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SYNCHRONIZATION OF CHAOTIC POWER SYSTEM WITH DELAY UNDER IMPULSIVE PERTURBATIONS

*S.P. Timoshenko Institute of Mechanics of NASU,
Nesterov str., 3, 03057, Kyiv, Ukraine; e-mail: ihorivanov2006@gmail.com*

Abstract. The sufficient conditions for exponential stability based on the Lyapunov-Razumikhin approach are obtained for the general class of nonlinear systems with delay and impulsive effects. These conditions depend on the maximum of how many times pulse effects may occur during the time delay. For the particular case when the minimum time between pulse effects is not less than doubled delay time, the results are shown in solving of complete synchronization problem of two identical electric power systems under impulsive perturbations where chaos may occur. An example of a parameter set for the power system model at which chaos is observed is given as well.

Key words: power system, impulsive systems with delay, exponential stability, chaos synchronization, Hopf bifurcation.

Introduction.

The Razumikhin approach, in addition to the Lyapunov method for the qualitative analysis of equations with delay, was developed in the works [16, 17]. Another approach to solving such problems is based on the Lyapunov functionals [19 – 21].

Lyapunov stability theory for differential systems with delay and impulsive effects is developed in [9, 11, 14, 22 – 24] (see recent reveiews [10, 12] as well). The Lyapunov-Razumikhin method is widely used while obtaining stability results [22, 23]. Such systems can be classified as hybrid systems: the dynamics of their solutions in different time points is determined by different laws of motion, each of which affects the solutions significantly [14]. For various applications, where impulsive perturbations are considered as destructive, it is advisable to assume that the time intervals between them are bounded from below, although they may be unbounded from above. Known conditions for exponential stability obtained by Lyapunov – Razumikhin method are not applicable for such impulsive perturbations [8, 26].

Electric power systems can be modeled by differential systems with delay and impulsive effects [4, 6, 15, 18]. Power systems can be considered as large-scale systems [2], for the stability study of which approaches based on the Lyapunov matrix function method may be appropriate [13]. A lot of works [4, 7, 25] dedicated to synchronization of power systems with chaotic motion in terms of master-slave interaction. Although chaotic dynamics is typical enough in power systems, the problem of their synchronization can be solved successfully by methods of Lyapunov stability theory that at first sight seems counterintuitive. In general, the problem of synchronization of power systems under impulsive perturbations remains poorly understood.

The structure of this article is as follows. Section 1 represents notations and assumptions. In Section 2, for general class of systems with delay and impulsive effects we give main theoretical results, where sufficient conditions for Lyapunov stability are established. Exponential estimation for solution of such class of systems is also given. These results are further used as a framework for synchronization analysis of models of power system. Section 3 represents a synchronization result for two power systems via system of algebraic inequalities, while the proof of which a specific form of discontinuous piecewise exponential Lyapunov function is constructed. It is illustrated in section 4 by numerical methods. A set of parameters at which a chaotic behavior of the power system occurs is given in this section too.

1. Notations and assumptions.

Let $\Delta x(t) = x(t+0) - x(t)$, $PC(X, Y)$ – space of functions $X \to Y$, that are left-continuous and have at most a countable set of the points of discontinuity of the first kind, where $X \subset \mathbf{R}$, $Y \subset \mathbf{R}^n$.

Let $\|.\|$ be Euclidean norm in $\mathbf{R}^n$ and for any compact interval $\Omega \subset \mathbf{R}$ and $\varphi \in PC(X, \mathbf{R}^n)$, $X \supseteq \Omega$ introduce the norm $\|\varphi\|_\Omega = \sup_{t\in\Omega} \|\varphi(t)\|$.

Definition 1. [3] *Function* $f : \mathbf{R}_+ \to \mathbf{R}_+$ *belongs to Hahn-class, if*

1) $f(0) = 0$;

2) $f(x) > 0$ when $x > 0$;

3) for all $x_2 > x_1 \geq 0$: $f(x_2) \geq f(x_1)$.

Definition 2. [22] *Function* $v(t,x)$ *belongs to the class* V_0*, if*

1) $v(t,x) \in C^1(E \times \mathbf{R}^n)$, where $E = \bigcup_{k=0}^{\infty} (\tau_k, \tau_{k+1})$, where $\tau_0 = 0$;

2) there exists at least one of limits:

$$\lim_{t\to\tau_k-0} v(t,x) = v(\tau_k, x); \quad \lim_{t\to\tau_k+0} v(t,x) = v(\tau_k, x), \ k \in \mathbf{N}.$$

Assumption 1. *For the function* $v(t,x)$ *there exists a function* a *from the Hahn-class such that* $a(\|x\|) \leq v(t,x), \ \forall (t,x) \in \mathbf{R}_+ \times \mathbf{R}^n$.

Assumption 2. *For the function* $v(t,x)$ *there exist functions* a, b *from the Hahn-class such that* $a(\|x\|) \leq v(t,x) \leq b(\|x\|), \ \forall (t,x) \in \mathbf{R}_+ \times \mathbf{R}^n$.

2. Main stability results.

Consider a hybrid system

$$\frac{dx}{dt} = f(t, x_t), \ t \neq \tau_k; \ \Delta x(t) = I_k(x), \ t = \tau_k, \ k \in N \tag{1}$$

and initial conditions for it

$$x(t) = \varphi_0(t), \ t \in [-r, 0], \tag{2}$$

where

$$x \in PC([-r, +\infty), \mathbf{R}^n), \ x_t \in PC([-r, 0), \mathbf{R}^n), \ x_t(\xi) = x(t+\xi), \ \xi \in [-r, 0],$$

$$f \in C\Big(\mathbf{R} \times PC([-r, 0), \mathbf{R}^n), \mathbf{R}^n\Big)$$

is Lipschitz function with respect to second argument, $0 < \tau_1 < \tau_2 < \tau_3 < \ldots, \tau_k \to \infty$, when $k \to \infty$, $I_k \in C(\mathbf{R}^n, \mathbf{R}^n)$, $\varphi_0 \in C([-r, 0), \mathbf{R}^n)$.

Suppose that initial problem (1), (2) has a unique solution. This solution can be also written more specifically as $x(t; \varphi_0)$.

Definition 3. *The solution* $x = 0$ *of the system of equations (1) is*

1) Lyapunov stable if for any $\varepsilon > 0$ there exist a number $\delta = \delta(\varepsilon) > 0$ such that $\sup_{t \ge t_0} \| x(t; \varphi_0) \| < \varepsilon$ for any φ_0 for which $\| \varphi_0 \|_{[-r,0]} \le \delta$;

2) asymptotically Lyapunov stable if it is Lyapunov stable and $x(t, \varphi_0) \to 0$ as $t \to \infty$ for any φ_0 .

Theorem 1. [22, 24] *Suppose that system (1) is such that there exists a function* $v(t, x)$ *from the class* V_0 *, that satisfies Assumption 1 and conditions*

$$D^- v(t, x(t))|_{(1)} \le 0 \text{ , whenever } v(t+0, x(t+0)) > v(t+\zeta, x(t+\zeta)) \text{ ,}$$

$$\zeta \in [\max\{-r', -t-r\}, 0] \text{ , where } D^- \text{ – left upper Dini derivative, } r' > 0 \text{ ;}$$

$$v(\tau_i + 0, x(\tau_i + 0)) \le v(\tau_i, x(\tau_i)), \ i \in \boldsymbol{N}.$$

Then equilibrium state $x = 0$ of the system (1) is stable.

From the Theorem 1 we can also derive an exponential estimation for the solutions of (1).

Corollary 1. *Suppose that system (1) is such that there exists a function* $v(t, x)$ *from the class* V_0 *, that satisfies Assumption 1 with* $a(\rho) = \rho^m$ *,* $m > 0$ *and a constant* $\gamma \in \mathbf{R}$ *such that:*

$(d/dt)v(t, x(t))|_{(1)} \le m\gamma v\ (t, x(t))$, whenever $v(t, x(t)) > e^{-m\gamma\zeta} v(t+\zeta, x(t+\zeta))$ for $\zeta \in [-r', 0)$ (Razumikhin condition);

$$v(\tau_k + 0, x(\tau_k + 0)) \le v(\tau_k, x(\tau_k)).$$

Then all the solutions of (1) from the vicinity of zero satisfy an estimation

$$\| x(t) \| \le Me^{\gamma t}, \tag{3}$$

where $M > 0$ depends only on initial conditions, and trivial solution is stable when $\gamma \le 0$ with Lyapunov exponent equals γ .

Indeed, for the function v of this corollary we can set $v_1 = ve^{-m\gamma t}$ that satisfies all the conditions of the Theorem 1, except, in general, the Assumption 1. Since the function v_1 is bounded by the Theorem 1, the function v for some $M' > 0$ may be estimated as $v(t, x(t)) \le M' e^{m\gamma t}$, from which we can get an estimation (3).

Define $\Delta\tau_k = \tau_{k+1} - \tau_k$, $\delta = \underline{\lim}_{k \to \infty} \Delta\tau_k$. Let $\chi(t)$ be a standard Heaviside step function:

$$\chi(t) = \begin{cases} 0, & if \ \ t < 0; \\ 1, & if \ \ t \ge 0. \end{cases}$$

Theorem 2. *Suppose that system (1) is such that there exist constants* $r' > 0$ *,* $l \in \mathbf{N}$ *,* $\delta_1 < \delta$ *such that* $r' < l\delta_1$ *, a constant* $\nu \ge 0$ *, a function* $v(t, x)$ *from the class* V_0 *, that satisfies Assumption 1 with* $a(\rho) = \rho^m$ *,* $m > 0$ *, and a constant* $\gamma \in \mathbf{R}$ *such that*

$$\frac{d}{dt} v(t, x(t))|_{(1)} \le (m\gamma - \nu) v(t, x(t))$$

whenever (Razumikhin condition)

$$v(t, x(t)) > \exp\left(-m\gamma\zeta + \nu(\zeta + \delta_1 \sum_k \chi(\tau_k - t - \zeta)) \right) v(t, x(t+\zeta)), \ \tau_k < t, \ \zeta \in [-r', 0); \tag{4}$$

$$v(\tau_k + 0, x(\tau_k + 0)) \le e^{\nu\delta_1} v(\tau_k, x(\tau_k)).$$

Then all the solutions of (1) from the vicinity of zero satisfy an estimation

$$\| x(t) \| \le Me^{\gamma t}, \tag{5}$$

where $M > 0$ depends only on initial conditions, and trivial solution is stable when $\gamma \le 0$ with Lyapunov exponent equals γ.

Proof. Denote $\tau_0 = -r$. Since a set

$$\{\Delta\tau_k : \Delta\tau_k < \delta_1,\ k \in \mathbf{N}\}$$

is finite, the function $v_1(t,x) = v(t,x)e^{\nu(t-\tau_k)}$, $t \in (\tau_k, \tau_{k+1}]$, $k \in \mathbf{N}_0$ satisfies the conditions of corollary 1 starting at some point in time $t = t_1$.

Indeed, take time moment t_1 for which for all τ_k such that $\tau_k > t_1 - r'$ condition $\Delta\tau_{k-1} < \delta_1$ holds. Check now the conditions of corollary 1.

Estimate the derivative of the function v_1 for $t \in [\tau_k, \tau_{k+1})$:

$$\frac{d}{dt} v_1(t, x(t))\,|_{(1)} = \frac{d}{dt} v(t,x)e^{\nu(t-\tau_k)}\,|_{(1)} \le$$

$$\le (m\gamma - \nu + \nu)e^{\nu(t-\tau_k)} v\big(t,\ x(t)\big)e^{\nu(t-\tau_k)} = m\gamma v_1\big(t,\ x(t)\big).$$

Consider now the Razumikhin condition. Take $\zeta \in [-r', 0)$ and note that the sum $\sum_k \chi(\tau_k - t - \zeta)\Delta\tau_k$ $(\tau_k < t)$ contains no more than l nonzero addends. Let the Razumikhin condition of corollary 1 is satisfied for the function v_1. We obtain:

$$v_1\big(t,\ x(t)\big) > e^{-m\gamma\zeta} v_1\big(t+\zeta,\ x(t+\zeta)\big);$$

$$e^{\nu(t-\tau_{k_2})} v\big(t,\ x(t)\big) > e^{-m\gamma\zeta + \nu(t+\zeta-\tau_{k_1})} v_1\big(t+\zeta,\ x(t+\zeta)\big);$$

$$v\big(t,\ x(t)\big) > e^{-m\gamma\zeta + \nu(\tau_{k_2} - \tau_{k_1} + \zeta)} v_1\big(t+\zeta,\ x(t+\zeta)\big)$$

for some $k_1, k_2 \in \mathbf{N}$, $k_1 \le k_2$. But for $t > t_1$

$$-m\gamma\zeta + \nu(\tau_{k_2} - \tau_{k_1} + \zeta) = -m\gamma\zeta + \nu\left(\zeta + \sum_{k:\tau_k<t} \chi(\tau_k - t - \zeta)\Delta\tau_k\right) >$$

$$> -m\gamma\zeta + \nu\left(\zeta + \delta_1 \sum_{k:\tau_k<t} \chi(\tau_k - t - \zeta)\right),$$

hence the Razumikhin condition of the proving theorem holds for the function v. Therefore when the Razumikhin condition of the proving theorem for the function v not holds, the Razumikhin condition of the corollary 1 for the function v_1 not holds too.

For the pulse effects we have

$$v_1\big(\tau_k + 0,\ x(\tau_k + 0)\big) = v\big(\tau_k + 0,\ x(\tau_k + 0)\big) \le$$

$$\le e^{\nu\delta_1} v\big(\tau_k,\ x(\tau_k)\big) < e^{\nu(\tau_k - \tau_{k-1})} v\big(\tau_k,\ x(\tau_k)\big) = v_1\big(\tau_k,\ x(\tau_k)\big).$$

Due to the continuous dependence of solutions of system (1) on the initial conditions the same is also true for point in time $t = 0$.

The theorem is proved.

3. Application: chaos synchronization in power systems under impulsive perturbations.

Consider a Single Machine Infinite Bus (SMIB) power system under impulsive perturbations in terms of rotating angle θ in a form:

$$M\ddot{\theta} + D\dot{\theta} + P_{\max}\sin\theta = P_m,\ t \neq \tau_k,\ k \in \mathbf{N};$$
$$\dot{\theta}(\tau_k + 0) = I(\theta(\tau_k),\ \dot{\theta}(\tau_k)),\ k \in \mathbf{N}, \tag{6}$$

where M – inertia moment; D – damping constant; $P_{\max}$ – maximum generator power, $P_m = A\sin wt$ – incoming power of machine, $0 = \tau_0 < \tau_1 < ... < \tau_k < ...$, I is linear function of its arguments responsible for impulse perturbations, A, w are constants.

System (6) can be represented as

$$\dot{x}_1 = x_2,\ \dot{x}_2 = -cx_2 - \beta\sin x_1 + f\sin wt,\ t \neq \tau_k,\ k \in \mathbf{N};$$
$$x_2(\tau_k + 0) = c_{k0} + c_{k1}x_1(\tau_k) + c_{k2}x_2(\tau_k),\ k \in \mathbf{N}, \tag{7}$$

where $x_1 = \theta$, $x_2 = \dot{\theta}$, $c = D/M$, $\beta = P_{\max}/M$, $f = A/M$. Set $x(t) = \left[x_1(t)\ \ x_2(t)\right]^T \in \mathbf{R}^2$, than taking into account outcome y we get the system in vector format

$$\dot{x} = Ax + f(x,t),\ t \neq \tau_k,\ k \in \mathbf{N};$$

$$x(\tau_k + 0) = C_{k0} + C_k x(\tau_k),\ k \in \mathbf{N};\ y = Cx,$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -c \end{bmatrix};\ f(x,t) = \begin{bmatrix} 0 \\ -\beta\sin(x_1) + f\sin wt \end{bmatrix};$$

$$C_{k0} = \begin{bmatrix} 0 \\ c_{k0} \end{bmatrix};\ C_k = \begin{bmatrix} 1 & 0 \\ c_{k1} & c_{k2} \end{bmatrix},\ C \in \mathbf{R}^{1\times 2}.$$

Solutions of this system are considered in a class $PC([-r,\infty),\mathbf{R}^2)$.

Definition 4. The pair of matrices (A,C), where $A \in \mathbb{R}^{n\times n}$, $C \in \mathbf{R}^{m\times n}$, $m, n \in \mathbf{N}$ is called observable, if the rank of a matrix

$$\begin{bmatrix} C \\ CA \\ CA^2 \\ ... \\ CA^{n-1} \end{bmatrix}$$

equals *n*.

Consider further two power systems with identical parameters. The task is to obtain conditions for synchronization for these two power systems under impulsive perturbations via feedback controller based on delayed data with lag $r > 0$. So, consider two systems: master

$$\dot{x}_m = Ax_m + f(x_m, t),\ t \neq \tau_k,\ k \in \mathbf{N}; x_m(\tau_k + 0) = C_{k0} + C_k x_m(\tau_k),\ k \in \mathbf{N};$$
$$y_m(t) = Cx_m(t - r) \tag{8}$$

and slave

$$\dot{x}_s = Ax_s + f(x_s,t) + L(y_s - y_m),\ t \neq \tau_k,\ k \in \mathbf{N};$$
$$x_s(\tau_k + 0) = C_{k0} + C_k x_s(\tau_k),\ k \in \mathbf{N},\ y_s(t) = Cx_s(t-r), \tag{9}$$

where state vectors are designated as x_m and x_s correspondingly. $C = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \in \mathbf{R}^{1\times 2}$ is input gain matrix, where the pair (A, C) is observable. $L \in \mathbf{R}^{2\times 1}$ – communication vector, constructed to achieve a synchronization between systems (8) and (9). If synchronization error between systems (9) and (8) is defined by $e = \begin{bmatrix} e_1 & e_2 \end{bmatrix}^T = \begin{bmatrix} x_{s1} - x_{m1} & x_{s2} - x_{m2} \end{bmatrix}^T$, then its behavior will obey the system

$$\dot{e} = Ae + f(x_s,t) - f(x_m,t) + LCe(t-r) = (A + F(t))e(t) + LCe(t-\tau),\ t \neq \tau_k,\ k \in \mathbf{N};$$
$$e(\tau_k + 0) = C_k e(\tau_k),\ k \in \mathbf{N}, \tag{10}$$

where

$$F(t) = \begin{bmatrix} 0 & 0 \\ \dfrac{-\beta(\sin x_{s1} - \sin x_{m1})}{x_{s1} - x_{m1}} & 0 \end{bmatrix}.$$

By the mean value theorem,

$$\sin x_{s1} - \sin x_{m1} = \cos\eta \cdot (x_{s1} - x_{m1});\ \eta \in [x_{s1}, x_{m1}] \text{ or } [x_{m1}, x_{s1}].$$

The function $F(t)$ can be represented as

$$F(t) = \begin{bmatrix} 0 & 0 \\ -\beta\cos\eta & 0 \end{bmatrix}.$$

Show further how to choose the feedback controller to achieve synchronization between systems (8) and (9), that is equivalent to fulfillment the condition $\| x_m(t) - x_s(t) \| \to 0$, $t \to \infty$ for any initial conditions if these systems.

The following lemma will be useful.

Lemma 1. *Let* $a, b : [0, 1] \to \mathbf{R}$, $c : [0,1] \to [0, \infty)$ *are integrable by Lebesgue,* $\lambda_1, \lambda_2 > 0$. *If for all* $s \in [0,1]$ $a(s) \le \lambda_1 c(s)$ *and* $a(s) + b(s) \le \lambda_2 c(s)$ *then for any* $t \in [0, 1]$

$$\int_0^1 a(s)ds + \int_t^1 b(s)ds \le \max\{\lambda_1, \lambda_2\} \int_0^1 c(s)ds.$$

The proof is trivial.

To represent the main result of this section we introduce some notations.

Let a, b are real constants. Denote further

$$\delta = \varliminf_{k\to\infty} (\tau_{k+1} - \tau_k);$$

$$F_{10} = L_1c_1 + aL_2c_1;\ F_{20} = L_1c_2 + 1 + a(L_1c_1 + L_2c_2 - c) + bL_2c_1;$$
$$F_{30} = a(L_1c_2 + 1) + b(L_2c_2 - c); \tag{11}$$

$$\tilde{D}_0 = b^2F_{10}^2 + F_{30}^2 + 2(2a^2 - b)F_{10}F_{30} + bF_{20}^2 - 2abF_{10}F_{20} - 2aF_{20}F_{30}; \tag{12}$$

$$\lambda_1 = \frac{b(L_1 + aL_2)^2 + (aL_1 + bL_2)^2 - 2a(L_1 + aL_2)(aL_1 + bL_2)}{b - a^2};$$

$$\lambda_{20} = \frac{b\beta^2c_2^2 + (c_1 - cc_2)^2 + 2a\beta c_2|c_1 - cc_2|}{b - a^2}; \tag{13}$$

$$\lambda_{30} = \left(L_1 c_1 + L_2 c_2\right)^2 \frac{b c_1^2 + c_2^2 - 2 a c_1 c_2}{b - a^2};$$

$$\lambda_{40} = \frac{b F_{10} + F_{30} - a F_{20} + \sqrt{\tilde{D}_0}}{b - a^2}. \tag{14}$$

For $k \in \mathbf{N}$ define

$$F_1 = F_{10} + \frac{c_2}{c_{k2}} c_{k1}(L_1 + a L_2);\ F_2 = F_{20} + \frac{c_2}{c_{k2}}\left(c_{k1}(a L_1 + b L_2) + (c_{k2} - 1)(L_1 + a L_2)\right);$$

$$F_3 = F_{30} + \frac{c_2}{c_{k2}}(c_{k2} - 1)(a L_1 + b L_2); \tag{15}$$

$$\tilde{D} = b^2 F_1^2 + F_3^2 + 2(2a^2 - b) F_1 F_3 + b F_2^2 - 2ab F_1 F_2 - 2a F_2 F_3; \tag{16}$$

$$\frac{dx}{dt} = f(t, x_t),\ t \neq \tau_k;\ \Delta x(t) = I_k(x),\ t = \tau_k,\ k \in \mathbf{N};$$

$$\lambda_{3k} = \frac{\left(c_1 L_1 c_{k2} - c_2 (L_1 c_{k1} - L_2)\right)^2}{c_2^2 c_{k2}^2 (L_1 c_1 + L_2 c_2)^2} \lambda_{30}; \tag{17}$$

$$\lambda_{4k} = \frac{b F_1 + F_3 - a F_2 + \sqrt{\tilde{D}}}{b - a^2}. \tag{18}$$

Theorem 3. *Suppose that in the system (10) an inequality* $2r < \delta$ *is satisfied, and control vector* L *is selected such that for some* $a, \gamma \in \mathrm{R}$, $b, \nu \in \mathrm{R}_+$ *the following conditions are hold*

(a) $b > a^2$; $g(\lambda_1, \lambda_{20}, \lambda_{30}, \lambda_{40}) < -\nu + 2\gamma$;

(b) $g(\lambda_1, \lambda_{20}, \lambda_{30}, \sup_{k \in N}\{\lambda_{4k}\}) < -\nu + 2\gamma$; $\sup_{k \in N}\left\{g(\lambda_1, \lambda_{2k}, \lambda_{3k}, \lambda_{4k})\right\} < -\nu + 2\gamma$,

where

$$g(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \left(\lambda_4 + 4\sqrt{\lambda_1}\left(\sqrt{\lambda_2} + \sqrt{\lambda_3} e^{\left(\frac{\nu}{2} - \gamma\right) r}\right) \frac{e^{\left(\frac{\nu}{2} - \gamma\right) r} - 1}{\nu - 2\gamma}\right);$$

(c) $(b - a^2) e^{2\nu\delta} + (-b(c_{k2}^2 + 1) + 2ab c_{k1}(c_{k2} - 1) - b^2 c_{k1}^2 + 2a^2 c_{k2}) e^{\nu\delta} + (b - a^2) c_{k2}^2 \geq 0$;

(d) $(1 + b) e^{\nu\delta} \geq 1 + 2a c_{k1} + b^2 c_{k1}^2 + b c_{k2}^2$.

Then for all $t > 0$ solutions of the system (10) may be exponentially estimated as

$$\|e(t)\| \leq M e^{\gamma t},$$

where $M > 0$ depends only on the initial conditions.

Proof. Take $\delta_1 > 0$, $2r < \delta_1 < \delta$. Consider system (10) and its auxiliary function $v(e)$ as

$$v(e) = e_1^2 + 2a e_1 e_2 + b e_2^2.$$

When condition (a) of this theorem is satisfied, function v belongs to the class V_0 and it fulfills the conditions of the assumption A. We will require the fulfilling by this function the conditions of Theorem 2 for $\delta_1 < \delta$, $m = 2$, $l = 1$, specified r and $r' = 2r$. Denote $\Delta\tau_k = \tau_{k+1} - \tau_k$. These conditions can be represented as:

$$\left.\frac{dv(e(t))}{dt}\right|_{(10)} \le (2\gamma - \nu)v(e(t)),\ t \ne \tau_k,\ k \in \boldsymbol{N}, \tag{19}$$

whenever

$$\begin{gathered} v\big(e(t)\big) > e^{(\nu-2\gamma)\zeta} v\big(e(t+\zeta)\big),\ \zeta \in [-2r,0), \text{ for } \tau_k - t \notin [-2r,0); \\ v\big(e(t)\big) > e^{-2\gamma\zeta + \nu(\zeta + \chi(\tau_k - t - \zeta)\delta_1)} v\big(e(t+\zeta)\big),\ \zeta \in [-2r,0), \text{ for } \tau_k - t \in [-2r,0) \end{gathered} \tag{20}$$

and

$$v(\tau_k + 0, e(\tau_k + 0)) \le e^{\nu\delta_1} v(\tau_k, e(\tau_k)),\ k \in \boldsymbol{N}. \tag{21}$$

respectively.

Further for definiteness we set $\tau_k \in [t-r,t)$.

Denote

$$a_{11} = L_1 c_1;\ a_{12} = L_1 c_2 + 1;\ a_{21} = L_2 c_1;\ a_{22} = L_2 c_2 - c,$$

so $A + LC = [a_{ij}]_{i,j=1,2}$.

Then derivative of the function v along the system (10) may be formulated as

$$\frac{d}{dt} v(e(t))|_{(10)} = 2(-a\beta\cos\eta + a_{11} + aa_{21})e_1^2 + 2(a_{12} +$$

$$+a(a_{11} + a_{22}) + b(a_{21} - \beta\cos\eta))e_1 e_2 + 2(aa_{12} + ba_{22})e_2^2 +$$

$$+2\big(c_1(e_1(t-r) - e_1(t)) + c_2(e_2(t-r) - e_2(t))\big)\big(e_1(L_1 + aL_2) + e_2(aL_1 + bL_2)\big).$$

By Newton – Leibnitz,

$$e_1(t) = e_1(t-r) + \int_{t-r}^{t} \big(e_2(s) + L_1 c_1 e_1(s-r) + L_1 c_2 e_2(s-r)\big) ds;$$

$$e_2(t) = e_2(t-r) + \int_{t-r}^{t} \big(-\beta\cos\eta e_1(s) - ce_2(s) + L_2 c_1 e_1(s-r) + L_2 c_2 e_2(s-r)\big) ds +$$

$$+\sum_{k=1}^{\infty} I_{[t-r,t)}(\tau_k)\big(c_{k1} e_1(\tau_k) + (c_{k2} - 1)e_2(\tau_k)\big),$$

where $I_\Omega(s)$ is an indicator function.

Therefore for $\tau_k \in [t-r,t)$ derivative if the function v along the system (10) may be written as

$$\frac{d}{dt} v(e(t))|_{(10)} = I_0 + I_{r0} + I_\delta,$$

where

$$I_0 = 2(a_{11} + a(a_{21} - \beta\cos\eta))e_1^2 +$$

$$+2(a_{12} + a(a_{11} + a_{22}) + b(a_{21} - \beta\cos\eta))e_1 e_2 + 2(aa_{12} + ba_{22})e_2^2;$$

$$I_{r0} = -2(e_1(L_1 + aL_2) + e_2(aL_1 + bL_2)) \times$$

$$\times \int_{t-r}^{t} \big(-\beta c_2 \cos\eta e_1(s) + (c_1 - cc_2)e_2(s) +$$

$$+(L_1c_1+L_2c_2)(c_1e_1(s-r)+c_2e_2(s-r)))\,ds;$$

$$I_\delta=-2c_2\left(e_1(L_1+aL_2)+e_2(aL_1+bL_2)\right)\left(c_{k1}e_1(\tau_k)+(c_{k2}-1)e_2(\tau_k)\right).$$

Note, that in the case $\tau_k \notin [t-r,t)$ in the expression for derivative of the function function v there will be no addend I_δ.

Consider I_δ. It can be represented as

$$I_\delta=-2c_2(e_1(L_1+a_2)+e_2(aL_1+bL_2))\left(\frac{c_{k1}}{c_{k2}}e_1(\tau_k^+)+\frac{c_{k2}-1}{c_{k2}}e_2(\tau_k^+)\right)=\delta I+\delta I_r(\tau_k,t),$$

where

$$\delta I=-2c_2\left(e_1(L_1+a_2)+e_2(aL_1+bL_2)\right)\left(\frac{c_{k1}}{c_{k2}}e_1(t)+\frac{c_{k2}-1}{c_{k2}}e_2(t)\right);$$

$$\delta I_r(\tau_k,t)=2c_2(e_1(L_1+a_2)+e_2(aL_1+bL_2))\times$$

$$\times\int_{\tau_k}^{t}\left(-\frac{\beta\cos\eta(c_{k2}-1)}{c_{k2}}e_1(s)+\frac{c_{k1}-c(c_{k2}-1)}{c_{k2}}e_2(s)+\right.$$

$$\left.+\frac{L_1c_1c_{k1}+L_2c_1(c_{k2}-1)}{c_{k2}}e_1(s-r)+\frac{L_1c_2c_{k1}+L_2c_2(c_{k2}-1)}{c_{k2}}e_2(s-r)\right)ds.$$

Define $I=I_0+\delta I$, $I_r(\tau_k,t)=I_{r0}+\delta I_r(\tau_k,t)$. We can estimate

$$\frac{d}{dt}v(e(t))|_{(10)}\le\max\left\{I_0+I_{r0},\ \sup_{\zeta\in[-r,0]}\{I+I_r(t+\zeta,t)\}\right\}. \qquad (22)$$

The quadratic forms may be estimated as

$$\left((L_1+aL_2)e_1+(aL_1+bL_2)e_2\right)^2\le\lambda_1v\left(e(t)\right);$$

$$\left(-\beta c_2\cos\eta e_1+(c_1-cc_2)e_2\right)^2\le\lambda_{20}v(e(t));$$

$$(L_1c_1+L_2c_2)^2(c_1e_1+c_2e_2)^2\le\lambda_{30}v(e(t));$$

$$\left(-\beta\frac{c_2}{c_{k2}}\cos\eta e_1+\left(c_1-\frac{c_2}{c_{k2}}(c_{k1}+c)\right)e_2\right)^2\le\lambda_{2k}v(e(t));$$

$$\left(\left(c_1^2L_1-c_1\frac{c_2}{c_{k2}}(L_1c_{k1}-L_2)\right)e_1+\left(c_1c_2L_1-\frac{c_2^2}{c_{k2}}(L_1c_{k1}-L_2)\right)e_2\right)^2\le\lambda_{3k}v(e(t)),$$

where $\lambda_1, \lambda_{20}, \lambda_{30}, \lambda_{2k}, \lambda_{3k}$ are defined in (13) and (17).

Further by Razumikhin condition (20) we get

$$\int_{t-r}^{t}\left(-\beta c_2\cos\eta e_1(s)+(c_1-cc_2)e_2(s)\right)ds\le\int_{t-r}^{t}\sqrt{\lambda_{20}v\left(e(s)\right)}ds\le$$

$$\leq \int_{t-r}^{t} \sqrt{e^{-2\gamma(t-s)+\nu(t-s-\delta_1\chi(\tau_k-s))}\lambda_{20}\nu(e(t))}ds \leq 2\frac{e^{\left(\frac{\nu}{2}-\gamma\right)r}-1}{\nu-2\gamma}\sqrt{\lambda_{20}\nu(e(t))};$$

$$\int_{t-r}^{t} \left(L_1c_1+L_2c_2\right)\left(c_1e_1\left(s-r\right)+c_2e_2\left(s-r\right)\right)ds \leq \int_{t-2r}^{t-r} \sqrt{\lambda_{30}\nu\left(e(s)\right)}ds \leq$$

$$\leq \int_{t-2r}^{t-r} \sqrt{e^{-2\gamma(t-s)+\nu(t-s-\delta_1\chi(\tau_k-s))}\lambda_{30}\nu(e(t))}ds \leq 2\frac{e^{(\frac{\nu}{2}-\gamma)r}-1}{\nu-2\gamma}e^{(\frac{\nu}{2}-\gamma)r}\sqrt{\lambda_{30}\nu(e(t))};$$

$$\int_{t-r}^{t}\left(-\beta\frac{c_2}{c_{k2}}\cos\eta e_1(s)+\left(c_1-\frac{c_2}{c_{k2}}(c_{k1}+c)\right)e_2(s)\right)ds \leq$$

$$\leq \int_{t-r}^{t}\sqrt{\lambda_{2k}\nu(e(s))}ds \leq \int_{t-r}^{t}\sqrt{e^{-2\gamma(t-s)+\nu(t-s-\delta_1\chi(\tau_k-s))}\lambda_{2k}\nu(e(t))}ds \leq 2\frac{e^{\left(\frac{\nu}{2}-\gamma\right)r}-1}{\nu-2\gamma}\sqrt{\lambda_{2k}\nu(e(t))};$$

$$\int_{t-r}^{t}\left(\left(c_1^2L_1-c_1\frac{c_2}{c_{k2}}(L_1c_{k1}-L_2)\right)e_1(s-r)+\left(c_1c_2L_1-\frac{c_2^2}{c_{k2}}(L_1c_{k1}-L_2)\right)e_2(s-r)\right)ds \leq$$

$$\leq \int_{t-2r}^{t-r}\sqrt{\lambda_{3k}\nu(e(s))}ds \leq \int_{t-2r}^{t-r}\sqrt{e^{-2\gamma(t-s)+\nu(t-s-\delta_1\chi(\tau_k-s))}\lambda_{3k}\nu(e(t))}ds \leq$$

$$\leq 2\frac{e^{\left(\frac{\nu}{2}-\gamma\right)r}-1}{\nu-2\gamma}e^{\left(\frac{\nu}{2}-\gamma\right)r}\sqrt{\lambda_{3k}\nu(e(t))}.$$

Note that when define for $\nu-2\gamma=0$ as r, the function $\left(e^{\left(\frac{\nu}{2}-\gamma\right)r}-1\right)\Big/\left(\nu-2\gamma\right)$ become an entire function of all its arguments, thus there is no need to consider the case $\nu-2\gamma=0$ specifically.

Thus, the following estimates hold:

$$\left|I_r(t,t)\right| = |I_{r0}| \leq 4\sqrt{\lambda_{10}}\left(\sqrt{\lambda_{20}}+\sqrt{\lambda_{30}}e^{\left(\frac{\nu}{2}-\gamma\right)r}\right)\frac{e^{\left(\frac{\nu}{2}-\gamma\right)r}-1}{\nu-2\gamma}\nu\left(e(t)\right);$$

$$|I_r(t-r,t)| \leq 4\sqrt{\lambda_{1k}}\left(\sqrt{\lambda_{2k}}+\sqrt{\lambda_{3k}}e^{\left(\frac{\nu}{2}-\gamma\right)r}\right)\frac{e^{\left(\frac{\nu}{2}-\gamma\right)r}-1}{\nu-2\gamma}\nu\left(e(t)\right). \tag{23}$$

Estimate now the expression I by the function ν. By taking into account the constants introduced in (11) we obtain characteristic equation for the regular bundle of forms $I-\lambda\nu(e(t))$ as

$$\begin{vmatrix} 2F_1-2a\beta\cos\eta-\lambda & F_2-b\beta\cos\eta-a\lambda \\ F_2-b\beta\cos\eta-a\lambda & 2F_3-b\lambda \end{vmatrix}=0.$$

The discriminant of this quadratic equation is defined by (12). The upper estimation of quadratic form is determined by bigger root of this equation and can be represented as

$$I_0 \le \lambda_{40} v(e(t)), \tag{24}$$

where λ_{40} is defined by the expression (14).

Similarly, we can obtain

$$I \le \lambda_{4k} v(e(t)), \tag{25}$$

with λ_{4k} defined by the expression (18).

Due to lemma 1, by using obtained estimates (23) – (25), from the estimate (22) it follows that

$$\frac{d}{dt} v(e(t))|_{(10)} \le \max\left\{\left(\lambda_{40} + 4\sqrt{\lambda_{10}}\left(\sqrt{\lambda_{20}} + \sqrt{\lambda_{30}}e^{\left(\frac{\nu}{2}-\gamma\right)r}\right)\frac{e^{\left(\frac{\nu}{2}-\gamma\right)r}-1}{\nu-2\gamma}\right)v(e(t));\right.$$

$$\left(\lambda_{4k} + 4\sqrt{\lambda_{10}}\left(\sqrt{\lambda_{20}} + \sqrt{\lambda_{30}}e^{\left(\frac{\nu}{2}-\gamma\right)r}\right)\frac{e^{\left(\frac{\nu}{2}-\gamma\right)r}-1}{\nu-2\gamma}\right)v(e(t));$$

$$\left.\left(\lambda_{4k} + 4\sqrt{\lambda_{1k}}\left(\sqrt{\lambda_{2k}} + \sqrt{\lambda_{3k}}e^{\left(\frac{\nu}{2}-\gamma\right)r}\right)\frac{e^{(\frac{\nu}{2}-\gamma)r}-1}{\nu-2\gamma}\right)v(e(t))\right\}.$$

The condition (19) leads to inequalities

$$\left(\lambda_{40} + 4\sqrt{\lambda_{10}}\left(\sqrt{\lambda_{20}} + \sqrt{\lambda_{30}}e^{\left(\frac{\nu}{2}-\gamma\right)r}\right)\frac{e^{(\frac{\nu}{2}-\gamma)r}-1}{\nu-2\gamma}\right) < -\nu + 2\gamma;$$

$$\left(\sup_{k\in N}\{\lambda_{4k}\} + 4\sqrt{\lambda_{10}}\left(\sqrt{\lambda_{20}} + \sqrt{\lambda_{30}}e^{\left(\frac{\nu}{2}-\gamma\right)r}\right)\frac{e^{(\frac{\nu}{2}-\gamma)r}-1}{\nu-2\gamma}\right) < -\nu + 2\gamma;$$

$$\sup_{k\in N}\left\{\left(\lambda_{4k} + 4\sqrt{\lambda_{1k}}\left(\sqrt{\lambda_{2k}} + \sqrt{\lambda_{3k}}e^{\left(\frac{\nu}{2}-\gamma\right)r}\right)\frac{e^{(\frac{\nu}{2}-\gamma)r}-1}{\nu-2\gamma}\right)\right\} < -\nu + 2\gamma,$$

that coincide with condition (b) of this theorem.

The condition (21) holds for all $k \in \mathbf{N}$, except for those belonging to at most a finite set, if the following condition is satisfied:

$$v(e(\tau_k + 0)) \le e^{\nu\delta_1} v(e(\tau_k)),\ k \in \mathbf{N}.$$

This leads to the inequality

$$e_1^2 + 2a(c_{ki1}e_1 + c_{ki2}e_2)e_1 + b(c_{k1}e_1 + c_{k2}e_2)^2 \le e^{\nu\delta_1}(e_1^2 + 2ae_1e_2 + be_2^2)\ .$$

Combining by all δ_1 the conditions, that guarantee the holding of this inequality between quadratic forms, we get the conditions (c) and (d) of the theorem.

The theorem is proved.

4. Numerical simulations.

Illustrate these results by using numerical methods.

Fig. 1 shows synchronization region estimation (8) and (9) (grey color) in a space of control parameters L_1, L_2 for given parameters of the initial system $c_1 = 1$; $c_2 = 1$; $c = 3$; $\delta = 3$; $r = 0,1$; $c_{k1} = -0,5$, $c_{k2} = 1$, $k \in \mathbf{N}$, $\beta = 1$. Exponential estimations for the error are also given.

Note that presented on fig. 1 level lines may be not smooth since the condition (b) of the theorem can be presented via max function that is not smooth.

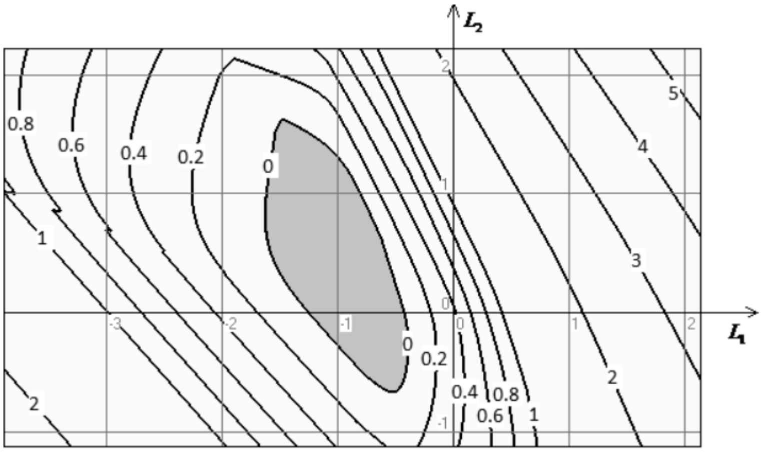
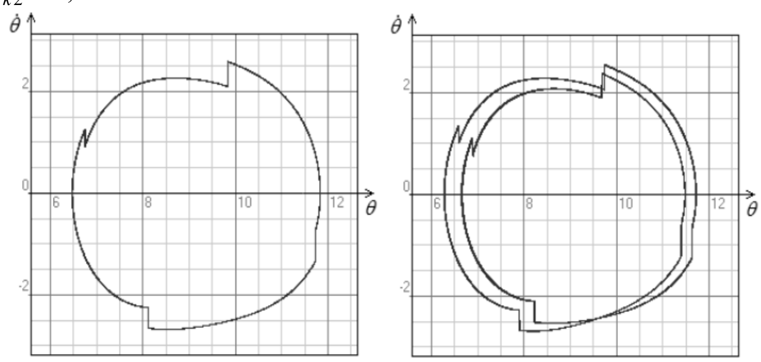


Fig. 1

A chaos in the system (7) is really possible.

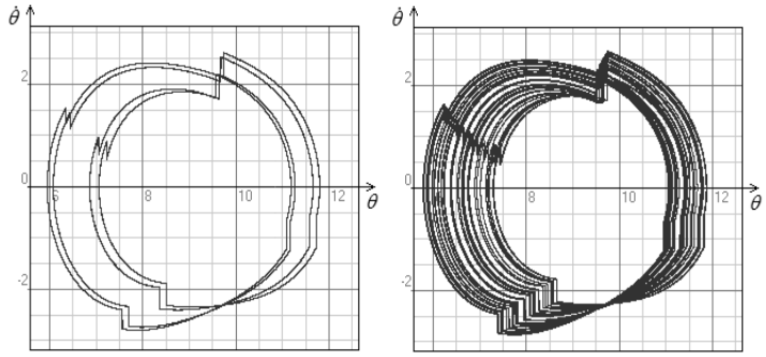
The possibility of a chaos in (7) without impulsive effects was established in paper [1] via numerical methods for the parameters $c = 0,5$; $\beta = 1$; $w = 1$; $f = 2,43$; $c_{k0} = 0$, $c_{k1} = 0$, $c_{k2} = 1$, $k \in N$.



a *b*

Fig. 2

Dynamics of solutions of the system (7) with nontrivial pulse effects in some special cases can be chaotic as well. Indeed, when interval between impulsive perturbations are constant and synchronized with external periodic impact, by taking $c = 0,5$; β=1; $w = 1$; f=3,65; δ=1,570796; $c_{k0} = 0$, $c_{k1} = 0,05 \cdot (-1)^{k+1}$, c_{k2}=1, $k \in \boldsymbol{N}$, we obtain an attractor that is similar to those in a work [1] (see Fig. 2, *a*). The origins of the chaos may be also traced: while decreasing of parameter f Hopf bifurcations are occurred, after each of which period of the solutions is doubling (see solution with doubled period for $f = 3{,}55$ on Fig. 2, *b*, and with quadruplicate period for $f = 3{,}449$ on Fig. 3, *a*).



a *b*

Fig. 3

Bifurcation values of parameter f are: $f^{(1)} = 3{,}58$; $f^{(2)} = 3{,}47$; $f^{(3)} = 3{,}449$; $f^{(4)} = 3{,}4453$ etc., where $f^{(i)}$ – bifurcation value of parameter that corresponds to bifurcation that switch between system with stable 2^{i-1}-periodic solutions and system with stable 2^i-periodic. By Feigenbaum universality [5] it may be suggested that beginning with $f^* \approx 3{,}44$ (up to 0,01) and lower the chaos in the system is observed. Fig. 3, *b* illustrates that for $f = 3{,}42$ it can be observed chaotic behavior without any stable periodic trajectories.

Conclusion.

Although in general case a condition 4 when represented by continuous functions can be interpreted as "hierarchy of Razumikhin conditions", that would lead to consideration of multiple cases in applications, but they are well comparable.

The established synchronization conditions and Lyapunov exponent estimations can be used to counter impulsive perturbations of electric power systems where chaos may appear (see also [4, 25]).

If r is considered as a small parameter that can be appropriate for various power systems controllers, the results obtained in Section 4 require the only restriction for the intervals between pulse effects: they should not have zero as a limit point. When $r \to 0$ the stability conditions become closer and closer to those for both nondelay system (10) with $r = 0$ and for the same system with $r = 0$ and with all phase variables that were delayed mapped by inverse pulse mapping.

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Р Е З Ю М Е. Отримано достатні умови експоненціальної стійкості на основі методу Ляпунова-Разуміхіна для загального класу нелінійних систем із запізненням та імпульсною дією. Ці умови залежать від максимальної кількості актів імпульсної дії, що можуть відбутись протягом часу запізнення. Для випадку, коли інтервали між актами імпульсної дії не є меншими подвоєного часу запізнення, розв'язано задачу повної синхронізації двох ідентичних електроенергетичних систем при імпульсних збуреннях, в яких може виникнути хаос. Крім того, наведено приклад набору параметрів для моделі енергосистеми, за якого може виникнути хаос.

К Л Ю Ч О В І С Л О В А: енергосистема, системи з імпульсною дією, експоненціальна стійкість, синхронізація хаосу, біфуркація Хопфа.

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