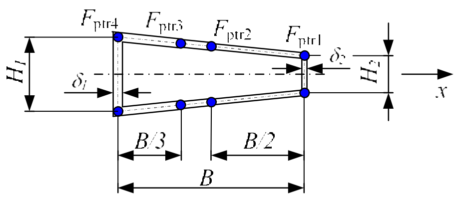

Fig. 10. Discrete model

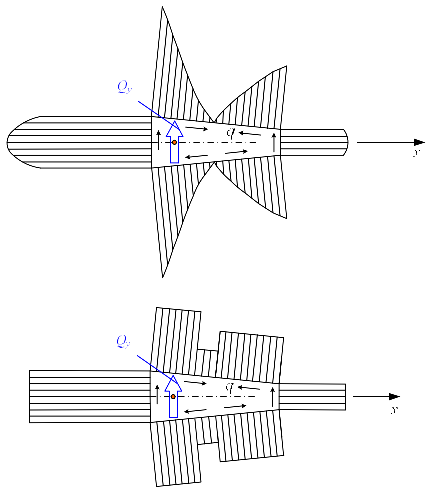

Fig. 11. Distribution of values of the flows of tangential forces

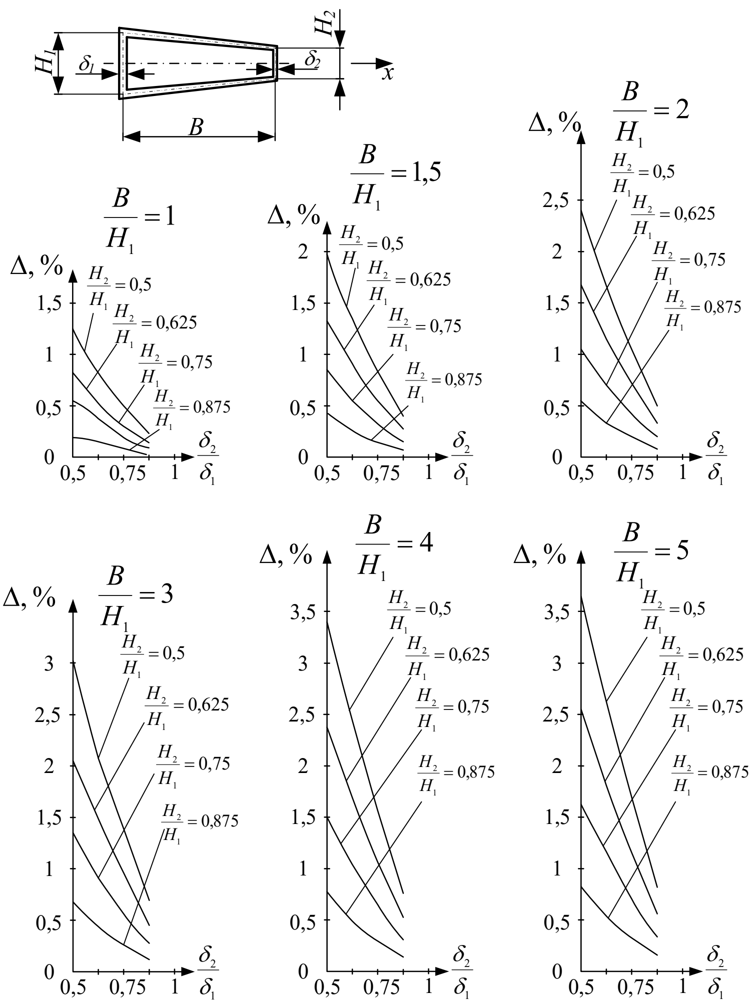

Fig. 12. Error Δ of discrete model in determining the position of the center of rigidity

Conclusion

As shown in the given data, the proposed discretization allows to use a simplified method of calculating tangential stresses for the analysis of thin-walled rods of an open contour, including branching, of a complex cross section with variable wall thickness without significant error. Thanks to this, the difficulty intensity at the design stages is reduced.

This is also true for sections of thin-walled rods with a closed cross-sectional contour, including those with closed regions and branches. Taking into account the variability of the wall thickness will allow to expand the possibilities of obtaining rational forms of the cross section of thin-walled rods.

The presented discrete model can be very beneficial for use at the design stage of sections of thin-walled structures to assess the location of the center of bending or stiffness (depending on the section type) in order to place it in the most convenient position for the perception of static and dynamic loadings.

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Дискретизація тонкостінних перерізів зі змінною товщиною стінки

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На етапі проєктування тонкостінних авіаційних конструкцій для спрощення розрахунків їх поперечні перерізи піддають ідеалізації. Для цього переріз з обшивкою і поздовжніми елементами, що підкріплюють її, замінюють дискретним, що складається із зосереджених площ у характерних точках. При цьому зберігається рівність моментів інерції вихідного й дискретного перерізів. Така ідеалізація використовується при розрахунку тонкостінних стрижнів на нормальні й дотичні напруження (модель Вагнера). Для перерізів, що складаються з системи прямокутних смужок постійної товщини, дискретизація дозволяє встановлювати наближені значення нормальних і дотичних напружень і точно визначати місцезнаходження особливих точок центру згинання (у відкритому контурі) і центру жорсткості (у закритому). Дискретна модель смужки складається з трьох зосереджених площ: двох на краях і однієї в центрі. У роботі запропоновано розширити дискретну модель на перерізи, в яких товщина обшивки за контуром змінюється за лінійним законом. Зауважено, що на додаток до прямокутної смужки можна використовувати витягнуті трикутники і трапеції, які замінюються трьома й чотирма зосередженими площами відповідно. Розглянуто можливість застосування дискретної моделі для розрахунку деяких тонкостінних перерізів відкритого й закритого контурів. Досліджено переріз відкритого контуру – задача про поперечне згинання без кручення швелера, що має полиці з лінійно змінюваною товщиною. Показані відмінності в потоках дотичних сил, підрахованих за точною й дискретною моделями. Встановлено збіг результатів щодо положення центру згинання за двома моделям. При вивченні застосування дискретної моделі до замкнутого контуру запропоновано спрощений варіант. Розглядалася задача про поперечне згинання без кручення і пошуку центру жорсткості в перерізі з контурною лінією у вигляді трапеції з пе-

редньою й задньою стінками постійної товщини та верхньою й нижньою обшивками змінної товщини за контуром, а також в аналогічному перерізі з контурною лінією у вигляді прямокутника. Встановлено відмінності в потоках дотичних сил, підрахованих за точними й дискретними моделями. Для замкнутого перерізу у вигляді прямокутника окремо досліджено зниження моменту інерції на кручення від перерозподілу матеріалу у поперечному перерізі. З'ясовано, що при знаходженні положення центру жорсткості розходження в результатах точної і дискретної моделей склало в перерізах з геометричними параметрами, близькими до реальних, для прямокутного контуру менше 1%, а для трапецієподібного – 4%. Результати свідчать про можливість розширення застосування дискретної моделі тонкостінного поперечного перерізу на проєктувальні розрахунки тонкостінних стрижнів зі змінною товщиною обшивки, що представляють практичні конструкції.

***Ключові слова**: дискретизація тонкостінних перерізів, змінна товщина стінки поперечного перерізу, модель Вагнера, центр жорсткості, центр згинання.*

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