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MULTICRITERIA OPTIMIZATION OF STOCHASTIC ROBUST CONTROL OF THE TRACKING SYSTEM

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A multicriteria optimization of stochastic robust control with two degrees of freedom of a tracking system with anisotropic regulators has been developed to increase accuracy and reduce sensitivity to uncertain object parameters. Such objects are located on a moving base, on which sensors for angles, angular velocities and angular accelerations are installed. Improvements in the accuracy of control with two degrees of freedom include closed-loop feedback control and open-loop feedback control through the use of reference and perturbation effects. The multicriteria optimization of the stochastic robust control tracking system with two degrees of freedom with anisotropic controllers is reduced to the iterative solution of a system of four coupled Riccati equations, the Lyapunov equation, and the determination of the anisotropy norm of the system by an expression of a special form, which is numerically solved using the homotopy method, which includes vectorization matrices and iterations according to Newton's method. The objective vector of robust control is calculated in the form of a solution of a vector game, the vector gains of which are direct indicators of the quality that the system should achieve in different modes of its operation. The calculation of the vector gains of this game is related to the simulation of a synthesized system with anisotropic regulators for different modes of operation with different input signals and object parameter values. The solutions of this vector game are calculated on the basis of a set of Pareto-optimal solutions taking into account the binary relations of preferences on the basis of the metaheuristic algorithm of multi-swarm Archimedes optimization. Based on the results of the synthesis of stochastic robust control of a tracking system with two degrees of freedom with anisotropic controllers, it is shown that the use of synthesized controllers made it possible to increase the accuracy of system control, reduce the time of transient processes by 3–5 times, reduce the variance of errors by 2.7 times, and reduce the sensitivity of the system to the change of object parameters compared to typical regulators.

Keywords: *discrete-continuous plant, nonlinear robust control, Hamilton-Jacobi-Isaacs equation, multicriteria parametric optimization, stochastic metaheuristic optimization algorithm.*

Introduction

Tracking systems are one of the main elements of technological processes in mechanical engineering, energy, metallurgy, transport and other industries. The vast majority of control theory methods were created and tested in relation to such systems. The theory of automatic control has come a long way from the simplest systems to modern robust digital controllers capable of providing high control accuracy under conditions of disturbances in a wide frequency range. Solving the problem of ensuring high accuracy of tracking systems is hindered by two main factors, namely, the presence of elastic elements in mechanical transmissions from the executive engine to the working body, which necessitates the presentation of the model of the engine-working mechanism system as a two-, three- and multi-mass [1], and a load with a nonlinear characteristic, which causes difficulties in the implementation of smooth movement of executive mechanisms [2]. Such tracking systems form a class of systems with interval uncertainty of part of the parameters, which leads to the need to consider a whole class of systems whose parameters are in a given area instead of one system [3]. A characteristic feature of this class of tracking systems is also the uncertainty of the parameters of external disturbances [4–6]. In addition, there are many possible modes of operation of tracking systems, which indicates their multi-purpose usage. Another feature of the considered class of tracking systems is the presence of restrictions on state and control variables, which do not allow to achieve the high control accuracy [7–9].

The presence of many different control goals leads to the need to set appropriate requirements for the quality criteria of controlled processes [10]. As a rule, certain requirements are put forward when the system works in transient processes with small movements. The quality criteria of transient processes are the time of first adjustment, time of adjustment in general, re-adjustment, etc. In transient processes with large displacements, the same criteria for the quality of transient processes as for small displacements are used, but their values may differ significantly.

In addition, in stable modes when processing or compensating harmonic and random signals, as well as when moving at low speed, own criteria for the quality of control systems are used. Thus, the next feature of the problems of synthesis of the control system under consideration is the multicriteria assessment of the quality of controlled processes.

A characteristic mode of operation of many control systems is the elimination of random influences, or the compensation of random external disturbing influences of a wide range of frequencies. Recently, the theory of stochastic robust control has been found under intensive development [9–10]. At the current stage, the basis of this theory is the methods of minimizing the anisotropy norm, which is an effective indicator of system quality in the presence of uncertainties in the description of the control object. Stochastic robust control systems have a number of advantages. First of all, they are robustly stable, that is, they maintain stability when the parameters of the control object change within the limits. Secondly, they are significantly less sensitive to changes in the parameters of the control object compared to optimal stochastic systems, despite the fact that the dynamic characteristics of stochastic robust systems may slightly differ from the corresponding characteristics of optimal stochastic systems. In view of this, the issue of designing control systems with uncertain parameters of the control object, which work under random and disturbing influences, is relevant. A significant increase in the accuracy of control of multi-mass electromechanical systems with uncertain parameters is possible with the use of special control algorithms, including robust methods that maintain their efficiency in the event of uncertainty of the parameters of the control object and external influences.

The aim of the paper is to develop a method of multicriteria optimization of stochastic robust control of the tracking system in such a way that the synthesized system meets all requirements during its operation in different modes.

Synthesis of stochastic robust control

The central problem of modern theory and practice of robust control is the creation of systems capable of functioning effectively in conditions of uncertainty of parameter values, and possibly of the structure of the control object models, as well as under influences that disturb control signals and have measurement obstacles. At the current stage of the development of the theory of automatic control systems, there is a need to find such control in conditions of incomplete, unclear and imprecise knowledge of the characteristics of the control object and the environment in which this object functions. This need arises due to the fact that the practice of design and operation of the control systems by industrial objects showed that systems synthesized according to the criteria of modular and symmetric optima, as well as according to the quadratic quality criterion, are sensitive both to changes in the parameters of the control object, input characteristics, disturbing influences, and the structure and parameters of the control object model used in control circuits.

A stochastic approach to H_∞ – optimization of automatic control systems based on the use of the quality criterion of the stochastic norm of the system. This norm quantitatively characterizes the sensitivity of the system output to random input disturbances, the probability distribution of which is not exactly known. The concretization of this approach was obtained by combining the concept of the stochastic norm of the system and the anisotropy of the average signal [9–10], which leads to a special option of the stochastic norm – the anisotropic norm.

Average anisotropy is a characteristic of the spatial-temporal coloring of a stationary Gaussian signal, which is closely related, on the one hand, to the information-theoretic approach to the quantitative description of chaos using the Kolmogorov entropy of the probability distribution, and on the other hand, to the principle of finite isotropy dimensional Euclidean space.

The anisotropic norm of the system characterizes its sensitivity to input Gaussian noise, the average anisotropy of which is bounded from above by some integral parameter. The synthesis of anisotropic controllers is related to the minimization of the anisotropic norm of the objective vector of robust control. In this case, a combination of the stochastic norm of the system and the average anisotropy of the random signal is used, which leads to one of the options of the stochastic norm, which is called the anisotropic norm. The problem of determining the anisotropic norm of the control system is reduced to solving the Riccati and Lyapunov equations, and the problem of synthesizing the system that minimizes the anisotropic norm – to the synthesis of two Riccati equations, the Lyapunov equation, and one algebraic equation.

Similarly, the state equation, the objective vector $\vec{z}(t)$ and the vector of the measured output $\vec{y}(t)$ are written in the standard form, which is accepted in the theory of robust control

$$\frac{d\vec{x}}{dt} = A\vec{x}(t) + B_1\vec{w}(t) + B_2\vec{u}(t); \tag{1}$$

$$\vec{z}(t) = C_1\vec{x}(t) + D_{11}\vec{w}(t) + D_{12}\vec{u}(t); \tag{2}$$

$$\vec{y}(t) = C_2\vec{x}(t) + D_{21}\vec{w}(t) + D_{22}\vec{u}(t). \tag{3}$$

where $\vec{w}(t)$ is the vector of external disturbances.

When describing the system in the state space in the form of matrices *A*, *B*, *C* and *D*, the anisotropic norm of the objective vector is determined by the expression [9–10]

$$\overline{A}(G) = -\frac{1}{2}\ln\det\left(\frac{m\Sigma}{Trace\left(LPL^T + \Sigma\right)}\right), \tag{4}$$

where $P \in R^{n\times n}$; *G* is the control gramian that satisfies the Lyapunov equation

$$P = (A + BL)P(A + BL)^T + B\Sigma B^T. \tag{5}$$

The matrices Σ and *L* are determined by the Riccati equation with respect to the matrix $R \in R^{n\times n}$

$$R = A^T RA + qC^T C + L^T \Sigma^{-1} L, \tag{6}$$

where

$$L = \Sigma\left(B^T RA + qD^T C\right);\ \Sigma = \left(I_m - B^T RB - qD^T D\right)^{-1}.$$

The anisotropic norm of the signal, that is, the numerical sequence, and the anisotropic norm of the system, firstly, are calculated according to different formulas, and secondly, have different physical meaning. The average signal anisotropy is zero if the discrete sequence is Gaussian "white noise" with a unit covariance matrix. The anisotropic norm of the system does not characterize the anisotropy of discrete sequences at the input and output of the system, but the equal sensitivity of the system on average to random input sequences with an average level of anisotropy. Moreover, the anisotropic norm of the system at zero anisotropy $a=0$ of the input discrete sequence is equal to the norm of the system H_2, and at infinite anisotropy $a\to\infty$ of the input discrete sequence – to the norm of the system H_∞.

Thus, if the value of the anisotropy of the input discrete system is within $0<a<\infty$, then the value of the anisotropic norm of the system is limited by the values H_2 and H_∞ of the system norms.

At zero mean anisotropy of the input signal, the synthesis of the optimal controller that minimizes the anisotropy norm of the system is reduced to the solution of two Riccati equations. Such an optimal anisotropic regulator corresponds to an optimal regulator that minimizes the dispersion of the output signal, and is actually a linear-quadratic regulator.

With zero average anisotropy of the input signal, which corresponds to a signal of the "white noise" type, the anisotropic controller is an optimal stochastic controller that minimizes the norm H_2.

With an infinite average input signal anisotropy corresponding to a fully determined deterministic signal, the anisotropic controller is an optimal deterministic robust controller that minimizes the norm. With the average anisotropy of the input signal in the range of $0<a<\infty$, the anisotropic control occupies an intermediate position between the controls that minimize the norms H_2 and H_∞.

Control of the system in the presence of external disturbances is considered as a differential game between two players - nature and the regulator. The optimal strategy of the first player, the regulator, is to obtain optimal control, and the optimal strategy of the second player, nature, is to obtain the "worst" disturbance in the form of parametric uncertainty of the control object [9–10]. In addition, each of the players knows the optimal strategy of their opponent. The second player implements their optimal strategy as follows: to create the "worst" perturbation, nature takes a copy of the system and organizes its own memory without feedback, which allows it to obtain information about the internal state of the closed system, which is formed by the internal state of the system and the internal state of the controller.

The first player, the regulator, behaves in a similar way. In an attempt to predict the optimal strategy of the second player, it evaluates the "worst" input based on the information it receives from observing the output of the system. If we assume that the "worst" signal has arrived at the input, the controller organizes dynamic feedback at the output, which provides the maximum coefficient of reduction of this interference.

In general, the synthesis problem of a robust anisotropic regulator and a robust anisotropic observer are minimax problems, and their solutions are the corresponding saddle points in the parameter space. In this case, it is necessary to find the "best" controllers and observers, with the help of which the anisotropic norms of the target vector of the closed system and the observer's error vector are minimized according to the gains of the controller and the observer.

To ensure robustness, these controllers and observers must be found for the "worst" cases in which the anisotropic norms of the closed-loop objective vector and the observer error vector are maximized according to the uncertainty vector of the output control object model and the noise vector in the measurement of the system output vector. Necessary conditions for the corresponding minima and maxima of these saddle points in the space of parameters and signals are the four Riccati equations [9–10].

The first Riccati equation is related to the search for the "worst" input of the system – the maximization of the anisotropic norm of the closed system using the input vector to implement the "worst" case using the vector of parametric uncertainty of the models of the original control object and external influences

$$\widetilde{Y} = A_t^T \widetilde{Y} A_t + L_t^T \Pi L_t + Q, \tag{7}$$

where

$$L_t = \Pi^{-1} F_t^T \widetilde{Y} A_t; \ \Pi = \Gamma - F_t^T \widetilde{Y} F_t .$$

The second Riccati equation is related to finding the "worst" signal in the synthesis of an anisotropic observer that maximizes the anisotropic norm of the closed system with respect to the signal vector to implement the worst case of uncertainty of the signal of the measurement noise vector of the output signal

$$R = A_\omega^T R A_\omega + q C_\omega^T C_\omega + L^T \Sigma^{-1} L, \tag{8}$$

where

$$L = \Sigma\left(B_\omega^T R A_\omega + q D_\omega^T C_\omega\right); \ \Sigma = \left(I_{m1} - B_\omega^T R B_\omega\right)^{-1} .$$

The third Riccati equation is related to the synthesis of an anisotropic observer, which minimizes the anisotropic norm of the observer's error vector from the observer's gain matrix

$$S = \widetilde{A}_{11} S \widetilde{A}_{11}^T + \widetilde{B}_1 \widetilde{B}_1^T - \Lambda \Theta \Lambda^T, \tag{9}$$

where

$$\Theta = \widetilde{C}_{21} S \widetilde{C}_{21}^T + \widetilde{D}\widetilde{D}^T; \ \Lambda = \left(\widetilde{A}_{11} S \widetilde{C}_{21}^T + \widetilde{B}_1 \widetilde{D}^T\right)\Theta^{-1}.$$

The fourth Riccati equation is related to the synthesis of an anisotropic controller that minimizes the anisotropic norm of the objective vector of the closed system using the gain matrix of the controller

$$T = A_u^T T A_u + C_u^T C_u - N^T \Psi N, \tag{10}$$

where

$$\Psi = B_u^T T B_u + D_{12}^T D_{12}, \ N = -\Psi^{-1}\left(B_u^T T A_u + D_{12}^T C_u\right).$$

The use of stochastic robust regulators, which are synthesized according to a mixed criterion that includes norms H_2 and H_∞, allows to obtain systems with sufficiently high dynamic characteristics with low sensitivity to changes in the parameters and structure of objects. However, the issue of choosing the tolerance parameter, which characterizes the relation between norms H_2 and H_∞, is resolved on an intuitive level. The closer the system is to the optimal norm, the more sensitive it is to changes in the parameters and structure of models of control objects and external influences. The closer the synthesized system is to the optimal one according to the norm H_∞, the less accuracy it has, because it shows excessive "caution" and is designed to work in the most unfavorable conditions.

One of the correct approaches to the justified choice of a mixed criterion, which includes norms H_2 and H_∞, is the construction of anisotropic regulators. In the stochastic approach to control synthesis, the stochastic norm of the system is used as a criterion for the system optimality.

The distribution principle in the problem of mixed robust control does not imply independence of the Riccati equations. Unlike classical optimization problems, the problem of evaluation synthesis and the problem of synthesis of an optimal static regulator in the form of feedback cannot be solved independently of each other. This generalized distribution principle allows to interpret the obtained results from the point of view of the differential games theory.

Mathematical model of the tracking system

The simplest model of the tracking system is a model of a two-mass electromechanical system, the diagram of which is shown in Fig. 1. At the same time, the kinematic connection from the drive motor shaft to the working body shaft is presented in the form of two moments of inertia of the motor and the working body, connected by a nonlinear elastic element. In addition, the presence of a nonlinear dependence of dry friction as a function of speed is taken into account on the shafts of the drive motor and the working body [11–14]. Despite the widespread use of AC drives, DC drives continue to be used in tracking systems due to their significantly lower cost compared to AC drives.

A mathematical model of a nonlinear electromechanical tracking system, taking into account the models of executive motors and sensors as objects of a robust control system with the state vector $x(t)$ is written in the standard form of the state equation

$$\frac{dx(t)}{dt} = f\big(x(t), u(t), \omega(t), \eta(t)\big), \quad (11)$$

where $u(t)$ is the control; $\omega(t)$ and $\eta(t)$ are vectors of the external signal and parametric disturbances; f is the nonlinear function.

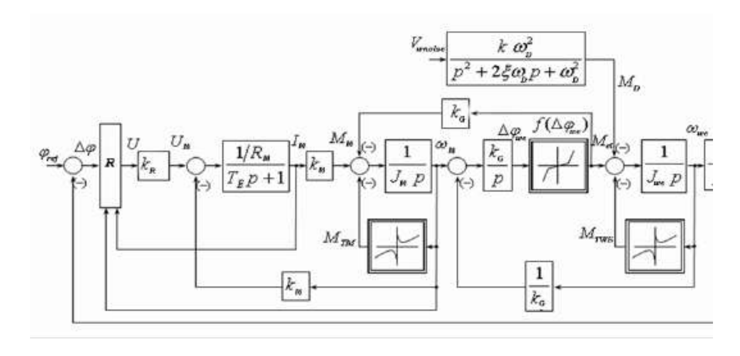


Fig. 1. Diagram-model of a two-mass electromechanical system

Mathematical model (11) takes into account nonlinear frictional dependences on the drive motor shafts, rotating parts of the gearbox and plant, the gap between the teeth of the driving and driven gears, control restrictions, current, torque and speed of the motor, as well as the moment of inertia of the object.

The measured output vector of the output system

$$y(t) = Y\big(x(t), \omega(t), u(t)\big) \quad (12)$$

is formed by various sensors that measure the angle, speed and acceleration of the object [3].

The objective vector of robust control

$$z\big(x(t), u(t), \eta(t)\big) = Z\big(x(t), u(t), \eta(t)\big) \quad (13)$$

where Z nonlinear function is introduced.

Multicriteria synthesis of anisotropic regulators

The dynamic properties of the nonlinear tracking system are determined by the mathematical model of the control object (11), measuring devices (12) and synthesized anisotropy regulators (7)–(10). Tracking systems are installed on a moving base. There are separate measuring systems for supporting and disturbing influences. Therefore, tracking systems are systems with two degrees of freedom with a combination of open-loop and closed-loop control principles. To calculate the control, information about reference and disturbing influences is used to obtain a minimum of errors in working out the set influences and compensation of disturbances by the system. The conditions of invariance of such systems are reduced to the minimization of the norms of the transfer functions of the system error according to standards and disturbing influences. Tracking systems have different requirements for their operation in different modes [1–3]. Certain limitations are imposed on the quality of transient processes – the time of the first adjustment, the time of adjustment in general, readjustment, etc. The variance of the tracking error is also limited. Constraints on state and control variables must be met. And one more requirement for tracking systems is the limitation of the error of working out the parameters and the compensation of disturbing influences in the form of harmonic signals of one frequency or several characteristic operating frequencies, as well as a range of operating frequencies in which certain conditions should be met. For tracking systems, the characteristic mode of operation is movement at low speeds or the implementation of small movements, for this mode, the smoothness of movement is set in the form of appropriate criteria. The reasons for the non-smooth movement of the working body at low speeds are the presence of nonlinearities of the "dry friction" type in drives and working bodies and elastic elements between the executive motor and the working body, which leads to jerky oscillations of the moving parts of the drive and the working body, which is accompanied by stops and disruptions of moving parts in relation to the position of stops [9–10].

The dynamic characteristics of the synthesized system, which includes a nonlinear object (11)–(12), which is closed by an anisotropic robust controller (10) and an anisotropic robust observer (9), are determined by the control system model, the parameters of measuring devices (12) and the objective vector (2). For the correct value of the objective vector (2), we introduce a vector of unknown parameters, which are matrices, with the help of which the objective vector (2) is calculated.

A multicriteria game is considered

$$J(R,G,\eta)=[[J_1(R,G,\eta),J_2(R,G,\eta),\ldots,J_m(R,G,\eta)]^T, \tag{14}$$

in which the components $J_i(R, G, \eta)$ of the game vector $J(R, G, \eta)$ are separate quality criteria put forward for the operation of the tracking system in different modes. The first player is a vector of matrix elements, with the help of which the objective vector (2) is calculated, and its strategy consists in minimizing the vector of game payoffs (14). The second player is the uncertainty vector of the system control object model (1), and its strategy is to maximize the same payoff vector of the game (14).

The payoff vector of the game (14) is calculated by modeling the original nonlinear system (11)–(12), which is closed by the synthesized anisotropic robust controller (10) and the anisotropic robust observer (9), in different modes of operation with different input signals and for different values of the tracking system parameters.

In such a closed-loop tracking system with two degrees of freedom, the stochastic robust control is calculated based on the object state vector, but the open-loop forward control is calculated based on the state vector of the task models and the state vector of the disturbance models. In addition, feedback and open control are calculated simultaneously on the basis of the iterative solution of the system of four connected Riccati equations (7)–(10), the Lyapunov equation (5) and the determination of the anisotropy norm of the system by the expression of the special form (4), which are numerically solved using the method of homotopies, which includes vectorization of matrices and iterations according to Newton's method.

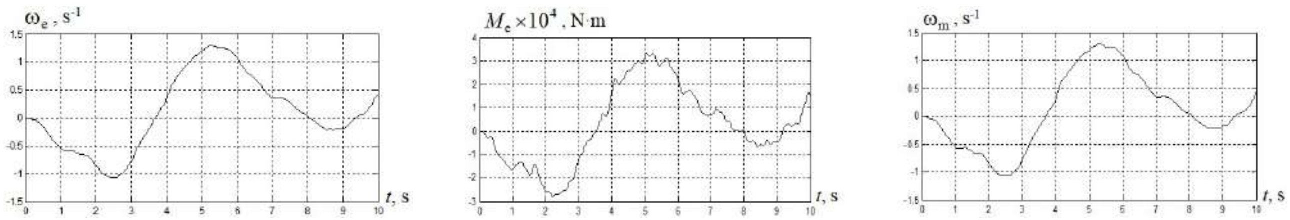
The solutions of the vector game (14) are calculated from Pareto optimal solutions [3] based on the multi-swarm stochastic metaheuristic optimization algorithm of Archimedes [15]. The number of particle swarms is equal to the number of components of the vector game (14). In fact, these heuristic algorithms are a first-order random search algorithm, since it uses only the velocity of the particle - the first-order derivative of the scalar objective function or the gradient of the vector objective function. To increase the search speed, not only the speed is used, but also the acceleration of the change of the objective function. In this case, the acceleration of the swarm particles is found as changes in velocities on neighboring iterations.

Special nonlinear algorithms of stochastic multi-agent optimization are used to find a solution to a multicriteria game from Pareto optimal solutions taking into account preference ratios. The application of the Archimedes algorithm [15] to calculate the solution of the vector game (14) made it possible to significantly reduce the time of calculating the solution of the vector game, which is very important, because the calculation of the components of the game's payoff vector is related to the iterative solution of a system of four connected Riccati equations (7)–(10), the Lyapunov equation (5) and the determination of the anisotropy norm of the system by the expression of the special form (4), which are numerically solved with the help of the homotopy method, which includes vectorization of matrices and iterations according to the Newton method, and with simulation of system operation in various modes and under various external influences, which requires significant computing resources.

Results of computer modelling

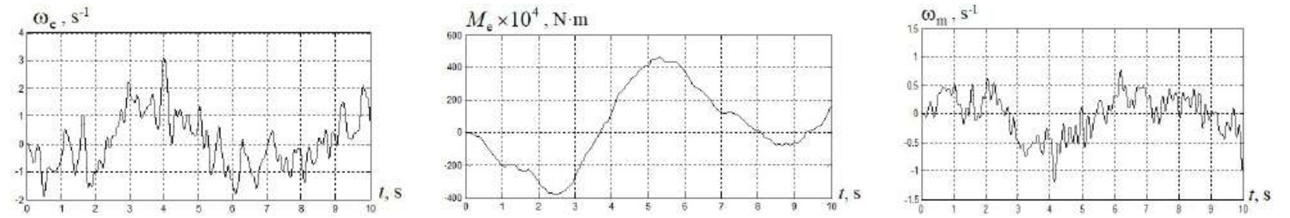
The dynamic characteristics of synthesized robust systems of stochastic control with a two-mass electromechanical tracking system are considered. Fig. 2 shows implementations of random processes of state variables of a two-mass electromechanical system with a typical controller, and Fig. 3 – implementation of random processes of the same state variables of the system with a stochastic robust regulator. The results of the synthesis showed that the use of robust regulators made it possible to reduce the error of adjusting the rotation speed of the mechanism by approximately two times.

Due to the application of the synthesis of stochastic robust control of a two-mass electromechanical tracking system with two degrees of freedom with anisotropic regulators, it was established that the use of synthesized regulators made it possible to increase the accuracy of electromechanical system control by reducing the time of transient processes by 3–5 times.



a b c

Fig. 2. Implementation of state variables in a system with a typical regulator:
a – engine rotation speed; b – moment of elasticity; c – mechanism rotation speed



a b c

Fig. 3. Implementation of state variables in a system with an anisotropy controller:
a – engine rotation speed; b – moment of elasticity; c – mechanism rotation speed

Results of experimental studies

To conduct experimental studies, a two-mass electromechanical tracking system stand, shown in Fig. 4, was developed.

Fig. 5 shows the diagram of the stand control system.

The first motor M1 is controlled from the converter C1 using the position regulator PR of the first motor M1 based on the signal from the PS1 or PS2 position sensors of the first or second motors.

The second motor M2, controlled by the converter C2, creates a load moment. To simulate a random load moment, a random signal from the random signal generator RSG is fed to the input of the second converter C2 through the shaping filter SF.

Fig. 4. The stand of the two-mass electromechanical tracking system

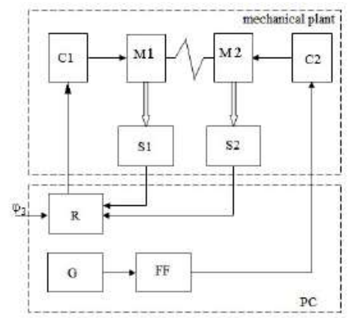
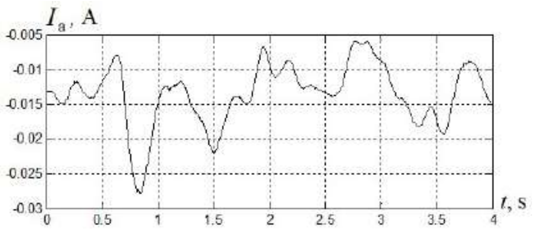
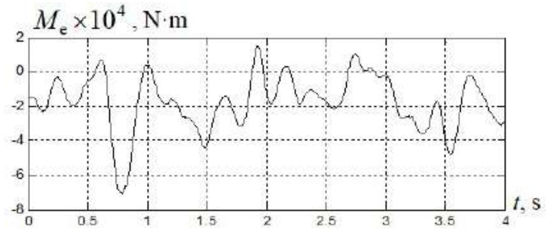


Fig. 5. Scheme of the stand control system

At the stand, experimental studies of the dynamic characteristics of a two-mass electromechanical system with typical regulators and with synthesized anisotropy regulators were carried out. As an example, experimental implementations of random processes of variable states of the motor shaft rotation angle control systems with a typical regulator and an anisotropy regulator are shown in Fig. 6 and Fig. 7.



a



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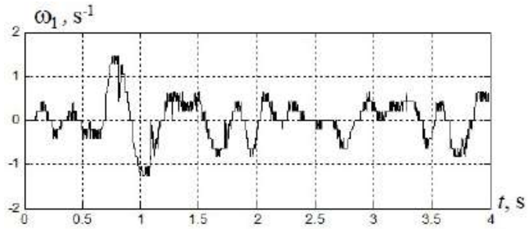

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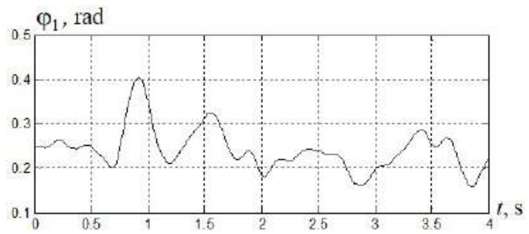
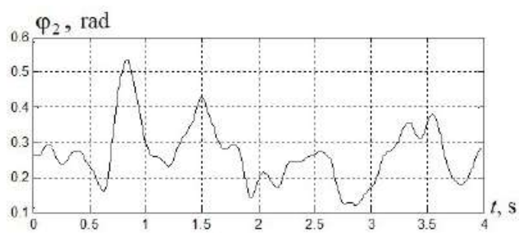
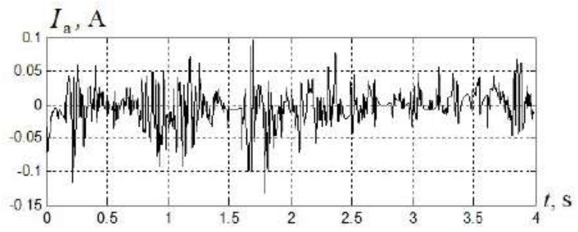


d

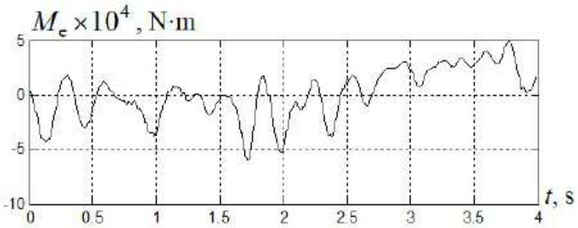


e

Fig. 6. Experimental implementations of system state variables with a typical regulator:
a – anchor current of the first motor; b – moment of elasticity; c – rotation speed of the first motor; d – angles of rotation of the shafts of the first and e – second motors



a



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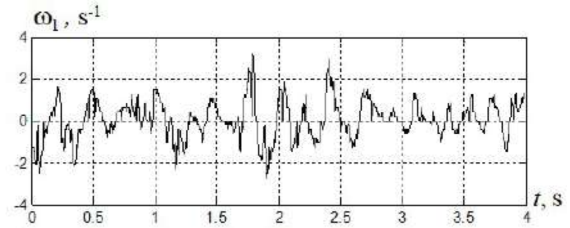

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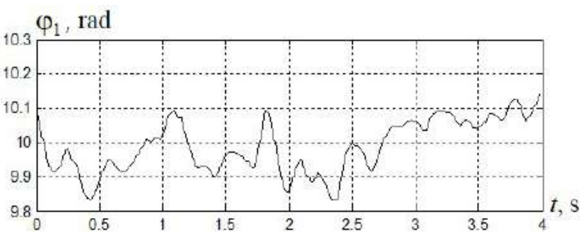
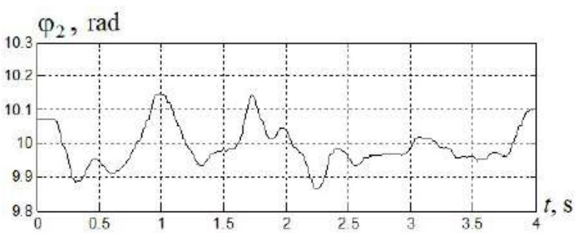


d



e

Fig. 7. Experimental implementations of the state variables of the system with an anisotropic regulator:
a – anchor current of the first motor; b – moment of elasticity; c – rotation speed of the first motor; d – angles of rotation of the shafts of the first and e – second motors

As can be seen from these figures, the maximum deviation of the angle of rotation of the shaft of the second motor in the control system with a typical regulator is $\Delta\varphi_2$=0.55 rad, and in the control system with an anisotropic regulator, the maximum deviation of the angle of rotation of the shaft of the second motor is $\Delta\varphi_2$=0.2 rad. Thus, the use of an anisotropy regulator in the motor shaft rotation angle control system with a random change in the load moment makes it possible to reduce the adjustment error by more than 2 times.

The adequacy of the developed mathematical models and the correctness of the results of theoretical studies of the method of multicriteria synthesis of robust control of tracking systems were experimentally confirmed on the developed experimental plant of a two-mass electromechanical tracking system with an elastic connection between the executive element and the working body. It was established that with the help of stochastic robust regulators, it was possible to increase control accuracy by reducing the time of the first adjustment of the experimental plant of a two-mass electromechanical tracking system by 3.1 times compared to a typical regulator.

Conclusions

1. A multicriteria optimization method for stochastic robust control of a tracking system with two degrees of freedom with anisotropic controllers is developed to increase accuracy and reduce sensitivity to uncertain parameters of the control object.

2. The multicriteria optimization of the stochastic robust control of the tracking system with two degrees of freedom with anisotropic controllers is reduced to the iterative solution of the system of four coupled Riccati equations, the Lyapunov equation and the determination of the anisotropy norm of the system by an expression of a special form, which are numerically solved using the method of homotopies, which includes vectorization of matrices and iteration according to Newton's method. The objective vector of robust control is chosen in the form of a vector game solution. The winnings of this vector game are the quality indicators that the system should achieve when working in different modes. The calculation of the winnings of this vector game is related to the simulation of a synthesized tracking system with anisotropic controllers for different modes of operation, with different input signals and object parameter values. The solution of this vector game is calculated on the basis of the multi-swarm stochastic metaheuristic optimization algorithm of Archimedes.

3. Based on the developed method, stochastic robust control of a tracking system with two degrees of freedom with anisotropic regulators was synthesized, and it was shown that the use of synthesized anisotropic regulators made it possible to increase control accuracy and reduce the sensitivity of the system to changes in object parameters compared to existing systems.

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Багатокритеріальна оптимізація стохастичного робастного керування системою стеження

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Розроблено багатокритеріальну оптимізацію стохастичного робастного керування з двома ступенями свободи системою стеження з анізотропійними регуляторами для підвищення точності й зниження чутливості до невизначених параметрів об'єкта. Такі об'єкти розташовані на рухомій основі, на якій встановлені датчики кутів, кутових швидкостей і кутових прискорень. Підвищення точності керування з двома ступенями свободи включає керування із зворотним зв'язком і замкнутим контуром і керування із прямим зв'язком і розімкненим контуром за допомогою використання задаючих та збурюючих впливів. Багатокритеріальна оптимі-

зація стохастичного робастного керування системи стеження з двома ступенями свободи із анізотропійними регуляторами зведена до ітеративного рішення системи з чотирьох пов'язаних рівнянь Ріккаті, рівняння Ляпунова та визначення анізотропійної норми системи по виразу спеціального вигляду, який чисельно вирішується за допомогою методу гомотопій, що включає векторизацію матриць та ітерацій за методом Ньютона. Вектор цілі робастного керування обчислюється в вигляді рішення векторної гри, векторні виграші якої – це прямі показники якості, яких має досягти система в різних режимах її роботи. Розрахунок векторних виграшів цієї гри пов'язаний із моделюванням синтезованої системи з анізотропійними регуляторами для різних режимів роботи з різними вхідними сигналами і значеннями параметрів об'єкта. Рішення цієї векторної гри розраховуються на основі множини Парето-оптимальних рішень з урахуванням бінарних відношень переваг на основі метаевристичного алгоритму багатороєвої оптимізації Архімеда. На основі результатів синтезу стохастичного робастного керування системи стеження з двома ступенями свободи з анізотропійними регуляторами показано, що використання синтезованих регуляторів дозволило підвищити точність керування системою, зменшити час перехідних процесів у 3–5 разів, зменшити дисперсію помилок у 2,7 рази, знизити чутливість системи до зміни параметрів об'єкта у порівнянні з типовими регуляторами.

Ключові слова *система, стеження, стохастичне робастне керування, багатокритеріальна оптимізація.*

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