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COMPUTER MODEL OF PUMP-WELL-RESERVOIR SYSTEM BASED ON THE NEW CONCEPT OF IMITATIONAL MODELING OF DYNAMIC SYSTEMS*

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Abstract. A new concept of simulation modeling of dynamic systems is developed. The basic notions and terms of the concept are outlined, as well as the principles of creating a simulation model of a physical process. The proposed concept is applied to the problem of modeling the process of developing a volatile oil formation operated by a rod pump in the "pump-drill hole-formation" system. The developed computer simulation model, taking into account the presence of the annular space and the fluid flow between the annular space and the lifting pipe, is used to study the operation process. Full filling of the rod pump cylinder does not always coincide with the maximum flow rate of the drill hole. The coincidence of the maxima of filling the cylinder and pump delivery is a condition for the optimal mode. The optimal mode can only be achieved by switching to the periodic mode of operation of the pump. It is established that in order to switch to a periodic mode, a condition is necessary when, at maximum flow, the maximum filling of the pump cylinder cannot be achieved.

Key words: integrated modeling, simulation, imitational modeling, volatile oil, sucker-rod pump, pump-well-reservoir system.

Introduction.

Let's get acquainted with the fundamental concepts and terms of the imitational modeling theory for "Pump-well-reservoir" systems in the proposed concept, but first we need to clarify the terms "computer modeling" and "imitational modeling" within the new concept. In the ordinary way "Computer modeling" is the application (or, realization) of a mathematical model on a computer. This basically means a computer implementation of numerical methods over differential equations which describe physical processes [7]. Therefore, before creating a computer model, it is necessary to have a mathematical model. As a result of the application or implementation of equations describing certain physical processes we obtain simulator that simulates the process, i.e. "Imitates" the mathematical model on the basis of which the simulator is built. We see the simulator has a more comprehensive meaning than the model. In English, the term "simulation" is synonymous with "imitation". However, the concept of imitation as we know it has a much broader meaning than simulation. Within the framework of the new imitation concept is understood as a certain methodology for the mathematical description of a complex physical process that represents a dynamic system, by division the system into constituents and connection the latter with input

* Згідно рішення редколегії дана стаття професора Ф.А.Алієва, як члена Міжнародної редакційної ради журналу «Прикладна механіка», публікується без рецензування та редактування.

and output parameters within an time quantum. Whereas the use of an imitational model of a dynamic system in general, and the "pump-well-reservoir" system in particular, without a computer is impossible (or, on practical grounds, it is meaningless), we often call it a computer-imitational model.

In imitational modeling, the system is divided into separate primitive objects. In such a case each object is interconnected by a causal relationship according to the system approach methodology [1, 4 – 6, 12]. Each of these objects can return a result to the next object and return some data generated as a result of an event occurring in this object to the previous object. Data transmission by an object to the previous object is called feedback. Besides, each object can be exposed to external actions, i.e. getting the data from the outside is called control. Objects are characterized by some parameters that determine their local state (see Fig. 1). Let us denote these parameters as state parameters of the object. Objects must be described in order to enable process modeling.

Object description is the definition of mathematical relations between the object's state parameters and time quantum.

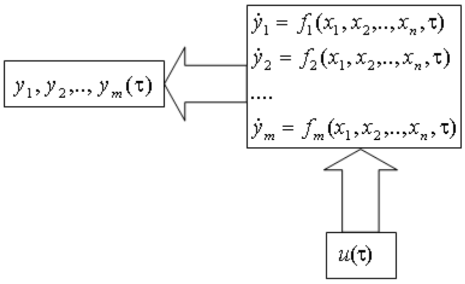
Events inside objects happen simultaneously (synchronously), they are discredited in compliance with the system time, which varies on the time quantum. At each time quantum, there happen events that lead to state changes, i.e. state parameters values of the system objects and, therefore, the system itself in general (in terms of information science). The greater the number of system objects, the more accurate the model reflects the real physical process, compared to a similar system with a smaller object. The division of a dynamic system into a larger number of objects leads to objects' simplicity, the model complexity and an enhanced accuracy of the physical process imitation. Thus, fragmentation of the process can bring the model closer to the real physical process.

In order to generalize the new imitational modeling concept, we tried to make a mathematical formulation of the presented imitation concept, the results are illustrated in Fig. 1. For comparison, a mathematical model is shown at the top, a generalized imitational model is shown in the middle, and an approximate model of the pump-well-reservoir system is shown at the bottom. Thus, while math modeling (see Fig. 1, upper diagram), the physical process is represented by one object described by a differential equation or by a system consisting of m number of differential equations controlled by a function $u(t)$ [9]. Analytical or numerical solution of the system of differential equations $\dot{y}_1, \dot{y}_2, ..., \dot{y}_m$ under certain initial and boundary conditions for each of the equations of the system gives the object state parameters $\dot{y}_1, \dot{y}_2, ..., \dot{y}_m$ for any time point t.

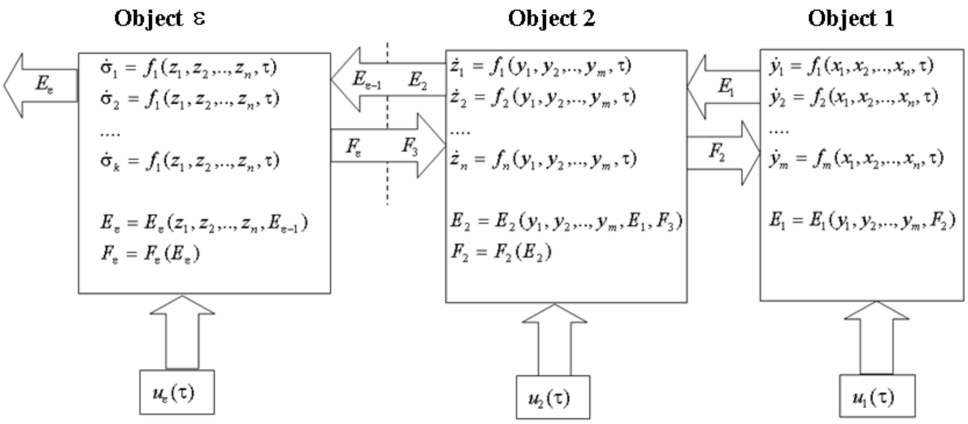
When imitational modeling, the identical system is represented as a system consisting of ε number of objects, each of which is described by its own systems of differential equations (diagram in the middle in Fig. 1): Object 1 – by equations $\dot{y}_1, \dot{y}_2, ..., \dot{y}_m$, Object 2 - by equations $\dot{z}_1, \dot{z}_2, ..., \dot{z}_n$, and Object ε – by equations $\dot{\sigma}_1, \dot{\sigma}_2, ..\dot{\sigma}_k$. Each object has *event functions* $E_1, E_2, ..E_\varepsilon$ and (except for Object 1) can have feedback functions $F_2, F_3, ..., F_\varepsilon$ with which within the time quantum Δt the objects exchange information. It is possible objects are controlled by functions $u_1, u_2, ..., u_\varepsilon$. The information E_ε at the output of the last object (in this case the Object ε) is the value of the required function to the time $t = \sum_{i=0}^{\delta} \Delta t$, where δ is the current value of the system time counter.

According to the above methodology, an approximate scheme of imitational modeling for the pump-well-reservoir system has been compiled [10]. It is shown in Fig. 1 (lower diagram), where $\dot{p}_t, \dot{s}_t, \dot{\Omega}_t, q$ are differential equations of reservoir pressure, pore oil saturation, total reservoir pore volume and the flow rate of oil influx, respectively; p_w – is bottom hole pressure; q_N – is pump rate; h, h_N – are height of the liquid column in the well and the height of the liquid column pumped by the plunger pump during the complete stroke cycle, respectively; ρ, g – are specific gravity of the reservoir liquid and gravity acceleration; q_d, ν – are well flow rate and crank rotation frequency, respectively; Δt – is time quantum.

Diagram of the mathematical modeling conception



Generalized diagram of the imitational modeling concept



The imitational modeling concept on the example of the pump-well-reservoir system

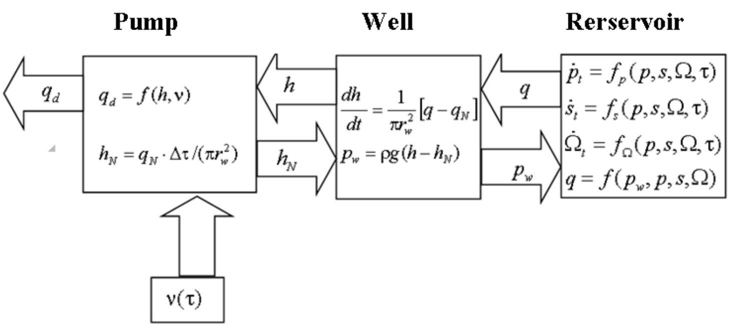


Fig. 1. Graphic illustration of the concepts of mathematical modeling and imitational modeling.

Summarizing the above, the essence of the proposed approach can be expressed as follows: The physical system is divided into some simple objects, described by their own differential equations. The use of quantization (the concept of time within a given object is lost) leads to the fact that in order to determine the current state (i.e., the current values of state parameters) of objects, it is only necessary to determine the increment of the corresponding function. And the time of the system counted with the time counter and "remembered" separately. Thus, the problem is reduced to determining the scale of the quantum based on the resolution of the most sensitive object of the system.

The imitational model is essentially discrete and the internal state of its objects changes synchronously. Therefore, the implementation of imitational models on high-efficiency parallel computing systems is natural and does not require non-standard actions, which opens up wide opportunities for modeling the most complex physical processes, where standard mathematical methods are powerless.

İmitational modeling of the "Pump-Lift-Annular space-Reservoir" system.

Based on the above principles of the proposed imitational modeling concept, the "pump-well-reservoir" system, taking into consideration the flow between the annulus and the tubing, can be schematically depicted as in Fig. 2. As the diagram shows, the "Well" object is represented consisting of two objects – the "Annular space" and "Lift". Below, we will take a detailed look at all the objects that make up the system separately.

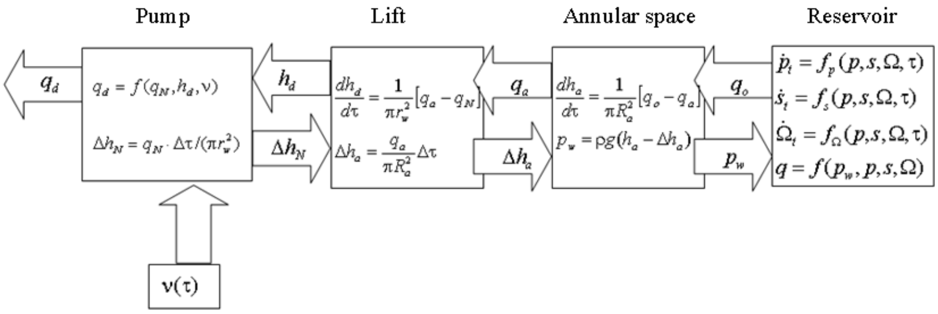


Fig. 2. Scheme of the dynamic system "Pump-Well-Reservoir", with availability of annular space, based on the principles of the proposed imitational modeling concept.

§1. Description of the "Reservoir" object.

The results of [6] allows us to write out the equations for the non-steady-state radial flow of light oils in the well drainage area based on a Binary model in the following form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\varphi(p,s)\frac{\partial p}{\partial r}\right] = -\frac{\partial}{\partial t}\left[f(p,s)\right]; \tag{1.1}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\varphi_g(p,s)\frac{\partial p}{\partial r}\right] = -\frac{\partial}{\partial t}\left[f_g(p,s)\right], \tag{1.2}$$

where

$$\varphi(p,s) = \left[\frac{k_{ro}(s)}{\mu_o(p)B_o(p)} + \frac{k_{rg}(s)p\beta c(p)}{\mu_g(s)z(p)p_{at}}\right]k(p); \tag{1*}$$

$$\varphi_g(p,s) = \left[\frac{k_{rg}(s)\,p\beta[1-c(p)\,\overline{\gamma}(p)]}{\mu_g(p)z(p)p_{at}} + \frac{k_{ro}(s)S(p)}{\mu_o(p)B_o(p)}\right]k(p); \tag{2*}$$

$$f(p,s) = \left[\frac{s}{B_o(p)} + (1-s)\frac{p\beta c(p)}{z(p)p_{at}}\right]m(p);$$

$$f_g(p,s) = \left[\frac{(1-s)p\beta[1-c(p)\,\overline{\gamma}(p)]}{z(p)p_{at}} + s\frac{S(p)}{B_o(p)}\right]m(p).$$

Here, k_{ro} is the relative phase permeabilities for the liquid and gas phases, respectively; s is the pore saturation with the liquid phase; z, β are the real gas factor and temperature correction coefficient for the gas phase; c is the content of hydrocarbon liquids in the gas phase; μ_o, μ_g are the liquid and gas phase viscosities, respectively; B_o is formation volume factor; m is the current reservoir porosity; S is the amount of oil-dissolved gas; $\bar{\gamma} = \dfrac{\gamma_o(p)}{\gamma_g(p)}$ is the ratio of the specific weights of the liquid and gas phases at reservoir pressure p; p_{at} is atmospheric pressure; k is current value of reservoir permeability; r is radial coordinate and t is time.

Equation (1.1) describes the motion of liquid hydrocarbons and potential vapors of lighter oil components, and (1.2) – describes the motion of gas and vapors of lighter oil components. To determine the oil production rate it is necessary to solve equation (1.1) under the following boundary conditions [8, 11, 14]:

$$r = r_e; \quad p = p_e(t); \quad r = r_w; \quad p = p_w(t) \quad \text{and} \quad t = 0, \quad p = p_0 . \tag{1.3}$$

Here, we use the averaging method for the linearization of the equation. Let us introduce the pseudo pressure H function in the following form:

$$H = \int \varphi(p, s)\, dp + \text{const} . \tag{1.4}$$

If we average the reservoir pressure along the coordinate r, the right side of equation (1.1) will depend only on time. With allowance for this, we equate the right part of equation (1.1) with a certain function $\Phi(t)$. Taking into account (1.4), we rewrite (1.1) in the following form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{ r\frac{\partial H}{\partial r} \right\} = -\Phi(t) . \tag{1.5}$$

Under boundary conditions $r = R_e$; $H = H_e(t)$; $r = r_w$; $H = H_w(t)$ and $\left.\dfrac{\partial H}{\partial r}\right|_{r=R_e} = 0$ equation (1.5) has the following general solution:

$$H = \frac{H_w - H_e}{R_e^2\left(2\ln\dfrac{R_e}{r_w} - 1\right) + r_w^2} \cdot \left[R_e^2\left(2\ln\frac{R_e}{r} - 1\right) + r^2 \right] + H_e , \tag{1.5*}$$

where from we can obtain the following expression for the gradient of the function H:

$$\frac{\partial H}{\partial r} = \frac{H_e - H_w}{\dfrac{R_e^2}{2}\left(2\ln\dfrac{R_e}{r_w} - 1\right) + \dfrac{r_w^2}{2}} \left(\frac{R_e^2 - r^2}{r} \right) . \tag{5*}$$

According to Darcy's law, we have: $q_o = Sv = 2\pi r_w h \cdot \left.\dfrac{\partial H}{\partial r}\right|_{r=r_w}$, where S, h, v are the area of the drilled part of the reservoir, the reservoir thickness and influx rate. Given (1.5*), we obtain the final form of the expression for the influx rate:

$$q_o = \frac{2\pi h\left(H_e - H_w\right)\left(1 - \dfrac{r_w^2}{R_e^2}\right)}{\ln\dfrac{R_e}{r_w} - \dfrac{1}{2}\left(1 - \dfrac{r_w^2}{R_e^2}\right)} \quad \text{or since } \frac{r_w^2}{R_e^2} << 1\text{, then } q_o = \frac{2\pi h(H_e - H_w)}{\ln\dfrac{R_e}{r_w} - \dfrac{1}{2}} , \tag{1.6}$$

where R_e, r_w – supply contour radius and well radius.

For practical application of (1.6), according to (1.4) a transition from pseudo depression $(H_e - H_w)$ to true depression $(p_e - p_w)$ is required. For this purpose, we investigated integrand $\varphi(p,s)$ and found that it is well approximated by a logarithmic function in the form:

$$\varphi = a^* \ln(\overline{p}) - b^*, \tag{1.7}$$

where $\overline{p} = p/p_0$, the coefficients a^* and b^* are found from the boundary values of the function φ.

The $(H_e - H_w)$ – reservoir pressure graphs calculated by (1.4) using the trapezoid method and the expression

$$\int_{p_w}^{p_e} \left[a^* \ln(\overline{p}) - b^* \right] d\overline{p}, \tag{1.8}$$

were compared in order to evaluate the accuracy of the accepted approximation.

A curve comparison showed that an average difference between the curve values makes 0,5%. The initial reservoir pressure was 40,0 MPa. When the reservoir pressure drops below 10,0 MPa, the deviation starts to increase but does not exceed 8%.

With regard to the approximation (1.7), we integrate (1.4) within the limits $[\overline{p}_w, \overline{p}_e]$ and get the expression $(H_e - H_w)$ as follows:

$$H_e - H_w = p_0 a^* \left[\overline{p}_e \ln \overline{p}_e - \overline{p}_e - \overline{p}_w \ln \overline{p}_w + \overline{p}_w \right] - p_0 b^* (\overline{p}_e - \overline{p}_w), \tag{1.9}$$

where are the relations for calculating the coefficients a^* and b^* are obtained from (1.7), given the corresponding boundary values φ in the following form:

$$a^* = \frac{\varphi_e - \varphi_w}{\ln \dfrac{p_e}{p_w}}; \quad b^* = \frac{\varphi_e - \varphi_w}{\ln \dfrac{p_e}{p_w}} \ln \overline{p}_e - \varphi_e. \tag{1.10}$$

Here, φ_e, φ_w, are φ values under pattern and bottom-hole pressures p_e and p_w, respectively; $\overline{p}_e = p_e/p_0$, $\overline{p}_w = p_w/p_0$.

To calculate the instantaneous well rate value from (1.6) and (1.9), we get the expression in the following form:

$$q_o = \frac{2\pi h \left[p_0 a^* \left(\overline{p}_e \ln \overline{p}_e - \overline{p}_e - \overline{p}_w \ln \overline{p}_w + \overline{p}_w \right) - p_0 b^* (\overline{p}_e - \overline{p}_w) \right]}{\ln \dfrac{R_e}{r_w} - \dfrac{1}{2}}. \tag{1.11}$$

Considering (1.10), formula (1.11) allows us to calculate the oil flow rate (well stream) at specific values of reservoir pressure p, bottom-hole pressure p_w and saturation s. This is the output parameter of the "Reservoir" object. It is generated by an entry of a new value p_w from the object "Annular space " as a feedback, based on the "Reservoir" state changes.

§2. State of the "Reservoir" object.

State of the "Reservoir" object is determined by the current values of the pore pressure p_e, oil saturation s, and hydrocarbon saturated pore volume Ω. A "Reservoir" object state change leads to generation at the output q_o. Below we get an algorithm to calculate the values of reservoir pressure, oil saturation, and the total reservoir pore volume. With this in view, we will use the material-balance equations of the liquid and gas phases of the hydrocarbon system:

$$q_o = -\frac{d}{dt}\left[\frac{s}{B(p)} + (1-s)\frac{p\beta c(p)}{z(p)p_{at}}\right]\Omega; \tag{2.1}$$

$$q_g = -\frac{d}{dt}\left[\frac{(1-s)p\beta}{z(p)p_{at}}\left[1-c(p)\bar{\gamma}\right] + s\frac{S(p)}{B(p)}\right]\Omega. \tag{2.2}$$

Based on equations system (2.1) and (2.2), we obtain a system of differential equations describing temporal variation in reservoir pressure and oil saturation state:

$$\frac{dp}{dt} = -\frac{\dfrac{q_o}{\Omega_0\overline{\Omega}}(\alpha_4 + G\alpha_2) - (\alpha_2\alpha_3 + \alpha_1\alpha_4)\dfrac{1}{\overline{\Omega}}\dfrac{d\overline{\Omega}}{dt}}{(\alpha_5+\alpha_6)\alpha_4 + (\alpha_7+\alpha_8)\alpha_2}; \tag{2.3}$$

$$\frac{ds}{dt} = -\frac{\dfrac{q_o G}{\Omega_0\overline{\Omega}} + (\alpha_7+\alpha_8)\dfrac{dp}{dt} + \alpha_3\dfrac{1}{\overline{\Omega}}\dfrac{d\overline{\Omega}}{dt}}{\alpha_4}. \tag{2.4}$$

Here oil production rate (q_o) is calculated by (11); total pore volume –

$$\Omega(p,t) = \pi(r_e^2 - r_w^2)hm(p); \quad \overline{\Omega} = \frac{\Omega}{\Omega_0};$$

$$\alpha_1 = (1-s)\frac{p\beta c(p)}{z(p)p_{at}} - s\frac{1}{B(p)}; \quad \alpha_2 = \frac{p\beta c(p)}{z(p)p_{at}} - \frac{1}{B(p)};$$

$$\alpha_3 = s\frac{S(p)}{B(p)} - (1-s)\frac{p\beta}{z(p)p_{at}}\left[1-c(p)\bar{\gamma}(p)\right]; \quad \alpha_4 = \frac{S(p)}{B(p)} - \frac{p\beta}{z(p)p_{at}}\left[1-c(p)\bar{\gamma}(p)\right];$$

$$\alpha_5 = (1-s)\left\{\frac{p\beta c(p)}{z(p)p_{at}}\right\}'; \quad \alpha_6 = s\left[\frac{1}{B(p)}\right]'; \quad \alpha_7 = s\left[\frac{S(p)}{B(p)}\right]';$$

$$\alpha_8 = (1-s)\left[\frac{p\beta}{z(p)p_{at}}[1-c(p)\bar{\gamma}(p)]\right]';$$

$\bar{\mu}(p)$ is viscosity ratio of liquid and gas phases; $\psi(s)$ is the ratio of the relative phase permeabilities of the gas and liquid phases; G is the gas-oil ratio, defined by the following expression:

$$G = \frac{\dfrac{\bar{\mu}(p)B(p)p\beta}{z(p)p_{at}}\left[1-c(p)\bar{\gamma}(p)\right] + \dfrac{S(p)}{\psi(s)}}{\dfrac{1}{\psi(s)} + \dfrac{\bar{\mu}(p)B(p)p\beta c(p)}{z(p)p_{at}}}. \tag{2.5}$$

As it's known a drop in pore pressure results in deformation of the reservoir rocks which leads to a change in the reservoir porosity and permeability [3]. The formation deformation can be elastic, elastic-viscous or viscous depending on the rheological properties of the rocks. This paper considers an elastic formation. The changes in porosity and permeability under elastic deformation obey the exponential law and are determined by the following expressions [2]:

$$m = m_0 \exp\left[a_m\left(p - p_0\right)\right] \text{ and } k = k_0 \exp\left[\beta_k\left(p - p_0\right)\right].$$

Here a_m and, β_k are permeability and porosity change coefficients, respectively; m_0, k_0, m, k – are initial and current values of porosity and permeability, respectively; p_0 is initial reservoir pressure. The porosity expression allows us to get:

$$\bar{\Omega} = \frac{\Omega}{\Omega_0} = \exp\left[a_m(p - p_0)\right] \text{ and } \frac{d\bar{\Omega}}{dt} = a_m \exp\left[a_m(p - p_0)\right]\frac{dp}{dt}. \qquad (2.6)$$

The system of ordinary differential equations (2.3), (2.4) and (2.6) allows us to determine the change in the state of the "Reservoir" object p, s, Ω for the next time quantum. The latter leads to the generation of a new parameter q_o in the output.

§3. Description of the "Well" object.

In the existence of an annulus the "Well" object is represented as consisting of two objects (see Fig. 2) – objects of the annular space and the lifting pipe.

The object "Annular space" can be described by the following differential equation:

$$\frac{dh_a}{dt} = \frac{1}{\pi R_a^2}\left[q_o(t) - q_a\right], \qquad (3.1)$$

where q_o is rate of fluid influx into the annulus, and q_a is the rate of fluid flow from the annulus to the lifting pipe. The object state is represented by parameter h_a – height of the liquid column in the annulus. The object returns parameter p_w, that is, the downhole pressure created by the column of liquid with height h_a, to the previous object (i.e., the "Reservoir" object). The parameter p_w is defined by the expression

$$p_w = \rho g h_a, \qquad (3.2)$$

where ρ, g – is the specific gravity of the fluid and gravity acceleration, respectively.

The object "Lift" is described by the following differential equation:

$$\frac{dh_d}{dt} = \frac{1}{\pi r_w^2}\left[q_a - q_N\right], \qquad (3.3)$$

where q_N is the pumping rate; r_w is the radius of the lifting pipe. The object state is represented by a parameter h_d – the height of the liquid column in the lifting pipe. (3.3) determines the change in the dynamic fluid level in the lift. Here, the height of liquid column pumped in a time quantum Δt can be determined by the following expression:

$$\Delta h_a = \frac{q_a}{\pi R_a^2}\Delta t \qquad (3.4)$$

(3.1) and (3.3) are the equations of the material balance between the "Annular space" and the "Lift" objects, which are communicating vessels, where R_a is the reduced (effective) annulus cross-section radius, the effective area of which is $S = \pi R_a^2$ and the rate of the liquid flow from the annular space to the lifting pipe can be determined by the formula:

$$q_a = S\sqrt{2g(h_a - h_d)}. \qquad (3.5)$$

It is worth noting that formula (3.5) is rather simplified. In this case, we will use it as an example. If necessary, you can obtain a more accurate equation of fluid flow from the annular space to the lift pipe for a specific case, taking into account fluid rheology, hydraulic resistance, etc.

§4. Description of the "Pump" object.

Note that the pumping rate (q_N) is determined depending on the pump type. For example, the oil volume lifted by sicker-rod pump during the complete stroke cycle is as follows [8]:

$$V_d = \pi r_w^2 \left(h_d - \tilde{l} + R_p \sin \frac{360 \cdot 3,1416}{60 \cdot 180} N\tau \right). \tag{4.1}$$

Here, the stroke speed N is measured in 1/min, and the time τ is measured in seconds. In (4.1), the third term expresses the position of the plunger relative to the pump suspension point $\tilde{l}$ under the static fluid level in the well.

Pumping rate during a time quantum:

$$q_d = \frac{V_d}{\Delta t}. \tag{4.2}$$

And the height of pumped liquid column during the same time:

$$\Delta h_N = \frac{V_d}{\pi r_w^2}. \tag{4.3}$$

§5. Application of the conception.

With the application of the proposed concept to the above equations, a computer-imitational model of the considered process was created. The algorithm of this model has been illustrated (in the imitation modeling style) in Fig. 3. The user interface of the program is demonstrated in Fig. 4 [13].

The existence of the annulus in the system had both a quantitative and a qualitative effect on the process parameters. Taking into consideration the annulus in the well and the flow between the annulus and the lifting pipe made corrections to the pump performance.

The dependence of the pumping rate and the pump fillage on the stroke speed have been investigated (Fig. 5). It was found that with an increase in stroke speed, the pumping rate and the pump fillage increase up to a certain stroke speed, after which the pumping rate almost does not change, and the pump fillage decreases sharply. Besides, the peaks of the noted parameters may not coincide in terms of the stroke speed, which will result in a decrease in efficiency. It was also found that in order to eliminate the noted effect, thereby increasing the pump efficiency, it is necessary to transfer the pump to intermittent mode.

Experiments executed on the developed model also showed that the pump should be hanged as close to the static level as possible, in order to reduce the power consumption of the pump. At the same time, the fluid influx should not be less than the pumping rate. Moreover, there is another condition that must be satisfied to maximize the flow rate. This is the provision of the maximum flow rate with the minimum pumping speed. We also need to satisfy the condition of the maximum pump efficiency - that is, to achieve the maximum flow rate at the maximum pump fillage. As you can see, satisfying all the conditions together is a non-trivial task.

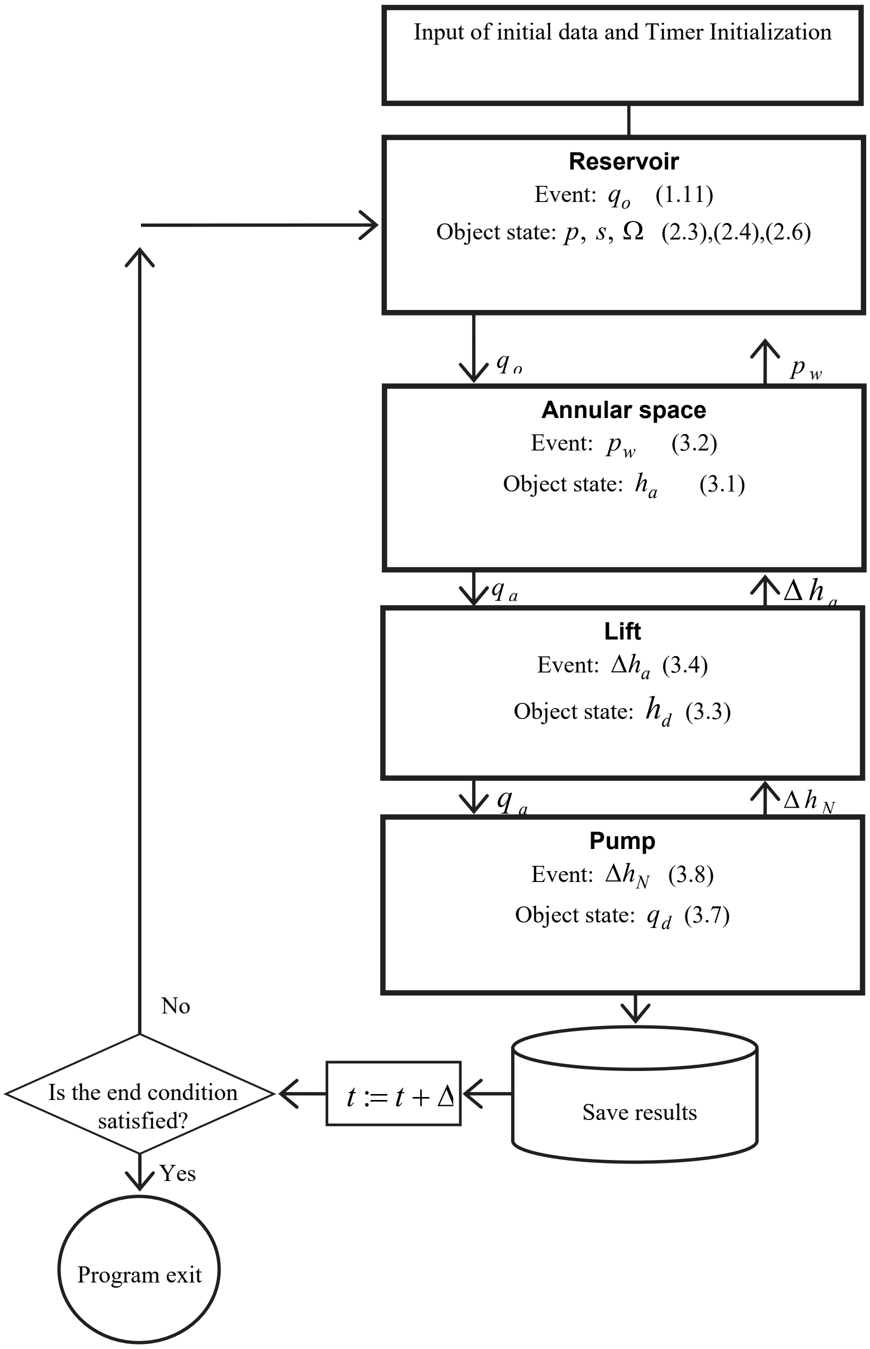


Fig. 3. Simplified flow chart of the imitation algorithm for the pump-well-reservoir system. The numbers of equations and mathematical expressions are in parentheses.

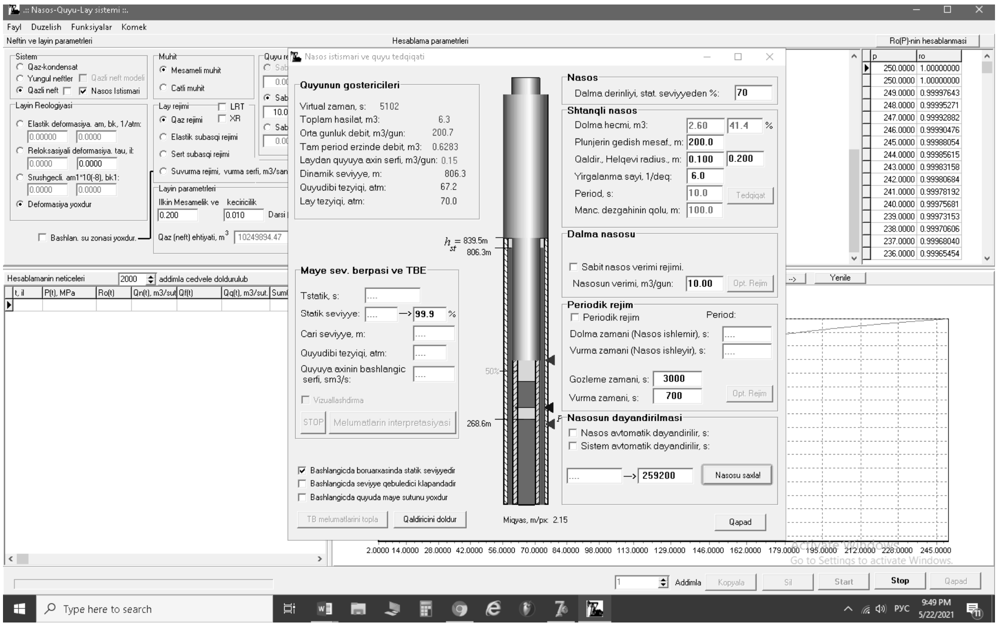


Fig. 4. User interface of the program.

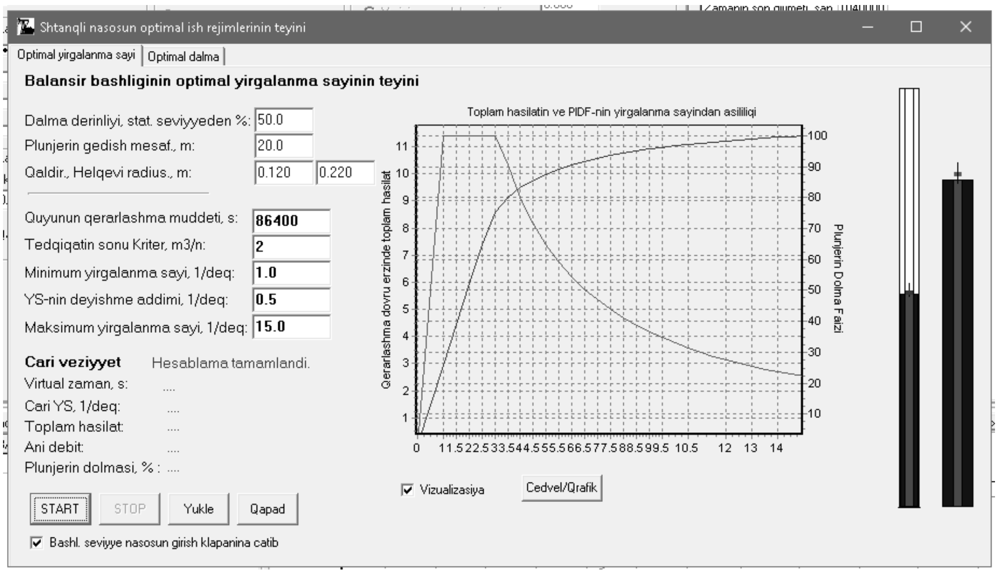


Fig. 5. The pump delivery (red line, left ordinate in m^3 / day) and pump cylinder filling percentage (green line, right ordinate in percent) versus the pumping speed.

Conclusion.

A new imitational modeling concept is presented. It is demonstrated the application of the concept in the example of the operation process of a pump-drill hole-reservoir system. Some computer experiments are made and the application of the concept is demonstrated in the example of the operation process of the pump-drill hole-reservoir system. The results confirmed the adequacy of the new concept for imitational modeling of dynamic systems.

Computer experiments have established that the condition of the maximum pumping rate at the minimum of the stroke speed is necessary, but not the only one for optimal good operation, since the maximum of the pump filling is also required to meet the optimal operating. It is also established that the pump filling does not always coincide with the maximum pumping rate with the minimum stroke speed. In such cases, the optimal operating can only be achieved by intermittent pumping.

РЕЗЮМЕ. Розвинуто нову концепцію імітаційного моделювання динамічних систем. Викладено основні поняття та терміни концепції, а також принципи побудови імітаційної моделі фізичного процесу. Запропонована концепція застосована до задачі моделювання процесу розробки пласта легкої нафти за допомогою штангової помпи в системі "помпа-свердловина-пласт". Розроблена комп'ютерно-імітаційна модель з урахуванням наявності міжтрубного простору та потоку рідини між міжтрубним простором і підйомною трубою використовується для дослідження процесу експлуатації системи. Повне заповнення циліндра штангової помпи не завжди збігається з максимальною витратою свердловини. Умовою оптимального режиму є збіг максимумів заповнення циліндра та подачі помпи. Оптимального режиму можна досягти лише шляхом переходу на періодичний режим роботи помпи. Встановлено, що для переходу на періодичний режим потрібна умова, коли при максимальній витраті не може бути досягнуто максимального наповнення циліндра помпи.

КЛЮЧОВІ СЛОВА: інтегроване моделювання, комп'ютерна симуляція, імітаційне моделювання, легка нафта, штангова помпа, система "помпа-свердловина-пласт".

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