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A MATHEMATICAL MODEL FOR THE DEFORMATION PROBLEM IN A GENERALIZED THERMOELASTIC MEDIUM UNDER MODIFIED GREEN-LINDSAY MODEL

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Abstract. The objective of this study is to examine the effect of non-local and hyperbolic two-temperature parameters on the thermostressed state using the modified Green – Lindsay generalized theory of thermoelasticity. At that, a ramp-type normal load/thermal source is used. The governing equations are transformed into a dimensionless form and simplified with the aid of Laplace and Fourier transforms. The transformed domain is used to obtain the physical field quantities such as stresses, displacement vector components, thermodynamic temperature, and conductive temperature. The numerical inversion technique is employed to recover the equations in the physical domain. The impact of non-local, hyperbolic two-temperature, and various theories of thermoelasticity on the material behavior is presented in the form of graphs. The results emphasize on the importance of considering different aspects and theories in the analysis of material behavior. The several unique cases are discussed and displayed.

Key word: non-local parameter, hyperbolic two-temperature parameter, modified Green – Lindsay theory, Laplace and Fourier transforms.

1. Introduction.

The main focus of thermoelasticity is the study of linear and nonlinear temperature-dependent behavior of materials, and the development of mathematical models to predict how materials will respond to thermal loads. This field is important in a variety of engineering applications, including the design of heat exchangers, turbines, and other high-temperature components. The first and second widely accepted generalised theories of thermoelasticity, respectively, were put forth by Lord and Shulman (L-S) [1] and Green and Lindsay (G-L) [2].

The two-temperature theory of thermoelasticity, as developed by Chen and Gurtin [3] and Chen et al. [4], aims to study the thermal behavior of materials under both temperature and stress. In this theory, it is assumed that the material has two separate temperatures: Thermodynamic temperature and conductive temperature. A generalised theory of two-temperature thermoelasticity was first presented by Youssef [5]. The hyperbolic two-temperature (HTT) generalized thermoelasticity theory is extensions of the basic two-temperature theory that take into account additional factors such as heat transfer across material interfaces and the influence of thermal waves on the material's response. The two temperature generalised theory was modified by Youssef and El-Bary [6] and provided the HTT generalised thermoelasticity theory.

Kumar et al. [7] investigated the thermal interactions between an infinite elastic medium and a cylindrical cavity using the HTT theory of thermoelasticity. The results of this study provide insights into the thermal behavior of materials under stress and temperature changes. Hobiny et al. [8] went further and introduced a photo-thermoelastic model within the framework of the HTT theory. This allowed them to analyze the thermal behavior of materials under both temperature and stress changes, as well as the effects of external radiation on the material.

The non-local theory of thermoelasticity is a modified form of the classical linear theory of thermoelasticity. This theory is particularly relevant in the analysis of materials with microstructures, such as composites and nano-materials, as well as materials with long-range interactions, such as biological tissues. In these materials, the non-local effects can play a significant role in determining the overall response of the material. The idea behind the non-local theory of thermoelasticity was created by Eringen [9, 10]. Sarkar and Sarkar [11] studied wave propagation in a non-local thermoelastic medium for the Green and Naghdi theory-II (without energy dissipation) of generalised thermoelasticity. The effect of a moving heat source on a magneto-thermoelastic rod was examined by Bayones et al. [12] within the framework of Eringen's non-local theory with three-phase lag and a memory-dependent derivative. By using the eigen value method, Saeed and Abbas [13] investigated the effects of the non-local thermoelastic parameters in the Green and Naghdi model without energy dissipation for a nanoscale material.

By adding strain rate terms to the Green – Lindsay model, Yu et al. [14] established a model of generalised thermoelasticity based on the extended thermodynamic principle. Quintanilla [15] described some qualitative results for the MG-L model of thermoelasticity. Farshad et. al. [16] presented a nonlinear numerical technique to solve the generalised thermoelasticity governing equations in a significant deformation zone of an elastic media subjected to thermal shock in the context of the MG-L theory of generalised thermoelasticity. Nihar and Sarkar [17] discussed the problem of reflection and propagation of thermoelastic harmonic plane waves on the stress-free and isothermal surface by using MG-L model. Sarkar and Mondal [18] provides analytical formulas for the coupled longitudinal wave incident on the free surface to obtain the amplitude ratios and their corresponding energy ratios for reflected thermoelastic waves.

The present investigation is concerned with interaction between stress and strain in a thermoelastic solid due to ramp type normal force/ thermal source, as it is very important due to its many applications in the fields of geophysics, plasma physics and related topics. Such mechanical and thermal loading may produce severe deformations and temperature rise in the medium, which causes excessive wear and even cracking near the contact zone. The dynamic response of homogeneous, isotropic generalized thermoelastic medium with hyperbolic two temperatures in the context of MG-L model is studied.

The variations of the normal stress, tangential stress, thermodynamic temperature and conductive temperature are illustrated graphically to compare the results for non-local and hyperbolic two temperature. The comparison of the results obtained from the MG-L model with those obtained from other theories of thermoelasticity, such as the L-S and G-L theories, highlights the importance of considering various aspects and theories in the analysis of material behavior and the potential impact they have on the results. This study can provide valuable information for the design and optimization of materials for various applications, particularly in high-temperature and dynamic environments.

2. Basic Equations.

Following Yu et. al. [14], Eringen [10], Youseff and El-Bary [6], the field equations and constitutive relations in the context of MG-L theory of thermoelasticity with non-local and hyperbolic two temperature in absence of body forces and heat sources can be written as:

$$\left(1+\eta_1\tau_1\frac{\partial}{\partial t}\right)\left[(\lambda+\mu)\nabla(\nabla\cdot\vec{u})+\mu\nabla^2\vec{u}\right]-\beta_1\left(1+\eta_2\tau_1\frac{\partial}{\partial t}\right)\nabla T=\rho\left(1-\xi_1{}^2\nabla^2\right)\frac{\partial^2\vec{u}}{\partial t^2}\,; \quad (1)$$

$$K^*\nabla^2\varphi=\left(1+\eta_3\tau_0\frac{\partial}{\partial t}\right)\beta_1T_0\dot{u}_{k,k}+\left(1+\eta_4\tau_0\frac{\partial}{\partial t}\right)\rho C_e\dot{T}\,; \quad (2)$$

$$t_{ij}=\left(1+\eta_1\tau_1\frac{\partial}{\partial t}\right)\left[\lambda u_{k,k}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)\right]-\beta_1\left(1+\eta_2\tau_1\frac{\partial}{\partial t}\right)T\delta_{ij}\,; \quad (3)$$

$$T=\left(1-a\nabla^2\right)\varphi\,; \tag{4}$$

$$\ddot{\varphi}-\ddot{T}=\beta^*\nabla^2\varphi\,, \tag{5}$$

where $\lambda,\ \mu$ – Lame's constants; $\vec{u}$ – displacement vector; $\beta_1=\left(3\lambda+2\mu\right)\alpha_t$, α_t – coefficient of linear thermal expansion; ξ_1 – non-local parameter; t_{ij} – components of stress tensor; t – time; $\rho,\ C_e$ – density and specific heat respectively; K^* – thermal conductivity; ∇^2 – Laplacian operator; φ – conductive temperature; T – thermodynamic temperature; a – two temperature parameter; τ_0 – relaxation time, δ_{ij} – Kronecker's delta, β^* – HTT parameter.

The equations (1) – (5) reduces to the following cases

$\eta_1=\eta_2=\eta_3=\eta_4=1$, Modified Green – Lindsay, (MG-L), (2018);

$\eta_1=\eta_3=0,\ \eta_2=\eta_4=1$, Green – Lindsay, (G-L), (1972);

$\eta_1=\eta_2=0,\ \eta_3=\eta_4=1$, Lord – Shulman, (L-S), (1967);

$\eta_1=\eta_2=\eta_3=\eta_4=0$, Coupled thermoelasticity, (C-T), (1980).

3. Formulation and solution of the problem.

A thermoelastic half-space under MG-L thermoelastic model is taken into the account along with non-local and HTT parameter and the region $x_3\geq 0$ is considered, which is homogenous and isotropic. The two dimensional problem is in the plane (x_1-x_3), which is subjected to Ramp type normal force/ thermal source, therefore with these consideration, we have

$$\vec{u}=\left(u_1,\ 0,\ u_3\right). \tag{6}$$

Dimensionless quantities are:

$$\begin{gathered}\left(x_i',u_i'\right)=\frac{\omega_1}{c_1}\left(x_i,u_i\right);\quad t_{3i}'=\frac{1}{\beta_1T_0}t_{3i};\quad \left(\varphi',T'\right)=\frac{1}{T_0}\left(\varphi,T\right);\quad \xi_1'=\frac{\omega_1}{c_1}\xi_1;\quad a'=\frac{\omega_1^2}{c_1^2}a;\\ \left(t',\tau_0'\right)=\omega_1\left(t,\tau_0\right);\quad \beta^{*\prime}=\frac{1}{c_1^2}\beta^*;\quad F_1'=\frac{1}{\beta_1T_0}F_1\quad (i=1,3),\end{gathered} \tag{7}$$

where $c_1^2=\left(\dfrac{\lambda+2\mu}{\rho}\right)$ and $\omega_1=\left(\dfrac{\rho C_e c_1^2}{K^*}\right)$.

Equations (1) – (5) by taking into account (6) and (7) determines

$$\left(1+\eta_1\tau_1\frac{\partial}{\partial t}\right)\left[a_1\frac{\partial e}{\partial x_1}+a_2\nabla^2u_1\right]-a_3\left(1+\eta_2\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1}=\left(1-\xi_1^2\nabla^2\right)\frac{\partial^2u_1}{\partial t^2}; \tag{8}$$

$$\left(1+\eta_1\tau_1\frac{\partial}{\partial t}\right)\left[a_1\frac{\partial e}{\partial x_3}+a_2\nabla^2u_3\right]-a_3\left(1+\eta_2\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3}=\left(1-\xi_1^2\nabla^2\right)\frac{\partial^2u_3}{\partial t^2}; \tag{9}$$

$$T=\left(1-a\nabla^2\right)\varphi\,; \tag{10}$$

$$\ddot{\varphi}-\ddot{T}=\beta^*\nabla^2\varphi\,; \tag{11}$$

$$t_{33}=\left(1+\eta_1\tau_1\frac{\partial}{\partial t}\right)\left[a_5\frac{\partial u_1}{\partial x_1}+a_6\frac{\partial u_3}{\partial x_3}\right]-\left(1+\eta_2\tau_1\frac{\partial}{\partial t}\right)T; \tag{12}$$

$$t_{31}=a_7\left(1+\eta_1\tau_1\frac{\partial}{\partial t}\right)\left(\frac{\partial u_3}{\partial x_1}+\frac{\partial u_1}{\partial x_3}\right), \tag{13}$$

where

$$a_1=\frac{\lambda+\mu}{\rho c_1^2}\,;\quad a_2=\frac{\mu}{\rho c_1^2}\,;\quad a_3=\frac{\beta_1 T_0}{\rho c_1^2}\,;\quad a_4=\frac{\beta_1 c_1^2}{K^*\omega_1}\,;$$

$$a_5=\frac{\lambda}{\beta_1 T_0}\,;\quad a_6=\frac{\lambda+2\mu}{\beta_1 T_0}\,;\quad a_7=\frac{\mu}{\beta_1 T_0}\,.$$

Equations (8) – (11) are decoupled by using the dimensionless form of potential functions q and ψ as

$$u_1=\frac{\partial q}{\partial x_1}-\frac{\partial \psi}{\partial x_3}\,;\quad u_3=\frac{\partial q}{\partial x_3}+\frac{\partial \psi}{\partial x_1}\,. \tag{14}$$

Laplace and Fourier transforms are taken as

$$\hat f\left(x_1,\,x_3,\,s\right)=\int_0^\infty e^{-st}f\left(x_1,\,x_3,\,t\right)\,dt\,;\qquad \tilde f\left(\xi,x_3,s\right)=\int_{-\infty}^{\infty}e^{-i\xi x_1}\hat f\left(x_1,x_3,s\right)dx_1. \tag{15}$$

Applying (15) on (11) gives,

$$\tilde T=\tilde\varphi-\zeta\left(\frac{\partial^2}{\partial x_3^2}-\xi^2\right), \tag{16}$$

where

$$\zeta=\begin{cases}0 & \text{for } 1\,T;\\ a & \text{for } TT;\\ \dfrac{\beta^*}{s^2} & \text{for } HTT.\end{cases}$$

Equations (8) – (10) in the accompany of (14) – (16) yield the resulting expressions (suppressing the primes for convenience) after some algebraic calculation as

$$\left(B_1\frac{d^4}{dx_3^4}+B_2\frac{d^2}{dx_3^2}+B_3\right)\left(\tilde q,\,\tilde\varphi\right)=0\,; \tag{17}$$

$$\left(\frac{d^2}{dx_3^2}-\lambda_3{}^2\right)\tilde\psi=0\,, \tag{18}$$

where

$$B_1=\left(1+\zeta\,R_6\right)\left(R_1+\xi_1{}^2s^2\right)+\zeta\,R_3R_4R_5a_4\,;$$

$$B_2=\left(R_1\xi^2+R_2s^2\right)\left(1+\zeta\,R_6\right)+\left(R_1+\xi_1{}^2s^2\right)\left(R_6R_4+\xi^2\right)+a_4R_3R_4R_5+\zeta\,a_4R_3R_5\,\xi^2\,;$$

$$B_3=\left(R_6R_4+\xi^2\right)\left(R_1\xi^2+R_2s^2\right)+a_4R_5\xi^2R_3R_4\,;\quad R_1=1+\eta_1\tau_1 s\,;\quad R_2=1+\xi_1^2\xi^2\,;$$

$$R_3=a_3\left(1+\eta_2\tau_1 s\right);\quad R_4=1+\zeta\,\xi^2;\quad R_5=s\left(1+\eta_3\tau_0 s\right);\quad R_6=s\left(1+\eta_4\tau_0 s\right).$$

The bounded solution of equation (17) and (18) i.e. $\tilde{q}, \tilde{\psi}, \tilde{\varphi} \to 0$ as $x_3 \to \infty$ can be expressed as

$$\tilde{q} = A_1 e^{-\lambda_1 x_3} + A_2 e^{-\lambda_2 x_3}; \tag{19}$$

$$\tilde{\varphi} = d_1 A_1 e^{-\lambda_1 x_3} + d_2 A_2 e^{-\lambda_2 x_3}; \tag{20}$$

$$\tilde{\psi} = A_3 e^{-\lambda_3 x_3}, \tag{21}$$

where

$$d_i = \frac{\lambda^2{}_i (R_1 + \xi_1{}^2 s^2) - (R_1 \xi^2 + R_2 s^2)}{R_3 R_4 - \zeta R_3 \lambda_i{}^2} \qquad (i = 1,\, 2)$$

λ_l $(l = 1,\, 2)$ being the roots of the characteristic equation $\left(B_1 \dfrac{d^4}{dx_3^4} + B_2 \dfrac{d^2}{dx_3^2} + B_3 \right) = 0$.

4. Boundary conditions.

Here, we explore the impact of ramp type normal force and Ramp type thermal source as

$$(i)\ t_{33} = -F_1(t)\delta(x); \quad (ii)\ t_{31} = 0; \quad (iii)\ \frac{\partial \varphi}{\partial x_3} = F_2(t)\delta(x) \text{ at } x_3 = 0, \tag{22}$$

where

$$\{F_1(t),\, F_2(t)\} = (F_{10}(t),\, F_{20}(t)) \begin{cases} 0 & 0 \le t < t_0; \\ \dfrac{t}{t_0} & 0 \le t < t_0; \\ 1 & t > t_0, \end{cases} \tag{23}$$

F_{10} is the magnitude of force, F_{20} is the constant temperature applied on the boundary. Invoking (15) on (22) – (23), we have

$$(i)\ \tilde{t}_{33} = \tilde{F}_1(\xi,\, s); \quad (ii)\ \tilde{t}_{31} = 0; \quad (iii)\ \frac{\partial \tilde{\varphi}}{\partial x_3} = \tilde{F}_2(\xi, s) \text{ at } x_3 = 0, \tag{24}$$

where

$$\left(\tilde{F}_1(\xi,\, s),\ \tilde{F}_2(\xi, s) \right) = (F_{10},\, F_{20}) \left\{ \frac{\left(1 - e^{-st_0}\right)}{t_0 s^2} \right\}. \tag{25}$$

Invoking (14) – (16) (after suppressing the primes) in (12) – (13) along with (19) – (21), the expressions of displacements, stresses, conductive temperature and thermodynamic temperature are obtained by considering the boundary conditions defined by (22) – (23) as

$$\tilde{u}_1 = \frac{1}{\Delta}\left[F_1 \left(-i\xi \sum_{i=1}^{2} e^{-\lambda_i x_3} \Delta_{i1} + \lambda_3 e^{-\lambda_i x_3} \Delta_{31} \right) + F_2 \left(-i\xi \sum_{i=1}^{2} e^{-\lambda_i x_3} \Delta_{1i} + \lambda_3 e^{-\lambda_3 x_3} \Delta_{32} \right) \right]; \tag{26}$$

$$\tilde{u}_3 = \frac{-1}{\Delta}\left[F_1 \left(\sum_{i=1}^{2} \lambda_i e^{-\lambda_i x_3} \Delta_{i1} + i\xi e^{-\lambda_3 x_3} \Delta_{31} \right) + F_2 \left(\sum_{i=1}^{2} \lambda_i e^{-\lambda_i x_3} \Delta_{1i} + i\xi e^{-\lambda_3 x_3} \Delta_{32} \right) \right]; \tag{27}$$

$$\hat{t}_{33} = \frac{1}{\Delta}\left[F_1 \sum_{i=1}^{3} H_i \Delta_{i1} e^{-\lambda_i x_3} + F_2 \sum_{i=1}^{3} H_i \Delta_{i2} e^{-\lambda_i x_3} \right]; \tag{28}$$

$$\hat{t}_{31}=\frac{1}{\Delta}\left[F_1\sum_{i=1}^{3}H_{i+3}\Delta_{i1}e^{-\lambda_i x_3}+F_2\sum_{i=1}^{3}H_{i+3}\Delta_{i2}\,e^{-\lambda_i x_3}\right]; \tag{29}$$

$$\hat{\varphi}=\frac{1}{\Delta}\left[F_1\sum_{i=1}^{2}d_i\Delta_{i1}e^{-\lambda_i x_3}+F_2\sum_{i=1}^{2}d_i\Delta_{i2}e^{-\lambda_i x_3}\right]; \tag{30}$$

$$\hat{T}=\frac{1}{\Delta}\left[F_1\sum_{i=1}^{2}H_{i+6}\Delta_{i1}\,e^{-\lambda_i x_3}+F_2\sum_{i=1}^{2}H_{i+6}\Delta_{i2}\,e^{-\lambda_i x_3}\right], \tag{31}$$

where

$$\Delta=H_6\left(H_6H_7-H_1H_8\right)+H_3\left(H_4H_8-H_5H_7\right);\quad \Delta_{11}=H_6H_8,\quad \Delta_{12}=H_2H_6-H_3H_5;$$

$$\Delta_{21}=-H_6H_7;\quad \Delta_{22}=H_3H_4-H_1H_6;\quad \Delta_{31}=H_5H_7-H_4H_8;\quad \Delta_{32}=H_1H_5-H_2H_4;$$

$$H_i=R_1(a_5\lambda_i^2-\xi^2a_6)-(1+\eta_2\tau_1 s)(R_4d_i-a\lambda_i^2d_i);\quad H_3=R_1(i\xi\lambda_3a_5-i\xi a_6\lambda_3);$$

$$H_{i+3}=2a_7R_1i\xi\lambda_i;\quad H_6=-a_7R_1(\lambda_3{}^2+\xi^2)\quad H_{i+6}=d_1\left[1-\zeta\left(\lambda_i^2-\xi^2\right)\right],\quad (i=1,2).$$

5. Special Cases.

5.1. Modified Green-Lindsay model with two temperature:

Let $\xi_1\to 0$ in equations (26) – (31), we obtain the resulting expression for MG-L theory of thermoelasticity along with two temperature effect.

5.2. Non local Modified Green-Lindsay model:

As two temperature parameter vanishes i.e. $a=0$ in equations (26) – (31), we obtain the results for MG-L model involving non-local impact.

5.3. Non local G-L generalized thermoelastic model with two temperature:

Taking $\eta_1=\eta_3=0$, $\eta_2=\eta_4=1$, reduces the system of equation defined by (26) – (31) for G-L model having non-local and two temperature effect.

5.4. Non local L-S generalized thermoelastic model with two temperature:

Putting, $\eta_1=\eta_2=0$, $\eta_3=\eta_4=1$ in equations (26) – (31) will yield the expression for L-S model involving non-local and two temperature.

5.5. Coupled thermoelastic model with non local and two temperature:

Let $\eta_1=\eta_2=\eta_3=\eta_4=0$, i.e. in absence of relaxation time, equations (26) – (31) gives the corresponding expression for CT model along with non-local and two temperature.

6. Inversion of the transforms.

The components of displacement, stresses, temperature distribution and conductive temperature are the functions of s, x_3 and ξ which are parameters of Laplace transform and Fourier transform respectively. To invert these quantities into physical domain, we invert the transforms by applying the method explained by Kumar et al. [19].

7. Numerical result and discussion.

By performing numerical calculations for different cases, including the non-local, hyperbolic two-temperature, ramp type normal force, and ramp type thermal source parameters, and comparing the results with different theories of thermoelasticity, such as the MG-L, L-S, and G-L theories, it is possible to gain a comprehensive understanding of the behavior of the material under various loading conditions.

Following Dhaliwal and Singh [20], we take the case of magnesium crystal, the physical constants used are

$\lambda = 2{,}17\times10^{10}$ Nm^{-2} ; $\mu = 3{,}278\times10^{10}$ Nm^{-2} ; $K^* = 1{,}7\times10^2$ $\mathrm{Wm}^{-1}\mathrm{deg}^{-1}$;

$\omega_1 = 3{,}58\times10^{11}$ S^{-1} ; $\beta_1 = 2{,}68\times10^6$ $\mathrm{Nm}^{-2}\mathrm{deg}^{-1}$; $\rho = 1{,}74\times10^3$ Kgm^{-3} ;

$C_e = 1{,}04\times10^3$ $\mathrm{JKg}^{-1}\mathrm{deg}^{-1}$; $T_0 = 298k$; $t_0 = 2s$

and non-dimensional relaxation times are taken as $\tau_0 = 0{,}03s$.

7.1. Non Local and Two Temperature. In this case, we consider non-local parameter $\xi_1 = 0{,}50$ and $\xi_1 = 0$ and hyperbolic two temperature $\zeta = 0{,}75$ and $\zeta = 0{,}0$ for the range $0 \le x_1 \le 10$. The solid line corresponds to ($\xi_1 = 0{,}50;\ \zeta = 0{,}75$), solid line having centre symbol diamond ‘ ◊ ’ corresponds to the case of ($\xi_1 = 0;\ \zeta = 0{,}75$), small dashed line represents the case of ($\xi_1 = 0{,}50;\ \zeta = 0$), whereas small dashed line having centre symbol circle ‘o’ displays the case of ($\xi_1 = 0;\ \zeta = 0$).

7.1.1. Normal Force. Fig. 1 shows the behavior of t_{33} with respect to x_1. It can be observed that for all cases considered, the values of t_{33} increase sharply in the range $0 \le x_1 \le 2$ and follow an oscillatory pattern in the remaining interval. The magnitude of the values of t_{33} for $\xi_1 = 0{,}5$ and $\zeta = 0$ is larger than in the other cases.

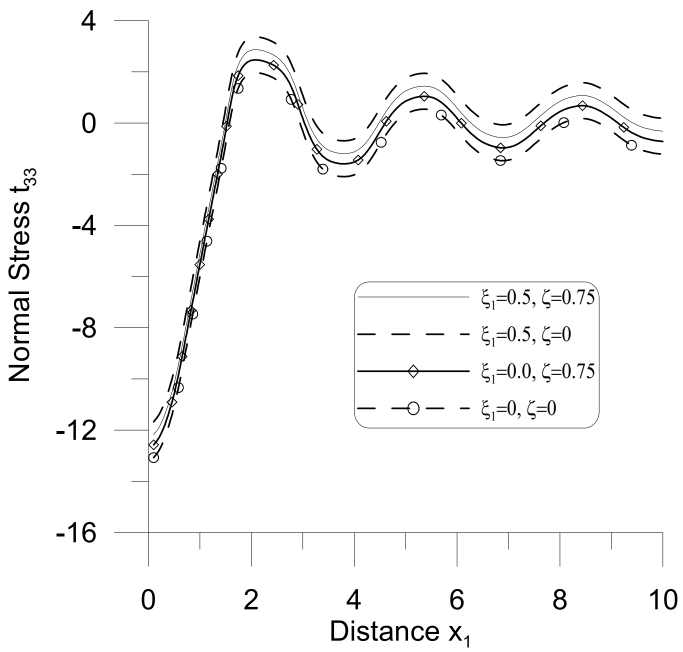

Fig. 1. Variation of t_{33} vs x_1
(Normal Force)

Fig. 2 is plot of t_{31} vs x_1, which demonstrate that the behavior of t_{31} for $\xi_1 = 0{,}5$; $\zeta = 0$ are opposite in nature as noticed for $\xi_1 = 0$; $\zeta = 0{,}75$ whereas in absence of hyperbolic two temperature i.e. $\xi_1 = 0{,}5$; $\zeta = 0$, $\xi_1 = 0$, $\zeta = 0$ shows similar behavior in the entire range.

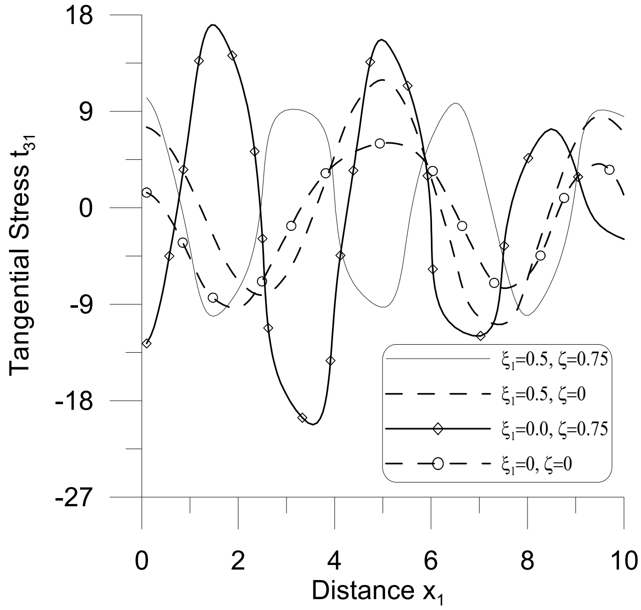

Fig. 2. Variation of t_{33} vs x_1
(Normal Force)

Fig. 3 predicts T vs x_1. It is found that behavior of T for $\xi_1 = 0,5;\ \zeta = 0$ are opposite in nature as observed for $\xi_1 = 0;\ \zeta = 0,5$ in the range $0 \le x_1 \le 3$ and with increase in x_1, T shows similar behavior. While in case of $\xi_1 = 0;\ \zeta = 0,75$ the values of T shows opposite behavior as noticed for $\xi_1 = 0;\ \zeta = 0$ throughout the range which reveals the impact of non-local and hyperbolic two temperature parameters.

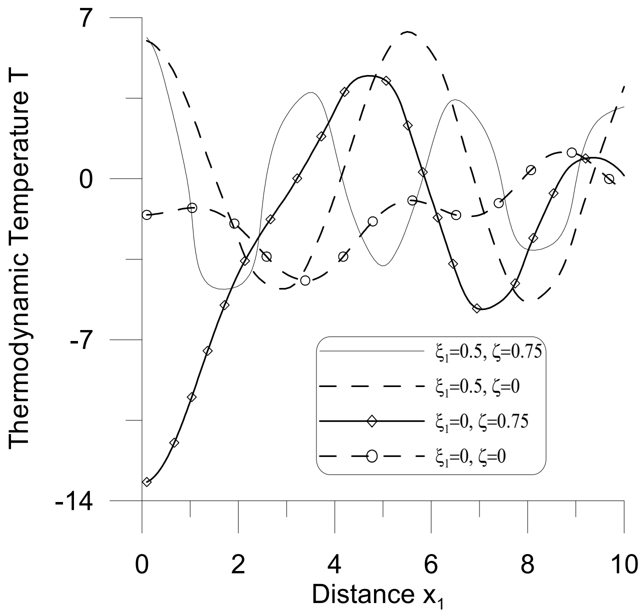

Fig. 3. Variation of T vs x_1
(Normal Force)

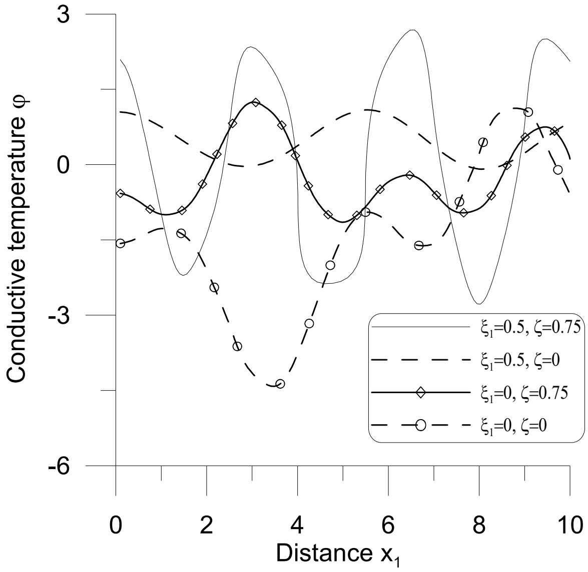

Fig. 4. Variation of φ vs x_1 (Normal Force)

The graphical representation of φ with x_1 is represented by fig. 4. It can be observed that the values of φ for $\xi_1 = 0;\ \zeta = 0,75$ is like mirror image of $\xi_1 = 0;\ \zeta = 0$ in the range $0 \le x_1 \le 8$ and shows similar behavior in the rest of the interval. It is also observed that the values of φ for $\xi_1 = 0,5;\ \zeta = 0$ decreases in the range $0 \le x_1 \le 3;\ 5 \le x_1 \le 8$ and vice versa in the remaining range. Whereas φ follows an oscillatory behavior for $\xi_1 = 0,5;\ \zeta = 0,75$ in the entire range, with increase in x1 the magnitude of oscillation also increases.

7.1.2. Thermal Source. Fig. 5 represents the variations of t_{33} vs x_1. It is examined that the trend of variation of t_{33} for $\xi_1 = 0,5;\ \zeta = 0,75$; $\xi_1 = 0;\ \zeta = 0,75$ are similar in nature in the entire range except in $3 \le x_1 \le 8$, where opposite behavior is observed. It is also observed that the variation of t_{33} in case of $\xi_1 = 0,5;\ \zeta = 0$ and $\xi_1 = 0;\ \zeta = 0$ are opposite in na-

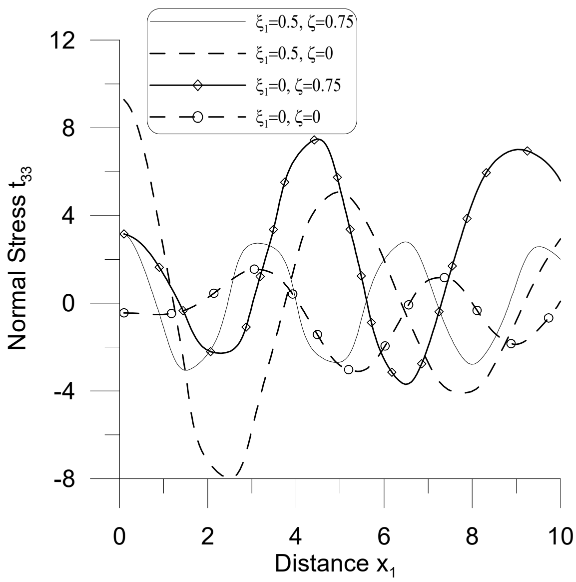

Fig. 5. Variation of t_{33} vs x_1 (Normal Force)

ture. It is seen that t_{33} has minimum value at $x_1 = 2,5$ when $\xi_1 = 0,5;\ \zeta = 0$.

The plot of t_{31} vs x_1 is represented by fig. 6. It is seen that t_{31} shows upward trend in $0 \le x_1 \le 2$ for all the considered cases except for $\xi_1 = 0;\ \zeta = 0$ and with increase in x_1 it shows oscillatory behavior for all the cases, magnitude of oscillation is greater in absence of non-local and HTT parameter.

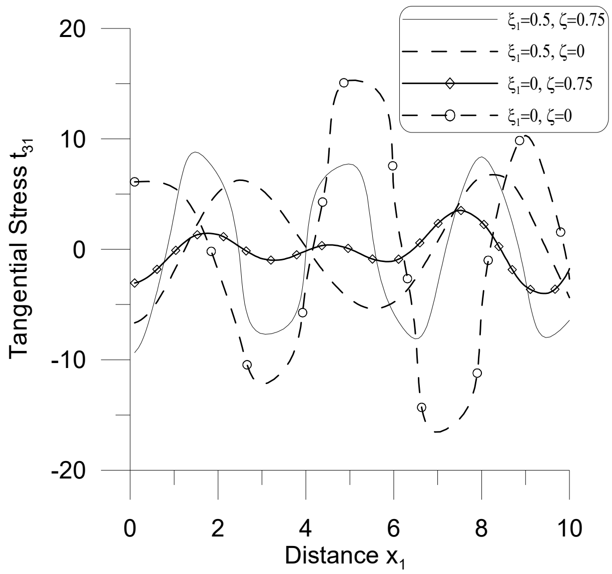

Fig. 6. Variation of t_{31} vs x_1 (Normal Force)

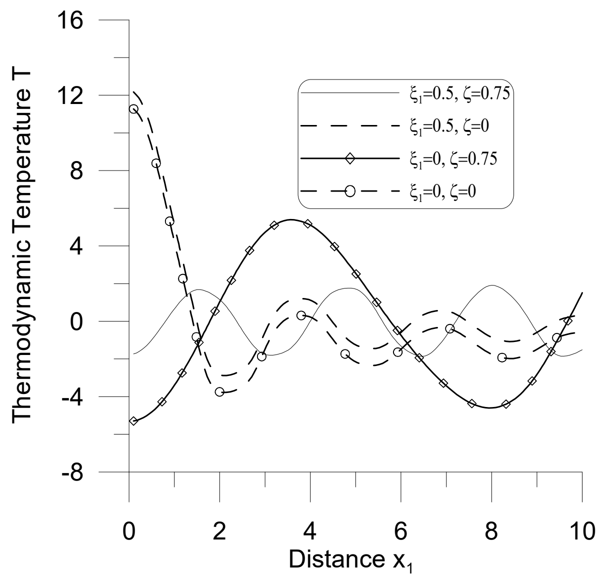

Fig. 7. Variation of T vs x_1 (Normal Force)

Fig. 7 shows the variations of T vs x_1. It is realized that the values of T in absence of HTT parameter (i.e. $\xi_1 = 0,5;\ \zeta = 0;\quad \xi_1 = 0;\ \zeta = 0$) shows downward trend in the range $0 \le x_1 \le 2$ whereas for the case of $\xi_1 = 0,5;\ \zeta = 0,75$ and $\xi_1 = 0;\ \zeta = 0,75$ increases. It is also observed that T for $\xi_1 = 0,5;\ \zeta = 0,75$ shows opposite behavior as observed for the cases of absence of HTT parameter (i.e. $\xi_1 = 0,5;\ \zeta = 0;\quad \xi_1 = 0;\ \zeta = 0$).

Fig. 8 exhibits the plot for φ vs x_1. It is discovered that the values of φ decreases in the range $0 \le x_1 \le 2$ for all the cases and attains minimum at $x_1 = 2$. With increase in x_1 φ shows an oscillatory behavior for all the cases. It is also observed that the magnitude of values of φ are smaller in case of $\xi_1 = 0;\ \zeta = 0$.

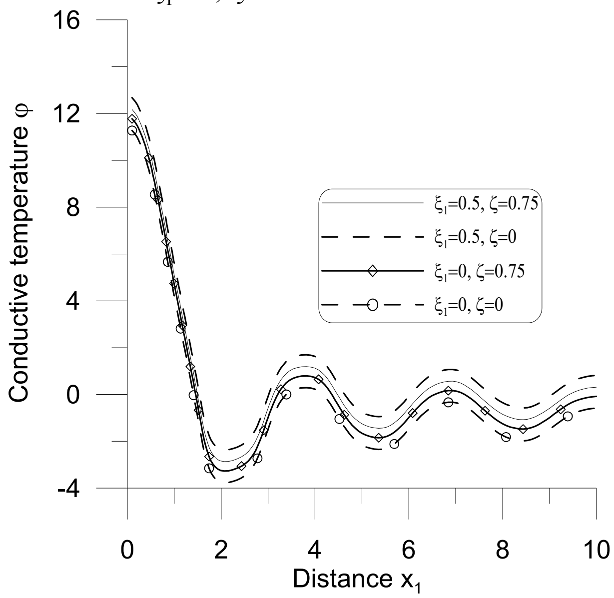

Fig. 8. Variation of φ vs x_1 (Normal Force)

7.2. Different theories of thermoelasticity. The solid line represents the MG-L model of thermoelasticity. The small dashed line shows the case of G-L model of thermoelasticity. The case of L-S theory of thermoelasticity is represented by big dashed line.

7.2.1. Normal Force. Fig. 9 demonstrate the variations of t_{33} vs x_1. It is checked that the values of t_{33} for MG-L, G-L and L-S theories of thermoelasticity increases in the range $0 \le x_1 \le 2$. With increase in x_1, t_{33} follows an oscillatory behavior. It is also observed that the magnitude of values of t_{33} remains higher for G-L theory of thermoelasticity.

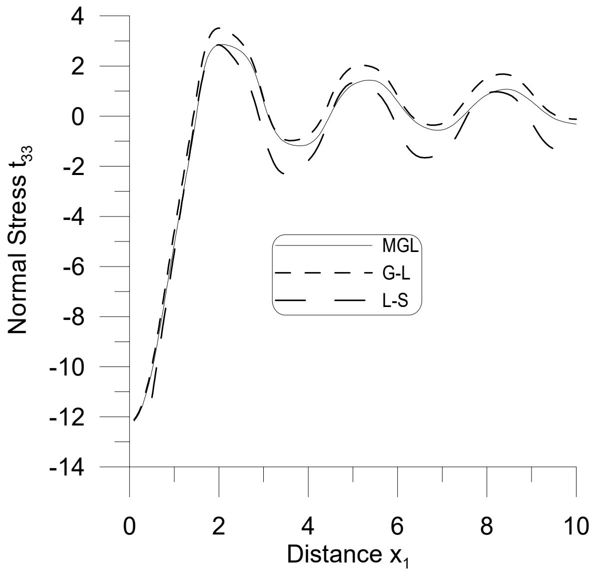

Fig. 9. Variation of t_{33} vs x_1 (Thermal Source)

The variation of t_{31} vs x_1 is represented in fig. 10. It is inspected that the values of t_{31} for MG-L theory and L-S theory shows similar behavior in the entire range whereas opposite behavior of t_{31} is noticed for G-L theory.

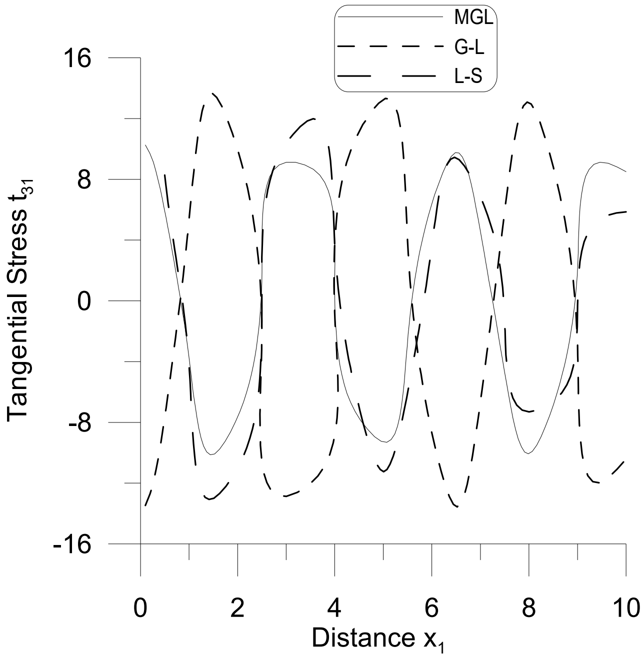

Fig. 10. Variation of t_{31} vs x_1 (Thermal Source)

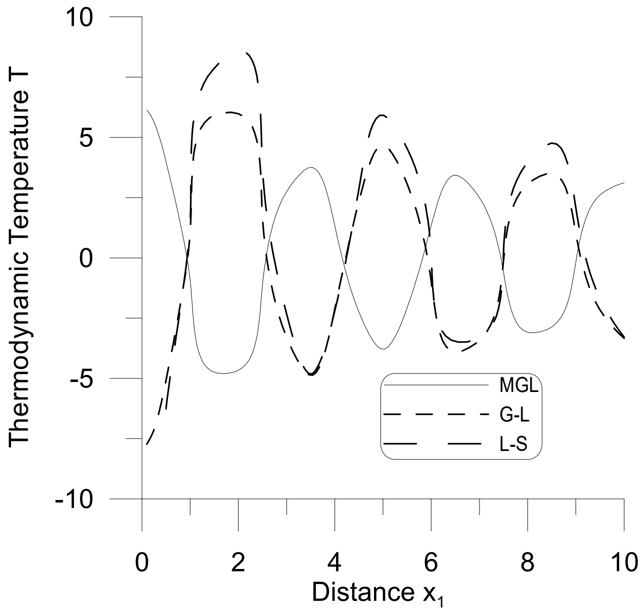

Fig. 11. Variation of T vs x_1 (Thermal Source)

Fig. 11 is a depicts the trend of T vs x_1. It is observed that the values of T for MG-L are like mirror image of values observed for L-S and G-L theory of thermoelasticity. Magnitude of values for L-S theory remains higher in the entire range.

The graphical presentation of φ vs x_1 is represented in fig. 12. It is scrutinized that the values of φ for G-L theory are opposite in nature as noticed for L-S and MG-L theory of thermoelasticity. It is also observed that the values of φ oscillates with greater magnitude for MG-L theory of thermoelasticity.

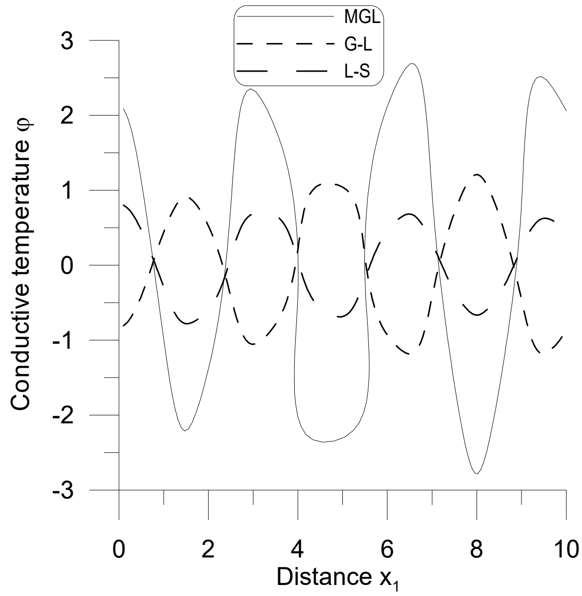

Fig. 12. Variation of φ vs x_1 (Thermal Source)

7.2.2. Thermal Source. Fig. 13 depicts the variations of t_{33} vs x_1. It is found that the values of t_{33} for L-S theory shows vice-versa behavior as observed for MG-L theory of thermoelasticity, magnitude of oscillation is greater for L-S model.

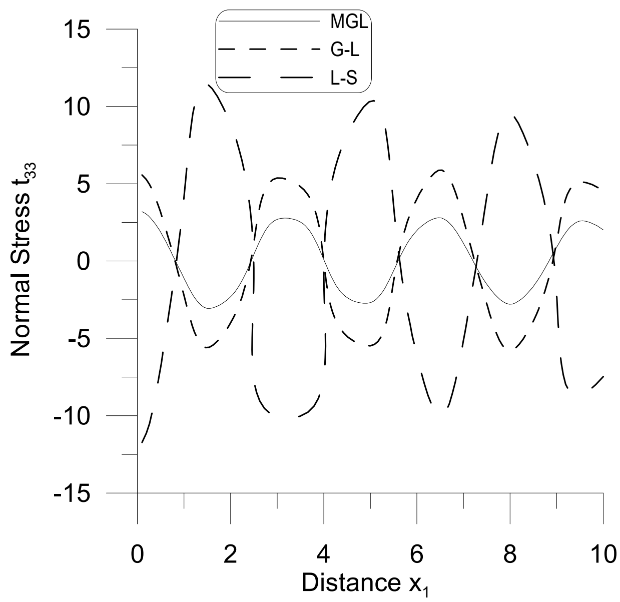

Fig. 13. Variation of t_{33} vs x_1 (Thermal Source)

Fig. 14 is a plot of t_{31} vs x_1. It is noticed that the behavior of t_{31} for all the considered cases are opposite in nature as observed for t_{31}.

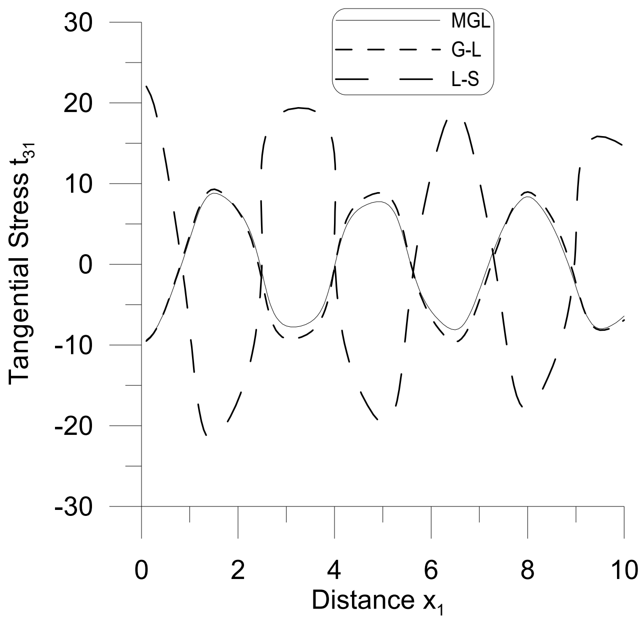

Fig. 14. Variation of t_{31} vs x_1 (Thermal Source)

Fig. 15 exhibits the plot for T vs x_1. It is acknowledged that the behavior of T for MG-L is contrast in nature as observed for G-L and L-S theory of thermoelasticity i.e. when the values of T for MG-L increases there is a decrease in the values of T for G-L and L-S model.

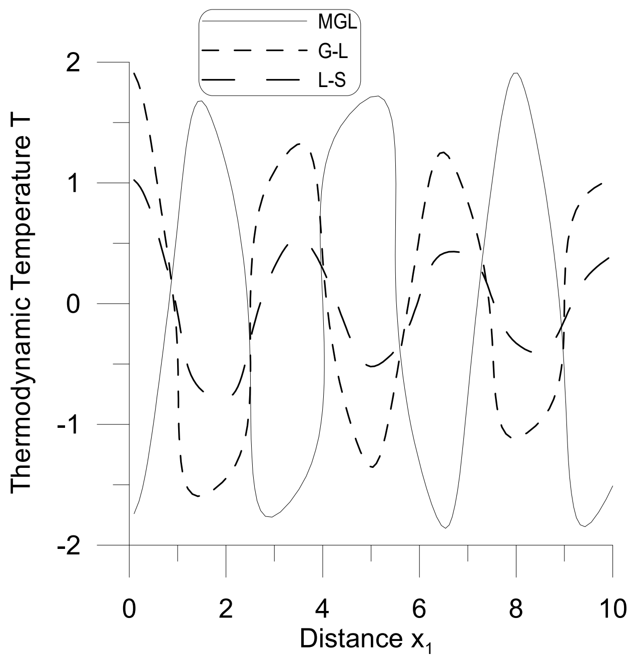

Fig. 15. Variation of T vs x_1 (Thermal Source)

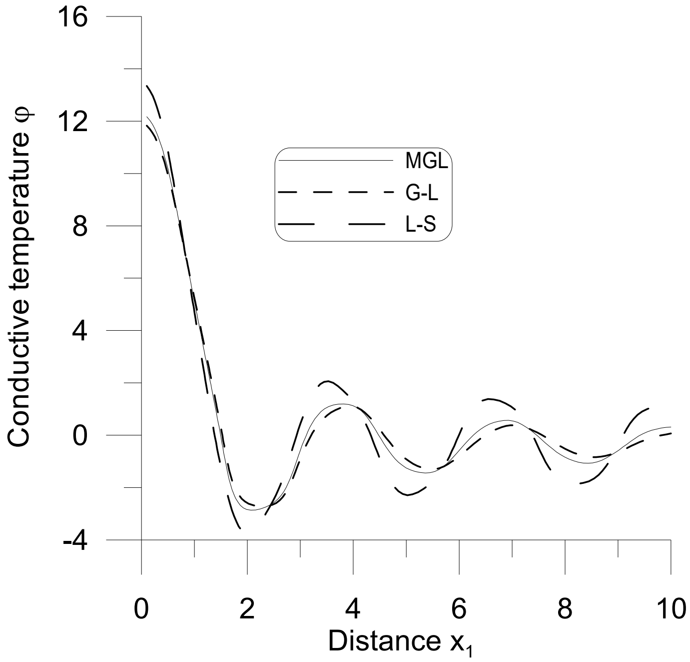

Fig. 16. Variation of φ vs x_1 (Thermal Source)

Fig. 16 is plot of φ vs x_1. It is noted that the values of φ for all the considered cases decreases sharply in the range $0 \le x_1 \le 2$ and attains minimum value at $x_1 = 2$. As x1 increases the values of φ for MG-L, G-L and L-S model follows an oscillatory behavior with decreasing magnitude of oscillation.

8. Conclusion.

These results are important for understanding the behavior of materials under various loads and temperatures and can be used to design and optimize materials for specific applications. In this work, we considered a thermomechanical problem based on the MG-L model having non-local, and hyperbolic two temperature parameters. The problem is further examined with Ramp type normal force/heat source**.** The results are displayed graphically to illustrate the effect of non-local parameter and hyperbolic two temperature along with different theories of thermoelasticity. The following observations are obtained from numerical computed result:

When normal distributed force is applied, it is observed that trend of t_{33} and t_{31} for $\xi_1 = 0,75$; $\xi_1 = 0,50$ are opposite in nature as observed for $\xi_1 = 0,20$ and $\xi_1 = 0$. It is also observed that T and φ shows oscillatory behavior, the magnitude of oscillation is higher for $\xi_1 = 0,50$.

In case of Ramp type thermal source, the value of t_{33} increases in most of the interval for all values of ξ_1, while t_{31}, T and φ shows oscillatory behavior in the entire range, magnitude of oscillation is higher in absence of non-local parameter.

It is observed that higher value of moving heat source parameter enhances the value of t_{33}, T and φ for normal distributed force as well as for Ramp type thermal source. It is also observed that the behavior of variation for t_{33}, t_{31}, T and φ for different value of v are similar in nature with difference in their magnitude of oscillations.

The hyperbolic two temperature effect enhances the magnitude of T and φ in contrast to t_{33} and t_{31}. It is also noticed that in most of the range, the behavior of all entities for higher value of hyperbolic two temperature parameter i.e. $\zeta = 0,75$ are opposite in nature as observed for two temperature parameter $(a = 0,104)$ and for classical one temperature $(a = 0,0)$ for normal distributed force as well as for Ramp type thermal source.

The physical applications of the model can be found in the mechanical engineering, geophysics. The present investigation provides a valuable contribution to the field of thermoelasticity and highlights the importance of considering non-local and hyperbolic two-temperature parameters in the analysis of material behavior. The present investigation be used to determine the optimal combination of parameters that lead to the desired material behavior, such as high strength, high thermal stability, or high resistance to thermal stress. This information can be useful in the design and optimization of materials for various engineering applications, particularly those subjected to rapidly changing temperature and load conditions.

РЕЗЮМЕ. Метою цього дослідження є вивчення впливу нелокальних і гіперболічних двотемпературних параметрів на термонапружений стан за допомогою модифікованої узагальненої теорії термопружності Гріна – Ліндсі. При цьому використовуються змінні нормальні навантаження / джерела тепла. Основні рівняння перетворюються в безрозмірну форму і спрощуються за допомогою перетворень Лапласа і Фур'є. Перетворена область використовується для отримання величин фізичного поля, таких як напруження, компоненти вектора зміщення, термодинамічна температура та температура провідності. Техніка чисельного оберненого перетворення використовується для відновлення рівнянь у фізичній області. Вплив нелокальної, гіперболічної двотемпературної та різноманітних теорій термопружності на поведінку матеріалу представлено у вигляді графіків. Результати підкреслюють важливість розгляду різних аспектів і теорій в аналізі поведінки матеріалу. Обговорено та показано кілька окремих випадків.

КЛЮЧОВІ СЛОВА: нелокальний параметр, гіперболічний двотемпературний параметр, модифікована теорія Гріна – Ліндсі, перетворення Лапласа та Фур'є.

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