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DYNAMIC MATHEMATICAL MODEL OF MODIFIED COUPLE STRESS THERMOELASTIC DIFFUSION WITH PHASE-LAG

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Abstract. Analysis of non-local, phase-lag, and temperature-dependent properties on modified couple stress thermoelastic diffusive medium is examined in conditions of exciting by thermomechanical sources. The governing equations are framed involving non-local, phase-lag, and temperature-dependent properties. These equations are simplified by using the potential functions and employing the Laplace and Fourier transforms for further study. The problem is solved by deploying suitable thermomechanical loads. A specific type of normal and thermal loading of the ramp-type is considered. The transformed components of the physical field like the displacements, stresses, temperature change, and chemical potential are derived. A numerical analysis is performed for these quantities using the numerical technique. The graphs of the resulting quantities are shown to analyze the impact of non-local, phase-lag, and temperature-dependent properties. The specific cases are also mentioned.

Key words: modified couple stress thermoelasticity, non-local, dual-phase-lag, Laplace and Fourier transforms, normal load, thermal source, ramp-type, variable properties.

1. Introduction.

The non-local theory of elasticity is based on the fact that the stress state at a point is a function of strain of all the points in the body, whereas the classical (local) elasticity describes the stress state at a given point by strain state at the same point. Non-local theory is derived by various researchers by adopting different assumptions, e.g. Eringen and Edelen [1], Edelen and Law [2], Eringen [3-8], McCay and Narsimhan [9], Narsimhan and McCay [10]. A comprehensive work on development of non-local theory is given by book of Eringen [11].

The non-local response and lagging response are same since earlier in space and as later in time. Tzou [12] combined the response of non-local with single phase-lag heat conduction and compared it with the model given by Cao & Guo [13] and Guo & Hou [14]. Tzou & Guo [15] demonstrate the union of non-local response with the dual-phase-lag model proposed by Tzou [16, 17] and which is known as the new theory comprising both effects.

Sharma [18] studied boundary value problems in generalized thermodiffusive elastic medium. Sharma and Sharma [19] studied influence of heat sources and relaxation time on temperature distribution in tissues. Sharma et al. [20] examined wave characteristics and some basic theorems in an electro-microstretch elastic solid. Yu et al. [21] analyzed the buckling of nanobeams under non uniform temperature based on non-local thermoelasticity. Abouelregal [22] showed the effect of thermal properties on non-local nanobeams.

Kumar et al. [23] investigated transient analysis of non-local microstretch thermoelastic thick circular plate with phase-lags. Danni Li and Tian-Hu He [24] studied the effect of non-local and temperature dependent properties in generalised piezoelectric thermoelastic problem. Kumar et al. [25] studied the effect of thermal and chemical potential sources in a thin beam in MCT with three-phase-lag thermoelastic diffusion model. Jin-Tao Ma and Tian-Hu He [26] explored response of a generalised thermoelastic problem with non-local effect and variable properties. Borjalilou et al. [27] presented an explicit relation for thermoelastic damping in non-local nanobeams considering dual-phase-lag effect. Ezzat [28] studied the effect of mechanical and thermal properties on tumorous tissue. Zenkour and Kutbi [29] investigated thermoelastic interactions due to continuous heat source in a hollow cylinder.

The novelty of the present study is that the effects of temperature, non-local and phase-lag are examined due to thermomechanical sources in MCT diffusion. Integral transform technique is applied to investigate the problem. Ramp-type normal and thermal loads are taken to show the approach. Resulting quantities are depicted graphically to show non-local and phase-lag effects. Such theory can describe the long range interactions between points in a continuum model. Such long range interactions occur between charged atoms or molecules in a solid. Long range forces may also be considered to propagate along fibers or laminae in a composite material.

2. Basic Equations.

Following Tzou, Guo [15], Sherief et al. [30] and Yu et al. [31], we have

(i) Constitutive Relations

$$t_{ij} = 2\mu e_{ij} - \frac{1}{2} e_{kij} m_{lk,l} + \delta_{ij}[\lambda_0 e_{kk} - \gamma_1 T - \gamma_2 P]; \tag{1}$$

$$m_{ij} = 2\alpha\chi_{ij}; \tag{2}$$

(ii) Equation of motion

$$\left(\lambda_0 + \mu + \frac{\alpha}{4}\Delta\right)\nabla(\nabla\cdot\vec{u}) + \left(\mu - \frac{\alpha}{4}\Delta\right)(\Delta\vec{u}) - \gamma_1\nabla T - \gamma_2\nabla P = \rho(1-\xi^2\Delta)\frac{\partial^2\vec{u}}{\partial t^2}; \tag{3}$$

(iii) Equation of heat conduction

$$\left(1 - \zeta^2\Delta + \tau_q\frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2}\right)(\gamma_1 T_0 \dot{e} + l_1 T_0 \dot{T} + T_0 d \dot{P}) = K\left(1 + \tau_t\frac{\partial}{\partial t}\right)\Delta T; \tag{4}$$

(iv) Equation of mass diffusion

$$\left(1 - \varsigma^2\Delta + \tau_u\frac{\partial}{\partial t} + \frac{1}{2}\tau_u^2\frac{\partial^2}{\partial t^2}\right)(\gamma_2 \dot{e} + d\dot{T} + n\dot{P}) = D\left(1 + \tau_p\frac{\partial}{\partial t}\right)\Delta P. \tag{5}$$

Here

$$\chi_{ij} = \frac{1}{2}(\omega_{i,j} + \omega_{j,i});\ \ \omega_i = \frac{1}{2}e_{ipq}u_{q,p},$$

where

$$\lambda_0 = \lambda - \frac{\beta_2^2}{b};\ \ \gamma_1 = \beta_1 + \frac{a}{b}\beta_2;\ \ \gamma_2 = \frac{\beta_2}{b};\ \ l_1 = \frac{\rho C_e}{T_o} + \frac{a^2}{b};\ \ d = \frac{a}{b};\ \ n = \frac{1}{b}.$$

In Eqs. (1) – (5), ξ, ζ, ς – non-local parameters, τ_q, τ_t – thermal relaxation times with $\tau_q, \tau_t \ge 0$ and τ_u, τ_p – diffusion relaxation times with $\tau_u, \tau_p \ge 0$, $\beta_1 = (3\lambda + 2\mu)\alpha_t$, $\beta_2 = (3\lambda + 2\mu)\alpha_c$. Here α_t, α_c correspond to the coefficient of linear thermal expansion and diffusion expansion respectively. Δ is the Laplacian operator, ∇ is nabla (gradient) operator. Other symbols have their usual meanings.

3. Statement of the Problem.

A homogeneous isotropic non-local MCT diffusive body with dual-phase-lag occupying the region $x_3 \ge 0$ is taken. Let the origin be on $x_3 = 0$ of plane Cartesian coordinate system (x_1, x_2, x_3). The half space is induced by ramp-type normal and thermal loading on the bounding plane $x_3 = 0$.

For two dimensional problems, we have

$$\vec{u} = (u_1(x_1, x_3, t), 0, u_3(x_1, x_3, t));\ \ T(x_1, x_3, t);\ \ P(x_1, x_3, t). \tag{6}$$

To examine the influence of temperature dependent material properties we consider

$$\lambda_0 = \lambda_{01} f(T);\ \mu = \mu_0 f(T);\ \gamma_1 = \gamma_{10} f(T);\ \gamma_2 = \gamma_{20} f(T);$$
$$K = K_0 f(T);\ D = D_0 f(T);\ f(T) = (1 - \alpha^* T_0), \tag{7}$$

where $\lambda_{01}, \mu_0, \gamma_{10}, \gamma_{20}, K_0, D_0$ are considered constants and α^* is an empirical material constant. For temperature independent material properties $f(T) = 1$.

Using (6), (7) in (3) – (5), recast the following equations

$$f(T)(\lambda_{01} + \mu_0)\frac{\partial e}{\partial x_1} + f(T)\mu_0 \Delta u_1 + \frac{\alpha}{4}\Delta\left(\frac{\partial e}{\partial x_1} - \Delta u_1\right) - \gamma_{10} f(T)\frac{\partial T}{\partial x_1} -$$
$$-\gamma_{20} f(T)\frac{\partial P}{\partial x_1} = \rho(1 - \xi^2 \Delta)\frac{\partial^2 u_1}{\partial t^2}; \tag{8}$$

$$f(T)(\lambda_{01} + \mu_0)\frac{\partial e}{\partial x_3} + f(T)\mu_0 \Delta u_3 + \frac{\alpha}{4}\Delta\left(\frac{\partial e}{\partial x_3} - \Delta u_3\right) - \gamma_{10} f(T)\frac{\partial T}{\partial x_3} -$$
$$-\gamma_{20} f(T)\frac{\partial P}{\partial x_3} = \rho(1 - \xi^2 \Delta)\frac{\partial^2 u_3}{\partial t^2}; \tag{9}$$

$$\left(1 - \zeta^2 \Delta + \tau_q \frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2 \frac{\partial^2}{\partial t^2}\right)\left(\gamma_{10} f(T) T_o \dot{e} + l_1 T_o \dot{T} + T_o d \dot{P}\right) = K_0 f(T)\left(1 + \tau_t \frac{\partial}{\partial t}\right)\Delta T; \tag{10}$$

$$\left(1 - \varsigma^2 \Delta + \tau_u \frac{\partial}{\partial t} + \frac{1}{2}\tau_u^2 \frac{\partial^2}{\partial t^2}\right)\left(\gamma_{20} f(T)\dot{e} + d \dot{T} + n \dot{P}\right) = D_0 f(T)\left(1 + \tau_p \frac{\partial}{\partial t}\right)\Delta P. \tag{11}$$

Following dimensionless quantities are used:

$$(\xi', \zeta', \varsigma', x', u_i') = \frac{\omega^*}{c_1}(\xi, \zeta, \varsigma, x_i, u_i);\ (t', \tau_t', \tau_q', \tau_u', \tau_p') = \omega^*(t, \tau_t, \tau_q, \tau_u, \tau_p);$$
$$t_{ij}' = \frac{1}{\gamma_{10} T_o} t_{ij};\ m_{ij}' = \frac{\omega^*}{\gamma_{10} T_o c_1} m_{ij};\ T' = \frac{\beta_1}{\rho c_1^2} T;\ P' = \frac{1}{\beta_2} P, \tag{12}$$

where

$$\omega^* = \frac{\rho C_e c_1^2}{K};\ c_1^2 = \frac{\lambda_{01} + 2\mu_0}{\rho}.$$

ω^* is the characteristic frequency and c_1 is the longitudinal wave velocity in the media.

Equations (8) – (11) with the aid of (12), reduce to the following equations after suppressing the primes

$$\frac{(\lambda_{01} + \mu_0)}{\rho c_1^2} f(T)\frac{\partial e}{\partial x_1} + \frac{\mu_0}{\rho c_1^2} f(T)\Delta u_1 + \frac{\alpha}{4}\frac{\omega^{*2}}{\rho c_1^4}\Delta\left(\frac{\partial e}{\partial x_1} - \Delta u_1\right) - f(T)\frac{\gamma_{10}}{\beta_1}\frac{\partial T}{\partial x_1} -$$
$$-\gamma_{20} f(T)\frac{\beta_2}{\rho c_1^2}\frac{\partial P}{\partial x_1} = (1 - \xi^2 \Delta)\frac{\partial^2 u_1}{\partial t^2}; \tag{13}$$

$$\frac{(\lambda_{01} + \mu_0)}{\rho c_1^2} f(T)\frac{\partial e}{\partial x_3} + \frac{\mu_0}{\rho c_1^2} f(T)\Delta u_3 + \frac{\alpha}{4}\frac{\omega^{*2}}{\rho c_1^4}\Delta\left(\frac{\partial e}{\partial x_3} - \Delta u_3\right) - f(T)\frac{\gamma_{10}}{\beta_1}\frac{\partial T}{\partial x_3} -$$
$$-\gamma_{20} f(T)\frac{\beta_2}{\rho c_1^2}\frac{\partial P}{\partial x_3} = (1 - \xi^2 \Delta)\frac{\partial^2 u_3}{\partial t^2}; \tag{14}$$

$$\left(1-\zeta^2\Delta+\tau_q\frac{\partial}{\partial t}+\frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2}\right)\left(\frac{\beta_1\gamma_{10}}{K_0\rho\omega^*}T_0\overset{\bullet}{e}+\frac{l_1c_1^2}{K_0\omega^*f(T)}T_0\overset{\bullet}{T}+\right.$$
$$\left.+\frac{\beta_2\beta_1}{K_0\rho\omega^*f(T)}T_0d\overset{\bullet}{P}\right)=\left(1+\tau_t\frac{\partial}{\partial t}\right)\Delta T; \tag{15}$$

$$\left(1-\varsigma^2\Delta+\tau_u\frac{\partial}{\partial t}+\frac{1}{2}\tau_u^2\frac{\partial^2}{\partial t^2}\right)\left(\frac{c_1^2\gamma_{20}}{D_0\beta_2\omega^*}\overset{\bullet}{e}+\frac{c_1^4d\rho}{D_0\omega^*\beta_1\beta_2f(T)}\overset{\bullet}{T}+\right.$$
$$\left.+\frac{nc_1^2}{D_0\omega^*f(T)}\overset{\bullet}{P}\right)=\left(1+\tau_p\frac{\partial}{\partial t}\right)\Delta P, \tag{16}$$

where

$$\Delta=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2};\; e=\frac{\partial u_1}{\partial x_1}+\frac{\partial u_3}{\partial x_3}.$$

4. Solution Procedure.

The displacement components $u_1(x_1, x_3, t)$ and $u_3(x_1, x_3, t)$ relate to scalar potentials $\varphi(x_1, x_3, t)$ and $\psi(x_1, x_3, t)$ in dimensionless form as

$$u_1=\frac{\partial\varphi}{\partial x_1}-\frac{\partial\psi}{\partial x_3};\; u_3=\frac{\partial\varphi}{\partial x_3}+\frac{\partial\psi}{\partial x_1}. \tag{17}$$

With the aid of (17), Eqs. (13) – (16) yield

$$(a_1+a_2)f(T)\Delta\varphi-a_4f(T)T-a_5f(T)P-(1-\xi^2\Delta)\frac{\partial^2\varphi}{\partial t^2}=0; \tag{18}$$

$$(a_2f(T)-a_3\Delta)\Delta\psi-(1-\xi^2\Delta)\frac{\partial^2\psi}{\partial t^2}=0; \tag{19}$$

$$\left(1-\zeta^2\Delta+\tau_q\frac{\partial}{\partial t}+\frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2}\right)\left(a_6f(T)\Delta\overset{\bullet}{\varphi}+a_7\overset{\bullet}{T}+a_8\overset{\bullet}{P}\right)=f(T)\left(1+\tau_t\frac{\partial}{\partial t}\right)\Delta T; \tag{20}$$

$$\left(1-\varsigma^2\Delta+\tau_u\frac{\partial}{\partial t}+\frac{1}{2}\tau_u^2\frac{\partial^2}{\partial t^2}\right)\left(a_9f(T)\Delta\overset{\bullet}{\varphi}+a_{10}\overset{\bullet}{T}+a_{11}\overset{\bullet}{P}\right)=f(T)\left(1+\tau_p\frac{\partial}{\partial t}\right)\Delta P, \tag{21}$$

where

$$a_1=\frac{(\lambda_{01}+\mu_0)}{\rho c_1^2};\; a_2=\frac{\mu_0}{\rho c_1^2};\; a_3=\frac{\alpha}{4}\frac{\omega^{*^2}}{\rho c_1^4};\; a_4=\frac{\gamma_{10}}{\beta_1};\; a_5=\frac{\gamma_{20}\mu_2}{\rho c_1^2};\; a_6=\frac{\beta_1\gamma_{10}T_0}{K_0\rho\omega^*};$$

$$a_7=\frac{l_1c_1^2T_0}{K_0\omega^*};\; a_8=\frac{\beta_2\beta_1T_0d}{K_0\rho\omega^*};\; a_9=\frac{c_1^2\gamma_{20}}{D_0\beta_2\omega^*};\; a_{10}=\frac{c_1^4d\rho}{D_0\omega*\beta_1\beta_2};\; a_{11}=\frac{nc_1^2}{D_0\omega^*}.$$

We define Laplace and Fourier transforms as

$$\overline{f}(x_1, x_3, s)=\int_0^\infty f(x_1, x_3, t)e^{-st}dt;\quad \widehat{f}(\xi_1, x_3, s)=\int_{-\infty}^\infty \overline{f}(x_1, x_3, s)e^{i\xi_1x_1}dx_1. \tag{22}$$

Using (22) on (18) – (21), determines following after simplification

$$(R_1 D_1^6 + R_2 D_1^4 + R_3 D_1^2 + R_4)(\hat{\varphi}, \hat{T}, \hat{P}) = 0\,; \tag{23}$$

$$(D_1^4 + R_5 D_1^2 + R_6)\hat{\psi} = 0, \tag{24}$$

where

$$R_1 = R_{11}R_{13}R_{15} - a_8 a_{10} s^2 R_{11}\zeta^2\varsigma^2 - a_4 a_6 s f^2(T) R_{15}\zeta^2 + a_4 a_8 a_9 s^2 f^2(T)\zeta^2\varsigma^2 -$$
$$- a_5 a_6 a_{10} s^2 f(T)\zeta^2\varsigma^2 + a_5 a_9 s f(T) R_{13}\varsigma^2;$$

$$R_2 = R_{01} - 3\xi_1^2 R_1;\ R_3 = 3\xi_1^4 R_1 - 2\xi_1^2 R_{01} + R_{02};\ R_4 = R_{01}\xi_1^4 - R_1\xi_1^6 - R_{02}\xi_1^2 - R_{03};$$

$$R_5 = -\frac{a_2 + \xi^2 s^2 + 2\xi_1^2 a_3}{a_3};\ R_6 = \frac{a_3\xi_1^4 + a_2 f(T)\xi_1^2 + s^2 + \xi_1^2\xi^2 s^2}{a_3};$$

$$R_{01} = -R_{11}R_{22}R_{15} - R_{11}R_{13}R_{26} - R_{13}R_{15}s^2 + a_8 s R_{11}R_{25}\zeta^2 + a_{10} s R_{23}R_{11}\varsigma^2 + a_8 a_{10}\zeta^2\varsigma^2 s^4 +$$
$$+ a_4 a_6 s f(T) R_{26}\zeta^2 + a_4 f(T) R_{21}R_{15} - a_4 a_8 s f(T) R_{24}\zeta^2 - a_4 a_9 s f(T) R_{23}\varsigma^2 + a_5 a_6 s f^2(T) R_{24}\zeta^2 +$$
$$+ a_5 a_{10} s f(T) R_{21}\varsigma^2 - R_{13}R_{24}a_5 f(T) - a_5 a_9 s f^2(T) R_{22}\varsigma^2;$$

$$R_{02} = R_{11}R_{22}R_{26} + R_{13}R_{26}s^2 + R_{22}R_{15}s^2 - R_{11}R_{23}R_{25} - a_8 R_{25}\zeta^2 s^3 - a_{10}R_{23}\varsigma^2 s^3 - a_4 f(T) +$$
$$+ R_{21}R_{26} + a_4 f(T) R_{23}R_{24} - R_{21}R_{25}a_5 f(T) + R_{22}R_{24}a_5 f(T);\ R_{03} = -s^2 R_{26}R_{22} + s^2 R_{23}R_{25};$$

$$R_{11} = a_1 f(T) + a_2 f(T) + \xi^2 s^2;\ R_{12} = 1 + s\tau_q + \frac{s^2}{2}\tau_q^2;\ R_{13} = a_7 s\zeta^2 + f(T) + f(T)s\tau_t;$$

$$R_{14} = 1 + s\tau_u + \frac{s^2}{2}\tau_u^2;\ R_{15} = f(T) + f(T)s\tau_p + s\zeta^2 a_{11};\ R_{21} = a_6 s f(T) R_{12};$$

$$R_{22} = a_7 s R_{12};\ R_{23} = a_8 s R_{12};\ R_{24} = a_9 s f(T) R_{14};\ R_{25} = a_{10} s R_{14};\ R_{26} = a_{11} s R_{14}.$$

The bounded solution of equations (23), (24) are

$$(\hat{\varphi}, \hat{T}, \hat{P})(x_3, \xi_1, s) = \sum_{i=1}^{3}(1, R_i^*, S_i^*) A e^{-m_i x_3}; \tag{25}$$

$$\hat{\psi}(x_3, \xi_1, s) = \sum_{i=4}^{5} A_i e^{-m_i x_3}. \tag{26}$$

Here $m_i (i = 1, 2, \ldots, 5)$ are the roots of Eq. (23), Eq. (24) and the coupling constants are given by

$$R_i^* = \frac{\begin{array}{c}(m_i^2 - \xi_1^2)^3(a_8 a_9 s^2 f(T)\zeta^2\varsigma^2 - a_6 s f(T) R_{15}\zeta^2) + (m_i^2 - \xi_1^2)^2(a_6 s f(T) R_{26}\zeta^2 + \\ + R_{21}R_{15} - a_8 s R_{24}\zeta^2 - a_{10} s R_{21}\varsigma^2) + (m_i^2 - \xi_1^2)(R_{21}R_{25} - R_{23}R_{24})\end{array}}{\begin{array}{c}(m_i^2 - \xi_1^2)^2(R_{13}R_{15} - a_8 a_{10} s^2\zeta^2\varsigma^2) + (m_i^2 - \xi_1^2)(-R_{22}R_{15} - R_{13}R_{26} + \\ + a_8 s R_{25}\zeta^2 + a_{10} s R_{23}\varsigma^2) + (R_{22}R_{26} - R_{23}R_{25})\end{array}};$$

$$S_i^* = \frac{\begin{array}{c}(m_i^2 - \xi_1^2)^3(a_5 a_9 s^2\zeta^2\varsigma^2 - a_9 s f(T) R_{13}\varsigma^2) + (m_i^2 - \xi_1^2)^2(a_9 s f(T) R_{22}\varsigma^2 + R_{13}R_{24} - \\ - a_6 s f(T) R_{25}\zeta^2 - a_{10} s R_{21}\varsigma^2) + (m_i^2 - \xi_1^2)(R_{21}R_{25} - R_{22}R_{24})\end{array}}{\begin{array}{c}(m_i^2 - \xi_1^2)^2(R_{13}R_{15} - a_8 a_{10} s^2\zeta^2\varsigma^2) + (m_i^2 - \xi_1^2)(-R_{22}R_{15} - R_{13}R_{26} + \\ + a_8 s R_{25}\zeta^2 + a_{10} s R_{23}\varsigma^2) + (R_{22}R_{26} - R_{23}R_{25})\end{array}}$$

$$(i = 1, 2, 3).$$

5. Thermomechanical Conditions.

Considering ramp-type normal loading and thermal source along with disappearing of tangential stress, tangential couple stress and chemical potential i. e

$$t_{33} = -F_1(t)\delta(x_1);\ t_{31} = 0;\ m_{32} = 0;\ T = F_2(t)\delta(x_1);\quad P = 0, \tag{27}$$

where

$$[F_1(t),\ F_2(t)] = (F_0,\ F_{01})\begin{cases} 0 & t \le 0; \\ t/t_0 & 0 < t \le t_0; \\ 1 & t > t_0, \end{cases} \tag{28}$$

where F_0 and F_{01} govern the magnitude of the force and stationary temperature and $\delta()$ denotes Dirac delta function and t_0 is ramp-type parameter.

Non dimensional stress components are given by

$$t_{33} = 2r_1\left(\frac{\partial u_3}{\partial x_3}\right) + r_2\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) - r_3T - r_4P; \tag{29}$$

$$t_{31} = r_1\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) - r_5\Delta\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right); \tag{30}$$

$$m_{32} = 2r_5\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right). \tag{31}$$

Using (22) on (27), (28) yield

$$\hat{t}_{33} = -\hat{F}_1(s);\ \hat{t}_{31} = 0;\ \hat{m}_{32} = 0;\ \hat{T} = \hat{F}_2(s);\ \hat{P} = 0;$$

$$\hat{F}_1(s) = \frac{F_0\left(1 - \exp(-st_0)\right)}{t_0 s^2};\ \hat{F}_2(s) = \frac{F_{01}\left(1 - \exp(-st_0)\right)}{t_0 s^2}. \tag{32}$$

Making use of (25), (26) in (32) along with (17), (22) and (29) – (31) givess

$$\hat{u}_1 = -\frac{i\xi_1}{\Delta}\sum_{i=1}^{3}\Delta_i \exp(-m_i x_3) + \frac{1}{\Delta}\sum_{i=4}^{5} m_i\Delta_i \exp(-m_i x_3); \tag{33}$$

$$\hat{u}_3 = -\frac{1}{\Delta}\sum_{i=1}^{3} m_i\Delta_i \exp(-m_i x_3) - \frac{i\xi_1}{\Delta}\sum_{i=4}^{5}\Delta_i \exp(-m_i x_3); \tag{34}$$

$$\hat{t}_{33} = \frac{1}{\Delta}\sum_{i=1}^{5} a_{1i}\Delta_i \exp(-m_i x_3); \tag{35}$$

$$\hat{t}_{31} = \frac{1}{\Delta}\sum_{i=1}^{5} a_{2i}\Delta_i \exp(-m_i x_3); \tag{36}$$

$$\hat{m}_{32} = \frac{1}{\Delta}\sum_{i=1}^{5} a_{3i}\Delta_i \exp(-m_i x_3); \tag{37}$$

$$\hat{T} = \frac{1}{\Delta}\sum_{i=1}^{3} R_i^*\Delta_i \exp(-m_i x_3); \tag{38}$$

$$\hat{P} = \frac{1}{\Delta}\sum_{i=1}^{3} S_i^*\Delta_i \exp(-m_i x_3). \tag{39}$$

Here

$$\Delta=(S_2^*R_1^*-S_1^*R_2^*)n_1+(S_1^*R_3^*-S_3^*R_1^*)n_2+(S_3^*R_2^*-S_2^*R_3^*)n_3\,;$$

$$n_1=a_{13}a_{24}a_{35}-a_{13}a_{34}a_{25}-a_{14}a_{23}a_{35}+a_{14}a_{33}a_{25}+a_{15}a_{23}a_{34}-a_{15}a_{33}a_{24};$$

$$n_2=a_{12}a_{24}a_{35}-a_{12}a_{34}a_{25}-a_{14}a_{22}a_{35}+a_{14}a_{32}a_{25}+a_{15}a_{22}a_{34}-a_{15}a_{32}a_{24};$$

$$n_3=a_{11}a_{24}a_{35}-a_{11}a_{32}a_{25}-a_{14}a_{21}a_{35}+a_{14}a_{31}a_{25}+a_{15}a_{21}a_{34}-a_{15}a_{31}a_{24};$$

$$a_{1i}=(2r_1+r_2)m_i^2-\xi_1^2r_2-r_3R_i^*-r_4S_i^*;\ a_{1j}=2i\xi_1r_1m_j;\ a_{2i}=2i\xi_1(-r_5m_i^3+r_1m_i+r_5\xi_1^2m_i);$$

$$a_{2j}=r_5m_j^4-r_1m_j^2-r_1\xi_1^2-r_5\xi_1^4;\ a_{3i}=4i\xi_1r_5m_i;\ a_{3j}=-2r_5(m_j^2+\xi_1^2);\ r_1=\frac{\mu_0f(T)}{\gamma_{10}T_0};$$

$$r_2=\frac{\lambda_of(T)}{\gamma_{10}T_0};\ r_3=\frac{\rho c_1^2}{\beta_1T_0};\ r_4=\frac{\gamma_{20}\beta_2f(T)}{\gamma_{10}T_0};\ r_5=\frac{\alpha}{4}\frac{\omega^{*^2}}{\gamma_{10}T_0c_1^2}\quad(i=1,\,2,\,3,\;\;j=4,\,5).$$

Putting $[-\widehat{F}_1(s),\,0,\,0,\,\widehat{F}_2(s),\,0)]'$ in i^{th} column of Δ respectively determine $\Delta_i\,(i=1,2,\ldots5)$.

6. Validation and Special cases.

(i) For ramp-type normal load $F_2=0$ in eqs. (33) – (39) yield the corresponding quantities.

(ii) For ramp-type thermal source $F_1=0$ in eqs. (33) – (39) determine the quantities.

Special cases.

By taking $\xi,\,\zeta,\,\varsigma=0$, we obtain the corresponding expressions given by (33) – (39) in absence of non-local parameters.

By taking $\tau_t,\,\tau_q,\,\tau_u,\,\tau_p=0$, we obtain the corresponding expressions given by (33) – (39) without dual-phase-lag.

7. Inversion of the Transformation.

To obtain the results in physical domain, invert the transforms in (33) – (39) by inverting the Fourier transform using

$$\overline{f}(x_{1,}x_3,s)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{f}(\xi_1,\,x_3,\,s)e^{i\xi_1x_1}d\xi_1=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\cos(\xi_1x)f_e-i\sin(\xi_1x)f_0\right|d\xi_1,\qquad(40)$$

where f_e and f_0 are respectively the odd and even parts of $\overline{f}(\xi_1,\,x_3,\,s)$. Following [32], the Laplace transform function $\hat{f}(\xi_1,\,x_3,\,s)$ can be inverted to $f(x_1,\,x_3,\,t)$ by

$$f(x_1,\,x_3,\,t)=\frac{1}{2\pi i}\int_{\nu-i\infty}^{\nu+i\infty}\hat{f}(\xi_1,\,x_3,\,s)\;e^{-st}ds.$$

The last step is to calculate the integral in eq. (40). The method for evaluating this integral is described by [33]. It involves the use of Romberg's integration with adaptive step size.

8. Numerical Implementation and Discussion.

For this section we follow [34] to consider the copper material (thermoelastic diffusion solid) as:

$$\lambda=7{,}76\times10^{10}\,\text{Kgm}^{-1}\text{s}^{-2};\ \mu=3{,}86\times10^{10}\,\text{Kgm}^{-1}\text{s}^{-2};\ T_0=0{,}293\times10^3\,\text{K};$$

$$C_e=0{,}3891\times10^3\,\text{JKg}^{-1}\text{K}^{-1};\ \alpha_t=1{,}78\times10^{-5}\,\text{K}^{-1};\ \alpha_c=1{,}98\times10^{-4}\,\text{m}^3\text{Kg}^{-1};$$

$$a=1{,}02\times10^4\,\text{m}^2\text{s}^{-2}\text{K}^{-1};\ b=9\times10^5\,\text{Kg}^{-1}\text{m}^5\text{s}^{-2};\ D=0{,}85\times10^{-8}\,\text{Kgm}^{-3}\text{s};$$

$$\rho=8{,}954\times10^3\,\text{Kgm}^{-3};\ K=0{,}386\times10^3\,\text{Wm}^{-1}\text{K}^{-1};\ \alpha=0{,}05\text{Kgms}^{-2};$$

$$t=0{,}01\text{s};\ t_o=0{,}2\text{s};\ \tau_t=0{,}6\text{s};\ \tau_q=0{,}7\text{s};\ \tau_p=0{,}8\text{s};\ \tau_u=0{,}9\text{s};$$

$$\xi=0{,}395\times10^{-9}\,\text{m};\ \zeta=0{,}2\times10^{-9}\,\text{m};\ \varsigma=0{,}15\times10^{-9}\,\text{m};\ F_0=1;\ F_{01}=1.$$

The software Matlab (R2016a) is used for computation for the following cases:

I. MCT diffusion with non-local and dual-phase-lag without temperature dependent $(\alpha^* = 0)$.

II. MCT diffusion with non-local and dual-phase-lag with moderate temperature $(\alpha^* = 0,5)$.

III. MCT diffusion with non-local and dual-phase-lag with high temperature $(\alpha^* = 0,75)$.

Figs.1 – 5, show the effect of ramp-type normal load for different temperatures for all the cases. Figs.6-10, show the effect of ramp-type thermal source for all the cases.

In all the fig. 1 – 10, line (——) corresponds to MCT diffusion with non-local and dual-phase-lag without temperature dependent (MNPWT), line (— —) corresponds to MCT diffusion with non-local and dual-phase-lag with moderate temperature (MNPMT), line (—*—) corresponds to MCT diffusion with non-local and dual-phase-lag with high temperature (MNPHT).

8.1. Normal Load with Varying Temperature. Fig. 1 demonstrates trend of t_{33} vs. x_1. Magnitude of t_{33} decreases for bounded region and increases beyond this region for MNPWT. t_{33} shows increasing trend near the source and after that decreasing trend for MNPMT. t_{33} shows oscillatory behavior for MNPHT.

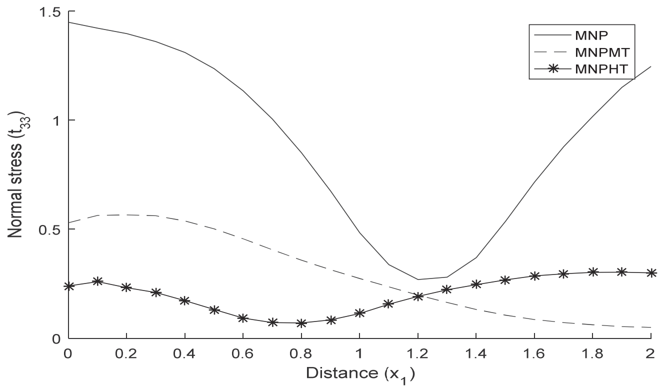

Fig. 1

Fig. 2 displays trend of t_{31} vs. x_1. t_{31} shows decreasing trend with x_1 for MNPWT. Magnitude of t_{31} decreases near the source and after that show oscillatory behavior for MNPMT and MNPHT.

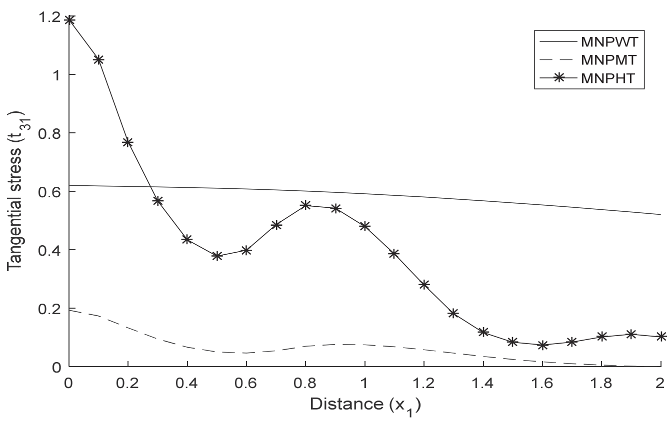

Fig. 2

Fig. 3 shows the trend of m_{32} vs. x_1. Magnitude of m_{32} decreases strictly with x_1 for MNPWT. Behavior of m_{32} is similar for MNPMT and MNPHT with difference in magnitude values.

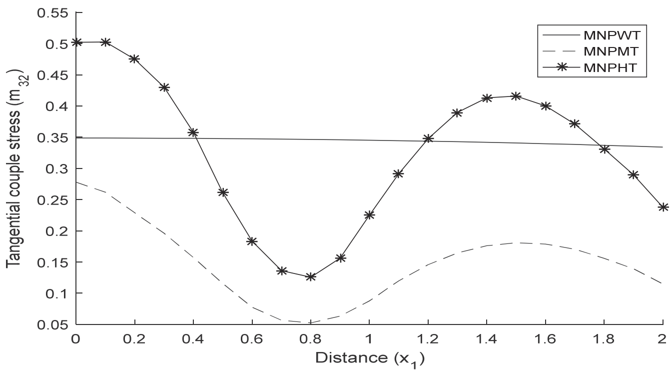

Fig. 3

Fig. 4. depicts trend of *T* vs. x_1. Magnitude of T shows decreasing trend near the source and away from the source increasing trend for MNPWT. *T* decreases for bounded region and increases beyond that region for MNPMT and MNPHT.

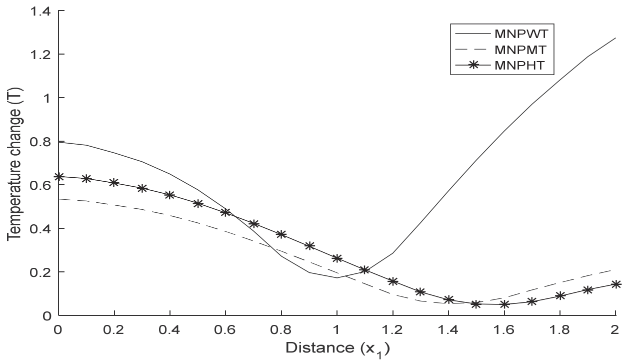

Fig. 4

Fig. 5. depicts the trend of P vs. x_1. P shows opposite trend for MNPWT and MNPHT. Magnitude of P increases near the source and after this shows oscillatory behavior for MNPMT.

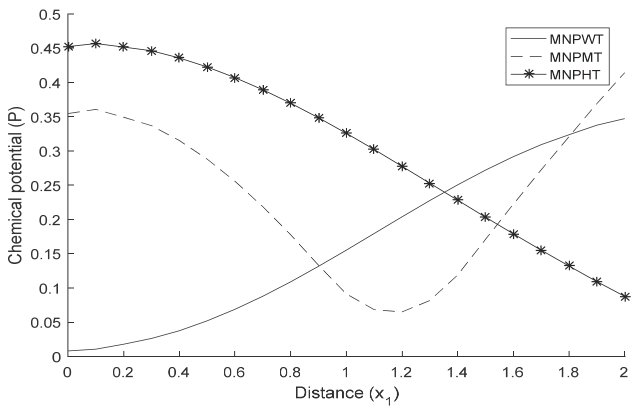

Fig. 5

8.2. Thermal Source with Varying Temperature. Fig. 6 displays t_{33} vs. x_1. For bounded region magnitude of t_{33} increases and after that decreases for MNPWT and MNPHT. t_{33} shows oscillatory behavior for MNPMT.

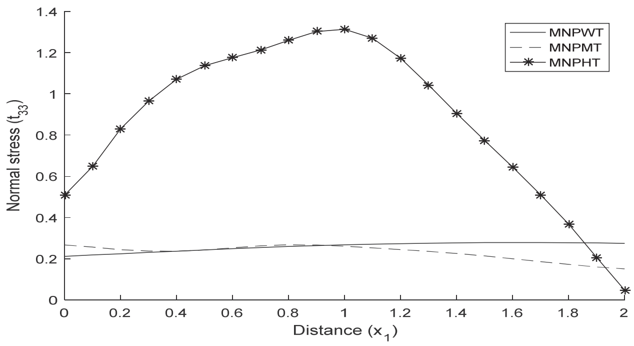

Fig. 6

Fig. 7 displays t_{31} vs. x_1. Magnitude of t_{31} decreases strictly for MNPWT. t_{31} shows opposite behavior for limited interval of x_1 and oscillatory behavior beyond that interval for MNPMT and MNPHT.

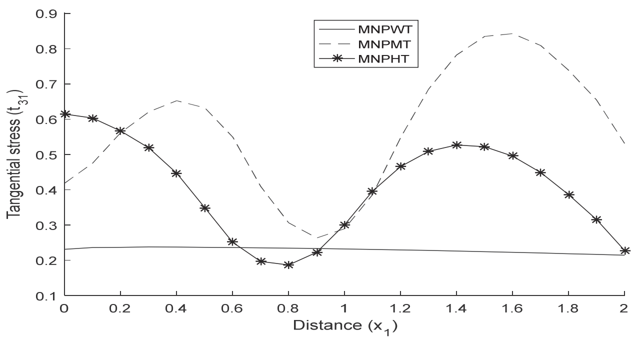

Fig. 7

Fig. 8 depicts m_{32} vs. x_1. Magnitude of m_{32} decreases near the source and increases away from the source for MNPWT. For bounded region m_{32} shows opposite trend and after that similar trend for MNPMT and MNPHT.

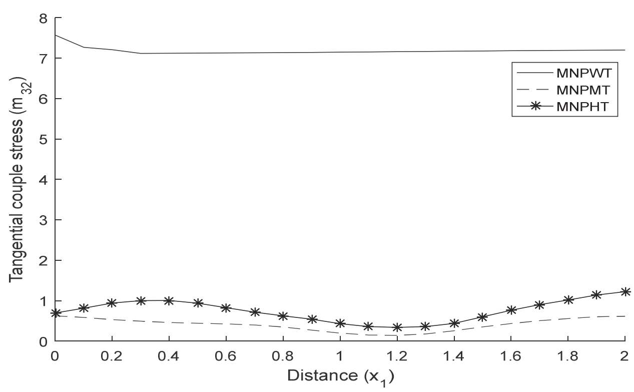

Fig. 8

Fig. 9 determines T vs. x_1. Trend of T is increasing near the source and after that decreasing for MNPWT. T shows similar behavior for MNPMT and MNPHT with difference in magnitude values.

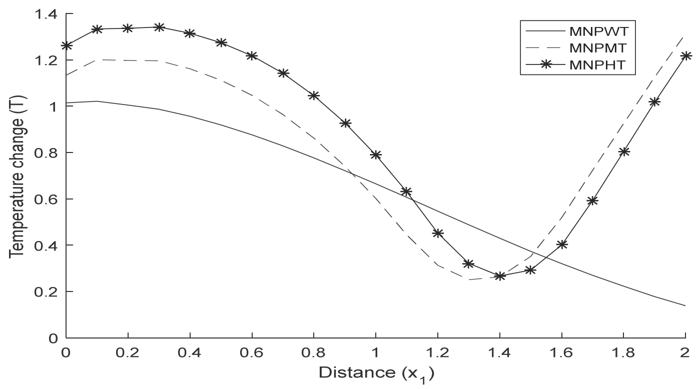

Fig. 9

Fig. 10 demonstrates P vs. x_1. P shows opposite trend for MNPWT and MNPHT. Magnitude of P decreases for bounded region and after that increases for MNPMT.

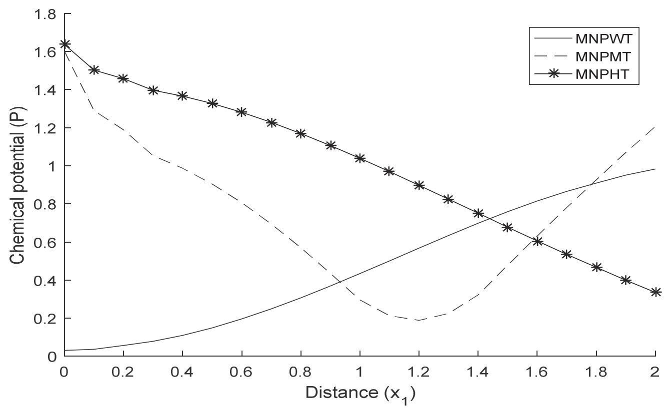

Fig. 10

Effect of non-local in MCT diffusion with fix temperature is shown by following cases:

(i) MCT diffusion with non-local and dual phase lag with fix temperature.

(ii) MCT diffusion with dual phase lag with fix temperature.

(iii) MCT diffusion with non-local and without diffusion phase lag with fix temperature.

(iv) MCT diffusion with non-local and without thermal phase lag with fix temperature.

(v) MCT diffusion with non-local with fix temperature.

Figs. 11 – 15, show the effect of ramp-type normal load for all the cases. Figs. 16 – 20, show the effect of ramp-type thermal source for all the cases.

In all the figures, line (——) corresponds to MCT diffusion with non-local and dual-phase-lag with fix temperature (MNPT), line (- - -) represents MCT diffusion with dual-phase-lag and with fix temperature (MPT), line (— —) corresponds to MCT diffusion with non-local and fix temperature and without diffusion phase-lag (MNWDPT), line (—*—) corresponds to MCT diffusion with non-local and fix temperature and without thermal phase-lag (MNWTPT), line (— o —) corresponds to MCT diffusion with non-local and fix temperature (MNT).

8.3. Normal Load with Fix Temperature. Fig. 11 demonstrates trend of t_{33} vs. x_1. Magnitude of t_{33} increases and decreases for bounded region for MNPT, MNWDPT and MNT. Trend of t_{33} is decreasing for MNWTPT. t_{33} shows decreasing trend for bounded region and oscillatory behavior for rest region for MPT.

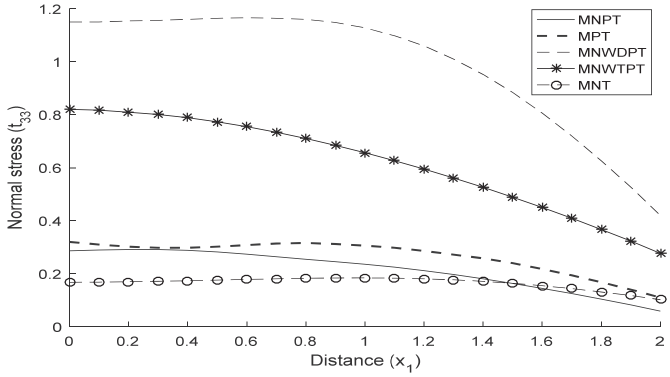

Fig. 11

Fig. 12 displays trend of t_{31} vs. x_1. t_{31} shows similar trend for all values of x_1 for MNPT, MPT and MNT. Magnitude of t_{31} decreases strictly for bounded region and increases far away from the source for MNWDPT and MNWTPT.

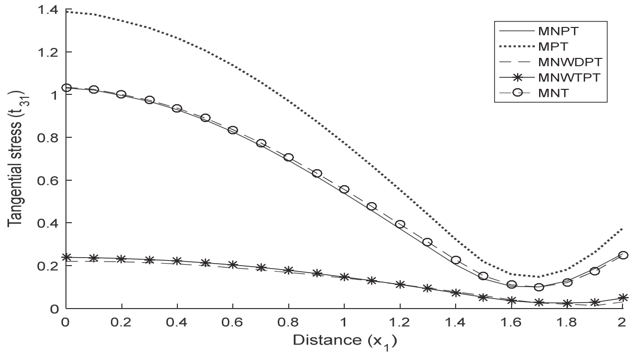

Fig. 12

Fig. 13 shows the trend of m_{32} vs. x_1. m_{32} shows increasing trend near the source and after that oscillatory behavior for MNPT. Trend of m_{32} is similar for MPT, MNWDPT, MNWTPT and MNT.

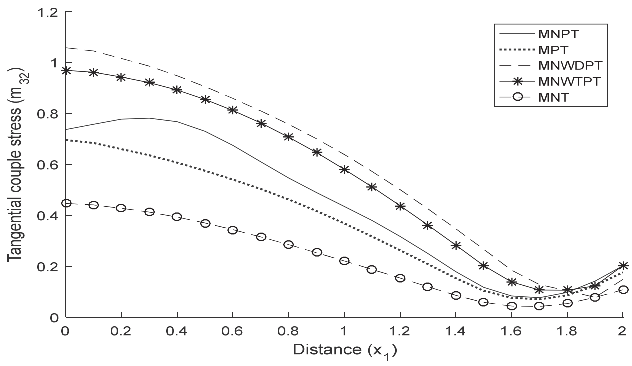

Fig. 13

Fig. 14. depicts trend of T vs. x_1. Trend of T is gradually decreasing with x_1 for all the cases. Trend of T is similar for all the cases with difference in magnitude values.

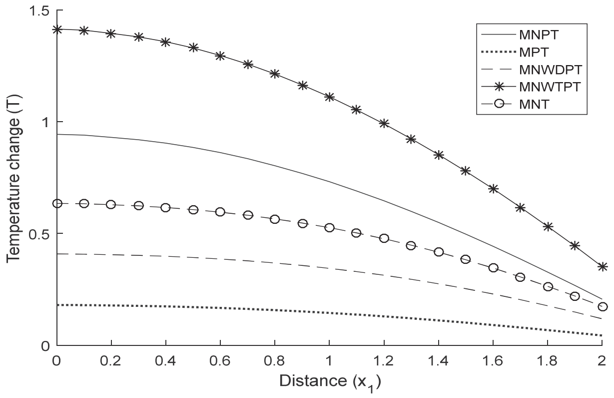

Fig. 14

Fig. 15. depicts the trend of P vs. x_1. Magnitude of P is strictly decreasing with x_1 for all the cases. P shows similar trend for MNPT, MPT and MNWTPT. Decrement in values of P in case of MNWDPT and MNT is more in comparison to other cases.

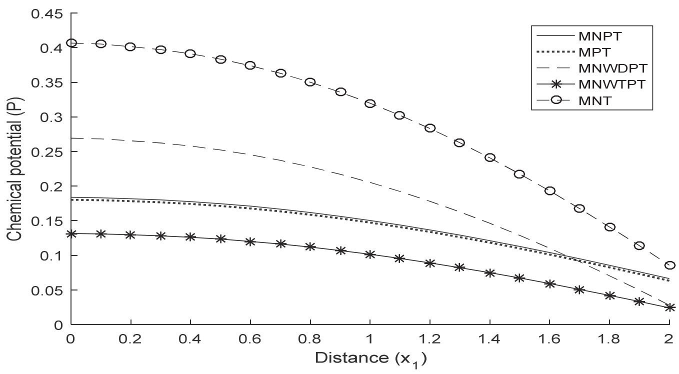

Fig. 15

8.4. Thermal Source with Fix Temperature. Fig.16 displays t_{33} vs. x_1. t_{33} shows similar trend for MNPT and MPT. t_{33} shows opposite behavior for limited interval of x_1 and beyond this interval similar trend for MNWDPT and MNWTPT. t_{33} shows oscillatory behavior for MNT.

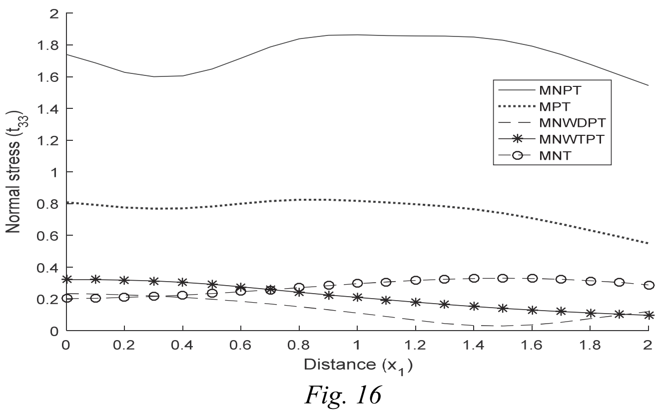

Fig. 16

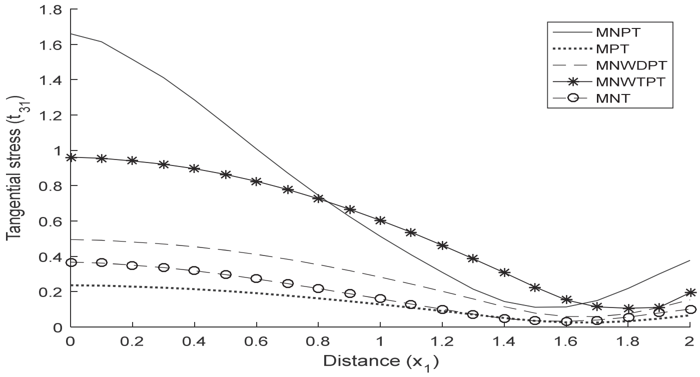

Fig. 17

Fig. 17 displays t_{31} vs. x_1. t_{31} shows similar trend for MNTP, MPT, MNWDPT and MNT with difference in magnitude values. t_{31} decreases strictly for limited interval and increases after this for MNWTPT.

Fig. 18 depicts m_{32} vs. x_1. m_{32} shows oscillatory behavior for MNPT. Magnitude of m_{32} decreases for bounded region and far away from the source increases for all the remaining cases with difference in magnitude values.

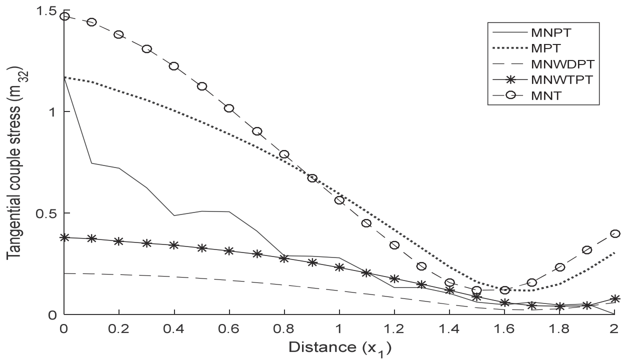

Fig. 18

Fig. 19 determines T vs. x_1. T shows reverse trend for MNPT and MNWTPT. T shows increasing and decreasing trend for bounded region for MPT and MNWDPT. Magnitude of T decreases near the source and after that increases for MNT.

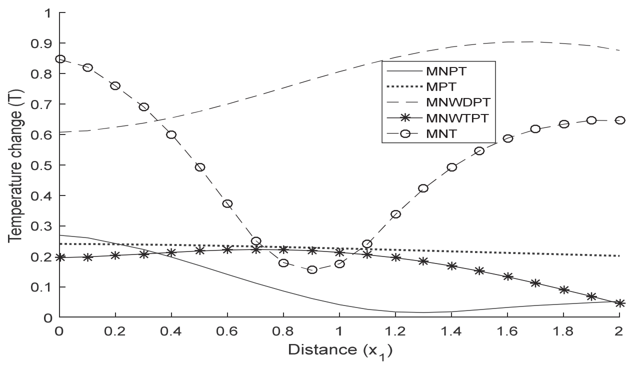

Fig. 19

Fig. 20 demonstrates P vs. x_1. P shows decreasing trend for all values of x_1 for all the cases with difference in magnitude values. Trend of P is similar for MNPT and MPT.

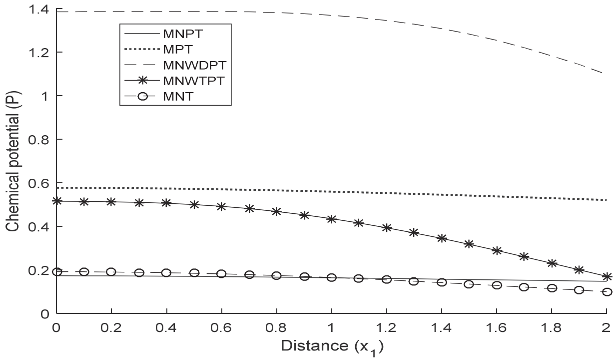

Fig. 20

9. Conclusion.

In this problem normal and thermal load are accounted to determine the physical quantities by using integral transform techniques. Integral transform involve Laplace transform with respect to time variable and Fourier transform with space variable. Effect of temperature, non-local and phase-lag parameter is significant on the resulting quantities. It is observed that these quantities are very sensitive towards these parameters. For temperature dependent properties the resulting quantities show high oscillatory behavior for ramp-type thermal source in comparison to ramp-type normal load.

It is observed that for temperature independent properties (in case of ramp-type normal load), magnitude values of resulting quantities show decreasing trend with increasing distance except tangential stress and tangential couple stress which show increasing trend away from the source. It is also observed that there is high oscillation for all the resulting quantities due to therrmal source in contrast to normal load except for chemical potential which shows decreasing behavior. The results of this model are distinctly different for different cases. The results obtained are important for the researchers working in the modified couple stress thermoelastic with diffusion and non-local parameters with temperture dependent properties.

РЕЗЮМЕ. Проаналізовано властивості нелокальності, залежності від запізнення фази та температури модифікованого моментного термопружного дифузійного середовища (КТР), збудженого термомеханічними джерелами. Базові рівняння утворюються з використанням властивостей нелокальності, залежності від фази та температури. Ці рівняння спрощуються за допомогою потенціалів, а далі застосовуються перетворення Лапласа та Фур'є. Задача розв'язується при відповідних термомеханічних навантаженнях. Розглянуто специфічний вид нормального та теплового навантаження рампового типу. Виведені трансформовані компоненти фізичних полів – переміщень, напружень, температури та хімічного потенціалу. Щодо цих величин приведено чисельний аналіз за допомогою чисельної методики. Наведені графіки отриманих величин, які показують вплив властивостей нелокальності, залежності від фази та температури. Також коментуються конкретні випадки.

КЛЮЧОВІ СЛОВА: модифікована моментна термопружність, нелокальність, дуальне запізнення фази, перетворення Лапласа і Фур'є, нормальне навантаження, теплове джерело, пандусний тип. змінні властивості.

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