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CONTACT PROBLEM FOR FINITE POROELASTIC CYLINDER - EXACT ANALYSIS

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Abstract. The novelty of the presented results is the deriving of exact solutions of axisymmetric poroelasticity problems for finite solid and hollow cylinders. The statement of these contact problems is given under the assumptions of Biot’s model for poroelastic materials. A simulation of the poroelastic response of a cylinder is implemented using the apparatus of boundary problems. By the method of integral transforms, the original boundary problem is reduced to a one-dimensional boundary problem in the transform domain. Based on the exact solution of the latter one, it is manageable to derive the explicit formulas for displacements, stress, and pore pressure. The investigation of contact stress and pore pressure distribution inside the cylinder regarding its geometrical ratios, type of applied loading, and poroelastic material is carried out.

Key words: contact problem, poroelastic finite cylinder, integral transform, vector boundary problem, exact solution.

Introduction.

Throughout the development of poroelasticity from its inception [3, 21] to nowadays, poroelastic cylinders have been a frequently used research model. However, most of the derived solutions were realized numerically, which is explained by the mathematical complexity of problems.

The finite-element method, Petrov – Galerkin method, pseudo-transient numerical method etc. are most well-known numerical methods for solving of such type of problems. A mathematical framework was developed in [14] using finite element method to investigate the effect of mechanical stimulus on the bone in-growth into undegradable scaffolds. The algorithm based on the spectral method solving corresponding equations as a generalized eigenvalue problem was used in [11] for the dispersion equation for cylindrical poroelastic structures. The dynamic analysis of cylinders made of fully saturated porous materials with considering uncertainties in the constitutive mechanical properties was done in [12] using the stochastic meshless local Petrov – Galerkin method.

There are much more problems for poroelastic cylinders, which were solved numerically. However, these methods do not allow to establish some important qualitative characteristics of the stress and pore pressure distribution. That is why analytical solving methods are necessary in order to reveal important features of the behavior of cylinders and serve as reference in the development of new numerical methods.

Some papers present a combination of analytical and numerical methods’ application. The solution for the problem of a cylinder under plane strain conditions after sudden impact of a constant fluid pressure was obtained in [5] explicitly for the Laplace transform of the various quantities, and the solution showing the dependence of parameters on time was then calculated using numerical inversion technique. The general poroelastic solution for axisymmetrical plane strain problems with time dependent boundary conditions was developed in Laplace domain in [10], and the Laplace transform was inverted numerically. The problem of long bone-like or borehole sample specimen probed by low frequency sound was solved in [6] using the approximation of the formulation, based on the Biot's model by the equivalent elastic solid model. The analytical solution for the problem of transient response of a poroelastic cylinder to sudden fluid injection was derived in [2] for a partial case, and numerical solutions were derived for different cases there. The semi-analytical results of the poromechanical fields in a hollow cylinder subjected to an asymmetric loading conditions was developed in [16]. There the heterogeneous hollow cylinder was approximated by a multilayer structure in which each cylindrical layer is assumed to be homogeneous. The analytical results of poromechanical field were derived in Laplace transform space, then they were numerically inverted.

The pure analytical solutions are less widespread. The phenomenon of porous elastic media makes it possible to use analytical methods used in the theory of elasticity, thermoelasticity, rheology, hydromechanics in order to study the mutual influence of coupled fields [7, 8, 15, 17, 19, 22]. These methods usually are applied for some partial cases. Plane-strain vibrations in a fluid-loaded poroelastic hollow cylinder surrounded by a fluid were investigated employing Biot's theory of wave propagation in poroelastic media in [20], and the solutions were derived for several particular cases. Generalized solutions for the differential equations of three-dimensional consolidation were derived in [13] with the help of Laplace transformations for strains and stresses in cylindrical bodies. Theoretical study of acoustic scattering of spherical waves generated by a monopole point source in a perfect compressible fluid by a fluid-saturated porous cylinder of infinite length was provided in [9]. The phenomena of mechanical creep and deformation in rock formations, coupled with the hydraulic effects of fluid flow was investigated in [1]. The problem for a porous saturated medium was solved in [18] using the Laplace time transform. The analytical solution for a poroelastic axisymmetric solid cylinder under the loading applied by the cylindrical surface was found in [23], where the loading was applied along the cylindrical surface.

In the present paper the authors propose a new analytical approach for deriving the exact solution for poroelastic problems for solid and hollow cylinders in axisymmetric statement.

§1. The statement of the problem.

The problem for a poroelastic hollow cylinder (Fig. 1) is considered in the cylindrical coordinate system $0 < a_0 < R < a_1$, $-\pi < \varphi < \pi$, $0 < Z < h$. In the dimensionless form the cylinder's surface can be described as $0 < b < r < 1$, $-\pi < \varphi < \pi$, $0 < z < d$, where $d = h / a_1$, $b = a_0 / a_1$.

At the boundary $z = 0$ the mechanical and fluid pressure loadings are given

$$\sigma_z^F\Big|_{z=0} = -L(r);\quad \tau_{rz}^F\Big|_{z=0} = T(r);\ p\big|_{z=0} = P(r), \tag{1.1}$$

where $\sigma_r^F(r,z), \tau_{rz}^F(r,z)$, $p(r,z)$ are dimensionless normal and shear full stress, and a pore pressure correspondently (the normalization was done by division of all characteristics by G, where G is shear modulus).

According to the relation between full and effective stress [24] conditions (1.1) can be rewritten as

$$\sigma_z\big|_{z=0} = -L(r) - \alpha P(r);\quad \tau_{rz}\big|_{z=0} = T(r);\ p\big|_{z=0} = P(r), \tag{1.2}$$

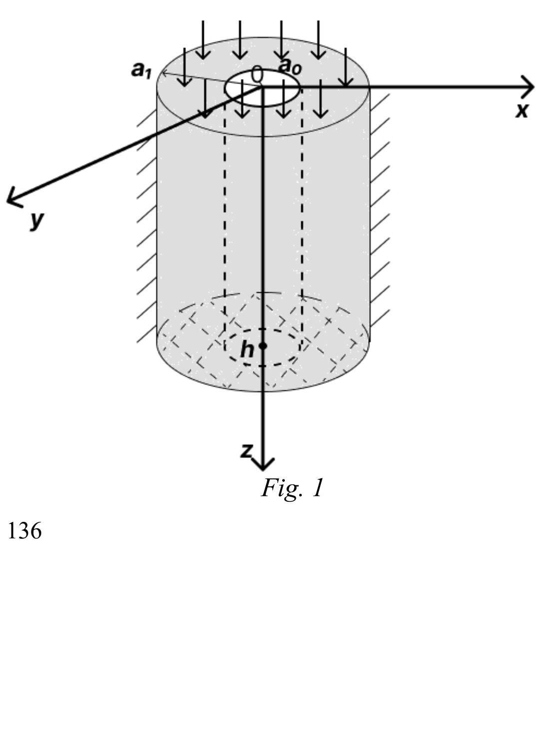


Fig. 1

where $\sigma_z(r, z), \tau_{rz}(r, z)$ are dimensionless normal and shear effective stress, α is Biot's coefficient.

The contact conditions of fixing are given at the boundary $z = h$

$$u\big|_{z=h} = 0;\ w\big|_{z=h} = 0 \tag{1.3}$$

under the assumptions of a permeability

$$p\big|_{z=h} = 0 \tag{1.4}$$

or an impermeability

$$\left.\frac{\partial p}{\partial z}\right|_{z=h} = 0. \tag{1.5}$$

Ideal contact conditions assuming the impermeability of the surface are given on cylindrical surfaces

$$u\big|_{r=1} = 0;\ \tau_{rz}\big|_{r=1} = 0;\ \left.\frac{\partial p}{\partial r}\right|_{r=1} = 0\,; \tag{1.6}$$

$$u\big|_{r=b} = 0;\ \tau_{rz}\big|_{r=b} = 0;\ \left.\frac{\partial p}{\partial r}\right|_{r=b} = 0\,, \tag{1.7}$$

where $u(r, z) = u_r(r, z)/a_1$; $w(r, z) = u_z(r, z)/a_1$ are dimensionless displacements of the solid skeleton.

The poroelastic cylinder's contact stress state satisfying boundary conditions (1.2) – (1.7) and following government equations of equilibrium [24] should be found

$$\begin{gathered}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)-\frac{1}{r^2}u+\frac{\kappa-1}{\kappa+1}\frac{\partial^2 u}{\partial z^2}+\frac{2}{\kappa+1}\frac{\partial^2 w}{\partial r\partial z}-\alpha\frac{\kappa-1}{\kappa+1}\frac{\partial p}{\partial r}=0;\\ \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{\kappa+1}{\kappa-1}\frac{\partial^2 w}{\partial z^2}+\frac{2}{\kappa-1}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial z}\right)-\alpha\frac{\partial p}{\partial z}=0;\\ \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)+\frac{\partial^2 p}{\partial z^2}-\frac{\alpha}{K}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{\partial w}{\partial z}\right]-\frac{S_P}{K}p=0,\end{gathered} \tag{1.8}$$

where $\kappa = 3-4\mu$ is Muskhelishvili's constant, μ is Poisson ratio, S_p is storativity of the pore space, k is permeability, $K = a_1^2/Gk$, $S_P = S_pG$ are dimensionless values.

§2. The deriving of the explicit formulas for the cylinder's stress state.

The one-dimensional boundary value problem in transform's domain is derived due to the application of finite Bessel integral transforms applied with respect to variable r [17]

$$\begin{bmatrix}u_\beta(z)\\ w_\beta(z)\\ p_\beta(z)\end{bmatrix}=\int_b^1\begin{bmatrix}u(r,z)\\ w(r,z)\\ p(r,z)\end{bmatrix}\begin{bmatrix}X_1(r,\beta)\\ X_0(r,\beta)\\ X_0(r,\beta)\end{bmatrix}rdr, \tag{2.1}$$

where $X_i(r, \beta) = J_i(\beta r)N_1(\beta) - N_i(\beta r)J_1(\beta)$, $i = 0, 1$, $\beta_0 = 0, \beta_k$; $k = 1, 2, ...$ are positive roots of the equation $X_1(b, \beta) = J_1(\beta b)N_1(\beta) - N_1(\beta b)J_1(\beta) = 0$, $J_0(r), J_1(r)$ are Bessel functions, $N_0(r)$, $N_1(r)$ are Neumann functions. The one-dimensional boundary value problem is reformulated in terms of displacements transforms vector

$$\begin{gathered}L_2\vec{y}_\beta(z) = 0,\ \ 0<z<h;\\ A_{0\beta}\vec{y}\,'_\beta(0)+B_{0\beta}\vec{y}_\beta(0)=\vec{g}_\beta;\ A_{1\beta}\vec{y}\,'_\beta(h)+B_{1\beta}\vec{y}_\beta(h)=0.\end{gathered} \tag{2.2}$$

Here L_2 is linear differential operator of the second order, $\vec{y}_\beta(z)$ is the vector containing displacements and pore pressure transforms, $A_{i\beta},\ B_{i\beta},\ i=0,1$ are known matrices and $\vec{g}_\beta$ is known vector (all formulae for the linear differential operator L_2, vectors and matrices one can find in Appendix A).

The vector boundary value problem (2.2) is solved with the help of the matrix differential calculation apparatus analogically to method [23] (in the problem [23] the poroelastic finite cylinder was considered under the assumption that a normal load was applied to the cylindrical surface of a cylinder, the detailed description of the proposed method is given there). According to the proposed approach, the solution of the matrix homogenous equation (2.2) is written by a formula

$$Y_\beta(z)=\frac{1}{2\pi i}\oint_C e^{\xi z}M_\beta^{-1}(\xi)\,d\xi,$$

where $M_\beta^{-1}(\xi)$ is the inverse matrix to the matrix $M_\beta(\xi)$, which is given in Appendix A. The closed contour C covers all singularity points of the matrix $M_\beta^{-1}(\xi)$

$$\xi_{1,2}=\pm\beta;\ \xi_{3,4}=\pm\sqrt{\beta^2+\frac{\alpha^2(\kappa-1)+S_P(\kappa+1)}{K(\kappa+1)}}.$$

With the help of the residual theorem the system of four fundamental matrix solutions is derived $Y_i(z), i=\overline{1,4}$. The inverse transform formulae of integral transform (2.1) are applied to the solution of the vector boundary value problem (2.2)

$$\vec{y}_\beta(z)=\left(Y_1(z)+Y_3(z)\right)\begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix}+\left(Y_2(z)+Y_4(z)\right)\begin{pmatrix}c_4\\c_5\\c_6\end{pmatrix}, \tag{2.3}$$

where constants $c_i,\ i=\overline{1,6}$ are found from the boundary conditions (2.2).

Finally, the explicit formulae for displacements and pore pressure are derived:

$$u(r,z)=\sum_{k=1}^{\infty}u_\beta(z)\frac{X_1(r,\beta_k)}{\|X_1(r,\beta_k)\|^2};$$

$$\begin{bmatrix}w(r,z)\\p(r,z)\end{bmatrix}=\sum_{k=1}^{\infty}\begin{bmatrix}w_\beta(z)\\p_\beta(z)\end{bmatrix}\begin{bmatrix}X_0(r,\beta_k)\\X_0(r,\beta_k)\end{bmatrix}\frac{1}{\|X_0(r,\beta_k)\|^2}. \tag{2.4}$$

Here

$$\|f(r)\|^2=\int_0^1 r\left[f(r)\right]^2dr;\ \|X_1(r,\beta_k)\|^2=\|X_0(r,\beta_k)\|^2=\frac{2}{\pi^2}\frac{J_1^2(h\beta_k)-J_1^2(\beta_k)}{\beta_k^2J_1^2(h\beta_k)},$$

$$k=1,2,...;\ \|X_0(r,\beta_0)\|^2=\frac{2}{\pi^2}\left(1-h^2\right).$$

§3. The special case of the stated problem – a solid cylinder.

Let's consider the case when $a_0=0$. It corresponds to solid porous cylinder occupying the area $0<r<1,\ -\pi<\varphi<\pi, 0<z<d$. Stress state and pore pressure satisfy equilibrium equations with boundary conditions (1.2) – (1.6). The original problem can be reduced to a one-dimensional problem with the help of finite Bessel transform applied with respect to variable *r*. With this aim the formulae (2.1) is changed by formulae

$$\begin{bmatrix} u_\beta(z) \\ w_\beta(z) \\ p_\beta(z) \end{bmatrix} = \int_0^1 \begin{bmatrix} u(r,z) \\ w(r,z) \\ p(r,z) \end{bmatrix} \begin{bmatrix} J_1(\beta r) \\ J_0(\beta r) \\ J_0(\beta r) \end{bmatrix} r dr ,$$

where $\beta_0 = 0,\ \beta_k,\ k = 1,2,...$ are positive roots of the equation $J_1(\beta) = 0$.

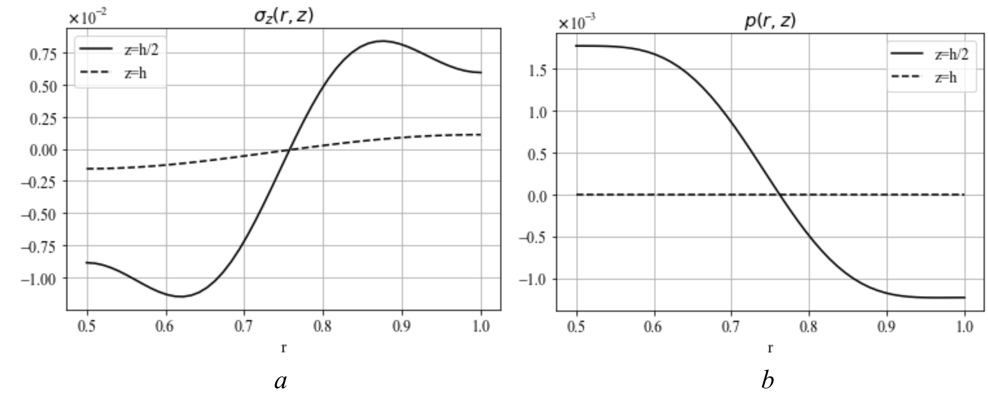
The boundary value problem (1.2) – (1.6) is reduced to the one-dimensional vector problem (2.2), which solution has the form (2.3). The application of inverse formula finalizes the construction of the stated problem's exact solution for a solid poroelastic cylinder.

§4. Results and discussion.

The derived exact formulae give possibility to conduct qualitative investigation of poroelastic stress state for cylinders made of a sandstone and granite porous materials, which are essentially important as building stones, computer chips, fiberglass, bridges, paving, monuments etc. Three loading types were considered: a concentrated mechanical load $\delta(r-(1/2))$ and $\delta(r-((b+1)/2))$ for solid and hollow cylinders respectively; distributed mechanical load $L(r) = \cos(\pi r/2)$ for solid and $L(r) = \cos\left[\left(\pi(r-b)\right)/\left(2(1-b)\right)\right]$ for hollow cylinders. The third type of the load is a fluid pressure which is supposed uniformly distributed with a unit intensity. For each type of load the normal stress (fig. *a*) and the pore pressure (fig. *b*) were studied in dependence either of *z* value, or of different poroelastic materials [4], which characteristics are presented in Table.

Material	G, N/m^2	μ	α	k, m^4/N×s	S_p, m^2/N
Charcoal granite	$1{,}87 \cdot 10^{10}$	0,27	0,242	$1 \cdot 10^{-16}$	$1{,}377 \cdot 10^{-11}$
Westerly granite	$1{,}5 \cdot 10^{10}$	0,25	0,449	$4 \cdot 10^{-16}$	$1{,}412 \cdot 10^{-11}$
Ruhr sandstone	$1{,}33 \cdot 10^{10}$	0,12	0,637	$2 \cdot 10^{-13}$	$2{,}604 \cdot 10^{-11}$

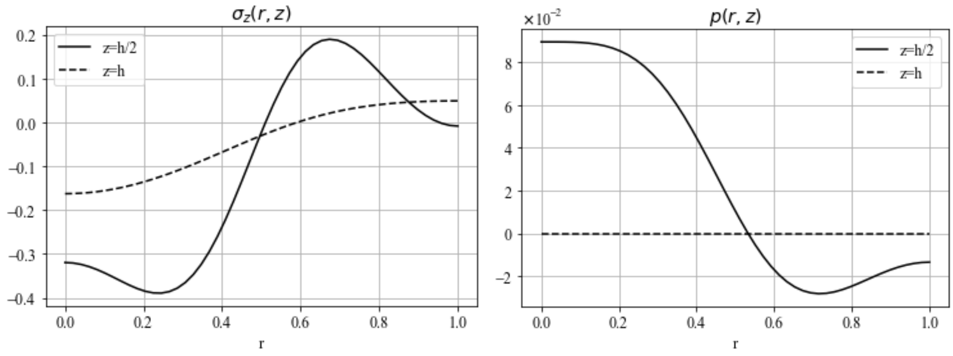
The change of normal stress and pore pressure for the case with concentrated mechanical loading is shown for Ruhr sandstone at fig. 2 – 3 for hollow and solid cylinders with permeable bottoms respectively. As it can be seen, the absolute values of normal stress and pore pressure are essentially larger for solid cylinder. Tensile stress is observed for values $r > 1/2$ and $r > (b+1)/2$ for solid and hollow cylinders respectively.



a *b*

Fig. 2

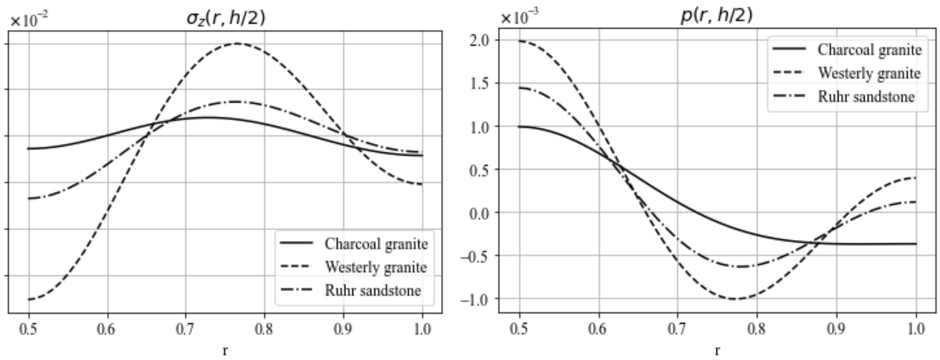
The pore pressure at the bottom surface is zero which completely coincides with permeable boundary conditions (1.4). The distribution of normal stress and pore pressure for the case with impermeable bottom has the similar pattern for hollow cylinder, but for solid cylinder the absolute values of normal stress and pore pressure are larger than in the case with permeable cylinder's bottom.



a *b*

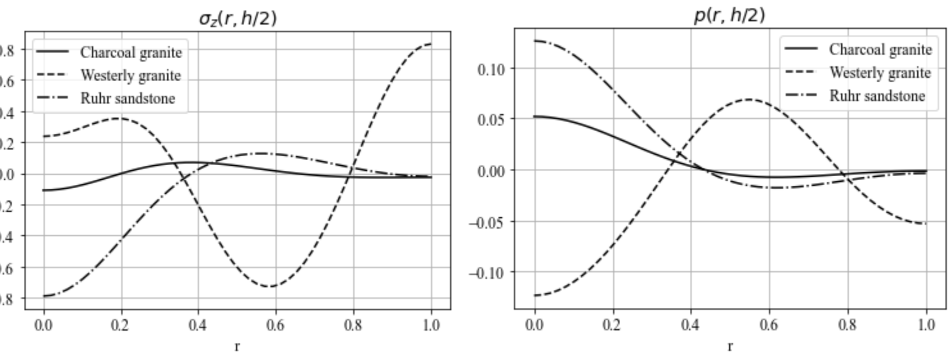
Fig. 3

Fig. 4 – 5 present the change of normal stress and pore pressure regarding the change of poroelastic materials for the case of the distributed mechanical loading for hollow and solid cylinders with permeable bottoms respectively. The least absolute values of normal stress and pore pressure are observed for the material with the smallest value of Biot's coefficient (Charcoal granite).



a *b*

Fig. 4

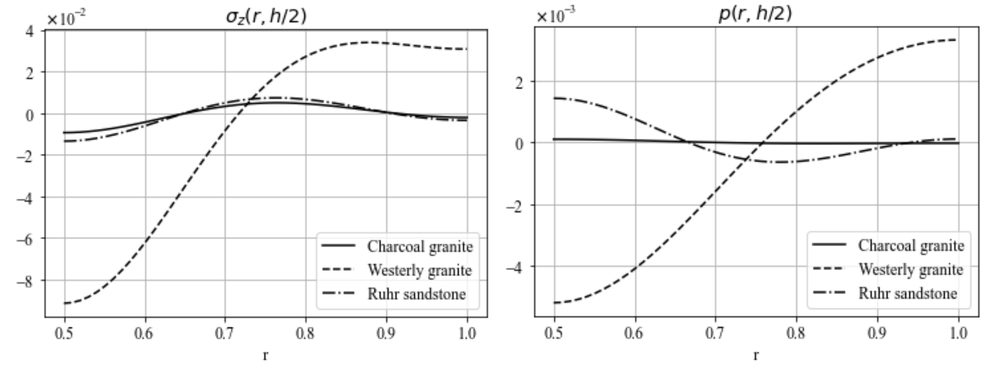


a *b*

Fig. 5

As for the previous case, the absolute values of stress and pore pressure are larger for solid cylinder. The growth of absolute values of normal stress and pore pressure are seen close to the center of the applied load for all poroelastic materials.

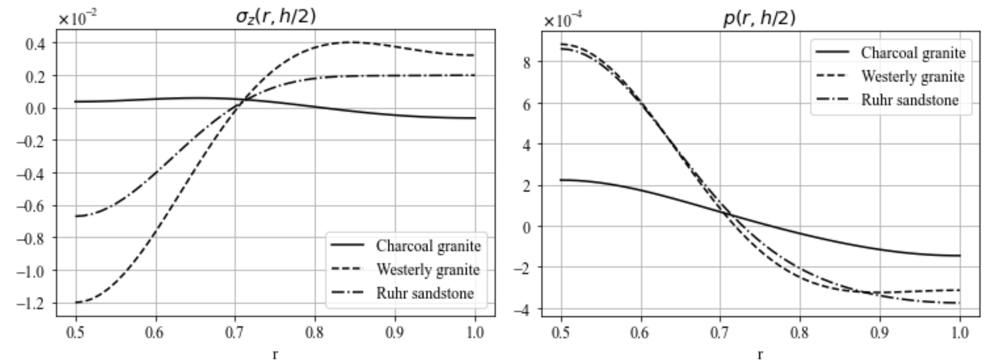
The case of impermeable cylinder’s bottom for distributed mechanical loading is shown at fig. 6 for hollow cylinder. In this case absolute values of stress and pore pressure are larger than for the case with permeable cylinder’s bottom. Also here there is no growth of the values close to the center of the applied load for Westerly granite, and a little grown of normal stress and pore pressure for other considered materials.



a *b*

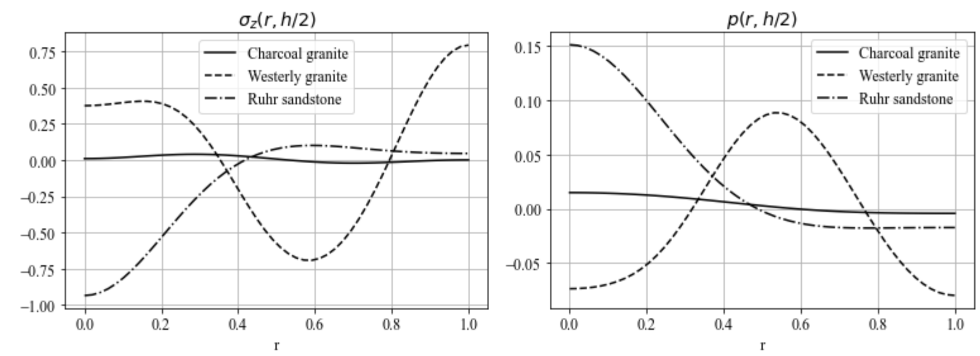
Fig. 6

The change of normal stress and pore pressure regarding the change of poroelastic material is presented at fig. 7 – 8 for hollow and solid cylinders with permeable bottom respectively.



a *b*

Fig. 7



a *b*

Fig. 8

The absolute values of stress and pore pressure for solid cylinder in this case are the largest among the considered loadings.

Conclusions.

The exact solution of a spatial axisymmetric contact poroelastic problem was firstly derived for a finite poroelastic cylinder. The cases of hollow and solid cylinders were considered under an influence of three different types of loads, different cases of a permeability of the cylinder's surface were considered. According to the derived formula for contact stress and pore pressure it was stated after numerical investigations that in the case of hollow cylinder the values of stress are noticeably less than the corresponding values of stress for a solid cylinder. The permeability has a crucial impact on the stress state of a cylinder. In the case of cylinder's bottom permeability the values of stress within a cylinder are significantly lower than in the case of the impermeability of the cylinder's bottom. The proposed results can be used in engineering for strength investigation of a cylindrical shape ceramical objects, a simulation of the bones' stress state under mechanical stimulus. The proposed method can be applied for the estimation of stress concentration near cracks inside cylindrical bodies.

Acknowledgments. The research is supported by European project funded by Horizon 2020 Framework Programme for Research and Innovation (2014-2020) (H2020-MSCA-RISE-2020) Grant Agreement number 101008140 EffectFact «Effective Factorisation techniques for matrix-functions: Developing theory, numerical methods and impactful applications», by the scholarship of the Cabinet of Ministers of Ukraine for young scientists.

Appendix A. The form of matrices and vectors at boundary vector problem (2.2) The matrices and vectors shown in (2.2) have the following form

$$L_2\vec{y}_\beta(z) = I\,\vec{y}''_\beta(z) + Q_\beta\vec{y}'_\beta(z) + R_\beta\vec{y}_\beta(z),\ I \text{ is unit matrix, } \vec{y}_\beta(z) = \begin{pmatrix} u_\beta(z) \\ w_\beta(z) \\ p_\beta(z) \end{pmatrix};$$

$$Q_\beta = \begin{pmatrix} 0 & -\dfrac{2\beta}{\kappa-1} & 0 \\ \dfrac{2\beta}{\kappa+1} & 0 & -\alpha\dfrac{\kappa-1}{\kappa+1} \\ 0 & -\dfrac{\alpha}{K} & 0 \end{pmatrix};\quad R_\beta = \begin{pmatrix} -\beta^2\dfrac{\kappa+1}{\kappa-1} & 0 & \alpha\beta \\ 0 & -\beta^2\dfrac{\kappa-1}{\kappa+1} & 0 \\ -\dfrac{\alpha\beta}{K} & 0 & -\beta^2-\dfrac{S_P}{K} \end{pmatrix};$$

$$A_{0\beta} = \begin{pmatrix} 0 & 1-\mu & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};\ B_{0\beta} = \begin{pmatrix} \beta\mu & 0 & 0 \\ 0 & -\beta & 0 \\ 0 & 0 & 1 \end{pmatrix};\ \vec{g}_\beta = \begin{pmatrix} (1-2\mu)(\alpha P_\beta - L_\beta)/2 \\ T_\beta \\ P_\beta \end{pmatrix}.$$

Here $\kappa = 3-4\mu$ is the Muskhelishvili's constant, μ is the Poisson ratio, S_p is storativity of the pore space, k is permeability. $S_P = S_pG$; $K = a_1^2/Gk$ are dimensionless values.

The matrices $A_{1\beta}$, $B_{1\beta}$ take the form

$$A_{1\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};\ B_{1\beta} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

for the permeable boundary conditions (1.3), and

$$A_{1\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix};\ B_{1\beta} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

for the impermeable boundary conditions (1.4).

$$M_{\beta}^{-1}(\xi) = d\begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix};$$

$$d = -\frac{Gk(\kappa+1)}{(\beta+\xi)^2(\beta-\xi)^2\left(\left((\beta-\xi)^2 k + S_p\right)(\kappa+1)G + \alpha^2(\kappa-1)\right)};$$

$$m_{11} = \frac{\left((-\kappa-1)\xi^2 + \beta^2(\kappa-1)\right)\left(\beta^2 k - k\xi^2 + S_p\right)G - \alpha^2\xi^2(\kappa-1)}{Gk(\kappa+1)};$$

$$m_{12} = -\frac{2\xi\beta\left(\left(\left(\beta^2-\xi^2\right)k + S_p\right)G + \alpha^2(\kappa-1)/2\right)}{Gk(\kappa-1)}; \quad m_{13} = \frac{\alpha\beta\left(\beta^2-\xi^2\right)(\kappa-1)}{G(\kappa+1)};$$

$$m_{21} = \frac{2\xi\beta\left(\left(\left(\beta^2-\xi^2\right)k + S_p\right)G + \alpha^2(\kappa-1)/2\right)}{Gk(\kappa+1)};$$

$$m_{22} = \frac{\left(\left(\beta^2-\xi^2\right)\kappa + \beta^2 + \xi^2\right)\left(\beta^2 k - k\xi^2 + S_p\right)G + \beta^2\alpha^2(\kappa-1)}{Gk(\kappa-1)};$$

$$m_{23} = -\frac{\alpha\xi\left(\beta^2-\xi^2\right)(\kappa-1)}{G(\kappa+1)}; \quad m_{31} = -\frac{\alpha\beta\left(\beta^2-\xi^2\right)(\kappa-1)}{k(\kappa+1)}; \quad m_{32} = -\frac{\alpha\xi\left(\beta^2-\xi^2\right)}{k};$$

$$m_{33} = \left(\beta^2-\xi^2\right)^2.$$

Р Е З Ю М Е. Новизна даної роботи полягає в отриманні точних розв’язків осесиметричних задач поропружності для скінчених суцільного та порожнистого циліндрів. Постановку запропонованих контактних задач подано у припущеннях моделі Біо для поропружних матеріалів. В даній роботі моделювання поропружного відгуку циліндру реалізовано з використанням апарату крайових задач. Вихідну задачу зведено до одновимірної крайової задачі у просторі трансформант за методом інтегральних перетворень. Спираючись на точний розв’язок останньої, можна отримати явні формули для переміщень, напружень та тиску рідини. Досліджено контактні напруження та тиск рідини всередині циліндру в залежності від геометричних співвідношень, типу прикладеного навантаження та параметрів поропружного матеріалу.

К Л Ю Ч О В І С Л О В А: контактна задача, поропружний скінченний циліндр, інтегральне перетворення, векторна крайова задача, точний розв’язок.

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Надійшла 03.08.2023 Затверджена до друку 12.12.2023