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NONLINEAR RANDOM VIBRATIONS OF MICRO-BEAMS RESTING ON VISCO-ELASTIC FOUNDATION VIA THE COUPLE STRESS THEORY

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Abstract. An analysis of nonlinear random vibration behaviors of micro-beams resting on the visco-elastic foundation is presented using the modified couple stress theory and the Euler-Bernoulli beam theory (EBT) with von-Karman nonlinearity. The equation of motion of the micro-beam is established based on Hamilton's principle. The input excitation is assumed to be a Gaussian process with zero means. The mean squares of the micro-beam's displacement are found by the regulated equivalent linearization method. A comparison of the obtained solutions with the published solutions and the classical solutions shows the accuracy of the method. The influences of the material length scale parameter (MLSP), the spectral density of the input excitation, and coefficients of the visco-elastic foundation on the nonlinear random vibration response of micro-beams are studied in detail.

Keywords: micro-beams, modified couple stress theory, nonlinear random vibration, equivalent linearization method.

1. Introduction.

The mechanical behaviors of micro-/nano-beams have always attracted the attention of researchers because of their wide applications in micro-/nano-electromechanical systems [1]. Several experimental studies have shown that the size-dependent effect plays a very important role on the mechanical behaviors of micro-/nano-structures, which is different from macro-structures [2, 3]. Some non-local continuum theories such as the nonlocal elasticity theory (NET) [4, 5], the strain gradient theory (SGT) [6, 7] and the modified couple stress theory (MCST) [8] were proposed to observe the size-dependent effect on micro-/nano-structures' mechanical behaviors. Many works related to analysis of mechanical behaviors of micro- and nano-beams are published using the above mentioned theories. Thai [9] emloyed the NET and the shear deformation beam theory (SDBT) to examine the bending, buckling and free vibration response of nano-beams. Refined models and the NET were used by Sayyad and Ghugal in analyzing the bending, instability and free vibration behaviors of functionally graded (FG) nano-beams [10]. The free vibration and static bending problems of Timoshenko micro-beams were investigated by Wang et al. [11] using the SGT. Based on the SGT and the SDBT, Akgöz and Civalek [12] studied the bending and free vibration behaviors of micro-beams. The dynamical pull-in instability behavior of micro-beams actuated by electrical loadings was studied by Shedighi [13] utilizing the NSGT and the EBT. The nonlinear free vibration and bending reponse of micro-beams resting on nonlinear elastic foundation were investigated by Şimşek [14] using the EBT and the MCST. Mollamahmutoğlu and Mercan [15] investigated the bending, buckling and free vibration characteristics of FG Timoshenko micro-beams based on the MCST by the mixed finite element method with a new novel functional. Additionally, the mechanical analysis of micro-/nano-beams was systematized by Farajpour et al. [16] in the review paper.

The equivalence linearization method (ELM) [17] is an efficient and convenient method for analyzing nonlinear stochastic problems. Replacing the original nonlinear system by an equivalent linear one is an advantage of this method. However, this replacement led to a significant change in the property of the original research system due to the solutions of the nonlinear and linear systems are very different in manner. The ELM is only suitable for weak nonlinear systems; but for strong nonlinear systems, the obtained solution is often inaccurate. Several improvements of this method have been proposed to achieve more accurate results [18-23]. Among the improvements of the ELM, it is necessary to mention the regulated equivalent linearization method (RELM) [24] suggested by Anh and Di Paola by modifying the replacement process. Instead of replacing the original nonlinear system by an equivalent linear one, the authors proposed to replace the original nonlinear system by a higher-order nonlinear system; the resulting nonlinear system is then replaced by a nonlinear system of the same order as the original nonlinear system; and the end of the replacement process is the replacement by a linear system. The results obtained by the RELM are much more accurate than those obtained by the classical method (namely, the ELM). The RELM is suitable for nonlinear systems with nonlinear terms of polynomial form. The RELM was extended by Elishakoff et al. [25] for some nonlinear random vibration systems by increasing the number of replacement steps from one step to two steps. The results show that the accuracy of the solutions obtained by one-step replacement or two-steps replacement depends on each individual nonlinear system. The nonlinear random vibrations of beams have been studied by several authors using the classical elasticity theory such as Elishakoff et al. [26], Crandall and Yildiz [27], Hieu et al. [28], Spanos and Malara [29]. Additionally, Burlon et al. [30] investigated the nonlinear random vibration of beams with in-span supports by using the statistical linearization technique. Nonlinear random vibrations of Euler–Bernoulli beam subjected to a harmonic random axial loading were studied by Karimi and Shadmani [31].

Recently, the random vibration behaviors of micro- and nano-beams have also investigated by some authors. Based on the nonlocal SGT, the linear vibration behavior of FG nanobeam resting on visco-elastic foundation and subjected to a stationary random loading was examined by Rastehkenari [32]. The author found that the mean square of displacement of the FG nanobeam increases by increasing the nonlocal parameter and the power-law index, or by reducing the MLSP, the damping and elastic coefficients of the visco-elastic foundation. The nonlinear random vibration response of FG porous Euler-Bernoulli nanobeams was investigated by Rastehkenari and Ghadiri [33] using the ELM. The obtained results showed that the mean square of response of the FG porous nanobeam increases as the porosity distribution factor increase but the effect of this parameter is very small compared to the effect of the power-law index.

Analysis of nonlinear random vibration of micro- and nano-beams is very limited. In this paper, author focuses on the analysis of the nonlinear random vibration of micro-beams resting on visco-elastic foundation by using the RELM for the first time. The MCST and EBT with von-Karman's assumption are used to derive the micro-beam's equation of motion. Two types of boundary conditions including the simply-supported (S-S) and clamped-clamped (C-C) are considered. The obtained results are more accurate than those obtained by the classical ELM. Numerical illustrations are performed to evaluate the impact of important parameters on the response of the system.

2. Model and formulations.

Figure 1 shows a model of a micro-beam resting on an visco-elastic foundation. The micro-beam has the length L and the cross-section area $A = b \times h$. The visco-elastic foundation based on the Kelvin–Voigt model has the damping and elastic coefficients c and k, respectively. $q(x,t)$ denotes the transverse distributed loading. The coordinate system is selected as shown in this figure, in which the x-axis is directed along the geometry middle axis of the micro-beam and the z-axis is directed along the thickness direction of the micro-beam.

The displacement components of the micro-beam based on the EBT can be given as:

$$
\begin{gathered}
u_x(x,z,t) = u(x,t) - z\frac{\partial w(x,t)}{\partial x};\\
u_y(x,z,t) = 0;\\
u_z(x,z,t) = w(x,t),
\end{gathered}
\tag{1}
$$

where $u(x,t)$ is the axial displacement and $w(x,t)$ is the transverse displacement. With von-Karman's assumption, the non-zero component of the strain tensor of the micro-beams has the following form [14, 33]:

$$
\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2. \tag{2}
$$

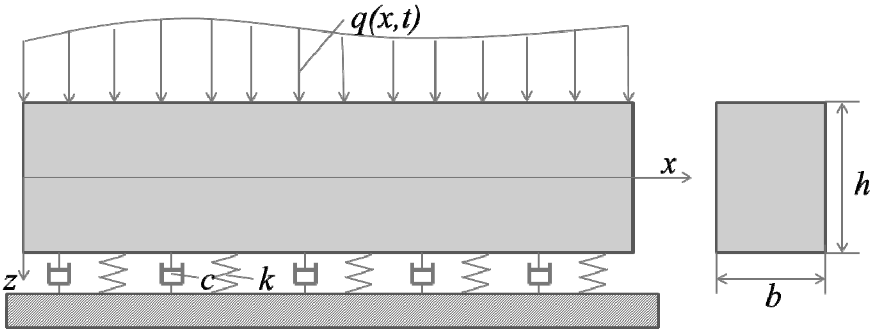

Fig. 1. Model of a micro-beam resting on an visco-elastic foundation.

To derive the equation of motion, the Hamilton's principle is utilized [14]:

$$
\delta\int_0^t \left[K_e - U + W_e\right]dt = 0\,, \tag{3}
$$

herein, K_e, U, and W_e respectively denote the kinetic energy, strain energy and work of the external forces. The virtual kinetic energy of the micro-beams is:

$$
\delta K_e = \int_0^L \rho A\left[\frac{\partial u}{\partial t}\delta\left(\frac{\partial u}{\partial t}\right) + \frac{\partial w}{\partial t}\delta\left(\frac{\partial w}{\partial t}\right)\right]dx\,, \tag{4}
$$

where ρ is the mass density of the micro-beams. The external forces acting on the micro-beam include the transverse distributed force $q(x,t)$ and the reaction due to the visco-elastic foundation ($-kw - c(\partial w/\partial t)$); thus, the virtual work of the external forces is calculated by:

$$
\delta W_e = \int_0^L \left[qw\delta w - kw\delta w - c\frac{\partial w}{\partial t}\delta\left(\frac{\partial w}{\partial t}\right)\right]dx. \tag{5}
$$

According to the MCST, the virtual strain energy of the micro-beam can be written as [8, 14]:

$$
\begin{aligned}
\delta U &= \int_V \left(\sigma_{ij}\delta\varepsilon_{ij} + m_{ij}\delta\chi_{ij}\right)dV = \int_V \left(\sigma_{xx}\delta\varepsilon_{xx} + 2m_{xy}\delta\chi_{xy}\right)dV = \\
&= \int_0^L \left\{N_{xx}\left[\delta\left(\frac{\partial u}{\partial x}\right) + \frac{\partial w}{\partial x}\delta\left(\frac{\partial w}{\partial x}\right)\right] - \left(M_{xx} + Y_{xy}\right)\delta\left(\frac{\partial^2 w}{\partial x^2}\right)\right\}dx,
\end{aligned}
\tag{6}
$$

where N_{xx}, M_{xx} and Y_{xy} respectively denote the axially force resultant, the bending moment and the couple moment, which are defined as [14]:

$$N_{xx} = \int_A \sigma_{xx} dA = EA\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]; \tag{7}$$

$$M_{xx} = \int_A z\sigma_{xx} dA = -EI\frac{\partial^2 w}{\partial x^2}; \tag{8}$$

$$Y_{xx} = \int_A m_{xy} dA = -\mu A l^2 \frac{\partial^2 w}{\partial x^2} \tag{9}$$

in which, I denotes the inertia moment of the cross-section of the micro-beam, l is the MLSP, and $\mu = E/2(1+\nu)$ is Lame's constant known as the shear modulus where ν is the Poisson's ratio.

Now, substituting the expressions (4) – (6) into the Hamilton's principle (4), the equation of motion can be otained:

$$\frac{\partial N_{xx}}{\partial x} - \rho A\frac{\partial^2 u}{\partial t^2} = 0; \tag{10}$$

$$\frac{\partial^2}{\partial x^2}\left(M_{xx} + Y_{xy}\right) + \frac{\partial}{\partial x}\left(N_{xx}\frac{\partial w}{\partial x}\right) - kw - c\frac{\partial w}{\partial t} - \rho A\frac{\partial^2 w}{\partial t^2} + q = 0. \tag{11}$$

If the in-plane inertia is ignored; from Eq. (10), the axially force resultant can be found [14]:

$$N_{xx} = \frac{EA}{2L}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx. \tag{12}$$

Therefore, the equation of motion of the micro-beam in terms of the transverse displacement w can be derived by substituting Eqs. (8), (9) and (12) into Eq. (11) as:

$$\left(EI + \mu A l^2\right)\frac{\partial^4 w}{\partial x^2} - \frac{EA}{2L}\left[\int_0^L \left(\frac{\partial w}{\partial x}\right) dx\right]\frac{\partial^2 w}{\partial x^2} + kw + c\frac{\partial w}{\partial t} + \rho A\frac{\partial^2 w}{\partial t^2} = q(x,t). \tag{13}$$

The kinematic boundary conditions (BCs) of the micro-beam are described by:
for the S-S micro-beam:

$$w(0,t) = w(L,t) = 0;\ \frac{\partial^2 w(0,t)}{\partial x^2} = \frac{\partial^2 w(L,t)}{\partial x^2} = 0; \tag{14}$$

for the C-C micro-beam:

$$w(0,t) = w(L,t) = 0;\ \frac{\partial w(0,t)}{\partial x} = \frac{\partial w(L,t)}{\partial x} = 0. \tag{15}$$

The following dimensionless variables are defined:

$$\bar{x} = \frac{x}{L};\ \bar{w} = \frac{w}{L};\ \bar{t} = t\sqrt{\frac{EI}{\rho A L^4}};\ \alpha = \frac{l}{h};\ B = 1 + \frac{6\alpha^2}{1+\nu};\ K = \frac{kL^4}{EI};\ C = c\sqrt{\frac{L^4}{\rho AEI}};\ \bar{q} = \frac{qL^4}{EIr}. \tag{16}$$

Using Eq. (14), the equation of motion (12) can be rewritten in the dimensionless form as:

$$B\frac{\partial^4 \overline{w}}{\partial \overline{x}^2}-\frac{1}{2}\left[\int_0^1\left(\frac{\partial \overline{w}}{\partial \overline{x}}\right)d\overline{x}\right]\frac{\partial^2 \overline{w}}{\partial \overline{x}^2}+K\overline{w}+C\frac{\partial \overline{w}}{\partial \overline{t}}+\frac{\partial^2 \overline{w}}{\partial \overline{t}^2}=\overline{q}(\overline{x},\overline{t}). \tag{17}$$

And the kinematic BCs (14) and (15) become:
for the S-S micro-beam:

$$\overline{w}(0,\overline{t})=\overline{w}(1,\overline{t})=0;\ \frac{\partial^2 \overline{w}(0,\overline{t})}{\partial \overline{x}^2}=\frac{\partial^2 \overline{w}(1,\overline{t})}{\partial \overline{x}^2}=0; \tag{18}$$

for the C-C micro-beam:

$$\overline{w}(0,\overline{t})=\overline{w}(1,\overline{t})=0;\ \frac{\partial \overline{w}(0,\overline{t})}{\partial \overline{x}}=\frac{\partial \overline{w}(1,\overline{t})}{\partial \overline{x}}=0. \tag{19}$$

3. Solution procedure.

For the purpose of studying the nonlinear random vibration problem; firstly, the Galerkin method is applied to reduce the equation of motion of the micro-beam (17) to the ordinary differential equation. The solution of Eq. (15) is assumed to have a form:

$$\overline{\mathrm{w}}(\overline{x},\overline{t})=Q(\overline{t})\cdot\varphi(\overline{x}), \tag{20}$$

where $Q(\overline{t})$ is the time-dependent function need to be determined, and $\varphi(\overline{x})$ is the basic function chosen so that the solution (20) satisfies the kinematic BCs of the micro-beam. For the S-S and C-C micro-beams, the basic functions can be chosen as in Table 1.

Table 1. The basic functions [14].

Types of BCs	Basic function
S-S	$\varphi(\overline{x})=\sin(\pi\overline{x})$
C-C	$\varphi(\overline{x})=\frac{1}{2}\left[1-\cos(2\pi\overline{x})\right]$

In this work, the external force $q(\overline{x},\overline{t})$ is considered to be a point load exerted at the point $\overline{x}=\overline{x}_0$ and it is random to time:

$$q(\overline{x},\overline{t})=\delta\left(\overline{x}-\overline{x}_0\right)\xi(\overline{t}), \tag{21}$$

where $\delta(\overline{x}-\overline{x}_0)$ is the Dirac-delta function, and $\xi(\overline{t})$ is a zero-mean Gaussian white noise excitation with the constant spectral density $S_\xi(\omega)=S_0$ and the correlation $R_\xi(\overline{t})$:

$$R_\xi(\overline{t})=E\left[\xi(\overline{t})\xi(\overline{t}+\tau)\right]=2\pi S_0\delta(\tau). \tag{22}$$

Applying the Galerkin method, Eq. (17) is reduced to:

$$\ddot{Q}(\overline{t})+\gamma_1\dot{Q}(\overline{t})+\gamma_2 Q(\overline{t})+\gamma_3 Q^3(\overline{t})=\gamma_4\xi(\overline{t}), \tag{23}$$

where the coefficients $\gamma_i\ (i=1\div 4)$ are determined by:

$$\gamma_1=C; \tag{24}$$

$$\gamma_2=\frac{B\int_0^1\varphi^{(4)}(\overline{x})\varphi(\overline{x})d\overline{x}}{\int_0^1\varphi^2(\overline{x})d\overline{x}}+K; \tag{25}$$

$$\gamma_3 = \frac{-\frac{1}{2}\left[\int_0^1 \left(\varphi'(\bar{x})\right)^2 d\bar{x}\right]\left[\int_0^1 \varphi''(\bar{x})\varphi(\bar{x})d\bar{x}\right]}{\int_0^1 \varphi^2(\bar{x})d\bar{x}}; \tag{26}$$

$$\gamma_4 = \frac{\int_0^1 \delta(\bar{x}-\bar{x}_0)\varphi(\bar{x})d\bar{x}}{\int_0^1 \varphi^2(\bar{x})d\bar{x}} = \frac{\varphi(\bar{x}_0)}{\int_0^1 \varphi^2(\bar{x})d\bar{x}}. \tag{27}$$

Now, the RELM will be used to find the mean square of response of Eq. (23). According to the RELM proposed by Anh and Di Paola [24], the nonlinear equation (23) is replaced by the following equivalent linear one:

$$\ddot{Q}(\bar{t}) + \gamma_1\dot{Q}(\bar{t}) + \left(\gamma_2 + \gamma_{3eq}\right)Q(\bar{t}) = \gamma_4\xi(\bar{t}). \tag{28}$$

The authors [24] proposed the scheme for replacing:

$$\gamma_3 Q^3(\bar{t}) \to \lambda_1 Q^5(\bar{t}) \to \lambda_2 Q^3(\bar{t}) \to \gamma_{3eq}Q(\bar{t}), \tag{29}$$

where the coefficients λ_1, λ_2 and γ_{3eq} are found from the minimum mean-square error criteria:

$$E\left[\left(\gamma_3 Q^3 - \lambda_1 Q^5\right)^2\right] \to \underset{\lambda_1}{Min}; \tag{30}$$

$$E\left[\left(\lambda_1 Q^5 - \lambda_2 Q^3\right)^2\right] \to \underset{\lambda_2}{Min}; \tag{31}$$

$$E\left[\left(\lambda_2 Q^3 - \gamma_{3eq} Q\right)^2\right] \to \underset{\gamma_{3eq}}{Min}\ , \tag{32}$$

herein, $E[\bullet]$ represents the mathematical expectation operation.

From the criteria (30) - (32), the coefficients λ_1, λ_2 and γ_{3eq} can be found as:

$$\lambda_1 = \gamma_3\frac{E[Q^8]}{E[Q^{10}]} = \gamma_3\frac{1}{9E[Q^2]}; \tag{33}$$

$$\lambda_2 = \lambda_1\frac{E[Q^8]}{E[Q^6]} = 7\lambda_1 E[Q^2]; \tag{34}$$

$$\gamma_{3eq} = \lambda_1\frac{E[Q^4]}{E[Q^2]} = 3\lambda_1 E[Q^2]. \tag{35}$$

Therefore, from Eqs. (33)-(35), the coefficient γ_{3eq} can be obtained:

$$\gamma_{3eq} = \frac{7}{3}\gamma_3 E[Q^2]. \tag{36}$$

By putting Eq. (36) into Eq. (28), the linear equation becomes:

$$\ddot{Q}(\bar{t}) + \gamma_1\dot{Q}(\bar{t}) + \left(\gamma_2 + \frac{7}{3}\gamma_3 E[Q^2]\right)Q(\bar{t}) = \gamma_4\xi(\bar{t}). \tag{37}$$

The solution of Eq. (37) has a form:

$$E[Q^2]=\frac{\pi S_0\gamma_4}{\gamma_1\left(\gamma_2+\frac{7}{3}\gamma_3 E[Q^2]\right)}. \tag{38}$$

From Eq. (38), the mean-square value $E[Q^2]$ using the RELM can be derived as:

$$E[Q^2]_{RELM}=\frac{-3\gamma_1\gamma_2+\sqrt{9\gamma_1^2\gamma_2^2+84\pi S_0\gamma_1\gamma_3\gamma_4}}{14\gamma_1\gamma_3}. \tag{39}$$

If the clasical ELM [17] is employed, the coefficient γ_{3eq} in Eq. (28) are determined from criterion:

$$E\left[\left(\gamma_3 Q^3-\gamma_{3eq}Q\right)^2\right]\to \underset{\gamma_{3eq}}{Min}. \tag{40}$$

Thus, the following result can be obtained:

$$\gamma_{3eq}=\gamma_3\frac{E\left[Q^4\right]}{E\left[Q^2\right]}=3\gamma_3 E\left[Q^2\right]. \tag{41}$$

Therefore, the mean-square value $E[Q^2]$ using the classical ELM can be achieved as:

$$E[Q^2]_{Classical}=\frac{-\gamma_1\gamma_2+\sqrt{\gamma_1^2\gamma_2^2+12\pi S_0\gamma_1\gamma_3\gamma_4}}{6\gamma_1\gamma_3}. \tag{42}$$

4. Numerical results.

For the classical beam, the obtained results in this paper are compared with those achieved by Hieu et al. [28]. Note that the results for the classical beam can be recovered from the present results by letting the MLSP equal to zero ($l=0$). The mean square value $E\left[Q^2\right]_{DC}$ is obtained by Hieu et al. [28] by using the dual criterion (DC) of the ELM, and the exact mean square value $E\left[Q^2\right]_{exact}$ is determined by applying the Fokker – Planck approach [28]. The input data is considered as $c=0{,}1$, $\omega_0=1$, $\varepsilon=1$, $\rho A=1$ and $S_0=1$; herein, $\omega_0^2=\frac{EI\pi^4}{\rho AL^4}$ and $\varepsilon=\frac{k_L}{\rho A\omega_0^2}$. It can be seen that the accuracy of the RELM compared with those of the DC and the classical ELM (Table 2).

Table 2. Comparison of the mean square values of displacement of the classical beam.

r	$E\left[Q^2\right]_{exact}$	$E\left[Q^2\right]_{DC}$ [28]	R. error (%)	$E\left[Q^2\right]_{Classical\ ELM}$	R. error (%)	$E\left[Q^2\right]_{RELM}$	R. error (%)
100	15,6895	15,6948	0,033781	15,6895	0	15,6936	0,026132
50	15,6349	15,6554	0,131117	15,6346	0,001919	15,6508	0,101696
20	15,2778	15,3907	0,738981	15,2707	0,046473	15,3637	0,562254
10	14,2613	14,5706	2,168806	14,1964	0,455078	14,4842	1,562971
5	11,9200	12,4086	4,098993	11,6419	2,333054	12,2223	2,536074
2	7,3952	7,7220	4,419083	6,8668	7,145175	7,5015	1,437419
1	4,4131	4,5614	3,36045	3,9581	10,31021	4,4024	0,24246
0,5	2,4261	2,4842	2,39479	2,1276	12,3037	2,3892	1,52096
0,2	1,0294	1,0463	1,641733	0,889	13,63901	1,0041	2,457742
0,1	0,5251	0,5322	1,352123	0,451	14,1116	0,5104	2,799467

In this below section, some numerical illustrations are given to study the effects of the MLSP ($\alpha = l/h$),the spectral density S_0 and the coefficients of visco-elastic foundation (*C* and *K*) on the nonlinear random vibration behavior of the micro-beams based on the MCST. The Possion's ratio is fixed $\nu = 0,3$.

The effects of the MLSP and the spectral density on the nonlinear random vibration behavior of the micro-beams are presented in Fig. 2. It can be concluded that the mean square value of displacement $E\left[Q^2\right]$ reduces when the MLSP increases and the spectral density decreases. The MLSP makes the micro-beams become stiffer, namely the displacement of the micro-beams decreases when the MLSP increases. Therefore, the mean square of displacement reduced when the MLSP increase is noticable. This figure also reveals that the mean square values $E\left[Q^2\right]$ of the C-C micro-beam are always smaller than those of the S-S micro-beam. The input data is selected to plot Fig. 2 includes $C = 0,1,\ K = 10$ and $\overline{x}_0 = 0,5$.

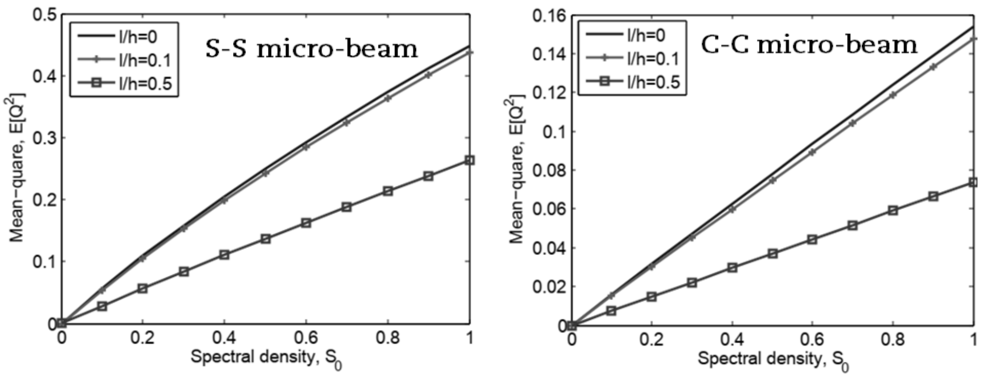

Fig. 2: The variation of the mean-square value $E\left[Q^2\right]$ to the spectral density S_0.

Fig. 3 and 4 show the influences of the damping and elastic coefficients of the visco-elastic foundation on the nonlinear random vibration response of the micro-beams. Fig. 3 is plotted with the following input data $K = 10,\ \overline{x}_0 = 0,5,$ and $S_0 = 0,2.$ and the selected input data to plot Fig. 4 includes $C = 0,1,\ \overline{x}_0 = 0.5,$ and $S_0 = 0,1.$ It can be seen that the mean-square value of displacement $E\left[Q^2\right]$ reduces when the damping and elastic coefficients increases. The mean square value of displacement reduces more strongly with small values of the damping coefficient and decreases linearly with the elastic coefficient.

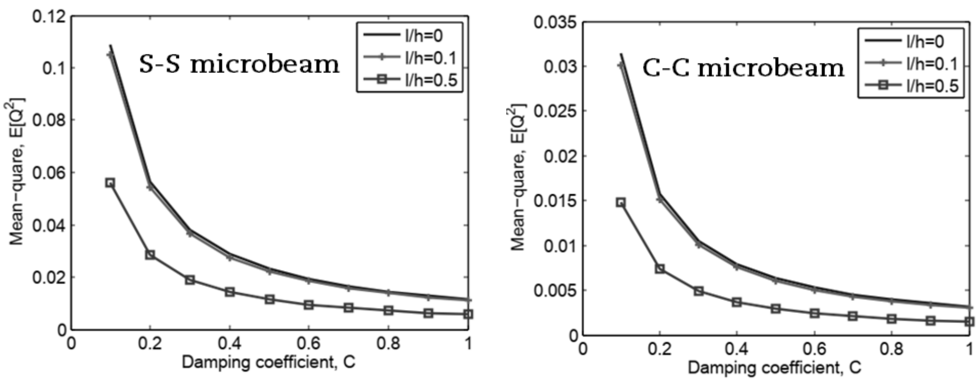

Fig. 3: The variation of the mean-square value $E\left[Q^2\right]$ to the damping coefficient *C*.

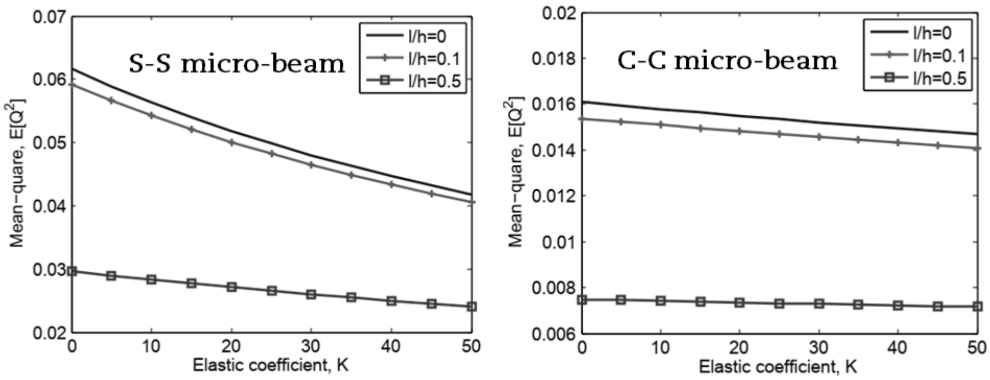

Fig. 4: The variation of the mean-square value $E\left[Q^2\right]$ to the elastic coefficient *K*.

Conclusion.

The nonlinear random vibration behaviors of micro-beams resting on visco-elastic foundation based on the MCST and the EBT are studied in this paper. The input excitation is assumed to be a Gaussian process with zero mean. The mean squares of the micro-beam's displacement are found by employing the RELM. The accuracy of the obtained results is verified by comparison with those of the DC of the ELM and the classical ELM. The influences of the MLSP, the spectral density of the input excitation and the coefficients of the visco-elastic foundation on the nonlinear random vibration response of micro-beams are examined. The obtained results show that the mean-square value of displacement reduces when the MLSP, damping and elastic coefficients increase; and increases when the spectral density of the input excitation increases.

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РЕЗЮМЕ. Аналіз поведінки нелінійних випадкових коливань мікробалок, що спираються на в'язкопружну основу, представлено з використанням модифікованої теорії моментних напружень і теорії балок Ейлера-Бернуллі з нелінійністю фон Кармана. Рівняння руху мікробалки отримано на основі принципу Гамільтона. Припускається, що вхідне збудження є процесом Гауса з нульовим середнім. Методом регламентованої еквівалентної лінеаризації знайдено середньоквадратичні зміщення мікробалки. Порівняння отриманих розв'язків з опублікованими розв'язками та класичними розв'язками свідчить про точність методу. Детально досліджено вплив параметра масштабу довжини матеріалу, спектральної щільності вхідного збудження та коефіцієнтів в'язкопружної основи реакції мікробалок на нелінійні випадкові коливання.

КЛЮЧОВІ СЛОВА: мікро-балка, модифікована теорія моментних напружень, нелінійні випадкові коливання, еквівалентний метод лінеаризації.

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