A. A. M a r t y n y u k

STABILITY AND CONTROL: THEORY, METHODS, AND APPLICATIONS (REVIEW)

Інститут механіки ім. С.П.Тимошенка НАНУ,
вул. Нестерова, 3, 03057, Київ, Україна; center@inmech.kiev.ua

Abstract. This paper presents a brief survey of the Series of Scientific Monographs "Stability and Control: Theory, Methods and Applications" published during 1995 – 2002 at the Gordon and Breach Science Publishers (UK) and the Taylor and Frances Publisher (USA). The Series comprises 22 volumes and is encyclopedic in the field of stability and control theories. Besides, in the presented survey, the history of appearance and realization of the idea of publishing the Series is outlined.

Key words: stability; control; optimization; nonlinear dynamics; qualitative, analytical, and asymptotic methods; viscoelastic systems; population dynamics; mathematical economics; resonance mechanics.

Introduction.

The problems of modern society are both complex and interdisciplinary. Despite the apparent diversity of problems, tools developed in one context are often adaptable to an entirely different situation. For example, consider Lyapunov's well-known second method. This interesting and fruitful technique has gained increasing significance and has given a decisive impetus for modern development of the stability theory of differential equations. A manifest advantage of this method is that it does not demand the knowledge of solutions and therefore has great power in application. It is now well recognized that the concept of Lyapunov-like functions and the theory of differential and integral inequalities can be utilized to investigate qualitative and quantitative properties of nonlinear dynamic systems. Lyapunov-like functions serve as vehicles to transform the given complicated dynamic systems into a relatively simpler system and therefore it is sufficient to study the properties of this simpler dynamic system. It is also being realized that the same versatile tools can be adapted to discuss entirely different nonlinear systems, and that other tools, such as the variation of parameters and the method of upper and lower solutions, provide equally effective methods to deal with problems of a similar nature. Moreover, interesting new ideas have been introduced which would seem to hold great potential.

Control theory, on the other hand, is that branch of application-oriented mathematics that deals with the basic principles underlying the analysis and design of control systems. To control an object implies the influence of its behavior so as to accomplish a desired goal. In order to implement this influence, practitioners build devices that incorporate various mathematical techniques. The study of these devices and their interaction with the object being controlled is the subject of control theory. There have been, roughly speaking, two main lines of work in control theory which are complementary. One is based on the idea that a good model of the object to be controlled is available and that we wish to optimize its behavior, and the other is based on the constraints imposed by uncertainty about the model in which the object operates. The control tool in the latter is the use of feedback in order to correct for deviations from the desired behavior. Mathematically, stability theory, dynamic systems and functional analysis have had a strong influence on this approach.

The overview proposed to the readers consists of 22 sections, each section is a brief survey of one of the volumes of this series (see [1 – 22]).*

1. Theory of Integro-Differential Equations [1].

This unique monograph investigates the theory and applications of Volterra integro-differential equations. Whilst covering the basic theory behind these equations, it also studies their qualitative properties and discusses a large number of applications. This comprehensive work presents a unified framework to investigate the fundamental existence of theory, treats stability theory in terms of Lyapunov functions and functionals, develops the theory of integro-differential equations with impulse effects, and deals with linear evolution equations in abstract spaces. Various applications of integro-differential equations, such as population dynamics, nuclear reactors, viscoelasticity, wave propagation and engineering systems, are discussed, making this book indispensable for mathematicians and engineers alike.

Integro-differential equations arise quite frequently as mathematical models in diverse disciplines. The origins of the study of integral and integro-differential equations may be traced to the work of N.H.Abel, A.J.Lotka, I.Fredholm, T.R.Maltus, P.F.Verhulst and V.Volterra on problems in mechanics, mathematical biology and economics.

The book [1] is devoted to the investigation of the theory and applications of Volterra integro-differential equations.

Let $R_+ = [0,\infty)$, R^n be an n – dimensional Euclidean space, $x(t) \in R^n$ be a state vector of the system for $t \in R_+$, $J = [t_0, t_0 + a]$, $a = \text{const} > 0$.

$$\frac{dx}{dt} = f(t, x(t)) + \int_{t_0}^{t} K\big(t, s, x(s)\big)\, ds; \tag{1.1}$$

$$x(t_0) = x_0, \tag{1.2}$$

where $x \in R^n$, $f \in C(J \times J \times R^n, R^n)$.

For the initial value problem (IVP) (1.1) – (1.2), the main problems of the theory are considered, namely, the existence, uniqueness, theory of inequalities, comparison results, continuous dependence and differentiability of solutions with respect to the initial data, linear and nonlinear variation of parameters, and monotone iterative techniques. An important method in the theory of integro-differential equations is concerned with estimating a function satisfying an integro-differential inequality by the extremal solutions of the corresponding integro-differential equations. One of the results of this type is the following theorem.

For the function $m(t) \in C(R_+, R)$, the following derivative is introduced:

$$D\_m(t) = \liminf \{[m(t+h) - m(t)]h^{-1} : h \to 0^-\}.$$

Theorem 1 (see [1], pp. 13 – 15) Assume that $g \in C(R_+^2, R)$, $H \in C(R_+^3, R)$, $H(t,s,n)$ is nondecreasing in u for each (t,s), and for $t \geq t_0$,

$$D\_m(t) \leq g(t, m(t)) + \int_{t_0}^{t} H(t,s,m(s))ds.$$

Suppose that $\gamma(t)$ is the maximal solution of

$$\frac{du}{dt} = g(t,u(t)) + \int_{t_0}^{t} H(t,s,u(s))\, ds; \quad u(t_0) = u_0$$

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* A series of books and monographs on the theory of stability and control. Edited by A.A.Martynyuk, S.P.Timoshenko Institute of Mechanics of National Academy of Sciences of Ukraine and V.Lakshmikantham, Florida Institute of Technology, USA.

existing on $[t_0, \infty)$. Then

$$m(t) \le \gamma(t), t \ge t_0$$

provided $m(t_0) \le u_0$.

Chapter 2 discusses the linear integro-differential systems

$$\frac{dx}{dt} = A(t)x(t) + \int_{t_0}^{t} K(t,s)x(s)ds, \tag{1.3}$$

under the initial conditions (1.2), where $A(t)$ is an $n \times n$ continuous matrix on $R_+, K(t,s)$ is an $n \times n$ continuous matrix for $0 \le s \le t < \infty$ and $F \in C(R_+, R^n)$.

This chapter addresses linear and weakly nonlinear systems for which fundamental properties, such as stability, boundedness and periodicity of solutions, are discussed. This chapter also covers equations with impulse effects, and difference equations resulting from Volterra integro-differential equations.

In Chapter 3, the Lyapunov stability theory for Volterra integro-differential equations is discussed. In addition, this chapter deals with stability analysis of nonlinear equations with impulse effects, construction of Lyapunov functions and functional, the Lyapunov-Razumikhin technique, equations with unbounded delay, and Lyapunov functions on product spaces.

Chapter 4 discusses integro-differential equations in Banach space E, that are of the form

$$\frac{dx}{dt} = H(t, x, Tx); \tag{1.4}$$

$$x(t_0) = x_0 \in \Omega, \tag{1.5}$$

where

$$Tx(t) = \int_{t_0}^{t} K(t,s)x(s)ds; \quad K \in C(J \times J \times R); \ |K(t,s)| \le K_1; \quad K_1 = \text{const} > 0;$$

$$H \in C(J \times \Omega^2, E), \ \Omega = B(0, N) = \{x \in E : |x| < N\}.$$

The main ingredients of this chapter are the basic existence theory, well-posedness of linear equations, semigroups, resolvents, linear evolution operators, asymptotic behaviour of solutions, and stability analysis.

Finally, Chapter 5 deals with the investigation of various qualitative properties of solutions of integro-differential equations that arise in problems of biological population, grazing systems, wave propagation, nuclear reactors and viscoelasticity. Stability analysis of engineering systems, such as input-output systems, multiloop systems and large scale systems, is also covered in this chapter.

The bibliography consists of 339 – 355 pages and 237 references.

2. Stability Analysis: Nonlinear Mechanics Equations [2].

This monograph describes the results of stability investigations of five types of solutions to nonlinear ordinary differential equations containing a small parameter. The averaging technique, the comparison principle and Lyapunov's direct method for scalar, vector and matrix-valued functions are employed for the construction of various sufficient conditions for the stability of solutions.

Chapter 1 treats a small-parameter system in general. It is represented by ordinary differential equations of the type

$$\frac{dy}{dt} = Y(t, y, \mu), \ y(t_0) = y_0, \tag{2.1}$$

where $t \in R_+$, $y \in \Omega \subset R^n$, $Y \in C(R_+ \times \Omega \times M, R^n)$, $\mu \in M \subseteq [0,1]$ is a scalar parameter which is assumed to be small. A "motion" is a solution of (2.1) with the initial value $(t_0, y_0) \in R_+ \times \Omega$.

The aim of the chapter is to present stability results which in some sense hold uniformly for the families of motions with parameter values μ from some interval $0 \le \mu < \mu_0$.

So, in the chapter, a variety of notions of μ-stability, μ-attraction (including those of a domain of t_0-uniform attraction with respect to R_+) and μ-asymptotic stability are introduced. These notions turn out to be of great use for estimating the deviations from certain "unperturbed motions" which, as a rule, are produced from limiting equations or via various asymptotic or averaging procedures going back to the classical approaches of N. M. Krylov, N. N. Bogolyubov and Yu. A. Mitropol'skij. These averaging techniques are combined with the comparison method and with Lyapunov's direct method using scalar, vector or matrix Lyapunov functions.

For the system of equations (2.1) in the compact-open topology, a limiting system of equations is considered and conditions of the proximity of solutions of the original and limiting systems are formulated. Moreover, conditions of uniform μ-boundedness, eventual μ-stability and eventual asymptotic μ-stability are established.

The final section of the chapter formulates the principle of contraction of system (2.1) to a system of lower dimension, for which

$$x(t, t_0, x_0, \mu) = Uy(t, t_0, y_0, \mu)$$

for all $t \in R_+$ and $\mu < \mu_0$. Here, $x(t, \cdot)$ is a solution of the system of lower dimension and U is a constant $n \times m$ – matrix with full raw rank.

Chapter 2 is devoted to standard systems according to N.N.Bogolyubov, i.e., to systems of the type

$$\frac{dy}{dt} = \mu Y(t, y, \mu). \tag{2.2}$$

For a system of the form (2.2), conditions for the stability of solutions on finite and infinite interval are established based on the principle of comparison with a scalar Lyapunov function. Also, the conditions of closeness are indicated here for the solutions of the standard integro-differential system and those of the corresponding averaged system.

Here, among others, conditions for stability on a finite interval $J = [t_0, t_0 + L\mu^{-1}]$, $L > 0$, are obtained.

Chapter 3 deals with motions with smooth drift and fast oscillations. Here, the conditions for the closeness of solutions on a finite interval of the original and averaged systems are given based on the comparison principle. The conditions for the stability of relatively slow variables on a finite time interval are established.

In Chapter 4, systems with small perturbing forces,

$$\frac{dy}{dt} = Y(t, y) + \mu R(t, y, \mu), \tag{2.3}$$

are considered, and conditions for stability and boundedness of solutions and for the existence of limit cycles and nested attractors are obtained.

Chapter 5 deals with singularly perturbed systems of the type

$$\frac{dx}{dt} = f(t, x, y, \mu); \quad \mu\frac{dy}{dt} = g(t, x, y, \mu), \tag{2.4}$$

where $(x^T, y^T)^T$ is the state vector of the whole system (2.4), $x \in R^n$, $y \in R^m$, $f \in C(R_+ \times R^n \times R^m \times M, R^n)$, $g \in C(R_+ \times R^n \times R^m \times M, R^m)$, $\mu \in (0,1] = M$.

For systems (2.4), using the method of matrix Lyapunov functions, their properties are determined and theorems on the stability and instability of the state $(x^T, y^T)^T = 0$ of system (2.4) are obtained.

For large-scale singularly perturbed systems, a method of matrix Lyapunov functions for uniform and non-uniform graduating the time line are proposed. In particular, conditions of absolute stability are obtained for singularly perturbed Lur'e-Postnikov systems with one nonlinear element in the fast part and one in the slow part.

The bibliography consists of 235 – 242 pages and 129 references.

3. Stability of Motion of Nonautonomous Systems (Method of Limiting Equations)[3].

This book is devoted to the stability analysis of the solutions of nonautonomous ordinary differential equations, differential-delay systems, integro-differential equations, and infinite-dimensional evolution equations. Special emphasis is laid on the use of the so-called method of limiting systems. This method is frequently applied in conjunction with other techniques such as the method of comparison equations and the use of the Lyapunov function.

Chapter 1 is devoted to the analysis of the stability of non-autonomous systems of ordinary differential equations of perturbed motion. Here, we introduce the limiting equations for the original systems of equations and study their properties. Sufficient conditions for the uniform asymptotic stability, eventual uniform stability, and Lipschitz stability are established. Using the direct Lyapunov method, conditions for the localization of compact sets of solutions are obtained as well as conditions for uniform asymptotic stability.

Chapter 2 discusses systems of equations with aftereffect. Here, we introduce the phase B -space for systems with unlimited delay and formulate the main provisions of the theory of limiting equations for these systems. Based on the established relationship between stability in B -space and that in R^n -space, sufficient conditions are set out for various types of stability of systems with aftereffect. The conditions are presented for the eventual stability under perturbations of the right-hand side of the system of equations are presented.

Chapter 3 examines the stability of motion of non-autonomous systems under the action of small forces. These are systems of the form

$$\frac{dx}{dt} = f(t,x) + \mu r(t,x)\,; \tag{3.1}$$

$$x(t_0) = x_0\,, \tag{3.2}$$

where $x \in R^n$, $f \in C(R_+ \times R^n, R^n)$, $r \in C(R_+ \times R^n, R^n)$, $r(t,0) \neq 0$ for all $x \in R_+$ and μ is a small parameter. For $\mu = 0$, we consider the limiting system

$$\frac{dx}{dt} = g(t,x), x(t_0) = x_0\,, \tag{3.3}$$

where $g \in C(R_+ \times R^n, R^n)$ such that $\lim \|f(t,x) - g(t,x)\| = 0$ for $t \to \infty$ uniformly in $x \in B \subset R^n$.

In terms of the direct Lyapunov method, conditions for various types of stability of system (3.1) are derived based on the properties of solutions to system (3.3). The chapter also covers loosely coupled non-autonomous systems of equations and established conditions for their stability and instability.

In Chapter 4, to study the stability of non-autonomous systems of ordinary differential equations, the principle of comparison is applied in combination with the method of limiting equations and the method of Lyapunov functions. Here, the stability conditions are established as well as the conditions of uniform asymptotic stability in all variables and with respect to some of the variables.

Chapter 5 establishes sufficient conditions for μ-stability of systems of integro-differential equations. The obtained conditions are based on knowledge of the exact solution of the limiting system and the Lyapunov function $V(t,x) \in C(R_+ \times \Omega, R_+)$ constructed for the original system at $\mu = 0$. Also, a model of population dynamics and stability of viscoelastic rod motions, which are described by the integro-differential equation, are considered.

Chapter 6 treats the feedback control problem

$$\frac{dy}{dt} = Y(t,y,u);\;\; y(t_0) = y_0, \tag{3.4}$$

where $y \in R^n$, $Y \in C(R_+ \times R^n \times R^m, R^n)$, and $u = u^0(t, y)$ is to be determined so as to render the origin $y = 0$ asymptotically stable. It is assumed that $Y(t, 0, 0) = 0$. A theorem is formulated and proved stating that if (3.9) admits a function of Lyapunov type with respect to a comparison equation

$$\frac{dw}{dt} = g(t, w, v), \tag{3.5}$$

and if an appropriate transversality condition is satisfied by at least one pair consisting of a limiting equation for (3.4) with $u = u^0(t, y)$ and a limiting equation for (3.5), then $y = 0$ is asymptotically stabilized by $u = u^0(t, y)$. One can go further and determine integral cost functionals $I = \int_{t_0}^{t} W(t, x[t], u[t])\,dt$ with the property that an asymptotically stabilizing control can be found which minimizes I.

Chapter 7 deals with stability theory for dynamical processes where motions are continuous with respect to the coordinates only; the solutions of partial differential equations in general are processes posessing these properties.

Chapter 8 addresses limiting processes for abstract dispersive dynamical processes on a Frechet convergence space, and the generalized Lyapunov direct method proposed by J.M. Ball. In the chapter the authors consider the generalized Lyapunov direct method for D^+ systems on a convergence space X in terms of Ball-type Lyapunov functional.

Finally, Chapter 9 utilizes the results of Chapter 8 to study the stability-like properties of solutions to asymptotically autonomous dynamical processes generated by nonautonomous evolutionary equations in a Banach space.

The bibliography consists of 239 – 251 pages and 225 references.

4. Control Theory and Applications [4].

Modern *"control theory"* or *"theory of optimal control"* started more or less in the 1950s and developed explosively in the 1960s. It was to a large extent a continuation of the classical *"calculus of variations"*. But it actually contained much more since it included many problems not related at all to optimality. In this more general sense, control theory can be seen as an extension of the theory of differential equations or of dynamical systems, indeed as a study of global properties of certain families of dynamical systems. From the very beginning, control theory leant strongly towards applications. Selected examples are as follows:

* *Physical systems*: stable performance of motors and machinery, optimal guidance of rockets.

* *Management*: optimal exploitation of natural resources.

* *Economics*: optimal investment or production strategies.

* *Biology and medicine*: regulation of physiological functions, fight against insects, epidemics.

Given almost any natural system evolving in time, it can be converted into a control system by introducing some way of acting upon it and influencing its evolution. For some systems, this may be quite unrealistic, for example, we would not set up equations for controlling planetary motion. But in most cases, the interaction between a system and some controlling device is not unthinkable. In a wider outlook than just mathematics, we can argue, for instance, that at the present level of their activities, humans have got the power of inducing worldwide ecological changes, large scale pollution, climatic changes and so on, hence the corresponding natural systems have become control systems. From an ethical point of view, this implies that, if we wish to be responsible for our actions, we should study such processes as control systems.

Any system described, for example, by differential equations of the evolution type, can be converted into a control system by adding an input variable representing the action of some 'controller' upon the system. The way this new variable enters in the equation will determine what this ‘controller’ can actually achieve. The interpretation of this ‘control’ as a willful action of a person is, of course, just one possibility. The added input can also be called ‘noise’ and can represent many factors about which we may not have any influence nor even knowledge. We could also add several such control variables and think that each of them is manipulated by a different person, called a ‘player’, thus converting the whole system into a 'differential game'. It sometimes happens that a mathematical problem is difficult to solve in its original setting, but it becomes easier when the problem is extended into a more general context. This is contrary to the expectation that a more general problem must be harder. The best known example is the case of finding certain areas: this problem is difficult to treat when we think of the area as of an unknown number, but when we pose a (more ambitious) problem of finding the area as a function of its end-point, we find that this is just the antiderivative, hence easy to solve for a large class of functions. Something similar sometimes happens in the context of control theory: given a problem concerning dynamical systems, it may become more accessible when we extend the dynamical system to become a control system since for the latter, we can then use many results about attainability, controllability and so on, which are absent in the theory of dynamical systems. Such possibilities are the main focus of this book. The content of the book is as follows.

Chapter 1 discusses briefly the relation between the real world and the task of making mathematical models of it. While to a large extent this is just common knowledge, it seems appropriate for the purpose of motivating to pose reasonable control problems for given non-control processes. In addition, this chapter may also be some lightly entertaining reading.

Chapter 2 introduces the basic concepts of control theory that are used in the following chapters. As a short introduction to the subject, this chapter is useful for people not very familiar with it. For those already familiar with control theory, it is useful to establish terminology and notations that are used later, as well as for references. The way in which an object is presented may also be of interest since most properties are based on the “attainability” relationship. This approach is first used in this book.

Chapter 3, a continuation of Chapter 2, presents more results from control theory, for example, related to optimization, and other results only marginally used in the remainder of the book.

Chapter 4 presents “typical control system behavior” for reasonably well-functioning control systems. Ignoring the esoteric cases when a researcher intends to create a control system (for example, from a given dynamical system), the chapter shows what can be obtained from a given dynamical system.

Chapter 5 discusses several techniques for transforming dynamical systems into control systems. It actually is somewhat more general since it includes the possibility of enlarging a given control system by introducing more control possibilities.

Chapter 6, finally, presents several applications. They go from very basic, straightforward cases of a didactic nature to a few more elegant and sophisticated applications. Some of them are just open-ended ideas, worthwhile to be looked more carefully into. After seeing them, it is hoped that the reader may get more fruitful ideas of his (or her) own and decide to work on them.

All the above is presented within the framework of n-dimensional space (a few examples, in Chapter 5, are on manifolds, but also n-dimensional). Indeed, the purpose is just to communicate the main ideas, which is best achieved in a rather simple framework. For a more advanced reader, this book may be useful in suggesting new lines of research, either in n-space or in a more general setting. For example, similar problems for partial or functional differential equations make perfect sense, but in many cases have not been systematically studied.

The bibliographic references include 151 – 156 pages and 100 references.

5. Advances in Nonlinear Dynamics [5].

This volume consists of 39 papers contributed by 57 experts from 15 countries, and addresses a wide range of problems of modern nonlinear dynamics focused on stability and control.

The first group of papers dealing with stability, discuss the oscillation of solutions, dissipation and almost-periodicity, global stability, tracking, dynamic uncertain systems, population stability, etc.

The second group of papers on control, deal with the processes of control in cardiology, nonlinear system controllability and its index, etc.

Finally, miscellaneous papers include discussions of the problems of existence of solutions for new classes of systems of equations and boundary problems, proofs of basic theorems of comparison principle, etc.

Contents: A. A. Martynyuk and S. Sivasundaram, A survey of the work of S. Leela (l – l0); Ravi P. Agarwal, S. Pandian and E. Thandapani, Oscillatory property for second-order nonlinear difference equations via Lyapunov's second method (11 – 21); Stephen R. Bernfeld and Pierre A. Vuillermot, Asymptotic behavior of solutions of limiting differential equations (23 – 32); Klaus Deimling, Dissipation and almost-penodicity (33 – 40); K. Gopalsamy and Pingzhou Liu, Global stability in a hyperbolic logistic map with eventually fading memory (41 – 50); Ljubomir T. Grujic, Exponential quality of time-varying dynamical systems: stability and tracking (51 – 61); L. Hatvani, Asymptotic constancy of the solutions of a nonlinear delay equation (63 – 70); George Karakostas, A local attractor for the planar system $x = y, \dot{y} = F(x, y)$ (71 – 77); V. Lakshmikantham and A. S. Vatsala, Stability of moving invariant sets (79 – 83); B. S. Lalli and S. R. Grace, Stability of n-dimensional inhomogeneous delay differential equations (85 – 92); Xinzhi Liu, Uniform boundedness for impulsive systems of integro-differential equations (93 – 98).

A. A. Martynyuk, A contraction principle (99 – 105); Juan J. Nieto, Monotone iterates and stability for first-order ordinary differential equations (107 – 115); N. H. Pavel, Periodicity and stability of semigroups via $\lambda^k \in \rho(C)$ (117 – 127); Wang Wendi and Ma Zhien, Asymptotic stability of linear delayed systems without instantaneous dissipative terms (129 – 136); J. S. Yu and B. G. Zhang, Uniform stability of a delay single population model (137 – 146); Jane Cronin. Control in cardiac components (147 – 155); E. N. Chukwu, Control of nonlinear delay differential economic systems in (157 – 170); S. G. Deo and S. Sivasundaram, Controllability of nonlinear integro-differential systems (171 – 178); Urszula Ledzewicz and Heinz Schattler, A second-order Dubovitskii-Milyutin theory and applications to control (179 – 192); Emilio O. Roxin, The index for control systems (193 – 196). N. U. Ahmed, Linear systems governed by operators beyond Hille-Yosida semigroups (197 – 207); Drumi Bainov and Emil Minchev, Monotone iterative methods for impulsive hyperbolic equations (209 – 216); C. Y. Chan and N. Ozalp, Beyond quenching for singular reaction-diffusion mixed boundary-value problems (217 – 227); C. Corduneanu. Neutral functional-differential equations with abstract Volterra operators (229 – 235); C. De Coster and P. Habets, Existence and multiplicity results for a Dirichlet problem with two parameters (237 – 244); Dajun Guo, Existence and uniqueness of solutions of impulsive integro-differential equations in Banach spaces (245 – 251); Chaitan P. Gupta, A nonlocal multipoint boundary-value problem at resonance (253 – 259); John R. Haddock, The "evolution" of invariance principles a la Liapunov's direct method (261 – 272); Seppo Heikkila, First-order discontinuous differential equations with functional boundary conditions (273 – 281): Johnny Henderson and William Yin, Focal boundary-value problems for singular ordinary differential equations (283 – 295).

Athanassios G. Kartsatos and William R. Zigler, Some properties of differential equations in the weak topology of a reflexive Banach space (297 – 306); Irena Lasiecka, Finite-dimensional attractors for von Karman systems with nonlinear dissipation and noncompact nonlinearity (307 – 317); C. V. Pao, Dynamics of weakly coupled parabolic systems with nonlocal boundary conditions (319 – 328); S. Siuvasunaaram, Convex dependence of solutions of dynamic systems on time scales relative to initial data (329 – 334); D. Trigiante. Multipoint methods for linear Hamiltonian systems (335 –- 48); Li Ta-Tsien and Kong De-Xing, Global existence of solutions to a class of nonlinear systems of functional-differential equations (349 – 353); John W. Lee and Donal O'Regan, Existence principles for nonlinear integral equations on semi-infinite intervals and half-open intervals (355 – 364); K. N. Murthy and P. V. S. Anand, Controllability and observability of continuous matrix Liapunov systems (365 – 379).

The results reported here are of great importance for further construction of stability and control theories using new models of real world phenomena. Many of the problems investigated are associated with the ideas and methods proposed and developed by Professor S. Leela. On the occasion of her 60th birthday, the editors and authors who contributed to this volume wish her success and prosperity.

6. Solving Differential Problems by Multistep Initial and Boundary Value Methods [6].

This book is Volume 6 of a series of books and monographs on the theory of stability and control. The numerical solution of differential equations continues to be an important concern of numerical analysis. The new generation of parallel computers, along with new classes of problems to be solved, has led to the reconsideration of numerical methods. Three aims of the book we can highlight are these: to generalize classical multistep methods for both initial and boundary value problems; to develop a theory which generalizes the classical stability theory of Dahlquist; and to select appropriate methods for general purpose software to solve efficiently a wide range of problems, possibly using parallel computers. The authors set up a generalized framework which is able to encompass the theory of numerical methods for continuous boundary-value problems, based on linear multistep formulae, and thus the treatment of initial and boundary-value problems is unified.

Next, we present a brief overview of the content of this monograph.

The first part (Chapters 1 – 4) presents the essential mathematical tools: 1. "Differential equations" introduces the stability concepts. 2. "Linear difference equations with constant coefficients" discusses the solutions of general linear difference equations with constant coefficients and the variation of the solutions under perturbations (stability theory). 3. ″Polynomials and Toeplitz matrices" is concerned with the location of the zeros of a polynomial with respect to the unit circle in the complex plane and with the Toeplitz matrices. A lower triangular Toeplitz matrix is associated with an initial value problem, while more generally, a banded matrix is associated with a boundary value problem. 4. "Numerical methods for initial value problems" is devoted to linear multi-step formulae for the solution of an initial value problem. For approximating the given continuous initial value problem, boundary value methods (BVMs) are defined. The initial value methods (IVMs) appear as a particular kind of BVMs. The convergence and several types of stability for IVMs and BVMs are discussed.

The second part of the book (Chapters 5 – 9) is devoted to the presentation of special families of methods. Chapter 5, "Generalized backward differentiation formulae", examines the BDF and GBDF (generalized backward differentiation formulae). 6. "Generalized Adams methods" is concerned with two families of boundary value methods obtained by generalization of Adams methods: reverse Adams methods and generalized Adams methods. Chapter 7, "Symmetric schemes", discusses the extended trapezoidal rule and the top order methods with their stability properties. An algorithm for numerically solving confluent Vandermonde systems, required for using the top order methods, is presented. Chapter 8, "Hamiltonian problems", examines the linear Hamiltonian systems and a discrete variational principle based on the BVMs. Chapter 9, "Boundary value problems", discusses the numerical solutions of continuous boundary value problems by means of BVMs, previously described.

The third part of the book (Chapters 10 – 13) deals with the practical implementation of the proposed methods. Chapter 10, "Mesh selection strategies", discusses in detail some strategies for mesh selection. For this reason, the authors give a classification of continuous problems and a more operative definition of stiffness. The equi-distribution of a monitor function is used for approximating continuous boundary value problems. Chapter 11, "Block BVMs", analyses the use of BVMs as block methods and the connections with the Runge-Kutta schemes and general linear methods. Chapter 12, "Parallel implementation of BVMs", demonstrates that efficient parallel solvers for ODEs can be derived from the block version of BVMs introduced before. Finally, Chapter 13, "Extensions and applications to special problems", contains applications to the important classes of problems: differential algebraic equations, delay differential equations, the method of lines, etc., with many numerical examples.

The bibliography consists of 399 – 412 pages and 296 references.

7. Dynamics of Machines with Variable Mass [7].

The book presents new results in the dynamics of machines with variable mass. The basic characteristic of these machines is that they all contain some parts in which the mass varies. Various types of machines are considered, such as centrifuges, sieves for sorting particles, transportation mechanisms, lifting mechanisms, cranes, automatic weight measuring instruments, and rotors used in the textile, cable and paper industries. The aim of modern industry is to increase productivity and to fully automate, resulting in various new challenges in the dynamics of machines with nonlinearities and mass variation.

This book is devoted to dynamic problems that arise in connection with mass variation in machines. The objective of the author is to acquaint the reader with certain concise and compact theories in the dynamics of systems with variable mass that can be used efficiently to find solutions to many problems in machines and mechanisms with time variable parameters. Well developed methods in theoretical and applied mechanics are extended and used for solving some real problems in the dynamics of machines with variable mass. Methods for calculating vibrations of machines with small and strong nonlinearity are demonstrated. Chaotic motion is considered and conditions of stability of motion are defined. Conservation laws and adiabatic invariants are also derived. Considerations are limited to those systems in which the mass variation is deterministic and given as time functions. In this way, a large variety of modern machines is encompassed.

The book contains seven chapters, for which brief descriptions are given below.

Chapter I describes various machines and rotors with time variable mass and parameters.

Chapter 2 introduces modelling of machines with variable mass. The classical laws of mechanics for constant mass are transformed for systems with variable mass. The theory of rigid bodies with variable mass is specially treated. The Lagrange procedure for forming differential equations of motion for systems with variable mass is developed. Some mathematical models are obtained by applying the suggested procedures, for example, for rotors with variable mass and mechanisms for transportation.

Chapters 3 and 4 consider the oscillations of machines with variable mass. Vibrations are a side effect of the required regime of motion of the machines and have adverse consequences. For example, working energy is dissipated on vibrations, and this has a negative effect on the working life of bearings, components of machines and on the equipment as a whole. Vibrations draw energy from the required motion and ultimately decrease the machine efficiency. Factors which transform energy into vibration modes are most often associated with the forces with variable parameters. Nonlinear oscillations of systems with time variable parameters are denoted analytically and compared with numerical results. Systems considered are not only those with small nonlinearity, but also those with strong nonlinearity. Differential equations with a complex deflection function are examined as they describe the motion of the majority of rotors with variable mass. The methods adopted for solving such equations are: the Bogolubov – Mitropolski method, the elliptic Krylov – Bogolubov method, the method of slow variable amplitude and phase, and the method of multiple scales. The influence of the reactive force on the motion of the machine is investigated. It can be seen that the reactive force has a fundamental influence on the dynamic behavior of rotors and mechanisms with variable mass. Therefore, it cannot be neglected.

Chapter 5 treats the problem of chaos in nonlinear systems with variable mass. The analytical Melnikov criteria are extended for such systems. Two examples of real machines are considered: chaos in the crane mechanism and chaos in the separator.

Chapter 6 is about the formulation of conservation laws and adiabatic invariants for systems with variable mass. The first integrals are obtained using the theory of Emmi Noether. The examples of a rotor with variable mass and of a sieve for separating particles are also discussed. The adiabatic invariants for systems with one and two degrees of freedom for different types of nonlinearity are also considered. The adiabatic invariants are applied for finding approximate analytical solutions for nonlinear differential equations of motion.

Chapter 7 deals with problems of stability in systems with variable mass. The direct Lyapunov theorems of stability, asymptotic stability and instability are applied. Some examples of the Lyapunov function formation for systems with variable mass are shown (for various types of rotors and mechanisms with variable mass). In addition, some new theories of stability based on the Lyapunov theory are presented and, in spite of some limitations, they are simpler than the previous procedures and thus much more convenient for engineers. The influence of the reactive force on the stability properties of the system is also analyzed.

The bibliography consists of 217 – 232 pages and 320 references.

8. Optimization of Linear Control Systems: Analytical Methods and Computational Algorithms [8].

In the book, the authors present analytical methods for synthesis of linear stationary and periodical optimal controlled systems, and create effective computational algorithms for synthesis of optimal regulators and filters. The procedures of Youla-Jabr-Bongiorno (1976) and Desoer-Lin-Murray-Saeks (1980) are special cases of this procedure. The monograph also includes original computational algorithms (solutions of usual and generalized Lyapunov and Riccati equations, polynomial matrix factorization) and illustrates the effectiveness of these algorithms by examples in the field of numerical methods for optimization of linear controlled systems.

The main body of this book is divided into four chapters. The first two chapters are devoted to analytical solutions of LGQ and H2 optimization problems. In the last two chapters, the authors go into details of the computational aspects of these problems, focusing on numerical techniques for solving generalized Algebraic Riccati Equations (AREs) and various types of spectral factorizations.

In Chapter 1, asymptotic solutions of the AREs are analyzed when control weighting or noise covariance tend to zero, as well as in the case where the Hamiltonian has zero eigenvalues or eigenvalues near the imaginary axis. In this chapter, the linear quadratic optimization problem is also studied in detail for periodic systems.

Chapter 2 begins with a special parameterization of all stabilizing controllers, originally introduced in the Russian literature in the early 1970s, and its relation to the other well-known controller parameterizations. Then, a frequency domain solution of the standard H2 problem is presented in terms of this controller parameterization. Analytical solutions of special types for two and three degree of freedom systems of H2 optimal control are also discussed in Chapter 2.

Computational methods for the solution of Lyapunov and Riccati equations can be found in Chapter 3 for the discrete and continuous-time cases. Numerical stability of various algorithms is discussed here. In the same chapter, an alternative to the standard singular-value decomposition-based balancing algorithm is also presented.

Chapter 4 deals with spectral and J-spectral factorizations appearing in the solution of H2 and H1 optimal control problems, and with the problem of decomposing a transfer matrix into its stable and anti-stable parts. Similar types of matrix factorizations have been discussed in other books as well. But the main goal of Chapter 4 is to numerically derive attractive ways to do these factorizations.

The bibliography consists of 247 – 257 pages and 222 references.

9. Dynamics and Control [9].

This multi-authored volume presents selected papers from the Eighth Workshop on Dynamics and Control. Many of the papers represent significant advances in this area of research, and cover the development of control methods, including the control of dynamical systems subject to mixed constraints on both the control and the state variables, and also include the development of a control design method for flexible manipulators with mismatched uncertainties. Advances in dynamic systems are presented, particularly in game-theoretic approaches, as well as the applications of dynamic systems methodology to social and environmental problems, for example, the concept of virtual biospheres in modeling climate change in terms of dynamical systems.

This volume consists of 19 papers contributed by 28 experts from 6 countries, and addresses a wide range of problems of modern nonlinear dynamics and control theory.

The first group consists of papers dealing with the development of control methods.

Control Methodology. F.L. Chernousko, Control of Dynamical Systems Subject to Mixed Constraints (1 – 8); E. Reithmeier and G. Leitmann, Pole Placement via Lyapunov for Constrained Control of Mismatched Systems (9 – 16); D.H. Kim and Y.H. Chen, Control Design for Flexible Joint Manipulators: Mismatched Uncertainty and Practical Stability (17 – 34); A.N. Krasovskii, Control Under Lack of Information (35 – 42); N.Yu. Lukoyanov, A Differential Game with Hereditary Information (43 – 54); E. Gyurkovics, Receding Horizon Control for the Stabilization of Discrete-Time Uncertain Systems with Bounded Controllers (55 – 70); R.R. Mower and Alex Y. Khapalov, On Global Controllability of Time-Invariant Nonhomogeneous Bilinear Systems (71 – 80); B.S. Mordukhovich and Kaixia Zhang, Robust Suboptimal Control of Constrained Parabolic Systems Under Uncertainty Conditions (81 – 92);

The second group of papers deals with advances in dynamical systems, and especially in game-theoretic approaches.

Dynamical Systems. A. B. Kurzhanski, Two Approaches to Viability and Set-Membership State Estimation (93 – 100); L. Akos, Time Constants for the Quadratic Lyapunov Functions (101 – 108); A. A. Martynyuk, Qualitative Analysis with Respect to Two Measures for Population Growth Models of Kolmogorov Type (109 – 118); B. Tibken, E.P. Hofer and C. Demir, Guaranteed Regions of Attraction for Dynamical Polynomial Systems (118 – 128); A.V. Kryazhimskii and G. Sonnevend, Dynamics for Bimatrix Games via Analytic Centers (129 – 138); A.M. Tarasyev, Analytic Solutions for Evolutionary Nonzero Sum Games (139 – 150);

Finally, the third group of papers deals with the applications of dynamical systems methodology to social and environmental problems.

Applications to Social and Environmental Problems. G. Feichtinger, Optimal Control of Law Enforcement (151 – 160); Y. M. Svirezhev and W. von Bloh, The Climate Change Problem and Dynamical Systems: Virtual Biospheres Concept (161 – 174); N. Raju and F. E. Udwadia, Ecological Modeling and Coupled Exponential Maps (175 – 186); L.A. Petrosjan, Electing the Directorial Council (187 – 194); A. F. Kleimenov, An Approach to Building Dynamics for Repeated Bimatrix 2x2 Games Involving Various Behavior Types (195 – 204).

10. Volterra Equations and Applications [10].

This volume consists of 52 papers contributed by 76 experts from 17 countries, presented at the Volterra Centennial Symposium and is dedicated to Volterra and his contribution to the study of systems - an important concept in modern engineering. Vito Volterra began his study of integral equations at the end of the nineteenth century and this was a significant development in the theory of integral equations and nonlinear functional analysis. Volterra series are of interest and use in pure and applied mathematics and engineering.

The contents of this volume is as follows. The 52 contributions can be grouped into several main research directions.

History. M. Schetzen, Retrospective of Vito Volterra and His Influence on Nonlinear System Theory (1-14); R.K. Miller, Volterra Integral Equations at Wisconsin (15 – 26);

Stability Theory. N. Azbelev, Stability and Asymptotic Behavior of Solutions of Equations with Aftereffect (27 – 38); V. Lakshmikantham and A.S. Vatsala, The Present Status of UAS for Volterra and Delay Equations (83 – 94); B. Cahlon and D. Schmidt, Algorithmics Stability Tests for Certain Delay Integral Equations of Volterra Type (163 – 172); Y. Hamaya, Global Attractivity in a Nonlinear Difference Model (235 – 240); Y. Lin, J.H. Liu and C. Ma, Exponential Decay and Stability of Volterra Diffusion Equations (299 – 308);

Stochastic Processes. P. Clement and G. Da Prato, Stochastic Convolutions with Kernels Arising in Some Volterra Equations (57 – 66); R.I.P. DeFigueiredo and Y. Hu, On Nonlinear Filtering of Non-Gaussian Processes through Volterra Series (196 – 202); Z.C. Okonkwo, Stochastic Functional Differential Equations with Abstract Volterra Operators l: Existence of Solutions (372 – 384); D. O'Regan, Sample Solutions of Stochastic Integral Equations (391 – 398); V.D. Potapov, The Stability of Some Stochastic Integro-Differential Equations (417 – 426);

Classical Volterra Equations. E. Buckwar, On a Nonlinear Volterra Integral Equation (157 – 162); M. Larrieu and N.H. Pavel, Volterra Integral Equations, Differential Equations and Flow-Invariance (291 – 298); W.E. Olmstead, Blow-up Solutions of Volterra Equation (385 – 390); M.K. Nestell and M. Ghandehari, A Quadratic Volterra Integral Equation and Its Solution for Various Kernels (357 – 366); C.A. Roberts, A Method to Determine Growth Rates of Nonlinear Volterra Equations (427 – 432);

Numerical Problems. D.M. Bedivan and G.J. Fix, Finite Element Approximation of Volterra Integral Equations (141 – 148); C.T.H. Baker and A. Tang, Generalized Halanay Inequalities for Volterra Functional Differential Equations and Discretized Versions (39 – 56); A.Makroglou, W.Harper, and B.Smith, Computational Treatment of the Integro-differential Equations of Collective Non-ruin; The Ultimate Non-ruin Case (341 – 350);

Periodic Solutions. T. Furumochi, Periodic Solutions of an Integral Equation (203 – 210); X. Huang, Stable Periodic Solutions of a Class of Delay Differential Equations via Monotone Dynamical System Methods (251 – 260);

Control Theory. S.A. Belbas, Optimal Control of Volterra Equations with Special Constraints on the Control (135 – 140); L.R. Hunt and G. Meyer, Flight Control Using Nonlinear System Inversion (261 – 268); W.H. Schmidt, Volterra Integral Processes with Delay: Necessary Optimality Conditions (441 – 448); M.A. Shubov, Nonself-Adjoint Spectral Operators and Their Application to Control Theory (457 – 464);

Infinite-Dimensional Systems. S. Aizicovici and Q. Yan, Continuous Dependence for a Doubly Nonlinear Volterra Integral Equation (125 – 134); B.P. Belinskiy and J.P Dauer, Wave Phenomena in a Wave Guide with Elastic Walls: Abstract Differential Equation Formulation (149 – 156); H. Engler, An Example of L^p – Regularity for Hyperbolic Integro-Differential Equations (67 – 82); K. Ito and E. Kappel, Locally Quasi-Dissipative Evolution Equations and Applications to Delay Differential Equations (269 – 282); O.J. Staffans, State Space Theory for Abstract Volterra Operators (111 – 124);

Integro-Differential Equations. P. Colli and M. Grasselli, Degenerate Nonlinear Volterra Integro-Differential Equations (187 – 196); J.H. Liu, Commutativity of Resolvent Operators in Integro-Differential Equations (309 – 316); X. Liu, Extremal Solutions of Impulsive Integro-Differential Equations of Volterra Type (317 – 324); C. Mihalyko, Z. Laszlo and E.O. Mihalyko, On the Qualitative Properties of the Analytical Solution of an Integro-Differential Equation (351 – 356); S. Saitoh, Linear Integro-Differential Equations and the Theory of Reproducing Kernels (433 – 440); A. Stulov, Solution of Nonlinear Integro-Differential Equations with Boundary Conditions Using an Algebraic Method (475 – 482); T. Wang, Wazewski's Inequality in a Linear Volterra Integro-Differential Equation (483 – 492);

Approximation Methods. I.W. Sandberg, Myopic Maps and Volterra Series Approximations (95 – 110); I. W. Sandberg and L. Xu, Uniform Approximation and Myopic Maps (448 – 456). These concern very general types of (not necessarily causal) maps and are of interest in connection with, for example, their applications to the equalization of nonlinear communication channels and to the image processing.

Abstract Volterra Operators and Equations. M. Mahdavi and Y. Li, Linear and Quasi-Linear Equations with Abstract Volterra Operators (325 – 330). The paper by N. Azbelev, in the Stability Theory group, could have been included here.

Problems in Physics and Engineering. D. Censol; I. Gurvvich and M. Sonnenschein, Volterra's Functionals Series and Wave Propagation in Weakly Nonlinear Media: The Problematic of First Principles of Physical Modeling (173 – 186); A. Haji-Sheikh and C. Aviles-Ramos, Volterra Equation in Heat Conduction (221 – 234); C.A. Marinov, Monotone Operators Approach of Electrical Circuit Modeling (330 – 340); S. Nomura, Effective Medium of Heterogeneous Materials Using Integral Equations (367 – 372); H. Poorkarimi, Asymptotic Behavior for Some Nonlinear Hyperbolic Equations (411 – 416); J.Su, The Inertial Manifold Property of a Boundary Control Type Problem (465 – 474).

Varia. S. Gogonea, Some Generalizations of the Boundary-Value Problem of Volterra (211 – 220); A.F. Ivanov, Several Remarks on Symmetric Delay Differential Equations (283 – 290); C. Outlaw, A Fourth Order Bel'tyukov Formula with Third Order Continuously Embedded (399 – 402); F.R. Payne, Does "Chaos" Exist Mostly in Computing Machinery? (403 – 410).

11. Nonlinear Problems in Aviation and Aerospace [11].

Aviation and aerospace nonlinear phenomena have stimulated cooperation among engineers and scientists in a variety of disciplines. Developments in computer technology have additionally allowed for solutions of nonlinear problems, while industrial recognition of using nonlinear mathematical models for solving technological problems is increasing. For example, today's high performance aircrafts often operate in regimes where the angle of attack is high and the angular rates are large. In other cases, the trajectory of the aircraft may cover a large flight envelope. In these situations, nonlinearities become a predominant feature of the aircraft's dynamics. The flight control system needs to respond to these nonlinearities to achieve satisfactory performance and stability. The nonlinear maneuver models become a more sophisticated alternative to the existing linear models. It is therefore clear that the attempt to understand the nonlinear nature will dominate a large part of mathematical sciences in several disciplines in the future.

This volume consists of 25 papers contributed by 55 experts from 10 countries.

Contents: M. Ignatiev, S. Sivasundaram and N. Simatos, Aircraft as Adaptive Nonlinear System Which Must Be in the Adaptation Maximum Zone for Safety (1 – 10); S.R. Vadali, H. Jung and K.T. Alfriend, Orbit Determination of a Tethered Satellite System Using Laser and Radar Tracking (11 – 24); E. Cherkashin and S. Vassilyev, Application of Automatic Theorem Proving (ATP) Approach to the Telescope Guidance (25 – 42); S.F. Asokanthan and X.-H. Wang, Attitude Stability of an Asymmetric Dual-Spin Spacecraft with Stochastic Rotor Speed Fluctuations (43 – 56); C.P. Mracek and J.R. Cloutier, Full Envelope Missile Longitudinal Autopilot Design Using the State-Dependent Riccati Equation Method (57 – 76); R.R. Mohler and R.R. Zakrzewski, Intelligent Control of Agile Aircraft (77 – 88); I. Lasiecka and R. Triggiani, Feedback Noise Control in an Acoustic Chamber: Mathematical Theory (89 – 112); H-K. Lee and K-W. Han, Estimation of Asymptotic Stability Regions of Nonlinear Systems by Use of Eigen-Vectors (113 – 122); L. Tian, P. Lu and J.J. Burken, Flight Control with and without Control Surfaces: A Nonlinear Look (123 – 138); A. Miele and S. Mancuso, Optimal Ascent Trajectories for a Single-Stage Suborbital Spacecraft (139 – 152); A.K. Misra, M.S. Nixon and V.I. Modi, Nonlinear Dynamics of Two-Body Tethered Satellite Systems (153 – 166); V. Casulli and P. Zanolli, Two- and Three-Dimensional Numerical Methods for Free Surface Hydrodynamics (167 – 178); A. V. Balakrishnan, Control of Structures with Self-Straining Actuators: Coupled Euler/Timoshenko Model: 1 (178 – 194); R.L. Carino, C.E Cox, J. Zhu and P. Cinnella, Building a Parallel Version of a "Real Gas" Flow Solver (195– 208); E. Duflos, P. Vanheeghe, P. Penel and P. Borne, A Probabilistic Method to Estimate a Missile Target (209 – 220); A. Cavallo and G. De Maria, Reentry Control for Low L/D Vehicles (221 – 236); F.R. Payne, Exact Euler Aerodynamics via a Novel Method (237 – 250); D. S. Naidu, Singular Perturbations and Time Scales in Aerospace Systems: An Overview (251 – 264); G. Leitmann and C.S. Lee, Planning for R and D Manpower in Aviation and Aerospace (265 – 280); J. Rohacs and P. Granasy, Effects of Nonlinearities in Aerodynamic Coefficients on Aircraft Longitudinal Motion (281– 296); L.T. Gruyitch, Vector Lyapunov Function Synthesis of Aircraft Control (297 – 316); S.A. Do-ganovsky, N.N. Maksimkin, S. Sivasundaram, S. Sliwa and S.N. Vassilyev, Control Systems with Parametrical and Structural Reconfiguration (317 – 328); S. Sivasundaram, Unified Control Systems (329 – 348); S.S. Sritharan and P. Sundar, Ergodic Control of Stochastic Navier-Stokes Equation (349 – 358); V.M. Matrosov, M.E Reshetnev, VA. Rayevsky and El. Somov, Nonlinear Methods and Software for Dynamic Investigations of Fail-Safe Gyromoment Attitude Control Systems of Spacecrafts (359 – 375).

12. Stabilization of Programmed Motion [12].

This volume presents a particular aspect of control theory - stabilization of programmed motion. Methods of the construction and synthesis of stabilizing controls are introduced together with original results and useful examples. The problem of optimal stabilization control synthesis is solved for linear systems of difference equations with quadratic quality criterion.

The book consists of four chapters. In Chapter 1, the methods of constructing the stabilizing controls are set out for the case of continuous receiving of information about the state of the system. Namely, in Section 1.1, the statement of the problems under consideration is presented. Section 1.2 gives an account of well-known results on stationary control systems stabilization, while the rest of the chapter deals with non-stationary systems. In Sections 1.2 – 1.4, the construction of stabilizing controls is carried out based on the reduction of the system to a simpler canonical form, whereas in Sections 1.4 – 1.8, the method of Lyapunov functions is applied to this end. In Sections 1.2 – 1.8, linear stabilizing controls are constructed as well as the relay stabilizing controls in Sections 1.5 – 1.8. In Sections 1.2 – 1.5, the coefficients in the right-hand parts of the differential equation systems are assumed to be known precisely, however those in Sections 1.6 – 1.8 are considered with errors which do not seem to be small. In Sections 1.2 – 1.6, the stabilizing controls are constructed based on the principle of complete feedback, while in Sections 1.7 – 1.8, the principle of partial feedback is incorporated.

Chapter 2 deals with the synthesis of stabilizing controls for the case of discrete receiving of information about the state of the system. Section 2.1 contains the statements of stabilization problems for the so-called hybrid systems, namely, systems with continuous time and discrete information coming in about its states. Moreover, in this section the problems of the stabilization of hybrid systems are reduced to those of stabilization of special-type discrete systems and also, some results from the theory of linear discrete systems are cited. In Section 2.2, stabilization methods are set out for both stationary and non-stationary discrete systems in the cases of complete and partial feedback. In Section 2.3, the problems of hybrid systems stabilization stated in Section 2.1 are solved.

In Chapter 3, the problems of optimal stabilization are solved for linear systems with a quadratic quality criterion in the cases of continuous and discrete receiving of information about the state of the system. In Section 3.1, the problems of stabilizing control synthesis are posed for linear systems with continuous and discrete time, being optimal in the sense of quadratic functionals of a certain type. Furthermore, the problem of optimal stabilizing control synthesis for a hybrid system with an integral quality criterion is reduced to a similar problem for a discrete system with a corresponding quadratic functional. Section 3.2 provides a solution of the problem of the optimal stabilizing control synthesis for a linear system of differential equations with a quadratic functional of the integral type in the case of information coming in continuously for control construction. Various necessary and sufficient existence conditions are formulated for the optimal stabilizing control and the method of successive approximations is suggested to specify it. Moreover, the optimal stabilization problem in question is shown to be solvable only when the initial functional with an arbitrary quadratic form in the quality criterion in the solutions of the differential equation system under consideration closed by a stabilizing control with the precision up to the term which is dependent only on the initial data of the system, coincides with a functional with a positive definite quadratic form in the quality criterion. In Section 3.3, the problem of optimal stabilizing control synthesis is solved for the linear system of difference equations with quadratic quality criterion.

In Chapter 4, the control synthesis problem is solved for the case of continuous receiving of information on the state of the system. However, in contrast to Chapter 1, the other type of stabilization is treated here, namely, the orbit stabilization for a wider class of controlled systems referred to in the book as the systems of variable structure. This class is included into the family of the so-called transforming systems which are presently of increasing interest. Section 4.1 presents the orbit stabilization problem for programmed motions of variable structure systems for the class of piece-wise constant controls. Also, a criteria of orbit asymptotical stability of programmed motion is determined, and the synthesis technique for the desired stabilizing controls is described, namely, the method of construction of control switching surfaces. Furthermore, the solution of the initial problem on the orbital stabilization of the programmed motion of the variable structure system with continuous time is reduced to the solution of the stabilization problem for a special linear system with discrete time discussed in Chapter 2. Auto-oscillations with a prescribed period are constructed in Section 4.2 based on the synthesis technique for stabilizing controls developed in Section 4.1. Also, this section addresses the systems containing the transistor keys and thyristor transformers in the control contour. It seems that there are no books of this type in the existing literature. Most of the material is original and presented in the monograph for the first time. For instance, Chapter 2 proposes the method of constructing stabilizing controls for linear non-stationary discrete systems based on the reduction of the initial system to the canonical form most suitable for the solution of the stabilization problem. In Chapter 4, a method of solution of the orbit stabilization problem for programmed motions of variable structure systems is suggested reducing this problem to that of stabilization of linear discrete system with a scalar control discussed in Chapter 2.

The bibliography consists of 257 – 262 pages and 77 references.

13. Advances in Stability Theory at the End of the 20th Century [13].

This volume presents surveys and research papers on various aspects of modern stability theory, including discussions on modern applications of the theory, all contributed by experts in the field. The volume consists of four sections that explore the following directions in the development of stability theory: progress in stability theory by first approximation; contemporary developments of Lyapunov's idea of the direct method; the stability of solutions to periodic differential systems; and selected applications. Advances in Stability Theory at the End of the 20th Century will be of interest to postgraduates and researchers in engineering fields as well as in mathematics.

The development of stability theory in the twentieth century has been closely connected with the solution of major problems of science and engineering and, also, with the modelling and investigation of more complex phenomena of the real world. The peculiar features of progress in this field of the natural sciences are:

* the variety of engineering and scientific problems whose solution by the methods of motion stability theory has allowed numerous projects to be carried out in aviation, rocket engineering, submarine dynamics, economics, traffic, construction, etc.

* the intensive development of the ideas and methods proposed by the creators of stability theory, such as Euler, Poincare, and Lyapunov, within the framework of modern achievements of the analytical and qualitative theory of equations;

* the integration of the efforts of scientists world-wide in solving specific scientific, engineering and general problems of stability theory.

The main idea of this volume of the International Series of Scientific Monographs is to present surveys and research papers from the various branches of the modern theory of stability written by scientists from around the world. Meanwhile, the application areas of stability theory are very diverse and an attempt to embrace as many of them as possible would inevitably result in the creation of several volumes. So, this volume presents only some of the applications of motion stability theory. The papers collected in this volume are written by the scientists who are deeply involved in current research and they provide a general insight into the present-day state of stability theory. This volume consists of four sections presenting the following areas of the development of stability theory.

Part 1. Progress in Stability Theory by the First Approximation. B. Aulbach and T. Wanner, Invariant Foliations for Caratheodory Type Differential Equations in Banach Spaces (1 – 14); C. Corduneanu and Yizeng Li, On Exponential Asymptotic Stability for Functional Differential Equations with Causal Operators (15 – 24); N.A. Izobov, Lyapunov Problems on Stability by Linear Approximation (25 – 48).

Part 2. Contemporary Development of Lyapunov's Idea of the Direct Method. P. Borne, M. Dambrine, W. Perruquetti and J.P. Richard, Vector Lyapunov Functions: Nonlinear, Time-Varying, Ordinary and Functional Differential Equations (49 –74); A. D'Anna, Some Results on Total Stability Properties for Singular Systems, (75 – 88); F. Dannan, S. Elaydi and P. Li , Stability Theory of Volterra Difference Equations (89 – 106); Ly. T. Gruyitch, Consistent Lyapunov Methodology for Exponential Stability: PCUP Approach (107 – 120); V.lakshmikantham, and S.Leela, Advances in Stability Theory of Lyapunov: Old and New (121 – 134); A. A. Martynyuk, Matrix Liapunov Functions and Stability Analysis of Dynamical Systems (135 – 152); A.Martynyuk, J.H.Shen, and I.P.Stavroulakis, Stability Theorems in Impulsive Functional Differential Equations with Infinite Delay (153 – 174); T. Taniguchi, The Asymptotic Behaviour of Solutions of Stochastic Functional Differential Equations with Finite Delays by Liapunov-Razumikhin Method (175 – 188); V.A. Vujicic, A Non-Standard Approach to the Study of the Dynamic System Stability (189 – 200).

Part 3. Stability of Solutions to Periodic Differential Systems. Yu.A. Mitropol'skii, A.A. Martynyuk and V.I. Zhukovskii, A Survey of Starzhinskii's Works on Stability of Periodic Motions and Nonlinear Oscillations (201 – 216); J.S. Muldowney, Implications of the Stability of an Orbit for Its Omega Limit Set (217 – 230); V.N. Pilipchuk, Some Concepts of Periodic Motions and Stability Originated by Analysis of Homogeneous Systems (231 – 242); A. A. Zevin, Stability Criteria for Periodic Solutions of Autonomous Hamiltonian Systems (243 – 254).

Part 4. Selected Applications. H.I.Freedman, M.Solomonovich, L.P.Apedaile, and A.Hailu, Stability in Models of Agriculture-Industry-Environment Interactions (255 – 266); V.I.Guliaev, Bifurcations of Periodic Solutions of the Three Body Problem (267 – 288); A.Yu.Ishlinsky, V.A.Storozhenko, and M.E.Temchenko, Complex Mechanical Systems: Steady-State Motions, Oscillations, Stability (289 – 320); Xinzhi Liu, Progress in Stability of Impulsive Systems with Applications to Population Growth Models (321 – 338).

14. Dichotomies and Stability in Nonautonomous Linear Systems [14].

Linear nonautonomous equations arise as mathematical models in mechanics, chemistry, and biology. The investigation of bounded solutions to systems of differential equations involves some important and challenging problems of perturbation theory for invariant toroidal manifolds. This monograph is a detailed study of the application of Lyapunov functions with variable sign, expressed in quadratic forms, to the solution of this problem. The authors explore the preservation of invariant tori of dynamic systems under perturbation. This volume is a classic contribution to the literature on stability theory and provides a useful source of reference for postgraduates and researchers.

The investigation of bounded solutions to systems of differential equations involves some important toroidal manifolds. This is due to the fact that many stationary processes in mechanics, chemistry and biology are modelled by bounded solutions of differential equations. Also, establishing general conditions under which the system of differential equations possesses one or more solutions bounded on the whole axis is a typical problem. Moreover, it is of importance that this property be preserved when small changes in the system occur. This volume deals with the application of Lyapunov functions with alternating signs in the form of quadratic forms to the solution of the following problems:

* the existence of solutions to linear systems of differential equations with variable coefficients bounded in the whole axis;

* establishing the conditions for preserving the invariant tori of a dynamical system under perturbations;

* the separation of normal variables in the linear extensions of dynamical systems on a torus.

It should be noted that the investigations of perturbation theory of the invariant tori of dynamical systems presented in the second chapter required fascinating new problems to be solved in the theory of dichotomous linear systems of differential equations with variable coefficients.

In Chapter 1, a theorem is proved on the transformation on the whole axis of every linear nonautonomous system of differential equations which are exponentially dichotomous only on both semiaxes to a specific block-triangular form. This allowed the necessary and sufficient condition to be established for the inhomogeneous linear system to have at least one solution bounded on the whole axis. The criterion is formulated in terms of alternating sign, degenerate quadratic forms, which have a definite sign derivative by virtue of the conjugated system.

Chapter 2 presents a new idea for the investigation of the weakly regular linear expansions of dynamical systems on a torus which provides a complement to the regular systems. On this basis, new possibilities have been opened up for the intriguing analysis of smoothness and parameter dependence of the invariant tori of weakly regular linear expansions of dynamical systems.

Chapter 3 discusses the approach to qualitative problems of the separation of normal variables in linear extensions of the dynamical systems on a torus. The interest in these problems has been evoked by the extensive investigations of the possibility of splitting the linear exponentially dichotomous system with quasiperiodic coefficients by the quasiperiodic change of variables. These investigations revealed that the properties of sign-alternating Lyapunov functions, which provide the means to study the exponential dichotomy of linear expansions of a torus, influence to a great extent the possibility of splitting normal variables in these extensions to a great extent. There relationships are also discussed in Chapter 3.

Chapter 4 presents new results which were obtained recently by the authors after the publication of the Russian edition of the book, and which relate to the problems of the existence of Lyapunov functions with alternating signs, the structure of regular linear extensions on a torus, and the variation of the family of solutions to the dynamical systems starting on a torus. Direct and converse theorems on the exponential dichotomy on the whole axis of linear systems of differential equations with variable coefficients are proved for the first time in terms of non–degenerate Lyapunov functions with alternating signs in the form of quadratic forms. The properties of weak regularity on the whole axis of linear systems of differential equations are established by means of sign-alternating degenerate Lyapunov functions. An integral representation is first derived for all invariant tori of weakly regular linear extensions of dynamical systems. A deep relationship is stated between the properties of sign–definite Lyapunov functions and the ability to split linear extensions of dynamical systems on a torus. New structures are found for regular linear extensions. Criteria are proved for exponential dichotomy of linear extensions on a torus. New theorems are first deduced on the existence of Lyapunov functions with alternating signs.

The bibliography consists of 355 – 366 pages and 168 references.

15. Almost Periodic Solutions of Differential Equations in Banach Spaces [15].

This monograph presents recent developments in spectral conditions for the existence of periodic and almost periodic solutions of inhomogenous equations in Banach spaces. Many of the results represent significant advances in this area. In particular, the authors systematically present a new approach based on the so-called evolution semigroups with an original decomposition technique. The book also extends classical techniques, such as fixed points and stability methods, to abstract functional differential equations with applications to partial functional differential equations. Almost Periodic Solutions of Differential Equations in Banach Spaces will appeal to anyone working in mathematical analysis.

Almost periodic solutions of differential equations have been studied since the very beginning of this century. The theory of almost periodic solutions has been developed in connection with problems of differential equations, dynamical systems, stability theory and its applications to control theory and other areas of mathematics. The classical books by C. Corduneanu (1968), A.M. Fink (1974), Yoshizawa (1975), L. Amerio and G. Prouse (1971), B.M. Levitan and V.V. Zhikov (1982) gave a very nice presentation of methods as well as results in the area. In recent years, there has been an increasing interest in extending certain classical results to differential equations in Banach spaces. In this book, we will make an attempt to gather systematically certain recent results in this direction.

We outline briefly the contents of the book. The main results presented here are concerned with conditions for the existence of periodic and almost periodic solutions and its connection with stability theory. In the qualitative theory of differential equations there are two classical results which serve as models for many works in the area. Namely,

Theorem A A periodic inhomogeneous linear equation has a unique periodic solution (with the same period) if I is not an eigenvalue of its monodromy operator.

Theorem B A periodic inhomogeneous linear equation has a periodic solution (with the same period) if and only if it has a bounded solution.

In the book, the main part is devoted to the discussion of the question of how to extend these results to the case of almost periodic solutions of (linear and nonlinear) equations in Banach spaces. To this end, the first chapter presents the introduction to the theory of semigroups of linear operators, its applications to evolution equations and the harmonic analysis of bounded functions on the real line.

Chapter 2 dwells on the results concerned with autonomous as well as periodic evolution equations, extending Theorems A and B to the infinite dimensional case. In contrast to the finite dimensional case, in general, one cannot treat the periodic evolution equations as the autonomous ones. This is due to the fact that in the infinite dimensional case, there is no Floquet representation, though one can prove many similar assertions to the autonomous case (see e.g. J.K.Hale (1977), D.Henry (1981), P.Kuchment (1993)). Sections l and 2 of this chapter are devoted to the investigation by means of evolution semigroups in translation invariant subspaces of BUC (R , X) of bounded uniformly continuous X -valued functions on the real line. A new technique of spectral decomposition is presented in Section 3. Section 4 presents various results extending Theorem B to the periodic solutions of abstract functional differential equations. In Section 5, we prove analogues of the results from Sections l, 2 and 3 for discrete systems and discuss an alternative method to extend Theorems A and B to periodic and almost periodic solutions of differential equations. In Sections 6 and 7, the method used in the previous sections is extended to semilinear and fully nonlinear equations. The conditions are given in terms of the dissipativeness of the equations under consideration.

In Chapter 3, the authors address the existence of almost periodic solutions of almost periodic evolution equations by using stability properties of nonautonomous dynamical systems. Sections I and 2 of this chapter extend the concept of skew product flow of processes to a more general concept which is called skew product flow of quasi-processes and investigate the existence of almost periodic integrals for almost periodic quasi-processes. For abstract functional differential equations with infinite delay, there are three kinds of definitions of stabilities. In Sections 3 and 4, we prove some equivalence of these definitions of stabilities and show that these stabilities fit in with quasi-processes. By using the results of Section 2, we discuss the existence of almost periodic solutions for abstract almost periodic evolution equations in Section 5. Concrete applications for functional partial differential equations are given in Section 6.

Chapter 4, Appendices, provides some information from mathematical analysis: Fredholm operators and closed range theorems; essential spectrum and measures of noncompactness; sums of commuting operators; Lipschitz operators.

The bibliography consists of 235 – 247 pages and 242 references.

16. Functional Equations with Causal Operators [16].

Functional equations encompass most of the equations used in applied science and engineering: ordinary differential equations, integral equations of the Volterra type, equations with delayed argument, and integro-differential equations of the Volterra type. The basic theory of functional equations includes functional differential equations with causal operators. Functional Equations with Causal Operators explains the connection between equations with causal operators and the classical types of functional equations encountered by mathematicians and engineers. It details the fundamentals of linear equations and stability theory and provides several applications and examples.

This book is dedicated to the investigation of functional or functional differential equations involving causal operators. These operators are also called nonanticipative, or abstract Volterra operators. The term "causal" is prevalent in the engineering literature. The definition of causal operators is very simple: an operator V, acting on a given function space $E([O,T],R^n)$, is called causal if for any pair of functions x, y of E, such that x and y coincide on an interval $[0,t], t < T$, Vx and Vy also coincide on that interval. In other words, the values of Vx up to a given point t are determined only by the values taken by the function x on the interval $[0,t]$. The idea of considering such operators appears implicitly in Volterra's work, but a sharp definition and further consideration appear in the paper of L. Tonelli (1930). In this paper, the functional equation

$$x(t) = f(t) + A(t, x_0^t(s))$$

is considered, where the second term in parantheses means the restriction of the function x to the interval $[0,t]$. The notation is obviously inspired by Volterra's work, and the above equation reminds us instantly of the Volterra integral equation

$$x(t) = f(t) + \int_0^t k(t,s,x(s))ds.$$

Tonelli's paper was dedicated to proving the existence and uniqueness of the solution of the functional equation he has devised by means of causal operators. The equation has been investigated in the space of continuous functions and the hypotheses are formulated in such a way that the compactness of the operator A is assured. This result of Tonelli is, very likely, the first existence result for equations with general causal operators. The next significant step in developing the theory of functional equations involving cauasal operators was made by A.N. Tychonoff (1938). The definition given by Tychonoff to causal operators is as formulated above, and besides the existence of solutions, the importance of these types of operators or equations for other fields is emphasized. In retrospect, it may appear somewhat strange that the concept of a causal (or, as both Tonelli and Tychonoff call it, Volterra) operator did not attract the immediate attention of researchers. A possible explanation may be that at the time this concept was advanced, the relatively new methods of functional analysis did not constitute the main tool for many investigators. Gradually, the theory of functional equations with causal operators has caught the attention of researchers. From the 1960s, the author mentions the papers by R. Driver (1962), C. Corduneanu (1966), and Z.B. Caljuk (1969). The last-quoted paper deals with functional inequalities with causal operators.

In the 1970s, there have been many authors dealing with causal operators/equations. For the first time in book form, these operators are discussed in the book by L. Neustadt (1976). Journal papers have been published by M. Kwapisz and his co-workers (1975), V.G. Kurbatov (1995), L.A. Zhivotovskii (1971). A group of researchers from Russia (Perm Technical University), under the leadership of N.V. Azbelev, has started the systematic investigation of linear functional differential equations with causal operators. The activity of this group continues nowadays, with its members in Russia, Israel and other countries. Also in the 1970s, a good deal of research work has been conducted with regard to the equations with infinite delay. The books by J.K. Hale (1977) and A.D. Myshkis (1972) have also contributed to making the subject of causal operators an attractive topic for researchers.

In the 1980s, the theory of functional or functional differential equations with causal operators made serious steps towards its maturity. A large number of papers were published during this decade in the USA, the former Soviet Union, Italy and other countries. Most of the basic problems of this theory, including stability theory, approximation procedures and other aspects, have been considered by many authors. Our list of references provides a large number of items from that period, without any claim to being complete. It is also important to notice that during the 1980s, a large number of engineering papers (system science) were produced. The books by W.J. Rugh (1981) and M. Schetzen (1980) cover some of these topics. Fundamental results concerning causal general operators have been obtained by I. W. Sandberg (1982, 1985, 1993) not necessarily related to the theory of functional equations.

The 1990s were characterized by an increasing interest in the theory of functional equations with causal operators, often related to applications. Several books including results concerning functional equations with causal operators have been published, or are in an advanced state of publication: G. Gripenberg, S.O. Londen and O. Staffans (1990), C. Corduneanu (1991), N.V. Azbelev, V.P. Maksimov and L.F. Rakhmatullina (1991), N.V. Azbelev and P.M. Simonov (2003) contain chapters or conspicuous sections dedicated to this theory. The journal literature is currently growing steadily, with at least one periodic publication called Functional Differential Equations being mostly dedicated to this theory. A serial publication with the same title has been published by N. V. Azbelev and his collaborators in Perm. Many other journals include contributions on this subject, authored by researchers from Russia, Ukraine, Israel, the USA, Italy, Ireland, Japan, Georgia, Australia, Poland, Germany and other countries.

This book evolved during the period 1991 – 1998, when the author and his former students held a weekly seminar at the University of Texas at Arlington. In particular, Dr. Mehran Mahdavi and Dr. Yizeng Li were active and wrote their PhD theses about functional differential equations with causal operators. After they graduated and went on to teach at other institutions of higher learning, the cooperation between us continued without interruption. The material contained in this book is in greatest part based on the work which was developed separately or jointly.

Briefly, the structure of the book is as follows: Chapter 1 is an introduction to the concept of functional equations, in general, with particular concern for equations with causal operators; Chapter 2 contains some auxiliary material, mostly pertaining to functional analysis (both linear and nonlinear), as a preparation of the reader for understanding the rest of the material presented in the book; Chapter 3 deals entirely with the existence theory for functional or functional differential equations with causal operators, emphasizing the fact that these equations contain as particular cases several classes of functional equations encountered in the literature; Chapter 4 is dedicated to the theory of linear and quasi–linear equations with causal operators, including the global character of existence results and the integral representation of solutions; Chapter 5 contains an introduction to stability theory for equations with causal operators, featuring both the method of "first approximation" and the "comparison method" based on Lypunov functionals and differential inequalities; Chapter 6 is completely dedicated to the theory of neutral functional equations with causal operators, and it is based on the work of the author and M. Mahdavi (1992); Chapter 7 contains some applications of the general theory to problems in optimal control, some generalizations of the existence results by means of the Leray-Schauder principle, as well as a review of certain results available in the literature.

The bibliography consists of 160 – 166 pages and 169 references.

17. Optimal Control of the Growth of Wealth of Nations [17].

Students and researchers in applied mathematics and applied economics can use this introductory-level graduate text. It looks at the current problems of the development of the global economy by studying the dynamics of key economic variables, such as gross national product, interest rates, employment, value of capital stock, prices (inflation) and balance of payments. Validation of the model is attempted using the economic time series of several countries. The constructed models explain the macroeconomic data of nations as dynamic games of pursuit, which are equivalent to "control" problems and are used to study mathematical optimal control of the growth of the wealth of nations. This invaluable reference for graduates and researchers compares the extent of government intervention in the economy with private firms to ensure the controllability of the economy.

The aim in this research monograph is to identify realistic dynamic models of the growth of wealth of nations. The dynamics are then examined for stability, controllability, and optimality when either time or effort (some time integral of the control) is to be minimized. The key fundamental idea in this book is controllability: from any initial state of gross domestic (national) product, interest rate, employment, value of capital stock, prices and cumulative balance of payment, is it possible to attain another desirable target state?

The model of ordinary differential equations is considered in Chapters 1 – 5, and the hereditary dynamic models are treated in Chapter 6. Chapter 1 is an introductory mathematical theory of competition and cooperation: the social environment for economic activity.

Chapter 2 derives the ordinary differential equation of national income, interest rate, employment, prices, and balance of payment. It is based on the economic market principle that the rate of growth of the economic variable is proportional to the excess demand over supply.

In Chapter 3, the economic system's controllability is explored using the Kalman-Hermes-LaSalle Theory (1969). The economic models derived in Chapter 2 are tested for stability and controllability. Some predictions are attempted using the predict command of MATLAB.

Neck's (1992) experiments with the problem of evaluating past macroeconomics policies by using "optimality" measures can now be read with great interest, and can be sharpened with methods of the book in Chapter 4.

Because of the apparent oscillation of both the model and the time series, Chapter 5 considers conditions for oscillation of linear ordinary dynamics systems. They are applied and interpreted to the dynamics of income. Policies are deduced which prevent oscillation or economic depression and boom. We extend our studies to cover nonlinear models. For a system of national income, interest rate, and employment, matters may be arranged such that a chaotic behavior appears. MATLAB commands and outcome are included.

In Chapter 6, as advocated by Gandolfo, economic models described by functional differential equations are constructed. Delays emerge because of the lag between the time economic decisions are made and the time the decisions bear fruit. By assuming "the rational expectations hypothesis" in the Fair (1976, 1984) formulation that expected future values of a variable to be functions of the current and past values of the variable, hereditary systems are constructed.

The bibliography is given in the text of the book to separate sections and chapters, 343 titles in total.

18. Stability and Stabilization of Nonlinear Systems with Random Structure [18].

Nonlinear systems with random structures arise quite frequently as mathematical models in diverse disciplines. This monograph presents a systematic treatment of stability theory and theory of stabilization of nonlinear systems with random structure in terms of new developments in the direct Lyapunov method. The analysis focuses on dynamic systems with random Markov parameters. This high-level research text is recommended for all those researching or studying in the fields of applied mathematics, applied engineering, and physics, particularly, in the areas of stochastic differential equations, dynamical systems, stability, and control theory.

Probability models are widely used in the investigation of real processes occurring in nature and technology. This involves consideration of ordinary differential equations, the parameters of which are random time functions. Since stability is one of the main conditions of the process physical realization, the analysis of probability models has resulted in the corresponding development of a general theory of motion stability known as stochastic stability theory.

The Lyapunov function (functionals) method has proved to be a most versatile and powerful approach in the investigation of the stability of determined systems. The main advantage of this method is that the stability of the system can be assessed without immediate integration of the differential motion equations. The insuperable difficulties associated with the integration of stochastic systems mean that the stability theory for such systems is based on the fundamental ideas of the Lyapunov functions method.

To construct an efficient stability theory of stochastic systems the following issues are of prime importance:

* consideration of systems possessing Markov properties (absence of aftereffects);

* concept of strong probability stability of the asymptotic behaviour of the process realizations;

* use of the auxiliary Lyapunov functions and, in particular, the notion of the auxiliary function derivative by virtue of the system whose computation does not require integration of the stochastic perturbed motion equations.

The abovementioned approach for systems with random Markov parameters was proposed by Krasovskii in 1960 and set forth by Kats and Krasovskii. The ensuing intensive development of this approach related mainly to investigation of the stability of solutions to stochastic differential Ito equations. The results have been presented in many well–known and important monographs and textbooks. However, many experts in this area were unaware of investigations on the stability of systems with random Markov parameters because the papers were published in different journals. This has also been the case for problems on the stabilization of controlled motions for which the results for systems with random parameters were also published in the early 1960s in journal papers by Krasovskii and Lidskii.

The aim of the book is to fill this gap and help experts and beginners alike. This book consists of six chapters. Chapter 1 covers background material and the ideas of the theory of probability processes. Some examples from mechanics show the essence of the process of modelling real phenomena using systems of ordinary differential equations with random structure.

Chapter 2 sets out the method of analysis of the stochastic stability of systems with random structure. The method is based on scalar Lyapunov functions and the idea of an averaged derivative of an auxiliary function along solutions of the stochastic system.

Chapter 3 describes the analysis of stochastic stability based on Lyapunov's matrix function method. Here, stochastic singularly perturbed systems are studied together with systems of ordinary differential equations with random parameters.

Chapter 4 presents the results of the stability analysis of systems with random structure using the Lyapunov function constructed for the first approximation of a nonlinear system.

Chapter 5 discusses the problem of constructing systems with a prescribed type of motion stability for the optimal of the transient process. The concluding chapter shows how efficient stability criteria for solutions of stochastic models of real systems can be obtained using the method of matrix valued Lyapunov functions.

Thus, this monograph develops the theory of stochastic stability and stabilization for objects modelled using differential equations the parameters of which are Markov functions of a quite general nature. The assumption that at random times of jump-like change of system parameters, the phase vector of its state can also change in a jump-like way is new. Such systems are called systems with random structure. This term is different to that used in some publications on the theory of automatic control and describes a wider class of systems. The authors give an approach to discontinuous phase trajectories which differs from the model of generalized stochastic differential Ito or Poisson equations, and is an efficient one.

The bibliography consists of 219 – 234 pages and 240 references.

19. Lyapunov Functions in Differential Games [19].

A major step in differential games is determining an explicit form of the strategies of players who follow a certain optimality principle. To do this, the associated modification of Bellman dynamic programming problems has to be solved; for some differential games this could be Lyapunov functions whose "arsenal" has been supplied by stability theory. This approach, which combines dynamic programming and the Lyapunov function method, leads to coefficient criteria, or ratios of the game math model parameters with which optimal strategies of the players not only exist but their analytical form can be specified. In this book, coefficient criteria are derived for numerous new and relevant problems in the theory of linear–quadratic multi-player differential games. Those criteria apply when the players formulate their strategies independently (non co-operative games) and use non-Nash equilibria or when the game model recognizes noise, perturbation and other uncertainties of which only their ranges are known (differential games under uncertainty). This text is useful for researchers, engineers and students of applied mathematics, control theory and the engineering sciences.

In the famous Russian opera “The Queen of Spades” by Tchaikovsky, Tomsky sings "What is our life? A game!", to which the author added "A differential game!" since it is the differential game that allows us to analyse the variety of life conflicts in view of their change in time. Usually, the family tree of each scientific direction is represented as a tree (in the sense of graph theory) which is branching from a single "root". For differential games, the book Differential Games by R. Isaacs (1965) is such a root. It still feeds the huge crown formed by numerous modern works on differential games (the bibliography in differential games with several traders (see Zhukovskiy and Dochev (Eds.) (1981), Zhukovskiy and Ukhobotov (Eds.) (1995), Zhukovskiy and Ushakov (Eds.)(1990) has attracted more than 5000 papers up to 1994). Conventionally, these papers can be divided into five directions: non-cooperation, coalition, cooperative, hierarchical and quality games. Each direction possesses spray and fruitful interwoven branches. The author of this book makes an attempt to graft on these branches a new sprout being on the boundary of the theory of differential games and stabilization theory. This is a solution of dynamical games problems under uncertainty by means of the Bellman-Lyapunov function.

Game problems under uncertainties have not yet been studied. The only exception is Chapter 4 of the book by Zhukovskiy and Chikriy (1994), where the concept of the Nash equilibrium and the analogue of the vector saddle point are considered. What is the difficulty of the investigation of game problems under uncertainty? Up to the present, "ideal games" have been studied. In these games a unique point in the space of payoff functions corresponds to each concrete situation (the set of player strategies). The presence of uncertainties results in the fact that a "cloud" in the space of payoff functions corresponds to the concrete situation. Such multivaluedness should already be taken into account when formalizing "a good" solution for the players, not to mention further theoretical investigations. In order to overcome the "curse of multivaluedness", two appropriate modifications of the vector saddle point and a vector maximin are given.

The author also indicates three facts.

First, the optimality concept in the book incorporates not the generally accepted Nash equilibrium, but the equilibrium of objections and counter-objections allowing us "to get rid" of some negative properties that are characteristic of the Nash equilibrium (such as the notion of the solution to non–coalition games).

Second, non–antagonistic games under uncertainty are a new direction in the general theory of games. Therefore, in this book, we consider the corresponding "static" version of the problem (not allowing for dynamics) prior to the investigation of the differential game.

Third, everyone who has been involved in consideration of concrete differential games is aware of how difficult (sometimes impossible) it is to find an explicit form of the player strategies forming the solution of the game. Linear quadratic positional differential games are the lucky exception. The contents of this book lie within the framework of these games. We have not strived for global generality, but have restricted ourselves to games of two or a maximum of three players, because the cumbersome generalizations sometimes obscure (or even conceal) the essence of the result and make it invisible. The volume is arranged as follows.

Chapter 1 describes the foundations of differential games under uncertainties and examples from economic dynamics necessary for the investigation are also presented. Attention is focused on the notion of the vector quarantee (Section 1.3), its properties and the methods of construction.

In Chapter 2 on differential linear quadratic games under uncertainty, new guaranteeing solutions are proposed which are based on the concept of the equilibrium of objections and counter–objections as well as the active equilibrium. A detailed comparison is carried out with similar guaranteeing Nash equilibria.

At the end of each chapter, exercises are set out which show the perspectives of the investigations and contain concrete applied problems (the solutions of these problems are placed at the end of the book).

The bibliography consists of 267 – 277 pages and 132 references.

20. Stability of Differential Equations with Aftereffect [20].

Stability of Differential Equations with Aftereffect presents stability theory for differential equations concentrating on functional differential equations with delay, integro-differential equations, and related topics. The authors provide background material on the modern theory of functional differential equations and introduce some new flexible methods for investigating the asymptotic behaviour of solutions to a range of equations. The treatment also includes some results from the authors' research group based at Perm and provides a useful reference text for graduates and researchers working in mathematical and engineering science.

This volume presents the problems of stability and asymptotic behaviour of solutions to the functional differential equation with linear or nonlinear Volterra operators, according to Tikhonov (see Azbelev, Maksimov and Rakhmatullina (1991, 1995), Corduneanu (1991) etc.).

Stability theory was being developed for a long time in the research direction and studies indicated and initiated by Lyapunov one hundred years ago. However, sometimes for the equations with delaying argument and their generalizations, the classical Lyapunov concepts and methods are not efficient and do not yield the desired results. This is due to the specific character of ordinary differential equations, on which some of Lyapunov ideas are based.

The volume consists of five chapters. Chapter 1 provides all the necessary information on functional differential equations. For a detailed presentation of the theory of these equations, see Azbelev, Maksimov and Rakhmatullina (1991, 1995), Azbelev and Rakhmatullina (1996).

Chapter 2 deals with examples of D-space constructions and D-stability test. For linear functional differential equations with delay, a number of D-stability tests are cited, expressed in terms of these equation parameters.

Chapter 3 treats the linear functional differential equation with delay, solved with respect to the derivative. The problem of representing general solution to such equation is scrutinized here as well. Based on the peculiarities of this representation, the best possible existence tests are proposed for the exponential estimate of the Cauchy functions for scalar equations and the refining tests for the existence of such estimate for the Cauchy matrices of some classes of systems.

In Chapter 4, the conditions are discussed under which D-stability of linear equation ensures more refined properties of solutions, i.e., a specific solvability of the Cauchy problem in a more narrow space $D_1 \subset D$. The development of this problem is associated with the names of Bohl, Perron, Halanay, and Tyshkevich. Chapter 4 is based on the specific sections of theory of semi-ordered spaces and may be omitted at first reading.

In Chapter 5, the quasilinear equation is studied. An analogue of the Lyapunov theorem on stability with reference to the first approximation is proposed, and the scheme of the proofs is set out for the theorems on existence of solutions satisfying the given a priori estimates. The scheme incorporates the theorems on integral functional inequalities of the type of the known theorems on integral inequalities.

The volume gives an account of some results that have been obtained during the years of study by a large group of mathematicians working at the Perm Seminar at the Scientific Research Centre "Functional Differential Equations" (Perm State Technical University) and at the International Laboratory of Constructive Methods of Dynamical Model Investigations (Perm State University).

The bibliography consists of 199 – 219 pages and 276 references.

21. Asymptotic Methods in Resonance Analytical Dynamics [21].

Asymptotic Methods in Resonance Analytical Dynamics presents new asymptotic methods for the analysis and construction of solutions (mainly periodic and quasiperiodic) of differential equations with small parameters. Along with some background material and theory behind these methods, the authors also consider a variety of problems and applications in nonlinear mechanics and oscillation theory. The methods examined are based on two types: the generalized averaging technique of Krylov – Bogolubov and the numerical-analytical iterations of Lyapunov – Poincare. This text provides a useful source of reference for postgraduates and researchers working in this area of applied mathematics.

Many important systems in analytical dynamics are described by nonlinear mathematical models, and the latter as a rule are represented by differential or integro-differential equations. The absence of exact universal methods for the investigation of nonlinear systems has driven the development of a wide range of approximate analytic and numerical-analytic methods that can be implemented in effective computer algorithms.

In Chapter 1, the basic theorems of mathematical analysis are given that are necessary for the statement of the contents of the later chapters, as well as the main symbols, properties of series asymptotic in the sense of Poincare's definition, and the main points of the classical Lyapunov-Poincare technique for differential equations with small parameter.

Chapter 2 describes the main presently known mathematical results of proving the applicability of asymptotic theory to multifrequency systems of ordinary differential equations typical for modern resonance dynamics. These are equations determined on tori or toroidal manifolds, with their right–hand members being multiperiodic functions with respect to fast angular variables, and the frequencies being functions of slow phase variables. Sufficiently well-investigated equations include those of Van der PoI, Mathieu, Duffing, and Hill, and one might say, simple cases of multifrequency systems defined on tori. For these equations, the phase space has dimension 2, and though the subspace of fast variables (torus) has dimension 1, here also occurs the problem of small denominators, like the Poincare problem of reflection of a circumference on itself. Generally speaking, this is sufficient to understand the difficulty of the mathematical problems of the construction of exact, but not asymptotic solutions of equations defined on many-dimensional tori.

Chapter 3 is devoted to some resonance problems of nonlinear mechanics. The previous chapter described the mathematical aspects of nonlinear differential equation theory, based on the averaging principles. The authors note that the main mathematical difficulties of this theory arise from the possible appearance of small denominators in asymptotic formulas. Problems with small denominators (or problems with frequency resonances, which is the same) are not particularly abstract, but reflect a real picture of the dynamic processes taking place in the micro- and macrocosm. In this chapter, the authors discuss some of the resonant problems of analytic dynamics that have some theoretical and practical implications.

The authors note that Chapter 4 does not claim to be a complete and consistent development of algorithms and their numerical-analytical implementation for problems of analytical resonant dynamics described by nonlinear differential equations. This is a kind of introduction to this development of events. But in any case, examples such as the construction of the Lyapunov and Krylov – Bogolyubov transformations, the construction of Hill solutions and Mathieu functions prove the high efficiency of numerical-analytical methods and the great possibility of implementing the corresponding algorithms by these methods in comparison with purely analytical techniques. The described variant of numerical-analytical methods, which mainly provides for the implementation of algorithms by operations on the corresponding structures with numerical (but not literal) coefficients, in some cases leads to fairly simple calculation schemes that require modest computer resources. (e.g. Hill solutions and Mathieu functions). Of course, in many problems, for example, in the three-body problem described above, to implement the algorithms, computers with a sufficiently large memory and operating speed are required. This is mainly due to the fact that the authors propose to focus on iterations with quadratic convergence. It is this computational process that allows, to some extent, to overcome the problem of small denominators, which is typical for problems of resonant analytical dynamics. It is also noted that for the further development of the proposed numerical-analytical methods, modern packages of computer analytical calculations, such as MAPLE, MATHEMATICA 3.0 and other similar programs, should be used.

The bibliography consists of 243 – 252 pages and 124 references.

22. Dynamical Systems and Control [22].

The 11th International Workshop on Dynamics and Control brought together scientists and engineers from diverse fields and gave them a venue to develop a greater understanding of this discipline and how it relates to many areas in science, engineering, economics, and biology. The event gave researchers an opportunity to investigate ideas and techniques from outside their own fields of expertise, enabling a cross-pollination of dynamics and control perspectives. Now there is a book that documents the major presentations of the workshop, providing a foundation for further research.

The range and diversity of papers in Dynamical Systems and Control demonstrate the remarkable reach of the subject. All of these contributed papers shed light on a multiplicity of physical, biological, and economic phenomena through lines of reasoning that originate and grow from this discipline.

The editors divide the book into three parts. The first covers fundamental advances in dynamics, dynamical systems, and control. These papers represent ideas that can be applied to several areas of interest. The second part deals with new and innovative techniques and their application to a variety of interesting problems, from the control of cars and robots, to the dynamics of ships and suspension bridges, and the determination of optimal spacecraft trajectories. The third section relates to social, economic, and biological issues. It reveals the wealth of understanding that can be obtained through a dynamics and control approach to issues such as epidemics, economic games, neo-cortical synchronization, and human posture control.

This volume consists of 24 papers contributed by 46 experts from 9 countries, and addresses a wide range of problems of modern nonlinear dynamics.

Part I Advances in dynamics, dynamical systems, and control. Luiz Bevilacqua, A geometric approach to the mechanics of densely folded media (3 – 20); Firdaus E. Udwadia, On a general principle of mechanics and its application to general non-ideal nonholonomic constraints (21 – 32); Marianna A. Shubov, Mathematical analysis of vibrations of nonhomogeneous filament with one end load (33 – 52); Henryk Flashner and Michael Golat, Expanded point mapping analysis of periodic systems (53 – 76); Debora Belato, Hans Ingo Weber and Jose Manoel Balthazar, A preliminary analysis of the phase portrait's structure of a nonlinear pendulum–mechanical system using the perturbed Hamiltonian formulation (77 – 90); Edson Cataldo and Rubens Sampaio, A review of rigid–body collision models in the plane (91 – 105).

Part II New and innovative techniques and their application. A. Miele, T. Wang and S. Mancuso, Optimal round-trip Earth-Mars trajectories for robotic flight and manned flight (109 – 126); N. Seube, R. Moitie and G. Leitmann, Aircraft take-off in windshear: a viability approach (127 – 144); N. U. Ahmed, Stability of torsional and vertical motion of suspension bridges subject to stochastic wind forces (145 – 162); Firdaus E. Udwadia, Hubertus F. von Bremen, Ravi Kumar and Mohamed Flosseini, Time delayed control of structural systems (163 – 206); Eduard Reithmeier, Robust real - and discrete-time control of a steer-by-wire system in cars (207 – 220); Vicente Lopes, Jr., Valder Steffen, Jr. and Daniel J. Inman, Optimal placement of piezoelectric sensor/ actuators for smart structures vibration control (221 – 236); J.M. Balthazar, R.M.L.R.F Brasil, H.I.Weber, A. Fenili, D. Belato, J.L.P. Felix and F.J. Garzelli, A review of new vibration issues due to non-ideal energy sources (237 – 258); Jose Joao de Espindola and Joao Morais da Silva Neto, Identification of flexural stiffness parameters of beams (259 – 269); Seyyed Said Dana, Naor Moraes Melo and Simplicio Arnaud da Silva, Active noise control caused by airflow through a rectangular duct (271 – 282); Helio Mitio Morishita and Jesse Rebello de Souza Junior, Dynamical features of an autonomous two-body floating system (283 – 297); Agenor de Toledo Fleury and Frederico Ricardo Ferreira de Oliveira, Dynamics and control of a flexible rotating arm through the movement of a sliding mass (299 – 317); Humberto Piccoli and Fernando Kokubun, Measuring chaos in gravitational waves (319 – 333).

Part III Social, economic, and biological issues. E. Cruck, N. Seube and G. Leitmann, Estimation of the attractor for an uncertain epidemic model (337 – 349); Ye-Hwa Chen, Liar paradox viewed by the fuzzy logic theory (351 – 361); Christophe Deissenberg and Francisco Alvarez Gonzalez, Pareto-improving cheating in an economic policy game (363 – 377); Gustav Feichtinger, Richard F. Hartl, Peter Kort and Vladimir Veliov, Dynamic investment behavior taking into account ageing of the capital goods (379 – 391); R. Stoop and D. Blank, A mathematical approach towards the issue of synchronization in neocortical neural networks (393 – 406); Luciano Luporini Menegaldo, Agenor de Toledo Fleury and Hans Ingo Weber, Optimal control of human posture using algorithms based on consistent approximations theory (407 – 429).

23. History reference.

The publication of this series was preceded by the creation in 1991 at the Academy of Sciences of Ukraine of the International Series of Scientific Monographs "Stability: Theory, Methods and Applications". Financial difficulties experienced at that time by the Academy of Sciences of Ukraine did not allow to implement this project, except for one volume.** And then, in the course of my visit to the Technological Institute in Florida (USA) in 1991, after discussion of the issue with Professor V.Lakshmikantham, we decided to offer this idea to one of the foreign publishers. This idea was adopted at the Gordon and Breach Science Publishers (UK) and the first volume of the series was published in 1995.

Volumes 1 – 12 were published by the Gordon and Breach Science Publishers, volumes 13 – 20 were published by the Taylor and Francis, and volumes 21 – 22 were published by the Chapman and Hall/CRC, which absorbed the two previous publishers.

All 22 volumes of this Series of Scientific Monographs have been abstracted in the abstract journals of Zentralblatt MATH and Mathematical Review.

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РЕЗЮМЕ. У статті наводиться короткий огляд серії наукових монографій: “Стійкість та керування: Теорія, методи та застосування”, виданих у 1995-2002 роках у видавництвах Гордон і Бріч (Велика Британія) та Тейлор і Френсіс (США). Дана серія складається з 22 томів і носить енциклопедичний характер у галузі теорій стійкості та керування. Крім того, в огляді наведено історичну довідку про виникнення ідеї видання даної серії та її реалізацію.

КЛЮЧОВІ СЛОВА: стійкість; керування; оптимізація; нелінійна динаміка; якісні, аналітичні та асимптотичні методи; в'язкопружні системи; динаміка популяцій; математична економіка; резонансна механіка.

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