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ON TIME IRREVERSIBILITY OF GENERALIZED HASSANI KINEMATICS

The original Hassani transforms were introduced in the works of Algerian physicist M. E. Hassani. Hassani's generalized (superluminal) kinematics appeared in the cohntext of generalization and development of Hassani's ideas. In the present paper, applying Theorem of non returning for universal kinematics, it is proven that Hassani's generalized kinematics with positive direction of time are certainly time irreversible. From the physical point of view the last result means that in any time-positive Hassani kinematics temporal paradoxes are impossible basically, that is there is no potential possibility to affect the own past by means of "traveling" and "jumping" between reference frames.

Key words: *universal kinematics, changeable sets, inertial reference frames, tachyons, temporal paradoxes, time irreversibility.*

Introduction. Subject of constructing the theory of super-light movement, had been initiated in the papers [1, 2] more than 55 years ago. Despite the fact that at present tachyons (i.e. objects moving at a velocity greater than the velocity of light) are not experimentally detected, this subject remains being actual. Initially, the theory of tachyons was considered in the framework of classical Lorentz transformations, and superlight speed for frames of reference was forbidden. But afterwards in the papers [22, 4, 21] and later in the papers of S. Medvedev [18] as well as J. Hill and B. Cox [17] the generalized Lorentz transforms for superluminal reference frames were deduced in the case of three-dimension space of geometric (non-time) variables. And in [14] the above generalized Lorentz transforms were extended to the more general case of arbitrary (in particular infinity) dimension of the space of geometric variables (namely to the case of real Hilbert space). M. E. Hassani in [16] proposed the another interesting, system of coordinate transforms for superluminal reference frames in the case of three-dimension space of geometric variables. In the paper [11] the above original Hassani transforms were generalized and extended to the case of arbitrary real Hilbert space. Also in [11] universal kinematics based on these generalized Hassani transformations were constructed, and it was shown that these generalized Hassani kinematics do not satisfy the principle of relativity in the general case. The main aim of this paper is to show that the generalized Hassani kinematics with positive direction of time, are time irreversible. This means that in these kinematics temporal paradoxes are impossible basically, that is there is not potential possibility to affect the own past by means of "traveling" and "jumping" between reference frames and therefore the principle of causality is not violated, despite the fact that these kinematics allow superluminal motion for material points and reference frames.

In Section 2 we recall definition of the generalized Hassani transforms over Hilbert space, introduced in [11]. In Section 3 we recall main definitions and some results of the theory of changeable sets and abstract kinematics, needed further, also we define generalized Hassani kinematics based on generalized Hassani transforms. In Section 4 we remind definition of time irreversibility and Theorem of non returning for universal kinematics. In Section 5 we obtain some criteria of positive or negative time direction between reference frames with affine mutual coordinate transform operator. Finally in Section 6, applying results of previous sections, we prove that generalized Hassani kinematics with

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positive direction of time are time irreversible, which is the main result of the paper.

1. Generalized Hassani transforms over Hilbert space. Let $(\mathfrak{H}, \|\cdot\|, \langle\cdot,\cdot\rangle)$ be a Hilbert space over the real field $\mathbb{R}$ such, that $\dim(\mathfrak{H}) \geq 1$, where $\dim(\mathfrak{H})$ is dimension of the space $\mathfrak{H}$. Emphasize, that the condition $\dim(\mathfrak{H}) \geq 1$ should be interpreted in a way that the space $\mathfrak{H}$ may be infinite-dimensional. Let $\mathcal{L}(\mathfrak{H})$ be the space of (homogeneous) linear continuous operators over the space $\mathfrak{H}$. Denote by $\mathcal{L}^{\times}(\mathfrak{H})$ the space of all operators of affine transformations over the space $\mathfrak{H}$, that is $\mathcal{L}^{\times}(\mathfrak{H}) = \{\mathbf{A}_{[\mathbf{a}]} \mid \mathbf{A} \in \mathcal{L}(\mathfrak{H}),\ \mathbf{a} \in \mathfrak{H}\}$, where $\mathbf{A}_{[\mathbf{a}]}x = \mathbf{A}x + \mathbf{a}$, $x \in \mathfrak{H}$. The ***Minkowski space*** over the Hilbert space $\mathfrak{H}$ is defined as the Hilbert space $\mathcal{M}(\mathfrak{H}) = \mathbb{R} \times \mathfrak{H} = \{(t,x) \mid t \in \mathbb{R}, x \in \mathfrak{H}\}$, equipped by the inner product and norm: $\langle \mathrm{w}_1, \mathrm{w}_2 \rangle = \langle \mathrm{w}_1, \mathrm{w}_2 \rangle_{\mathcal{M}(\mathfrak{H})} = t_1 t_2 + \langle x_1, x_2 \rangle$, $\|\mathrm{w}_1\| = \|\mathrm{w}_1\|_{\mathcal{M}(\mathfrak{H})} = \left(t_1^2 + \|x_1\|^2\right)^{1/2}$ (where $\mathrm{w}_i = (t_i, x_i) \in \mathcal{M}(\mathfrak{H})$, $i \in \{1,2\}$) ([14, 7]). In the space $\mathcal{M}(\mathfrak{H})$ we select the next subspaces: $\mathfrak{H}_0 := \{(t,0) \mid t \in \mathbb{R}\}$, $\mathfrak{H}_1 := \{(0,x) \mid x \in \mathfrak{H}\}$ with $\mathbf{0}$ being zero vector. Then, $\mathcal{M}(\mathfrak{H}) = \mathfrak{H}_0 \oplus \mathfrak{H}_1$, where $\oplus$ means the orthogonal sum of subspaces. Denote: $\mathbf{e}_0 := (1,0) \in \mathcal{M}(\mathfrak{H})$. We denote by $\mathbf{X}$ and $\hat{\mathbf{T}}$ the orthogonal projectors on the subspaces $\mathfrak{H}_1$ and $\mathfrak{H}_0$:

$$\mathbf{X}\mathrm{w} = (0, x) \in \mathfrak{H}_1; \quad \hat{\mathbf{T}}\mathrm{w} = (t, 0) = \mathcal{T}(\mathrm{w})\mathbf{e}_0 \in \mathfrak{H}_0,$$

where $\mathcal{T}(\mathrm{w}) = t \qquad (\mathrm{w} = (t,x) \in \mathcal{M}(\mathfrak{H}))$.

Definition 1. *The operator $S \in \mathcal{L}(\mathcal{M}(\mathfrak{H}))$ is referred to as **linear coordinate transform operator** if and only if there exists the continuous inverse operator $S^{-1} \in \mathcal{L}(\mathcal{M}(\mathfrak{H}))$.*

*Denote via $\mathbf{Pk}(\mathfrak{H})$ the set of all operators $\mathbf{S} \in \mathcal{L}^{\times}(\mathcal{M}(\mathfrak{H}))$, which has the continuous inverse operator $\mathbf{S}^{-1} \in \mathcal{L}^{\times}(\mathcal{M}(\mathfrak{H}))$. Operators $\mathbf{S} \in \mathbf{Pk}(\mathfrak{H})$ will be called as **(affine) coordinate transform operators**.*

Let $\mathbf{B}_1(\mathfrak{H}_1)$ be the unit sphere in the space $\mathfrak{H}_1$ ($\mathbf{B}_1(\mathfrak{H}_1) = \{x \in \mathfrak{H}_1 \mid \|\mathrm{x}\| = 1\}$). Any vector $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ generates the following orthogonal projectors, acting in $\mathcal{M}(\mathfrak{H})$:

$$\left.\begin{array}{l} \mathbf{X}_1[\mathbf{n}]\mathrm{w} = \langle \mathbf{n}, \mathrm{w} \rangle \mathbf{n} \quad (\mathrm{w} \in \mathcal{M}(\mathfrak{H})); \\ \mathbf{X}_1^{\perp}[\mathbf{n}] = \mathbf{X} - \mathbf{X}_1[\mathbf{n}]. \end{array}\right] \tag{1}$$

Recall, that an operator $U \in \mathcal{L}(\mathfrak{H})$ is referred to as ***unitary*** on $\mathfrak{H}$, if and only if $\exists\, U^{-1} \in \mathcal{L}(\mathfrak{H})$ and $\forall\, x \in \mathfrak{H}$ $\|Ux\| = \|x\|$. Let $\mathfrak{U}(\mathfrak{H}_1)$ be the set of all ***unitary*** operators over the space $\mathfrak{H}_1$. Fix some real number c such, that $0 < c < \infty$. Then for every $\lambda \in [0, c)$, $s \in \{-1, 1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ and $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$ we introduce the following operators in $\mathcal{M}(\mathfrak{H})$:

$$\mathbf{W}_{\lambda,c}[s,\mathbf{n},J]\mathrm{w} := \frac{\left(s\mathcal{T}(\mathrm{w}) - \frac{\lambda}{c^2}\langle \mathbf{n},\mathrm{w}\rangle\right)}{\sqrt{1-\frac{\lambda^2}{c^2}}}\mathbf{e}_0 + J\left(\frac{\lambda\mathcal{T}(\mathrm{w}) - s\langle \mathbf{n},\mathrm{w}\rangle}{\sqrt{1-\frac{\lambda^2}{c^2}}}\mathbf{n} + \mathbf{X}_1^{\perp}[\mathbf{n}]\mathrm{w}\right); \quad (2)$$

$$\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]\mathrm{w} := \mathbf{W}_{\lambda,c}[s,\mathbf{n},J](\mathrm{w}+\mathbf{a}) \qquad (\mathrm{w}\in\mathcal{M}(\mathfrak{H})). \quad (3)$$

Under the additional conditions $\dim(\mathfrak{H})=3$, $s=1$ the right-hand part of the formula (2) is equivalent to the same part of the formula (28b) from [19, page 43]. That is why, in this case we obtain the classical Lorentz transforms for inertial reference frame in the most general form (with arbitrary orientation of axes).

Denote by Υ the class of functions $\vartheta:[0,\infty)\to\mathbb{R}$, satisfying the following conditions:

$$\left.\begin{array}{lll} \vartheta(\lambda)\ge\lambda & \text{for} & \lambda\in[0,\infty), \\ \exists\ \eta>0 & \vartheta(\lambda)>\lambda & (\forall\lambda\in[0,\eta)). \end{array}\right\} \quad (4)$$

For any function $\vartheta\in\Upsilon$ we use the following notation:

$$\mathfrak{D}_*[\vartheta] := \{\lambda\in[0,\infty)\mid \vartheta(\lambda)>\lambda\}. \quad (5)$$

According to the conditions (4), we have, $\mathfrak{D}_*[\vartheta]\ne\varnothing$, and moreover,

$$[0,\eta)\subseteq\mathfrak{D}_*[\vartheta] \quad \text{for some} \quad \eta>0. \quad (6)$$

For each functional parameter $\vartheta\in\Upsilon$ (where Υ is the class of functions, satisfying (4)) we introduce classes of operators:

$$\mathfrak{O}(\mathfrak{H},[\vartheta]) := \left\{\mathbf{W}_{\lambda,\vartheta(\lambda)}[s,\mathbf{n},J] \mid s\in\{-1,1\},\ \lambda\in\mathfrak{D}_*[\vartheta],\ \mathbf{n}\in\mathbf{B}_1(\mathfrak{H}_1),\ J\in\mathfrak{U}(\mathfrak{H}_1)\right\}; \quad (7)$$

$$\begin{aligned} \mathfrak{O}_+(\mathfrak{H},[\vartheta]) &:= \left\{\mathbf{W}_{\lambda,\vartheta(\lambda)}[s,\mathbf{n},J]\in\mathfrak{O}(\mathfrak{H},[\vartheta]) \mid s=1\right\} = \\ &= \left\{\mathbf{W}_{\lambda,\vartheta(\lambda)}[1,\mathbf{n},J] \mid \lambda\in\mathfrak{D}_*[\vartheta],\ \mathbf{n}\in\mathbf{B}_1(\mathfrak{H}_1),\ J\in\mathfrak{U}(\mathfrak{H}_1)\right\}; \end{aligned} \quad (8)$$

$$\mathfrak{P}(\mathfrak{H},[\vartheta]) := \left\{\mathbf{W}_{\lambda,\vartheta(\lambda)}[s,\mathbf{n},J;\mathbf{a}] \mid \mathbf{W}_{\lambda,\vartheta(\lambda)}[s,\mathbf{n},J]\in\mathfrak{O}(\mathfrak{H},[\vartheta]),\ \mathbf{a}\in\mathcal{M}(\mathfrak{H})\right\}; \quad (9)$$

$$\mathfrak{P}_+(\mathfrak{H},[\vartheta]) := \left\{\mathbf{W}_{\lambda,\vartheta(\lambda)}[s,\mathbf{n},J;\mathbf{a}] \mid \mathbf{W}_{\lambda,\vartheta(\lambda)}[s,\mathbf{n},J]\in\mathfrak{O}_+(\mathfrak{H},[\vartheta]),\ \mathbf{a}\in\mathcal{M}(\mathfrak{H})\right\}, \quad (10)$$

where $\mathfrak{D}_*[\vartheta]$ is the set of ϑ***-allowed*** velocities, defined by (5). It is not hard to verify that for each $\vartheta\in\Upsilon$ we have $\mathfrak{O}(\mathfrak{H},[\vartheta])\subseteq\mathcal{L}(\mathcal{M}(\mathfrak{H}))\cap\mathbf{Pk}(\mathfrak{H})$ and $\mathfrak{P}(\mathfrak{H},[\vartheta])\subseteq\mathbf{Pk}(\mathfrak{H})\subseteq\mathcal{L}^{\times}(\mathcal{M}(\mathfrak{H}))$ (for details see [11]), moreover the following set-theoretic inclusions are performed:

$$\mathfrak{O}_+(\mathfrak{H},[\vartheta])\subseteq\mathfrak{O}(\mathfrak{H},[\vartheta])\subseteq\mathfrak{P}(\mathfrak{H},[\vartheta]); \quad (11)$$

$$\mathfrak{O}_+(\mathfrak{H},[\vartheta])\subseteq\mathfrak{P}_+(\mathfrak{H},[\vartheta])\subseteq\mathfrak{P}(\mathfrak{H},[\vartheta]). \quad (12)$$

According to [11], we call the class $\mathfrak{O}(\mathfrak{H},[\vartheta])$ by class of ***generalized Hassani transforms*** over Hilbert space $\mathfrak{H}$; we call $\mathfrak{O}_+(\mathfrak{H},[\vartheta])$ by class of ***time-positive generalized Hassani transforms*** over $\mathfrak{H}$; we call $\mathfrak{P}(\mathfrak{H},[\vartheta])$ by class of ***Poincare–Hassani transforms*** over $\mathfrak{H}$; we call $\mathfrak{P}_+(\mathfrak{H},[\vartheta])$ by class of ***time-positive Poincare–Hassani transforms*** over $\mathfrak{H}$. For $0<c<\infty$ we note:

$$\vartheta_c(\lambda) := \begin{cases} c, & 0\le\lambda<c, \\ \lambda, & \lambda\ge c. \end{cases} \quad (13)$$

It is easy to verify that $\vartheta_c \in \Upsilon$ and $\mathfrak{D}_*[\vartheta_c] = [0,c)$ (for each $c \in (0,+\infty)$). For the function $\vartheta_c \in \Upsilon$ we get, $\mathfrak{O}(\mathfrak{H},[\vartheta_c]) = \mathfrak{O}(\mathfrak{H},c)$, $\mathfrak{P}(\mathfrak{H},[\vartheta_c]) = \mathfrak{P}(\mathfrak{H},c)$, $\mathfrak{O}_+(\mathfrak{H},[\vartheta_c]) = \mathfrak{O}_+(\mathfrak{H},c)$, $\mathfrak{P}_+(\mathfrak{H},[\vartheta_c]) = \mathfrak{P}_+(\mathfrak{H},c)$, where the classes of operators $\mathfrak{O}(\mathfrak{H},c)$, $\mathfrak{O}_+(\mathfrak{H},c)$, $\mathfrak{P}(\mathfrak{H},c)$, $\mathfrak{P}_+(\mathfrak{H},c)$ are defined in [11, 7, 6, etc].

Remark 1. It can be proven that for any $c \in [0,+\infty)$ all four classes of operators $\mathfrak{O}(\mathfrak{H},c)$, $\mathfrak{O}_+(\mathfrak{H},c)$, $\mathfrak{P}(\mathfrak{H},c)$, $\mathfrak{P}_+(\mathfrak{H},c)$ are groups of operators (in algebraic sense) in the space $\mathcal{M}(\mathfrak{H})$ relatively the operation of multiplication (composition) of operators (see [6, Remark 4.1, Corollary 4.1]; see also [7, Assertion 2.17.1 and formula (2.94), Assertion 2.17.6, Corollary 2.19.5]). In particular $\mathfrak{O}(\mathfrak{H},c)$ coincides with the group of all linear coordinate transform operators over the space $\mathcal{M}(\mathfrak{H})$, leaving unchanged values of the functional $\mathrm{M}_c(\mathrm{w}) = \|\mathbf{X}\mathrm{w}\|^2 - c^2\mathcal{T}^2(\mathrm{w})$ $(\mathrm{w} \in \mathcal{M}(\mathfrak{H}))$, that is the set of all bijective operators $L \in \mathcal{L}(\mathcal{M}(\mathfrak{H}))$ such, that $\mathrm{M}_c(L\mathrm{w}) = \mathrm{M}_c(\mathrm{w})$ $(\forall \mathrm{w} \in \mathcal{M}(\mathfrak{H}))$. In the case $\mathfrak{H} = \mathbb{R}^3$ the group of operators $\mathfrak{O}_+(\mathfrak{H},c)$ coincides with with the full Lorentz group, being considered in [20]. In the case $\mathfrak{H} = \mathbb{R}^3$ the group of operators $\mathfrak{P}_+(\mathfrak{H},c)$ coincides with the famous Poincare group [7, Remark 2.19.1]. Moreover, in [11] it had been proven that if for some function $\vartheta \in \Upsilon$ one of the classes of operators $\mathfrak{O}(\mathfrak{H},[\vartheta])$, $\mathfrak{O}_+(\mathfrak{H},[\vartheta])$, $\mathfrak{P}(\mathfrak{H},[\vartheta])$, $\mathfrak{P}_+(\mathfrak{H},[\vartheta])$ is a group of operators in the space $\mathcal{M}(\mathfrak{H})$ then a number $c \in (0,\infty)$ exists such, that $\vartheta(\lambda) = \vartheta_c(\lambda)$ for every $\lambda \in (0,\infty)$, and in this case we have $\mathfrak{O}(\mathfrak{H},[\vartheta]) = \mathfrak{O}(\mathfrak{H},c)$, $\mathfrak{O}_+(\mathfrak{H},[\vartheta]) = \mathfrak{O}_+(\mathfrak{H},c)$, $\mathfrak{P}(\mathfrak{H},[\vartheta]) = \mathfrak{P}(\mathfrak{H},c)$, $\mathfrak{P}_+(\mathfrak{H},[\vartheta]) = \mathfrak{P}_+(\mathfrak{H},c)$.

2. Some facts from the theory of changeable sets and abstract kinematics. In this section we present some definitions and results from the theory of changeable sets and abstract kinematics, needed for statement of the main results. From an intuitive point of view, changeable sets are sets of objects which, unlike elements of ordinary (static) sets, may be in the process of continuous transformations, and which may change properties depending on the point of view on them (that is depending on the reference frame).

Definition of changeable set will be made in two steps. In the first step we formulate the definition of base changeable set.

Let $\mathbb{T} = (\mathbf{T},\leq)$ be any linearly (totally) ordered set (the sense of [3, p. 12]) and let $\mathcal{X}$ be any nonempty set. For any ordered pair $\omega = (t,x) \in \mathbf{T} \times \mathcal{X}$ we use the notations:

$$\mathsf{bs}(\omega) := x, \qquad \mathsf{tm}(\omega) := t.$$

Definition 2 ([8][1]). *The ordered triple of kind $\mathcal{B} = (\mathbf{B},\mathbb{T},\lhd)$, where $\mathbf{B} \subseteq \mathbf{T} \times \mathcal{X}$, is called by **base changeable set** if and only if the following conditions are satisfied:*

1. *$\mathbf{B} \neq \varnothing$ and $\lhd$ is reflexive binary relation on $\mathbf{B}$ (that is $\forall \omega \in \mathbf{B}$ $\omega \lhd \omega$);*
2. *for arbitrary $\omega, \omega_2 \in \mathbf{B}$ the conditions $\omega_2 \lhd \omega_1$ and $\omega_1 \neq \omega_2$ cause the inequality $\mathsf{tm}(\omega_1) < \mathsf{tm}(\omega_2)$, where $<$ is the strict order relation, generated by the non-strict order $\leq$ of linearly ordered set $\mathbb{T} = (\mathbf{T},\leq)$* [2].

[1] In some papers it can be found definition of base changeable set, that uses the notion of primitive changeable set, which is different from Definition 2 (see for example [7, 5]). As it was proven in [7, 6], the both definitions are equivalent.

Remark 2. *For an arbitrary base changeable set* $\mathcal{B} = (\mathbf{B}, \mathbb{T}, \lhd) = (\mathbf{B}, (\mathbf{T}, \leq), \lhd)$ *(where* $\mathbf{B} \subseteq \mathbf{T} \times \mathcal{X}$*) we use the following notations and terminology:*

$$\mathbb{B}\mathfrak{s}(\mathcal{B}) := \mathbf{B};\ \underset{\mathcal{B}}{\leftarrow} := \lhd;\ \mathbb{T}\mathbf{m}(\mathcal{B}) := \mathbb{T}; \mathbf{Tm}(\mathcal{B}) := \mathbf{T};\ \leq_{\mathcal{B}} := \leq$$

$$\mathfrak{B}\mathfrak{s}(\mathcal{B}) := \{x \in \mathcal{X} \mid \exists \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B}) (\mathsf{bs}(\omega) = x)\} = \{\mathsf{bs}(\omega) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\}. \quad (14)$$

- For $t, \tau \in \mathbf{Tm}(\mathcal{B})$ we write $t <_{\mathcal{B}} \tau$ if and only if $t \leq_{\mathcal{B}} \tau$ and $t \neq \tau$.
- The set $\mathfrak{B}\mathfrak{s}(\mathcal{B})$ is called by the basic set or the set of all elementary states of $\mathcal{B}$.
- The set $\mathbb{B}\mathfrak{s}(\mathcal{B})$ is called by the set of all elementary-time states of $\mathcal{B}$.
- The set $\mathbf{Tm}(\mathcal{B})$ is called by the set of time points of $\mathcal{B}$.
- The relation $\underset{\mathcal{B}}{\leftarrow}$ is called by the base of elementary processes of $\mathcal{B}$.

Note that from the definition (14) together with the above notations for any base changeable set $\mathcal{B}$ we deduce:

$$\mathbb{B}\mathfrak{s}(\mathcal{B}) \subseteq \mathbf{Tm}(\mathcal{B}) \times \mathfrak{B}\mathfrak{s}(\mathcal{B}) \quad (15)$$

Remark 3. In the cases, when the base changeable set $\mathcal{B}$ is evident we use the notations $\leftarrow, \leq, <$ instead of the notations $\underset{\mathcal{B}}{\leftarrow}, \leq_{\mathcal{B}}, <_{\mathcal{B}}$.

For the elements $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$ the noting $\omega_2 \leftarrow \omega_1$ should be interpreted as "the elementary-time state ω_2 is the result of transformations (or the transformation prolongation) of the elementary-time state ω_1".

We say that elementary-time states $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$ are ***united by fate*** in $\mathcal{B}$ if at least one of the correlations $\omega_2 \leftarrow \omega_1$ or $\omega_1 \leftarrow \omega_2$ is valid.

The main method of generation base changeable sets is connected with systems of abstract trajectories.

Definition 3. *Let* M *be an arbitrary set and* $\mathbb{T} = (\mathbf{T}, \leq)$ *be any linearly ordered set.*

1. *Any mapping* $r : \mathfrak{D}(r) \to M$*, where* $\mathfrak{D}(r) \subseteq \mathbf{T}$*, will be referred to as an* ***abstract trajectory*** *from* $\mathbb{T}$ *to* M *(here* $\mathfrak{D}(r)$ *is the domain of the abstract trajectory* r*).*
2. *Any set* $\mathcal{R}$*, which consists of abstract trajectories from* $\mathbb{T}$ *to* M *will be called* ***system of abstract trajectories*** *from* $\mathbb{T}$ *to* M*.*

Theorem 1 ([5], see [7])**.** *Let* $\mathcal{R}$ *be a system of abstract trajectories from* $\mathbb{T} = (\mathbf{T}, \leq)$ *to* M*. Then there exists a unique base changeable set* $\mathcal{B} = \mathcal{A}t(\mathbb{T}, \mathcal{R})$*, such, that:*

1) $\mathbb{T}\mathbf{m}(\mathcal{B}) = \mathbb{T}$*;*

2) $\mathbb{B}\mathfrak{s}(\mathcal{B}) = \bigcup_{r \in \mathcal{R}} r$*;*

3) *For arbitrary* $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$ *the condition* $\omega_2 \underset{\mathcal{B}}{\leftarrow} \omega_1$ *is satisfied if and only if* $\mathsf{tm}(\omega_1) \leq \mathsf{tm}(\omega_2)$ *and there exists an abstract trajectory* $r \in \mathcal{R}$ *such, that* $\omega_1, \omega_2 \in r$*.*

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[2] Recall [3] that the (non strict) linear order relation $\leq$ generates the strict order relation $<$ on $\mathbf{T}$ by the following rule:

$$t_1 < t_2 \text{ holds if and only if } t_1 \leq t_2 \text{ and } t_1 \neq t_2 (\forall t_1, t_2 \in \mathbf{T}).$$

Remark 4. Note, that in Theorem 1 any trajectory $r \in \mathcal{R}$ can be interpreted as the set: $r = \{(t, r(t)) \mid t \in \mathfrak{D}(r)\}$. So, in accordance with item 3) of this theorem, for the base changeable set $\mathcal{B} = \mathcal{A}t(\mathbb{T}, \mathcal{R})$ we have, $\mathfrak{B}\mathfrak{s}(\mathcal{B}) = \bigcup_{r \in \mathcal{R}} \mathfrak{R}(r) \subseteq M$, where $\mathfrak{R}(r)$ is the range of trajectory $r \in \mathcal{R}$.

Conversely, it can be proven, that any base changeable set can be generated by some system of abstract trajectories ([5], see also [7]).

Other further important method of generation new base changeable sets is creation of image of existing base changeable set.

Theorem 2 (theorem on image, published in [10], see also [7]). *Let $\mathcal{B}$ be a base changeable set, $\mathbb{T} = (\mathbf{T}, \leq)$ be a linearly ordered set, $\mathcal{X}$ be any set and U be a mapping from $\mathbb{B}\mathfrak{s}(\mathcal{B})$ into $\mathbf{T} \times \mathcal{X}$ ($U : \mathbb{B}\mathfrak{s}(\mathcal{B}) \to \mathbf{T} \times \mathcal{X}$). Then there exists only one base changeable set $\mathcal{B}_1 := U[\mathcal{B}, \mathbb{T}]$, satisfying the following conditions:*

1. $\mathbb{T}\mathbf{m}(\mathcal{B}_1) = \mathbb{T}$;
2. $\mathbb{B}\mathfrak{s}(\mathcal{B}_1) = U(\mathbb{B}\mathfrak{s}(\mathcal{B})) = \{U(\omega) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\}$;
3. *Let* $\tilde{\omega}_1, \tilde{\omega}_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B}_1)$ *and* $\mathsf{tm}(\tilde{\omega}_1) \neq \mathsf{tm}(\tilde{\omega}_2)$. *Then* $\tilde{\omega}_1$ *and* $\tilde{\omega}_2$ *are united by fate in* $\mathcal{B}_1$ *if and only if, there exist united by fate in* $\mathcal{B}$ *elementary-time states* $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$ *such, that* $\tilde{\omega}_1 = U(\omega_1)$, $\tilde{\omega}_2 = U(\omega_2)$.

$U[\mathcal{B}, \mathbb{T}]$ is called by ***image of base changeable set*** $\mathcal{B}$ relatively the mapping U and time scale $\mathbb{T}$. In the case where $\mathbb{T} = \mathbb{T}\mathbf{m}(\mathcal{B})$ we use the notation $U[\mathcal{B}]$ instead of $U[\mathcal{B}, \mathbb{T}]$:

$$U[\mathcal{B}] = U[\mathcal{B}, \mathbb{T}\mathbf{m}(\mathcal{B})].$$

Definition 4. *Let $\overleftarrow{\mathcal{B}} = (\mathcal{B}_\alpha \mid \alpha \in \mathcal{A})$ be any indexed family of base changeable sets (where $\mathcal{A} \neq \varnothing$ is the some set of indexes). The system of mappings $\overleftarrow{\mathfrak{U}} = (\mathfrak{U}_{\beta\alpha} \mid \alpha, \beta \in \mathcal{A})$ of kind $\mathfrak{U}_{\beta\alpha} : 2^{\mathbb{B}\mathfrak{s}(\mathcal{B}_\alpha)} \to 2^{\mathbb{B}\mathfrak{s}(\mathcal{B}_\beta)}$ $(\alpha, \beta \in \mathcal{A})$ is referred to as **unification of perception** on $\overleftarrow{\mathcal{B}}$ if and only if the following conditions are satisfied:*

1. $\mathfrak{U}_{\alpha\alpha} A = A$ *for any* $\alpha \in \mathcal{A}$ *and* $A \subseteq \mathbb{B}\mathfrak{s}(\mathcal{B}_\alpha)$.

(Here and further we denote by $\mathfrak{U}_{\beta\alpha} A$ the action of the mapping $\mathfrak{U}_{\beta\alpha}$ to the set $A \subseteq \mathbb{B}\mathfrak{s}(\mathcal{B}_\alpha)$, that is $\mathfrak{U}_{\beta\alpha} A := \mathfrak{U}_{\beta\alpha}(A)$.)

2. *Any mapping $\mathfrak{U}_{\beta\alpha}$ is a monotonous mapping of sets, i.e. for any $\alpha, \beta \in \mathcal{A}$ and $A, B \subseteq \mathbb{B}\mathfrak{s}(\mathcal{B}_\alpha)$ the condition $A \subseteq B$ assures $\mathfrak{U}_{\beta\alpha} A \subseteq \mathfrak{U}_{\beta\alpha} B$.*
3. *For any $\alpha, \beta, \gamma \in \mathcal{A}$ and $A \subseteq \mathbb{B}\mathfrak{s}(\mathcal{B}_\alpha)$ the following inclusion holds:*

$$\mathfrak{U}_{\gamma\beta} \mathfrak{U}_{\beta\alpha} A \subseteq \mathfrak{U}_{\gamma\alpha} A. \tag{16}$$

*In this case the mappings $\mathfrak{U}_{\beta\alpha}$ ($\alpha, \beta \in \mathcal{A}$) we call by **unification mappings**, and the triple of kind $\mathcal{Z} = (\mathcal{A}, \overleftarrow{\mathcal{B}}, \overleftarrow{\mathfrak{U}})$ we name by **changeable set**.*

Remark 5 (on notations). Let $\mathcal{Z} = (\mathcal{A}, \overleftarrow{\mathcal{B}}, \overleftarrow{\mathfrak{U}})$ be a changeable set, where $\overleftarrow{\mathcal{B}} = (\mathcal{B}_\alpha \mid \alpha \in \mathcal{A})$ is an indexed family of base changeable sets and $\overleftarrow{\mathfrak{U}} = (\mathfrak{U}_{\beta\alpha} \mid \alpha, \beta \in \mathcal{A})$ is an unification of perception on $\overleftarrow{\mathcal{B}}$. Further we will use the following terms and notations:

1) The set $\mathcal{A}$ will be called the ***index set*** of the changeable set $\mathcal{Z}$, and it will be denoted by $\mathcal{I}nd(\mathcal{Z})$.

2) For any index $\alpha \in \mathcal{I}nd(\mathcal{Z})$ the pair $\mathbf{lk}_\alpha(\mathcal{Z}) = (\alpha, \mathcal{B}_\alpha)$ will be referred to as ***reference frame*** of the changeable set $\mathcal{Z}$.

3) The set of all reference frames of $\mathcal{Z}$ will be denoted by $\mathcal{L}k(\mathcal{Z})$:

$$\mathcal{L}k(\mathcal{Z}) := \{(\alpha, \mathcal{B}_\alpha) \mid \alpha \in \mathcal{I}nd(\mathcal{Z})\} = \{\mathbf{lk}_\alpha(\mathcal{Z}) \mid \alpha \in \mathcal{I}nd(\mathcal{Z})\}.$$

Typically, reference frames will be denoted by small Gothic letters ($\mathfrak{l}, \mathfrak{m}, \mathfrak{k}, \mathfrak{p}$ and so on).

4) For $\mathfrak{l} = (\alpha, \mathcal{B}_\alpha) \in \mathcal{L}k(\mathcal{Z})$ we introduce the following denotations:

$$\mathrm{ind}(\mathfrak{l}) := \alpha; \qquad \mathfrak{l}^\wedge := \mathcal{B}_\alpha.$$

Thus, for any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$ the object $\mathfrak{l}^\wedge$ is a base changeable set. Further, when it does not cause confusion, for any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$ the symbol “ $^\wedge$ ” will be omitted in the denotations $\mathfrak{Bs}(\mathfrak{l}^\wedge)$, $\mathbb{Bs}(\mathfrak{l}^\wedge)$, $\mathbf{Tm}(\mathfrak{l}^\wedge)$, $\mathbb{T}\mathbf{m}(\mathfrak{l}^\wedge)$, $\underset{\mathfrak{l}^\wedge}{\leftarrow}$, $\le_{\mathfrak{l}^\wedge}$, $<_{\mathfrak{l}^\wedge}$ and the denotations $\mathfrak{Bs}(\mathfrak{l})$, $\mathbb{Bs}(\mathfrak{l})$, $\mathbf{Tm}(\mathfrak{l})$, $\mathbb{T}\mathbf{m}(\mathfrak{l})$, $\underset{\mathfrak{l}}{\leftarrow}$, $\le_{\mathfrak{l}}$, $<_{\mathfrak{l}}$ will be used.

5) For any reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ the mapping $\mathfrak{U}_{\mathrm{ind}(\mathfrak{m}),\mathrm{ind}(\mathfrak{l})}$ will be denoted by $\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \rangle$. Hence:

$$\left\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \right\rangle = \mathfrak{U}_{\mathrm{ind}(\mathfrak{m}),\mathrm{ind}(\mathfrak{l})}.$$

In the case, when the changeable $\mathcal{Z}$ set is known in advance, the symbol $\mathcal{Z}$ in the above notation will be omitted, and the denotation “$\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle$” will be used instead.

6) In the case, when it does not cause confusion, we will use the denotations $\leftarrow$, $\le$, $<$ instead of the denotations $\underset{\mathfrak{l}}{\leftarrow}$, $\le_{\mathfrak{l}}$, $<_{\mathfrak{l}}$.

7) For any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$ we reserve the terminology, introduced in Remark 2 (where the symbol $\mathcal{B}$ should be replaced by the symbol “$\mathfrak{l}$” and the phrase “base changeable set” should be replaced by the phrase “reference frame”).

Definition 5. *We say, that a changeable set $\mathcal{Z}$ is **precisely visible** if and only if for any reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ and for any element $\omega \in \mathbb{Bs}(\mathfrak{l})$ there exist a unique element $\omega' \in \mathbb{Bs}(\mathfrak{m})$ such, that $\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle \{\omega\} = \{\omega'\}$.* [3]

Let $\mathcal{Z}$ be any precisely visible changeable set and $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ be any reference frames of $\mathcal{Z}$. For any $\omega \in \mathbb{Bs}(\mathfrak{l})$ we denote by $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \rangle \omega$ (or by $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \omega$) the unique (in accordance with Definition 5) element $\omega' \in \mathbb{Bs}(\mathfrak{m})$ such, that $\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle \{\omega\} = \{\omega'\}$. Hence, we have $\forall \omega \in \mathbb{Bs}(\mathfrak{l})$ $\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle \{\omega\} =$ $= \{\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \omega\}$. The mapping $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle : \mathbb{Bs}(\mathfrak{l}) \to \mathbb{Bs}(\mathfrak{m})$ we call as the ***precise unification mapping*** of $\mathcal{Z}$.

[3] In some papers (see, for example, [7, Definition I.12.3]) it had been given another, different, definition of precisely visible changeable set notion. Using [7, Corollary I.12.5 and Assertion I.12.11] it can be proved, that Definition 5 is equivalent to the definition, given in [7].

Assertion 1 ([6], see also [7]). *Let $\mathcal{Z}$ be any precisely visible changeable set, and $\mathfrak{l},\mathfrak{m},\mathfrak{p}\in\mathcal{L}k(\mathcal{Z})$ be arbitrary reference frames of $\mathcal{Z}$. Then:*

1. $\forall\ \omega\in\mathbb{B}\mathfrak{s}(\mathfrak{l})\quad \left\langle !\ \ \mathfrak{l}\leftarrow\mathfrak{l}\right\rangle\omega=\omega$;

2. $\forall\ A\subseteq\mathbb{B}\mathfrak{s}(\mathfrak{l})\quad \left\langle \mathfrak{m}\leftarrow\mathfrak{l}\right\rangle=\left\{\left\langle !\ \ \mathfrak{m}\leftarrow\mathfrak{l}\right\rangle\omega\mid\omega\in A\right\}$;

3. $\forall\ \omega\in\mathbb{B}\mathfrak{s}(\mathfrak{l})\quad \left\langle !\ \ \mathfrak{p}\leftarrow\mathfrak{m}\right\rangle\left\langle !\ \ \mathfrak{m}\leftarrow\mathfrak{l}\right\rangle\omega=\left\langle !\ \ \mathfrak{p}\leftarrow\mathfrak{l}\right\rangle\omega$.

Below we present definition of universal kinematics. Universal kinematics are mathematical objects, in which changeable sets are equipped by different geometrical or topological structures (namely topological, linear, Banach, Hilbert and other spaces) together with some universal coordinate transforms between reference frames.

Definition 6. *Let $\mathcal{Z}$ be any precisely visible changeable set. The triple of kind $\mathcal{F}=\left(\mathcal{Z},\mathcal{G},\overleftarrow{\mathcal{Q}}\right)$ is called by* ***universal kinematic set*** *or, abbreviated, by* ***universal kinematics*** *if and only if:*

1. *$\mathcal{G}$ is an indexed family of kind $\mathcal{G}=\left(\left(\mathfrak{X}_{\mathfrak{l}},\|\cdot\|_{(\mathfrak{l})},k_{\mathfrak{l}}\right)\mid\mathfrak{l}\in\mathcal{L}k(\mathcal{Z})\right)$.*

2. *$\overleftarrow{\mathcal{Q}}$ is an indexed family of kind $\overleftarrow{\mathcal{Q}}=\left(\tilde{\mathcal{Q}}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m}\in\mathcal{L}k(\mathcal{Z})}$.*

3. *For any reference frame $\mathfrak{l}\in\mathcal{L}k(\mathcal{Z})$ the following conditions are satisfied:*

a) $\left(\mathfrak{X}_{\mathfrak{l}},\|\cdot\|_{(\mathfrak{l})}\right)$ is a linear normed space over real field $\mathbb{R}$ or complex field $\mathbb{C}$;

b) $k_{\mathfrak{l}}:\mathfrak{Bs}(\mathfrak{l})\to\mathfrak{X}_{\mathfrak{l}}$ is a mapping from $\mathfrak{Bs}(\mathfrak{l})$ to $\mathfrak{X}_{\mathfrak{l}}$.

4. *For any $\mathfrak{l},\mathfrak{m}\in\mathcal{L}k(\mathcal{Z})$ the following conditions are satisfied:*

a) $\tilde{\mathcal{Q}}_{\mathfrak{m},\mathfrak{l}}$ is a bijection (one-to-one mapping) from $\mathbf{Tm}(\mathfrak{l})\times\mathfrak{X}_{\mathfrak{l}}$ to $\mathbf{Tm}(\mathfrak{m})\times\mathfrak{X}_{\mathfrak{m}}$;

b) for any elementary-time state $\omega\in\mathbb{B}\mathfrak{s}(\mathfrak{l})$ the following equality is performed:

$$\left(\mathsf{tm}\left(\left\langle !\ \ \mathfrak{m}\leftarrow\mathfrak{l}\right\rangle\omega\right),k_{\mathfrak{m}}\left(\mathsf{bs}\left(\left\langle !\ \ \mathfrak{m}\leftarrow\mathfrak{l}\right\rangle\omega\right)\right)\right)=\tilde{\mathcal{Q}}_{\mathfrak{m},\mathfrak{l}}\left(\mathsf{tm}(\omega),k_{\mathfrak{l}}(\mathsf{bs}(\omega))\right);$$

5. *For any $\mathfrak{l},\mathfrak{m},\mathfrak{p}\in\mathcal{L}k(\mathcal{Z})$ and $\mathrm{w}\in\mathbf{Tm}(\mathfrak{l})\times\mathfrak{X}_{\mathfrak{l}}$ the following equalities are true:*

$$\tilde{\mathcal{Q}}_{\mathfrak{l},\mathfrak{l}}(\mathrm{w})=\mathrm{w};\qquad \tilde{\mathcal{Q}}_{\mathfrak{p},\mathfrak{m}}\left(\tilde{\mathcal{Q}}_{\mathfrak{m},\mathfrak{l}}(\mathrm{w})\right)=\tilde{\mathcal{Q}}_{\mathfrak{p},\mathfrak{l}}(\mathrm{w}). \tag{17}$$

From intuitive point of view we can imagine universal kinematics $\mathcal{F}$ in Definition 6 as evolutionary model of some system of material points in a some space-time environment, where evolution of the system is described by the changeable set $\mathcal{Z}$ in each reference frame.

Remark 6. Let $\mathcal{F}=\left(\mathcal{Z},\mathcal{G},\overleftarrow{\mathcal{Q}}\right)=\left(\mathcal{Z},\left(\left(\mathfrak{X}_{\mathfrak{l}},\|\cdot\|_{(\mathfrak{l})},k_{\mathfrak{l}}\right)\mid\mathfrak{l}\in\mathcal{L}k(\mathcal{Z})\right),\left(\tilde{\mathcal{Q}}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m}\in\mathcal{L}k(\mathcal{Z})}\right)$ be any universal kinematics. The sets $\mathcal{L}k(\mathcal{F}):=\mathcal{L}k(\mathcal{Z})$; $\mathcal{I}nd(\mathcal{F}):=\mathcal{I}nd(\mathcal{Z})$ will be called by the set of all ***reference frames*** and the set of ***indexes*** of universal kinematics $\mathcal{F}$ (correspondingly).

For each index $\alpha\in\mathcal{I}nd(\mathcal{F})=\mathcal{I}nd(\mathcal{Z})$ we use the notation:

$$\mathbf{lk}_{\alpha}(\mathcal{F}):=\mathbf{lk}_{\alpha}(\mathcal{Z}).$$

Further we use the following notations for arbitrary reference frames $\mathfrak{l},\mathfrak{m}\in\mathcal{L}k(\mathcal{F})=\mathcal{L}k(\mathcal{Z})$:

1. We keep all denotations, introduced for reference frames of changeable sets (namely $\operatorname{ind}(\mathfrak{l})$, $\mathfrak{l}^{\wedge}$, $\mathfrak{Bs}(\mathfrak{l})$, $\mathbb{Bs}(\mathfrak{l})$, $\underset{\mathfrak{l}}{\leftarrow}$, $\mathbf{Tm}(\mathfrak{l})$, $\mathbb{T}\mathbf{m}(\mathfrak{l})$, $\le_{\mathfrak{l}}$, $<_{\mathfrak{l}}$) together with abbreviated variants of these denotations, introduced in item 6) of Remark 5 and terminology, described in item 7) of Remark 5 (where the symbol "$\mathcal{Z}$" should be replaced by "$\mathcal{F}$").

2. For unification mappings and precise unification mappings we use the following notations:

$$\left\langle \mathfrak{m}\leftarrow\mathfrak{l},\ \mathcal{F}\right\rangle := \left\langle \mathfrak{m}\leftarrow\mathfrak{l},\ \mathcal{Z}\right\rangle, \quad \left\langle !\ \mathfrak{m}\leftarrow\mathfrak{l},\mathcal{F}\right\rangle\omega := \left\langle !\ \mathfrak{m}\leftarrow\mathfrak{l},\mathcal{Z}\right\rangle\omega \quad (\omega\in\mathbb{Bs}(\mathfrak{l})).$$

3. Denote: $\mathbf{Zk}(\mathfrak{l};\ \mathcal{F}) := \mathfrak{X}_{\mathfrak{l}}$, $\quad \mathbb{M}k(\mathfrak{l};\mathcal{F}) := \mathbf{Tm}(\mathfrak{l})\times\mathbf{Zk}(\mathfrak{l};\ \mathcal{F})$, $\|\cdot\|_{\mathfrak{l},\mathcal{F}} := \|\cdot\|_{(\mathfrak{l})}$,

$\mathfrak{q}_{\mathfrak{l}}(x,\mathcal{F}) := k_{\mathfrak{l}}(x)\in\mathfrak{X}_{\mathfrak{l}} = \mathbf{Zk}(\mathfrak{l};\ \mathcal{F}) \quad (x\in\mathfrak{Bs}(\mathfrak{l}))$.

The set $\mathbf{Zk}(\mathfrak{l};\ \mathcal{F})$ will be called by set of ***coordinate values*** for reference frame $\mathfrak{l}$ in universal kinematics $\mathcal{F}$.

4. In the cases, when the universal kinematics $\mathcal{F}$ is known in advance, we will use the ***abbreviated variants of denotations*** $\langle\mathfrak{m}\leftarrow\ \mathfrak{l}\rangle$, $\langle !\ \mathfrak{m}\leftarrow\ \mathfrak{l}\rangle\omega$, $\mathbf{Zk}(\mathfrak{l})$, $\mathbb{M}k(\mathfrak{l})$, $\|\cdot\|_{\mathfrak{l}}$ and $\mathfrak{q}_{\mathfrak{l}}(x)$ instead of $\langle\mathfrak{m}\leftarrow\ \mathfrak{l},\ \mathcal{F}\rangle$, $\langle !\ \mathfrak{m}\leftarrow\ \mathfrak{l},\ \mathcal{F}\rangle\omega$, $\mathbf{Zk}(\mathfrak{l};\ \mathcal{F})$, $\mathbb{M}k(\mathfrak{l};\mathcal{F})$, $\|\cdot\|_{\mathfrak{l},\mathcal{F}}$ and $\mathfrak{q}_{\mathfrak{l}}(x,\mathcal{F})$ (correspondingly). The set $\mathbb{M}k(\mathfrak{l})$ we call by ***Minkowski set*** or ***Minkowski space*** of reference frame $\mathfrak{l}$ in kinematics $\mathcal{F}$.

5. Also we use the following notations:

$$\mathbf{Q}^{\langle\mathfrak{l}\rangle}(\omega;\mathcal{F}) := \left(\mathsf{tm}(\omega),\mathfrak{q}_{\mathfrak{l}}(\mathsf{bs}(\omega))\right), \qquad \left[\mathfrak{m}\leftarrow\mathfrak{l},\mathcal{F}\right] := \tilde{Q}_{\mathfrak{m},\mathfrak{l}}.$$

In the cases, when the universal kinematics $\mathcal{F}$ is known in advance, we use the ***abbreviated variants of denotations*** $\mathbf{Q}^{\langle\mathfrak{l}\rangle}(\omega)$ and $[\mathfrak{m}\leftarrow\ \mathfrak{l}]$ instead of $\mathbf{Q}^{\langle\mathfrak{l}\rangle}(\omega;\mathcal{F})$ and $[\mathfrak{m}\leftarrow\ \mathfrak{l},\mathcal{F}]$ (correspondingly). The mapping $[\mathfrak{m}\leftarrow\ \mathfrak{l}]$ is called by ***universal coordinate transform*** between reference frames $\mathfrak{l}$ and $\mathfrak{m}$ in kinematics $\mathcal{F}$.

Let $\mathcal{F}$ be any universal kinematics and $\mathfrak{l},\mathfrak{m},\mathfrak{p}\in\mathcal{L}k(\mathcal{F})$ be any reference frames of $\mathcal{F}$. Then, according to Definition 6, Assertion 1 and notations, introduced in Remark 6, for any elements $\omega\in\mathbb{Bs}(\mathfrak{l})$, $A\subseteq\mathbb{Bs}(\mathfrak{l})$ and $\mathrm{w}\in\mathbb{M}k(\mathfrak{l})$ the following equalities are performed:

$$\left\langle !\ \mathfrak{l}\leftarrow\mathfrak{l}\right\rangle\omega=\omega;\ \left\langle !\ \mathfrak{p}\leftarrow\mathfrak{m}\right\rangle\left\langle !\ \mathfrak{m}\leftarrow\mathfrak{l}\right\rangle\omega=\left\langle !\ \mathfrak{p}\leftarrow\mathfrak{l}\right\rangle\omega;\ \left\langle \mathfrak{m}\leftarrow\mathfrak{l}\right\rangle A=\left\{\left\langle !\ \mathfrak{m}\leftarrow\mathfrak{l}\right\rangle\nu \mid \nu\in A\right\};$$

$$\begin{aligned}
\mathbf{Q}^{\langle\mathfrak{m}\rangle}\left(\left\langle !\ \ \mathfrak{m}\leftarrow\mathfrak{l}\right\rangle\omega\right) &= \left[\mathfrak{m}\leftarrow\mathfrak{l}\right]\mathbf{Q}^{\langle\mathfrak{l}\rangle}(\omega);\\
\left[\mathfrak{l}\leftarrow\mathfrak{l}\right]\mathrm{w} &= \mathrm{w};\\
\left[\mathfrak{p}\leftarrow\mathfrak{m}\right]\left[\mathfrak{m}\leftarrow\mathfrak{l}\right]\mathrm{w} &= \left[\mathfrak{p}\leftarrow\mathfrak{l}\right]\mathrm{w}.
\end{aligned}$$

The next aim is to formulate theorem on multi-image for universal kinematics, which is the powerful tool for construction examples of universal kinematics in particular generalized Hassani kinematics. Let $(\mathfrak{H},\|\cdot\|,\langle\cdot,\cdot\rangle)$ be a real Hilbert space such, that $\dim(\mathfrak{H})\ge 1$. Let $\mathcal{B}$ be any base changeable set such, that $\mathfrak{Bs}(\mathcal{B})\subseteq\mathfrak{H}$ and $\mathbb{T}\mathbf{m}(\mathcal{B})=\mathbb{R}_{\le}$, where $\mathbb{R}_{\le}=(\mathbb{R},\le)$ and $\le$ is the standard linear order over the real field[4]. Then, according to the inclusion (15) we have $\mathbb{B}\mathfrak{s}(\mathcal{B})\subseteq\mathbf{Tm}(\mathcal{B})\times\mathfrak{B}\mathfrak{s}(\mathcal{B})\subseteq\mathbb{R}\times\mathfrak{H}=\mathcal{M}(\mathfrak{H})$. Therefore for every mapping $\mathbf{U}:\mathcal{M}(\mathfrak{H})\to\mathcal{M}(\mathfrak{H})$ there exists the base changeable set $\mathbf{U}[\mathcal{B}]=\mathbf{U}[\mathcal{B},\mathbb{T}\mathbf{m}(\mathcal{B})]$ and, moreover, by Theorem 2, we get:

$$\left.\begin{array}{ll}\mathbb{T}\mathbf{m}(\mathbf{U}[\mathcal{B}])=\mathbb{T}\mathbf{m}(\mathcal{B})=\mathbb{R}_{\leq}, & \mathbf{Tm}(\mathbf{U}[\mathcal{B}])=\mathbb{R},\\ \mathbb{B}\mathfrak{s}(\mathbf{U}[\mathcal{B}])\subseteq\mathcal{M}(\mathfrak{H}), & \mathfrak{B}\mathfrak{s}(\mathbf{U}[\mathcal{B}])\subseteq\mathfrak{H}.\end{array}\right] \tag{18}$$

Definition 7. *Let $\left(\mathfrak{X}_1,\|\cdot\|_{(1)}\right)$, $\left(\mathfrak{X}_2,\|\cdot\|_{(2)}\right)$ be linear normed spaces over real or complex field and $\mathbb{T}_1=(\mathbf{T}_1,\leq_1)$, $\mathbb{T}_2=(\mathbf{T}_2,\leq_2)$ be linearly ordered sets. Any bijection $\mathcal{U}:\mathbf{T}_1\times\mathfrak{X}_1\to\mathbf{T}_2\times\mathfrak{X}_2$ between $\mathbf{T}_1\times\mathfrak{X}_1$ and $\mathbf{T}_2\times\mathfrak{X}_2$ is called by coordinate transform operator (CTO) from $(\mathbb{T}_1,\mathfrak{X}_1)$ to $(\mathbb{T}_2,\mathfrak{X}_2)$. The set of all CTO from $(\mathbb{T}_1,\mathfrak{X}_1)$ to $(\mathbb{T}_2,\mathfrak{X}_2)$ we denote by $\mathbb{P}\mathbf{k}(\mathbb{T}_1,\mathfrak{X}_1;\mathbb{T}_2,\mathfrak{X}_2)$.*

Note that the set $\mathbb{P}\mathbf{k}(\mathbb{T}_1,\mathfrak{X}_1;\mathbb{T}_2,\mathfrak{X}_2)$ in Definition 7 is nonempty if and inly if the sets $\mathbf{T}_1\times\mathfrak{X}_1$ and $\mathbf{T}_2\times\mathfrak{X}_2$ are equipotent (i.e. $\mathbf{card}(\mathbf{T}_1\times\mathfrak{X}_1)=\mathbf{card}(\mathbf{T}_2\times\mathfrak{X}_2)$, where $\mathbf{card}(\mathcal{A})$ means the **cardinality** of the set $\mathcal{A}$).

In the case where $\mathbb{T}_1=\mathbb{T}_2=\mathbb{R}_{\leq}$ and $(\mathfrak{H},\|\cdot\|,\langle\cdot,\cdot\rangle)$ is a real Hilbert space such, that $\dim(\mathfrak{H})\geq 1$ we use the notation $\mathbb{P}\mathbf{k}(\mathfrak{H}):=\mathbb{P}\mathbf{k}(\mathbb{R}_{\leq},\mathfrak{H};\mathbb{R}_{\leq},\mathfrak{H})$. It is apparently that $\mathbf{Pk}(\mathfrak{H})\subseteq\mathbb{P}\mathbf{k}(\mathfrak{H})$, but in the general case the inverse inclusion does not hold, because $\mathbb{P}\mathbf{k}(\mathfrak{H})$ contains all bijective operators over $\mathcal{M}(\mathfrak{H})=\mathbb{R}\times\mathfrak{H}$ (not only affine-continuous).

Theorem 3 (on multi-image for universal kinematics, [10][5]). *Let $(\mathfrak{H},\|\cdot\|,\langle\cdot,\cdot\rangle)$ be a real Hilbert space such, that $\dim(\mathfrak{H})\geq 1$ and $\mathcal{B}$ be a base changeable set such, that $\mathfrak{B}\mathfrak{s}(\mathcal{B})\subseteq\mathfrak{H}$ and $\mathbb{T}\mathbf{m}(\mathcal{B})=\mathbb{R}_{\leq}$ then any set of operators $\mathbb{S}\subseteq\mathbb{P}\mathbf{k}(\mathfrak{H})$ generates a unique universal kinematics*

$$\mathcal{F}=\mathfrak{K}\mathfrak{u}(\mathbb{S},\mathcal{B};\mathfrak{H}),$$

satisfying the following conditions:

1. $\mathcal{L}k(\mathcal{F})=\{(\mathbf{U},\mathbf{U}[\mathcal{B}])\mid\mathbf{U}\in\mathbb{S}\}$.
2. *For every reference frame $\mathfrak{l}=(\mathbf{U},\mathbf{U}[\mathcal{B}])\in\mathcal{L}k(\mathcal{F})$ (where $\mathbf{U}\in\mathbb{S}$) it is valid the equality $\left(\mathbf{Zk}(\mathfrak{l}),\|\cdot\|_{\mathfrak{l}}\right)=(\mathfrak{H},\|\cdot\|)$ (and therefore taking into account (18), we have $\mathbb{M}k(\mathfrak{l})=\mathbf{Tm}(\mathfrak{l})\times\mathbf{Zk}(\mathfrak{l})=\mathbf{Tm}(\mathbf{U}[\mathcal{B}])\times\mathfrak{H}=\mathbb{R}\times\mathfrak{H}=\mathcal{M}(\mathfrak{H})$).*
3. *For any reference frames $\mathfrak{l}=(\mathbf{U},\mathbf{U}[\mathcal{B}])\in\mathcal{L}k(\mathcal{F})$, $\mathfrak{m}=(\mathbf{V},\mathbf{V}[\mathcal{B}])\in\mathcal{L}k(\mathcal{F})$ (where $\mathbf{U},\mathbf{V}\in\mathbb{S}$) the following equalities are performed:*

$$\mathfrak{q}_{\mathfrak{l}}(\mathrm{x})=\mathrm{x}\quad(\forall\mathrm{x}\in\mathfrak{B}\mathfrak{s}(\mathfrak{l})=\mathfrak{B}\mathfrak{s}(\mathbf{U}[\mathcal{B}])\subseteq\mathfrak{H});$$

$$\left\langle !\ \mathfrak{m}\leftarrow\mathfrak{l}\right\rangle\omega=\mathbf{V}\left(\mathbf{U}^{-1}(\omega)\right)\quad(\forall\omega\in\mathbb{B}\mathfrak{s}(\mathfrak{l})=\mathbb{B}\mathfrak{s}(\mathbf{U}[\mathcal{B}])\subseteq\mathcal{M}(\mathfrak{H}));$$

[4] There exist infinitely many examples of base changeable sets $\mathcal{B}$, satisfying these conditions, because we may put $\mathcal{B}:=\mathcal{A}t(\mathbb{R}_{\leq},\mathcal{R})$, where $\mathcal{R}$ is any system of abstract trajectories from the linear ordered set $\mathbb{R}_{\leq}=(\mathbb{R},\leq)$ to the set $\mathfrak{H}$. And then according to Theorem 1 and Remark 4, we receive $\mathbb{T}\mathbf{m}(\mathcal{B})=\mathbb{R}_{\leq}$ and $\mathfrak{B}\mathfrak{s}(\mathcal{B})=\bigcup_{r\in\mathcal{R}}\mathfrak{R}(r)\subseteq\mathfrak{H}$.

[5] In fact Theorem 3 is the particular case of more general theorem on multi-image for universal kinematics, published in [10], see also [7].

$$\left[\mathfrak{m}\leftarrow\mathfrak{l}\right]\mathrm{w}=\mathbf{V}\left(\mathbf{U}^{-1}(\mathrm{w})\right)\quad(\forall\mathrm{w}\in\mathbb{M}k(\mathfrak{l})=\mathcal{M}(\mathfrak{H})),$$

where $\mathbf{U}^{-1}$ *is the* ***inverse*** *mapping to* $\mathbf{U}$.

Applying Theorem 3 to the classes of operators $\mathfrak{P}(\mathfrak{H},[\vartheta])$ and $\mathfrak{P}_+(\mathfrak{H},[\vartheta])$ (where $\vartheta\in\Upsilon$) we can introduce the following universal kinematics:

$$\mathfrak{UH}_0(\mathfrak{H},\mathcal{B},\vartheta):=\mathfrak{Ku}(\mathfrak{P}(\mathfrak{H},[\vartheta]),\mathcal{B};\mathfrak{H});$$

$$\mathfrak{UH}(\mathfrak{H},\mathcal{B},\vartheta):=\mathfrak{Ku}(\mathfrak{P}_+(\mathfrak{H},[\vartheta]),\mathcal{B};\mathfrak{H}). \tag{19}$$

In the case $\dim(\mathfrak{H})=3$, $\vartheta(\lambda)=\vartheta_c(\lambda)$ ($\lambda\in[0,\infty)$), where $c\in(0,+\infty)$ the universal kinematics $\mathfrak{UH}(\mathfrak{H},\mathcal{B},\vartheta_c)$ represents the simplest mathematically strict model of the kinematics of special relativity theory in inertial frames of reference. Universal kinematics $\mathfrak{UH}_0(\mathfrak{H},\mathcal{B},\vartheta)$ is constructed on the basis of general Poincare−Hassani transforms, and it includes apart from usual reference frames (with positive direction of time), which have understandable physical interpretation, also reference frames with negative direction of time relatively the given "zero" frame. So most natural generalization of the kinematics of special relativity theory represents the kinematics of kind $\mathfrak{UH}(\mathfrak{H},\mathcal{B},\vartheta)$. And the main aim of the article is to prove that the kinematics $\mathfrak{UH}(\mathfrak{H},\mathcal{B},\vartheta)$ is certainly time irreversible for each function $\vartheta\in\Upsilon$, where the strict definition of time irreversibility will be given in the next section of the article.

3. Theorem of non returning for universal kinematics. In this section we present the abstract notions and results, needed for derivation of main results of the paper.

Definition 8. *Let* $\mathcal{F}$ *be any universal kinematics,* $\mathfrak{l}\in\mathcal{L}k(\mathcal{F})$ *be any reference frame of* $\mathcal{F}$ *and* $\omega\in\mathbb{B}\mathfrak{s}(\mathfrak{l})$ *be any elementary-time state in the reference frame* $\mathfrak{l}$. *The set*

$$\omega^{\{\mathfrak{l},\mathcal{F}\}}=\left\{\left(\mathfrak{m},\left\langle !\ \mathfrak{m}\leftarrow\mathfrak{l}\right\rangle\omega\right)\ \middle|\ \mathfrak{m}\in\mathcal{L}k(\mathcal{F})\right\}$$

(*where* (x,y) *is the ordered pair, composed of* x *and* y) *is called by* ***elementary-time state of the universal kinematics*** $\mathcal{F}$, *generated by* ω *in the reference frame* $\mathfrak{l}$.

Remark 7. In the case, where the universal kinematics $\mathcal{F}$ is known in advance, we use the abbreviated denotation $\omega^{\{\mathfrak{l}\}}$ instead of the denotation $\omega^{\{\mathfrak{l},\mathcal{F}\}}$.

Assertion 2 ([15]). *Let* $\mathcal{F}$ *be any universal kinematics and* $\mathfrak{l},\mathfrak{m}\in\mathcal{L}k(\mathcal{F})$. *Then for arbitrary elementary-time states* $\omega\in\mathbb{B}\mathfrak{s}(\mathfrak{l})$ *and* $\omega_1\in\mathbb{B}\mathfrak{s}(\mathfrak{m})$ *the following statements are equivalent:*

1) $\omega^{\{\mathfrak{l}\}}=\omega_1^{\{\mathfrak{m}\}}$; 2) $\omega_1=\langle !\ \mathfrak{m}\leftarrow\ \mathfrak{l}\rangle\omega$.

Corollary 1 ([15]). *Let* $\mathcal{F}$ *be any universal kinematics. Then for every* $\mathfrak{l},\mathfrak{m}\in\mathcal{L}k(\mathcal{F})$ *and* $\omega\in\mathbb{B}\mathfrak{s}(\mathfrak{l})$ *the following equality holds:*

$$\omega^{\{\mathfrak{l}\}}=\left(\left\langle !\ \mathfrak{m}\leftarrow\mathfrak{l}\right\rangle\omega\right)^{\{\mathfrak{m}\}}.$$

Theorem 4 ([15]). *Let* $\mathcal{F}$ *be any universal kinematics. Then the set*

$$\mathbb{B}\mathfrak{s}[\mathfrak{l},\mathcal{F}]=\left\{\omega^{\{\mathfrak{l},\mathcal{F}\}}\ \middle|\ \omega\in\mathbb{B}\mathfrak{s}(\mathfrak{l})\right\} \tag{20}$$

does not depend of the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ *(i.e.* $\forall \mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ $\mathbb{B}\mathfrak{s}[\mathfrak{l}, \mathcal{F}] = \mathbb{B}\mathfrak{s}[\mathfrak{m}, \mathcal{F}]$ *).*

Definition 9. *Let* $\mathcal{F}$ *be any universal kinematics.*

1. *The set* $\mathbb{B}\mathfrak{s}(\mathcal{F}) = \mathbb{B}\mathfrak{s}[\mathfrak{l}, \mathcal{F}]$ *(* $\forall\, \mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ *) is called by the set of all elementary-time states of* $\mathcal{F}$ *.*

2. *Any subset* $\hat{\mathbf{A}} \subseteq \mathbb{B}\mathfrak{s}(\mathcal{F})$ *is called by the (common)* ***changeable system of the universal kinematics*** $\mathcal{F}$ *.*

Assertion 3 ([15]). *Let* $\mathcal{F}$ *be any universal kinematics and* $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ *be any reference frame of* $\mathcal{F}$ *. Then for every element* $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ *only one element* $\omega_0 \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ *exists such, that* $\hat{\omega} = \omega_0^{\{\mathfrak{l}\}}$ *.*

Definition 10. *Let* $\mathcal{F}$ *be any universal kinematics,* $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ *be any elementary-time state of* $\mathcal{F}$ *and* $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ *be any reference frame of* $\mathcal{F}$ *. Elementary-time state* $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ *is called by* ***image*** *of elementary-time state* $\hat{\omega}$ ***in the reference frame*** $\mathfrak{l}$ *if and only if* $\hat{\omega} = \omega^{\{\mathfrak{l}\}}$ *.*

In accordance with Assertion 3, every elementary-time state $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ always has only one image in any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$. Image of elementary-time state $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ in the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ will be denoted via $\hat{\omega}_{\{\mathfrak{l}, \mathcal{F}\}}$ (in the cases, where the universal kinematics $\mathcal{F}$ is known in advance, we use the abbreviated denotation $\hat{\omega}_{\{\mathfrak{l}\}}$).

Assertion 4 ([15]). *Let* $\mathcal{F}$ *be any universal kinematics and* $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ *be any reference frame of* $\mathcal{F}$ *. Then:*

1. *For arbitrary* $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ *the following equality holds:*

$$\left(\hat{\omega}_{\{\mathfrak{l}\}}\right)^{\{\mathfrak{l}\}} = \hat{\omega}. \tag{21}$$

2. *For each* $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ *the following equality is valid:*

$$\left(\omega^{\{\mathfrak{l}\}}\right)_{\{\mathfrak{l}\}} = \omega. \tag{22}$$

3. *The mapping* $(\cdot)^{\{\mathfrak{l}\}}$ *is bijection from* $\mathbb{B}\mathfrak{s}(\mathfrak{l})$ *onto* $\mathbb{B}\mathfrak{s}(\mathcal{F})$ *.*
4. *The mapping* $(\cdot)_{\{\mathfrak{l}\}}$ *is bijection from* $\mathbb{B}\mathfrak{s}(\mathcal{F})$ *onto* $\mathbb{B}\mathfrak{s}(\mathfrak{l})$ *.*
5. *The mapping* $(\cdot)_{\{\mathfrak{l}\}}$ *is inverse to the mapping* $(\cdot)^{\{\mathfrak{l}\}}$ *.*

Assertion 5 ([15]). *Let* $\mathcal{F}$ *be any universal kinematics and* $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ *be any reference frames* $\mathcal{F}$ *. Then the following statements are performed:*

1. *For every* $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ *the equality* $\hat{\omega}_{\{\mathfrak{m}\}} = \langle !\ \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \hat{\omega}_{\{\mathfrak{l}\}}$ *holds.*
2. *For each* $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ *the equality* $\left(\omega^{\{\mathfrak{l}\}}\right)_{\{\mathfrak{m}\}} = \langle !\ \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega$ *is performed.*

Let $\mathcal{F}$ *be any universal kinematics. The set* $\overset{\wedge}{\mathbf{A}}_{\{\mathfrak{l},\mathcal{F}\}} = \left\{\omega_{\{\mathfrak{l},\mathcal{F}\}} \mid \omega \in \overset{\wedge}{\mathbf{A}}\right\}$ *is called* ***image of changeable system*** $\overset{\wedge}{\mathbf{A}} \subseteq \mathbb{B}\mathfrak{s}(\mathcal{F})$ *in the reference frame* $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$. Any changeable system (i.e. subset) $A \subseteq \mathbb{B}\mathfrak{s}(\mathfrak{l})$ of the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ always generates the (common) changeable system $A^{\{\mathfrak{l},\mathcal{F}\}} := \left\{\omega^{\{\mathfrak{l},\mathcal{F}\}} \mid \omega \in A\right\} \subseteq \mathbb{B}\mathfrak{s}(\mathcal{F})$.

Remark 8. In the cases, where universal kinematics $\mathcal{F}$ is known in advance, we use the abbreviated denotations $\overset{\wedge}{\mathbf{A}}_{\{\mathfrak{l}\}}$ and $A^{\{\mathfrak{l}\}}$ instead of $\overset{\wedge}{\mathbf{A}}_{\{\mathfrak{l},\mathcal{F}\}}$ and $A^{\{\mathfrak{l},\mathcal{F}\}}$ (correspondingly).

Applying equalities (21) and (22), we obtain the equalities:

$$\left(\overset{\wedge}{\mathbf{A}}_{\{\mathfrak{l}\}}\right)^{\{\mathfrak{l}\}} = \overset{\wedge}{\mathbf{A}} \qquad \text{and} \qquad \left(A^{\{\mathfrak{l}\}}\right)_{\{\mathfrak{l}\}} = A$$

(for arbitrary universal kinematics $\mathcal{F}$, reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ and changeable systems $\overset{\wedge}{\mathbf{A}} \subseteq \mathbb{B}\mathfrak{s}(\mathcal{F})$ as well $A \subseteq \mathbb{B}\mathfrak{s}(\mathfrak{l})$).

A mathematical object (i.e. a set) $\tilde{\mathcal{B}}$ will be called by a ***base changeable set object*** if $\tilde{\mathcal{B}}$ satisfies one of the following conditions:

- $\tilde{\mathcal{B}}$ is a base changeable set,
- $\tilde{\mathcal{B}}$ is a reference frame, that is $\tilde{\mathcal{B}} \in \mathcal{L}k(\mathcal{Y})$, where $\mathcal{Y}$ is a changeable set or universal kinematics.

Definition 11. *Let* $\mathcal{B}$ *be a base changeable set object.*

Nonempty subset $N \subseteq \mathbb{B}\mathfrak{s}(\mathcal{B})$ *is referred to as* ***transitive*** *in* $\mathcal{B}$ *if for any* $\omega_1, \omega_2, \omega_3 \in N$ *such, that* $\omega_3 \leftarrow \omega_2$ *and* $\omega_2 \leftarrow \omega_1$ *we have* $\omega_3 \leftarrow \omega_1$.

The transitive subset $L \subseteq \mathbb{B}\mathfrak{s}(\mathcal{B})$ *is referred to as* ***chain*** *of* $\mathcal{B}$ *if for any* $\omega_1, \omega_2 \in L$ *at least one of the relations* $\omega_2 \leftarrow \omega_1$ *or* $\omega_1 \leftarrow \omega_2$ *is true. The set of all chains of* $\mathcal{B}$ *we denote by* $\mathbb{L}l(\mathcal{B})$*:*

$$\mathbb{L}l(\mathcal{B}) = \left\{\mathcal{L} \subseteq \mathbb{B}\mathfrak{s}(\mathcal{B}) \mid \mathcal{L} \text{ is a chain of } \mathcal{B}\right\}.$$

Definition 12. *Let* $\mathcal{F}$ *be any universal kinematics. Changeable system* $\mathbf{A} \subseteq \mathbb{B}\mathfrak{s}(\mathcal{F})$ *is called* ***piecewise chain*** *changeable system if and only if there exist the sequences of changeable systems* $\overset{\wedge}{\mathbf{A}}_1, \ldots, \overset{\wedge}{\mathbf{A}}_n \subseteq \mathbb{B}\mathfrak{s}(\mathcal{F})$ *and reference frames* $\mathfrak{l}_1, \ldots, \mathfrak{l}_n \in \mathcal{L}k(\mathcal{F})$ *(* $n \in \mathbb{N}$ *) satisfying the following conditions:*

(a) $\left(\overset{\wedge}{\mathbf{A}}_k\right)_{\{\mathfrak{l}_k\}} \in \mathbb{L}l(\mathfrak{l}_k) \quad \left(\forall k \in \overline{1,n}\right)$. [6]

(b) $\bigcup_{k=1}^{n} \overset{\wedge}{\mathbf{A}}_k = \overset{\wedge}{\mathbf{A}}$,

and, moreover, in the case $n \geq 2$ *the following additional conditions are satisfied:*

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[6] Further we denote via $\overline{m,n}$ ($m,n \in \mathbb{N}$, $m \leq n$) the set $\overline{m,n} = \{m, \ldots, n\}$.

(c) $\hat{\mathbf{A}}_k \cap \hat{\mathbf{A}}_{k+1} \neq \varnothing \quad \left(\forall k \in \overline{1, n-1}\right)$.

(d) *For each* $k \in \overline{1, n-1}$ *and arbitrary* $\omega_1 \in \left(\hat{\mathbf{A}}_k \setminus \hat{\mathbf{A}}_{k+1}\right)_{\{\mathfrak{l}_k\}}$, $\omega_2 \in \left(\hat{\mathbf{A}}_k \cap \hat{\mathbf{A}}_{k+1}\right)_{\{\mathfrak{l}_k\}}$ *the inequality* $\mathsf{tm}(\omega_1) <_{\mathfrak{l}_k} \mathsf{tm}(\omega_2)$ *holds.*

(e) *For every* $k \in \overline{2, n}$ *and arbitrary* $\omega_1 \in \left(\hat{\mathbf{A}}_{k-1} \cap \hat{\mathbf{A}}_k\right)_{\{\mathfrak{l}_k\}}$, $\omega_2 \in \left(\hat{\mathbf{A}}_k \setminus \hat{\mathbf{A}}_{k-1}\right)_{\{\mathfrak{l}_k\}}$ *the inequality* $\mathsf{tm}(\omega_1) <_{\mathfrak{l}_k} \mathsf{tm}(\omega_2)$ *is performed.*

In this case the ordered composition $\mathcal{A} = \left(\hat{\mathbf{A}}, \left(\hat{\mathbf{A}}_1, \mathfrak{l}_1\right), \ldots, \left(\hat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$ *will be called by the* ***chain path*** *of universal kinematics* $\mathcal{F}$.

Definition 13. Let $\mathcal{F}$ be any universal kinematics.

(a) *Changeable system* $A \subseteq \mathbb{B}\mathfrak{s}(\mathfrak{l})$ *is referred to as* ***geometrically-stationary*** *in the reference frame* $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ *if and only if* $A \in \mathbb{L}l(\mathfrak{l})$ *and for arbitrary* $\omega_1, \omega_2 \in A$ *the equality* $\mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}(\omega_1)\right) = \mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}(\omega_2)\right)$ *holds.*

(b) *The set of all* ***geometrically-stationary*** *changeable systems in the reference frame* $\mathfrak{l}$ *is denoted via* $\mathbb{L}g(\mathfrak{l}, \mathcal{F})$. *In the cases, where the universal kinematics* $\mathcal{F}$ *is known in advance, we use the abbreviated denotation* $\mathbb{L}g(\mathfrak{l})$.

(c) *The chain path* $\mathcal{A} = \left(\hat{\mathbf{A}}, \left(\hat{\mathbf{A}}_1, \mathfrak{l}_1\right), \ldots, \left(\hat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$ *in* $\mathcal{F}$ *(* $n \in \mathbb{N}$ *) is called by* ***piecewise geometrically-stationary*** *if and only if* $\forall k \in \overline{1, n} \left(\hat{\mathbf{A}}_k\right)_{\{\mathfrak{l}_k\}} \in \mathbb{L}g(\mathfrak{l}_k)$.

Definition 14. *Let* $\mathcal{F}$ *be any universal kinematics and let* $\mathcal{A} = \left(\hat{\mathbf{A}}, \left(\hat{\mathbf{A}}_1, \mathfrak{l}_1\right), \ldots, \left(\hat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$ *be arbitrary chain path in* $\mathcal{F}$.

1. *Element* $\hat{\omega}_s \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ *is called by* ***start*** *element of the path* $\mathcal{A}$, *if and only if* $\hat{\omega}_s \in \hat{\mathbf{A}}_1$ *and for every* $\hat{\omega} \in \hat{\mathbf{A}}_1$ *the inequality* $\mathsf{tm}\left(\left(\hat{\omega}_s\right)_{\{\mathfrak{l}_1\}}\right) \leq_{\mathfrak{l}_1} \mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}_1\}}\right)$ *is performed.*

2. *Element* $\hat{\omega}_f \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ *is called by* ***final*** *element of the path* $\mathcal{A}$, *if and only if* $\hat{\omega}_f \in \hat{\mathbf{A}}_n$ *and for every* $\hat{\omega} \in \hat{\mathbf{A}}_n$ *the inequality* $\mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}_n\}}\right) \leq_{\mathfrak{l}_n} \mathsf{tm}\left(\left(\hat{\omega}_f\right)_{\{\mathfrak{l}_n\}}\right)$ *holds.*

3. *The chain path* $\mathcal{A}$, *which owns (at least one) start element and (at least one) final element, is called by* ***closed***.

Assertion 6 ([15]). *Any chain path* $\mathcal{A}$ *of arbitrary universal kinematics* $\mathcal{F}$ *cannot have more, than one start element and more, than one final element.*

Further the start element of the chain path $\mathcal{A}$ of the universal kinematics $\mathcal{F}$ will be denoted via $\mathsf{po}(\mathcal{A}, \mathcal{F})$, or via $\mathsf{po}(\mathcal{A})$. The final element of the chain path $\mathcal{A}$ will be denoted via $\mathsf{ki}(\mathcal{A}, \mathcal{F})$, or via $\mathsf{ki}(\mathcal{A})$. Where the denotations $\mathsf{po}(\mathcal{A})$ and $\mathsf{ki}(\mathcal{A})$ are used in the cases when they do not cause misunderstanding. Thus, for every closed chain path $\mathcal{A}$ both start and final elements ($\mathsf{po}(\mathcal{A})$ and $\mathsf{ki}(\mathcal{A})$) always exist.

Definition 15. *Closed chain path $\mathcal{A}$ of universal kinematics $\mathcal{F}$ is referred to as **geometrically-cyclic** in the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ if and only if* $\mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{po}(\mathcal{A})_{\{\mathfrak{l}\}}\right)\right) = \mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{ki}(\mathcal{A})_{\{\mathfrak{l}\}}\right)\right)$.

Definition 16. *Universal kinematics $\mathcal{F}$ is called **time irreversible** if and only if for every reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ and for each chain path $\mathcal{A}$, geometrically-cyclic in the frame $\mathfrak{l}$ and piecewise geometrically-stationary in $\mathcal{F}$, it is performed the inequality* $\mathsf{tm}\left(\mathsf{po}(\mathcal{A})_{\{\mathfrak{l}\}}\right) \le_{\mathfrak{l}} \mathsf{tm}\left(\mathsf{ki}(\mathcal{A})_{\{\mathfrak{l}\}}\right)$.

*Universal kinematics $\mathcal{F}$ is called **time reversible** if and only if it is not time irreversible.*

The physical sense of time irreversibility notion is that in time irreversible kinematics there is not any process or object which returns to the begin of the own path at the past, moving by means of "jumping" from previous reference frame to the next frame. So, there are not temporal paradoxes in these kinematics.

Definition 17. *Let $\mathcal{F}$ be any universal kinematics.*

1. *We say that reference frame $\mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ is **time-positive** in $\mathcal{F}$ relatively the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ (denotation is $\mathfrak{m} \Uparrow^{+}_{\mathcal{F}} \mathfrak{l}$) if and only if for arbitrary $w_1, w_2 \in \mathbb{M}k(\mathfrak{l})$ such that $\mathsf{bs}(w_1) = \mathsf{bs}(w_2)$ and $\mathsf{tm}(w_1) <_{\mathfrak{l}} \mathsf{tm}(w_2)$ it is performed the inequality,* $\mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]w_1) <_{\mathfrak{m}} \mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]w_2)$.

2. *We say that reference frame $\mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ is **time-negative** in $\mathcal{F}$ relatively the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ (denotation is $\mathfrak{m} \Downarrow^{-}_{\mathcal{F}} \mathfrak{l}$) if and only if for arbitrary $w_1, w_2 \in \mathbb{M}k(\mathfrak{l})$ such that $\mathsf{bs}(w_1) = \mathsf{bs}(w_2)$ and $\mathsf{tm}(w_1) <_{\mathfrak{l}} \mathsf{tm}(w_2)$ it is performed the inequality,* $\mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]w_1) >_{\mathfrak{m}} \mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]w_2)$. [7]

3. *The universal kinematics $\mathcal{F}$ is called by **weakly time-positive** if and only if there exists at least one reference frame $\mathfrak{l}_0 \in \mathcal{L}k(\mathcal{F})$ such that the correlation $\mathfrak{l}_0 \Uparrow^{+}_{\mathcal{F}} \mathfrak{l}$ holds for every reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$.*

Remark 9. Apart from weak time-positivity we can introduce other, more strong, form of time-positivity. We say that universal kinematics $\mathcal{F}$ is **time-positive** if and only if for arbitrary reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ the correlation $\mathfrak{l} \Uparrow^{+}_{\mathcal{F}} \mathfrak{m}$ holds. It is not hard to prove that every kinematics of kind $\mathcal{F} = \mathfrak{UP}(\mathfrak{H}, \mathcal{B}, c) := \mathfrak{UH}(\mathfrak{H}, \mathcal{B}, \vartheta_c)$ ($0 < c < +\infty$) (connected with classical special relativity) is a time-positive.

Theorem 5 (on non returning, [15]). *Any weakly time-positive universal kinematics $\mathcal{F}$ is time irreversible.*

Definition 18. *We say that the universal kinematics $\mathcal{F}_1$ and $\mathcal{F}_2$ are **equivalent relatively coordinate transform** and write $\mathcal{F}_1 [\equiv] \mathcal{F}_2$ if and only if:*

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[7] We note by "$>_{\mathfrak{m}}$" the relation, inverse to $<_{\mathfrak{m}}$, this means that for $t, \tau \in \mathbf{Tm}(\mathfrak{m})$ the correlation $t >_{\mathfrak{m}} \tau$ holds if and only if $\tau <_{\mathfrak{m}} t$.

1. $\mathcal{I}nd(\mathcal{F}_1)=\mathcal{I}nd(\mathcal{F}_2)$;

2. *For every index* $\alpha\in\mathcal{I}nd(\mathcal{F}_1)=\mathcal{I}nd(\mathcal{F}_2)$ *the following equalities hold:*

$$\mathbb{T}\mathbf{m}(\mathbf{lk}_\alpha(\mathcal{F}_1))=\mathbb{T}\mathbf{m}(\mathbf{lk}_\alpha(\mathcal{F}_2)); \tag{23}$$

$$\left(\mathbf{Zk}(\mathbf{lk}_\alpha(\mathcal{F}_1);\mathcal{F}_1),\|\cdot\|_{\mathbf{lk}_\alpha(\mathcal{F}_1),\mathcal{F}_1}\right)=\left(\mathbf{Zk}(\mathbf{lk}_\alpha(\mathcal{F}_2);\mathcal{F}_2),\|\cdot\|_{\mathbf{lk}_\alpha(\mathcal{F}_2),\mathcal{F}_2}\right) \tag{24}$$

(note, that equalities (23) and (24) assure the equalities $\mathbf{Zk}(\mathbf{lk}_\alpha(\mathcal{F}_1);\mathcal{F}_1)=\mathbf{Zk}(\mathbf{lk}_\alpha(\mathcal{F}_2);\mathcal{F}_2)$ *and* $\mathbb{M}k(\mathbf{lk}_\alpha(\mathcal{F}_1);\mathcal{F}_1)=\mathbb{M}k(\mathbf{lk}_\alpha(\mathcal{F}_2);\mathcal{F}_2)$.

3. *For any indexes* $\alpha,\beta\in\mathcal{I}nd(\mathcal{F}_1)=\mathcal{I}nd(\mathcal{F}_2)$ *it is performed the equality:*

$$\left[\mathbf{lk}_\beta(\mathcal{F}_1)\leftarrow\mathbf{lk}_\alpha(\mathcal{F}_1),\mathcal{F}_1\right]=\left[\mathbf{lk}_\beta(\mathcal{F}_2)\leftarrow\mathbf{lk}_\alpha(\mathcal{F}_2),\mathcal{F}_2\right].$$

Assertion 7 ([9], see also [7]). *Binary relation* $[\equiv]$ *is an equivalence relation on any set* $\mathcal{M}$, *which consists of universal kinematics.*

Remark 10. From the point of view of physical intuition we may consider that universal kinematics $\mathcal{F}_1$ and $\mathcal{F}_2$ such, that $\mathcal{F}_1[\equiv]\mathcal{F}_2$ are two different scenarios of evolution, acting in the same space-time and coordinate-transform environment.

Definition 19. *We say that universal kinematics* $\mathcal{F}$ *is* ***certainly time irreversible*** *if and only if arbitrary universal kinematics* $\mathcal{F}_1$ *such, that* $\mathcal{F}[\equiv]\mathcal{F}_1$ *is time irreversible. In the opposite case we will say that universal kinematics* $\mathcal{F}$ *is* ***conditionally time reversible***.

Since, according to Assertion 7, for each universal kinematics $\mathcal{F}$ it is fulfilled the correlation $\mathcal{F}[\equiv]\mathcal{F}$, then we receive the following Assertion as a corollary from Definition 19:

Assertion 8. *Any certainly time irreversible universal kinematics* $\mathcal{F}$ *is time irreversible.*

The physical sense of certain time irreversibility notion is that in certainly time irreversible kinematics temporal paradoxes are impossible basically, that is there is not potential possibility to affect the own past by means of “traveling” and “jumping” between reference frames. Whereas, in time irreversible, but conditionally time reversible kinematics such potential possibility exists, but it is not realized in the scenario of evolution, acting in this kinematics.

Assertion 9 ([15]). *Let universal kinematics* $\mathcal{F}$ *be weakly time-positive. Then every universal kinematics* $\mathcal{F}_1$ *such that* $\mathcal{F}_1[\equiv]\mathcal{F}$ *is weakly time-positive also.*

Applying Assertion 9 as well as Theorem 5, we obtain the following (strengthened) variant of theorem of non returning:

Theorem 6 ([15]). *Any weakly time-positive universal kinematics* $\mathcal{F}$ *is* ***certainly*** *time irreversible.*

4. Criteria of Time-positivity for Affine Coordinate Transform Operators.

Definition 20. *Let* $(\mathfrak{X}_1,\|\cdot\|_{(1)}),(\mathfrak{X}_2,\|\cdot\|_{(2)})$ *be linear normed spaces and* $\mathbb{T}_1=(\mathbf{T}_1,\leq_1)$, $\mathbb{T}_2=(\mathbf{T}_2,\leq_2)$ *be linearly ordered sets. We say that CTO* $\mathcal{U}\in\mathbb{P}\mathbf{k}(\mathbb{T}_1,\mathfrak{X}_1;\mathbb{T}_2,\mathfrak{X}_2)$ *is:*

1. ***time-positive*** *if and only if for arbitrary* $t_1,t_2\in\mathbf{T}_1$ *and* $\mathrm{x}\in\mathfrak{X}_1$ *inequality* $t_1<_1 t_2$ *assures the inequality,* $\mathrm{tm}(\mathcal{U}(t_1,\mathrm{x}))<_2\mathrm{tm}(\mathcal{U}(t_2,\mathrm{x}))$, *where* $<_1$ *and* $<_2$ *are strict linear order relations, generated by* $\leq_1$ *and* $\leq_2$ *respectively;*

2. ***time-negative*** *if and only if for arbitrary* $t_1, t_2 \in \mathbf{T}_1$ *and* $\mathrm{x} \in \mathfrak{X}_1$ *inequality* $t_1 <_1 t_2$ *assures the inequality,* $\mathsf{tm}(\mathcal{U}(t_1,\mathrm{x})) >_2 \mathsf{tm}(\mathcal{U}(t_2,\mathrm{x}))$ *(i.e. the inequality* $\mathsf{tm}(\mathcal{U}(t_2,\mathrm{x})) <_2 \mathsf{tm}(\mathcal{U}(t_1,\mathrm{x}))$*).*

Directly from Definition 17 and Definition 20 we readily obtain the following assertion.

Assertion 10. *Let* $\mathcal{F}$ *be any universal kinematics and* $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ *be any reference frames of* $\mathcal{F}$. *Then the following statements hold:*

1. $\mathfrak{m} \Uparrow^{+}_{\mathcal{F}} \mathfrak{l}$ *if and only if the operator*

$[\mathfrak{m} \leftarrow \mathfrak{l}] \in \mathbb{P}\mathbf{k}(\mathbb{T}\mathbf{m}(\mathfrak{l}), \mathbf{Zk}(\mathfrak{l});\ \mathbb{T}\mathbf{m}(\mathfrak{m}), \mathbf{Zk}(\mathfrak{m}))$ *is time-positive.*

2. $\mathfrak{m} \Downarrow^{-}_{\mathcal{F}} \mathfrak{l}$ *if and only if the operator*

$[\mathfrak{m} \leftarrow \mathfrak{l}] \in \mathbb{P}\mathbf{k}(\mathbb{T}\mathbf{m}(\mathfrak{l}), \mathbf{Zk}(\mathfrak{l});\ \mathbb{T}\mathbf{m}(\mathfrak{m}), \mathbf{Zk}(\mathfrak{m}))$ *is time-negative.*

Assertion 10 shows that the question on time-positivity (time-negativity) of one reference frame relatively to other in some universal kinematics can be reduced to the question on time-positivity (time-negativity) of coordinate transform operator (CTO) between these reference frames. That is why further in this section we will focus on obtaining some needed results on time-positivity (time-negativity) of coordinate transform operators (CTOs), namely affine CTOs in the space $\mathbf{Pk}(\mathfrak{H})$ over some Hilbert space $\mathfrak{H}$.

For any real Hilbert space $(\mathfrak{H}, \|\cdot\|, \langle\cdot,\cdot\rangle)$ and operator $\mathbf{S} \in \mathbf{Pk}(\mathfrak{H})$ we have $\mathbf{S} \in \mathbb{P}\mathbf{k}(\mathfrak{H}) = \mathbb{P}\mathbf{k}(\mathbb{R}_{\le}, \mathfrak{H}; \mathbb{R}_{\le}, \mathfrak{H})$. So it is correct to say about time-positivity or time-negativity of the operator $\mathbf{S}$. For any operator $\mathbf{S} \in \mathbf{Pk}(\mathfrak{H})$ we introduce the following notation:

$$\mathbf{tsg}(\mathbf{S}) := \operatorname{sign}\left(\operatorname{tm}(\mathbf{S}\mathbf{e}_0 - \mathbf{S}0)\right) = \operatorname{sign}\left(\mathcal{T}(\mathbf{S}\mathbf{e}_0 - \mathbf{S}0)\right),$$

the number $\mathbf{tsg}(\mathbf{S})$ we call by ***time sign*** of the operator $\mathbf{S} \in \mathbf{Pk}(\mathfrak{H})$.

Remark 11. *If* $\mathbf{S} \in \boldsymbol{Pk}(\mathfrak{H}) \cap \mathcal{L}(\mathcal{M}(\mathfrak{H}))$ *then* $\mathbf{tsg}(\mathbf{S}_{[\mathbf{a}]}) = \mathbf{tsg}(\mathbf{S})$ *for each* $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$. (Where $\mathbf{S}_{[\mathbf{a}]}\mathrm{w} = \mathbf{S}\mathrm{w} + \mathbf{a}$ ($\mathrm{w} \in \mathcal{M}(\mathfrak{H})$).)

Indeed, by definition of $\mathbf{tsg}(\cdot)$, we have:

$$\mathbf{tsg}(\mathbf{S}_{[\mathbf{a}]}) = \operatorname{sign}\left(\mathcal{T}\left((\mathbf{S}\mathbf{e}_0 + \mathbf{a}) - (\mathbf{S}0 + \mathbf{a})\right)\right) = \operatorname{sign}\left(\mathcal{T}(\mathbf{S}\mathbf{e}_0 - \mathbf{S}0)\right) = \mathbf{tsg}(\mathbf{S}).$$

Remark 12. *If* $\mathbf{S} \in \mathbf{Pk}(\mathfrak{H}) \cap \mathcal{L}(\mathcal{M}(\mathfrak{H}))$ *then* $\mathbf{S}0 = \mathbf{0}$. So in this case definition of $\mathbf{tsg}(\mathbf{S})$ reduces to more simple form:

$$\mathbf{tsg}(\mathbf{S}) := \operatorname{sign}\left(\operatorname{tm}(\mathbf{S}\mathbf{e}_0)\right) = \operatorname{sign}\left(\mathcal{T}(\mathbf{S}\mathbf{e}_0)\right).$$

Assertion 11. *Operator* $\mathbf{S} \in \mathbf{Pk}(\mathfrak{H})$ *is:*

(a) *time-positive if and only if* $\mathbf{tsg}(\mathbf{S}) > 0$ *;*

(b) *time-negative if and only if* $\mathbf{tsg}(\mathbf{S}) < 0$.

P r o o f. For convenience we introduce the following operator $\mathbf{E} : \mathfrak{H} \to \mathcal{M}(\mathfrak{H})$:

$$\mathfrak{H} \ni x \mapsto \mathbf{E}x := (0, x) \in \mathcal{M}(\mathfrak{H}),$$

that is $\mathbf{E}$ is the embedding operator of space $\mathfrak{H}$ into $\mathcal{M}(\mathfrak{H})$. So, for every vector $\mathrm{w} = (t, x) \in \mathcal{M}(\mathfrak{H})$ we can write:

$$(t, x) = t\mathbf{e}_0 + \mathbf{E}x. \tag{25}$$

Every operator $\mathbf{S}\in\mathbf{Pk}(\mathfrak{H})$ can be represented by the form:

$$\mathbf{S}\mathrm{w}=\tilde{\mathbf{S}}\mathrm{w}+\mathbf{s}\qquad(\mathrm{w}\in\mathcal{M}(\mathfrak{H})), \tag{26}$$

where $\tilde{\mathbf{S}}\in\mathcal{L}(\mathcal{M}(\mathfrak{H}))$ and $\mathbf{s}\in\mathcal{M}(\mathfrak{H})$, so $\mathbf{s}=\mathbf{S0}$. Using (25) and (26), for arbitrary $t_1,t_2\in\mathbb{R}$ and $x\in\mathfrak{H}$ such that $t_1<t_2$ we obtain:

$$\begin{aligned}\mathbf{S}(t_2,x)-\mathbf{S}(t_1,x)&=\tilde{\mathbf{S}}(t_2,x)-\tilde{\mathbf{S}}(t_1,x)=\tilde{\mathbf{S}}(t_2\mathbf{e}_0+\mathbf{E}x)-\tilde{\mathbf{S}}(t_1\mathbf{e}_0+\mathbf{E}x)=\\&=\tilde{\mathbf{S}}((t_2-t_1)\mathbf{e}_0)=(t_2-t_1)\tilde{\mathbf{S}}(\mathbf{e}_0)=\\&=(t_2-t_1)(\mathbf{S}(\mathbf{e}_0)-\mathbf{s})=(t_2-t_1)(\mathbf{S}(\mathbf{e}_0)-\mathbf{S0}).\end{aligned}$$

Thence: $\mathrm{sign}\left(\mathrm{tm}(\mathbf{S}(t_2,x))-\mathrm{tm}(\mathbf{S}(t_1,x))\right)=\mathrm{sign}\left(\mathcal{T}\left(\mathbf{S}(t_2,x)-\mathbf{S}(t_1,x)\right)\right)=$
$=\mathrm{sign}\left((t_2-t_1)\mathcal{T}\left(\mathbf{S}(\mathbf{e}_0)-\mathbf{S0}\right)\right)=\mathbf{tsg}(\mathbf{S})$.

Therefore, in the case $\mathbf{tsg}(\mathbf{S})>0$ the inequality $t_1<t_2$ leads to $\mathrm{tm}(\mathbf{S}(t_2,x))-\mathrm{tm}(\mathbf{S}(t_1,x))>0$ and operator $\mathbf{S}$ is time-positive as well in the case $\mathbf{tsg}(\mathbf{S})<0$ operator $\mathbf{S}$ is time-negative. ♦

Assertion 12. *Let* $c\in(0,+\infty)$, $\lambda\in[0,c)$, $s\in\{-1,1\}$, $J\in\mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n}\in\mathbf{B}_1(\mathfrak{H}_1)$ *and* $\mathbf{a}\in\mathcal{M}(\mathfrak{H})$ *then:*

$$\mathbf{tsg}\left(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]\right)=s.$$

P r o o f. Since $\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]=\mathbf{W}_{\lambda,c}[s,\mathbf{n},J]_{\left[\mathbf{W}_{\lambda,c}[s,\mathbf{n},J]\,\mathbf{a}\right]}$ then, according to Remark 11, Remark 12 and formula (2), we deliver:

$$\mathbf{tsg}\left(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]\right)=\mathbf{tsg}\left(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J]\right)=\mathrm{sign}\left(\mathcal{T}\left(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J]\mathbf{e}_0\right)\right)=$$

$$=\mathrm{sign}\left(\frac{\left(s\mathcal{T}(\mathbf{e}_0)-\frac{\lambda}{c^2}\mathbf{n},\mathbf{e}_0\right)}{\sqrt{1-\frac{\lambda^2}{c^2}}}\right)=\mathrm{sign}\left(\frac{s}{\sqrt{1-\frac{\lambda^2}{c^2}}}\right)=s.\qquad\blacklozenge$$

5. On Time Irreversibility of Generalized Hassani Kinematics. Before investigating time irreversibility of generalized Hassani kinematics, we obtain some more general results, concerning time irreversibility of the kinematics of kind $\mathfrak{Ku}(\mathbb{S},\mathcal{B};\mathfrak{H})$.

Theorem 7. *Let* $(\mathfrak{H},\|\cdot\|,\langle\cdot,\cdot\rangle)$ *be a real Hilbert space such, that* $\dim(\mathfrak{H})\geq 1$ *and* $\mathcal{B}$ *be a base changeable set such, that* $\mathfrak{Bs}(\mathcal{B})\subseteq\mathfrak{H}$ *and* $\mathbb{T}\mathbf{m}(\mathcal{B})=\mathbb{R}_{\leq}$. *If in the set of operators* $\mathbb{S}\subseteq\mathbb{P}\mathbf{k}(\mathfrak{H})$ *there exists an operator* $\mathbf{U}_0\in\mathbb{S}$ *such that for every* $\mathbf{U}\in\mathbb{S}$ *the operator* $\mathbf{S}_{\mathbf{U}}(\mathrm{w})=\mathbf{U}_0\left(\mathbf{U}^{-1}(\mathrm{w})\right)$ $(\mathrm{w}\in\mathcal{M}(\mathfrak{H}))$ *is time-positive then the kinematics* $\mathcal{F}=\mathfrak{Ku}(\mathbb{S},\mathcal{B};\mathfrak{H})$ *is certainly time irreversible.*

P r o o f. Let $\mathbf{U}_0\in\mathbb{S}$ be an operator, which satisfies conditions of the theorem. Denote, $\mathfrak{l}_0:=\left(\mathbf{U}_0,\mathbf{U}_0[\mathcal{B}]\right)$. According to Theorem 3 (item 1), $\mathfrak{l}_0\in\mathcal{L}k(\mathcal{F})$ (i.e. $\mathfrak{l}_0$ is a reference frame of the kinematics $\mathcal{F}$). Consider any reference frame $\mathfrak{l}\in\mathcal{L}k(\mathcal{F})$. According to Theorem 3 (items 1 and 3) the frame $\mathfrak{l}$ can be represented in the form $\mathfrak{l}=(\mathbf{U},\mathbf{U}[\mathcal{B}])$, where $\mathbf{U}\in\mathbb{S}$ and for the operator $\left[\mathfrak{l}_0\leftarrow\mathfrak{l}\right]\in\mathbb{P}\mathbf{k}\left(\mathbb{T}\mathbf{m}(\mathfrak{l}),\mathbf{Zk}(\mathfrak{l});\ \mathbb{T}\mathbf{m}(\mathfrak{l}_0),\mathbf{Zk}(\mathfrak{l}_0)\right)=\mathbb{P}\mathbf{k}(\mathfrak{H})$ we get:

$$\left[\mathfrak{l}_0 \leftarrow \mathfrak{l}\right] \mathrm{w} = \mathbf{U}_0\left(\mathbf{U}^{-1}(\mathrm{w})\right) = \mathbf{S}_{\mathbf{U}}(\mathrm{w}) \quad (\forall \mathrm{w} \in \mathbb{M}k(\mathfrak{l}) = \mathcal{M}(\mathfrak{H})).$$

So, by conditions of the theorem, the operator $[\mathfrak{l}_0 \leftarrow \mathfrak{l}]$ is time-positive, and, by Assertion 10, we have $\mathfrak{l}_0 \Uparrow_{\mathcal{F}}^{+} \mathfrak{l}$ for each reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$. Therefore, by Definition 17, kinematics $\mathcal{F}$ is weakly time-positive. Thus, by Theorem 6, this kinematics is certainly time irreversible. ♦

Theorem 7 and Assertion 11 immediately imply the following corollary:

Corollary 2. *Let* $(\mathfrak{H}, \|\cdot\|, \langle\cdot,\cdot\rangle)$ *be a real Hilbert space such, that* $\dim(\mathfrak{H}) \geq 1$ *and* $\mathcal{B}$ *be a base changeable set such, that* $\mathfrak{Bs}(\mathcal{B}) \subseteq \mathfrak{H}$ *and* $\mathbb{T}\mathbf{m}(\mathcal{B}) = \mathbb{R}_{\leq}$. *If in the set of operators* $\mathbb{S} \subseteq \mathbf{Pk}(\mathfrak{H})$ *there exists an operator* $\mathbf{U}_0 \in \mathbb{S}$ *such that* $\mathbf{tsg}\left(\mathbf{U}_0\, \mathbf{U}^{-1}\right) > 0$ *for every* $\mathbf{U} \in \mathbb{S}$ *then the kinematics* $\mathcal{F} = \mathfrak{Ku}(\mathbb{S}, \mathcal{B}; \mathfrak{H})$ *is certainly time irreversible.*

Denote by $\mathbb{I}$ or by $\mathbb{I}_{\mathcal{M}(\mathfrak{H})}$ the identity operator over the space $\mathcal{M}(\mathfrak{H})$ that is the operator such, that $\mathbb{I}\mathrm{w} = \mathrm{w}$ $(\forall \mathrm{w} \in \mathcal{M}(\mathfrak{H}))$. From Corollary 2 we immediately deduce the following corollary:

Corollary 3. *Let* $(\mathfrak{H}, \|\cdot\|, \langle\cdot,\cdot\rangle)$ *be a real Hilbert space such, that* $\dim(\mathfrak{H}) \geq 1$ *and* $\mathcal{B}$ *be a base changeable set such, that* $\mathfrak{Bs}(\mathcal{B}) \subseteq \mathfrak{H}$ *and* $\mathbb{T}\mathbf{m}(\mathcal{B}) = \mathbb{R}_{\leq}$. *If the set of operators* $\mathbb{S} \subseteq \mathbf{Pk}(\mathfrak{H})$ *possesses the following properties:*

1. $\mathbb{I} \in \mathbb{S}$; 2. $\mathbf{tsg}\left(\mathbf{U}^{-1}\right) > 0$ *for every* $\mathbf{U} \in \mathbb{S}$,

then the kinematics $\mathcal{F} = \mathfrak{Ku}(\mathbb{S}, \mathcal{B}; \mathfrak{H})$ *is certainly time irreversible.*

Now we are near to obtain the main result of this article.

Theorem 8. *Let* $(\mathfrak{H}, \|\cdot\|, \langle\cdot,\cdot\rangle)$ *be a real Hilbert space such, that* $\dim(\mathfrak{H}) \geq 1$ *and* $\mathcal{B}$ *be a base changeable set such, that* $\mathfrak{Bs}(\mathcal{B}) \subseteq \mathfrak{H}$ *and* $\mathbb{T}\boldsymbol{m}(\mathcal{B}) = \mathbb{R}_{\leq}$. *Then for any function* $\vartheta \in \Upsilon$ *universal kinematics* $\mathfrak{UH}(\mathfrak{H}, \mathcal{B}, \vartheta)$ *is certainly time irreversible.*

P r o o f. According to definition (see formula (19)), for any function $\vartheta \in \Upsilon$ we have the equality:

$$\mathfrak{UH}(\mathfrak{H}, \mathcal{B}, \vartheta) = \mathfrak{Ku}\left(\mathfrak{P}_{+}(\mathfrak{H}, [\vartheta]), \mathcal{B}; \mathfrak{H}\right),$$

where (in accordance with [11, Properties 1]) the class of operators $\mathfrak{P}_{+}(\mathfrak{H}, [\vartheta]) \subseteq \mathbf{Pk}(\mathfrak{H})$ possesses the following properties:

1^0. $\mathbb{I} \in \mathfrak{P}_{+}(\mathfrak{H}, [\vartheta])$.

2^0. If $\mathbf{U} \in \mathfrak{P}_{+}(\mathfrak{H}, [\vartheta])$ then $\mathbf{U}^{-1} \in \mathfrak{P}_{+}(\mathfrak{H}, [\vartheta])$.

Moreover, the class of operators $\mathfrak{P}_{+}(\mathfrak{H}, [\vartheta])$ has also the following property:

3^0. $\mathbf{tsg}(\mathbf{U}) = 1$ for every $\mathbf{U} \in \mathfrak{P}_{+}(\mathfrak{H}, [\vartheta])$.

Indeed, if $\mathbf{U} \in \mathfrak{P}_{+}(\mathfrak{H}, [\vartheta])$ then, according to (10) and (8), operator $\mathbf{U}$ can be represented in the form $\mathbf{U} = \mathbf{W}_{\lambda, \vartheta(\lambda)}[1, \mathbf{n}, J;\ \mathbf{a}]$, where $\lambda \in \mathfrak{D}_{*}[\vartheta]$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$. And, by Assertion 12, we have, $\mathbf{tsg}(\mathbf{U}) = \mathbf{tsg}\left(\mathbf{W}_{\lambda, \vartheta(\lambda)}[1, \mathbf{n}, J;\ \mathbf{a}]\right) = 1$.

From properties 1^0, 2^0, 3^0, in accordance with Corollary 3 it follows that the kinematics $\mathfrak{UH}(\mathfrak{H}, \mathcal{B}, \vartheta) = \mathfrak{Ku}\left(\mathfrak{P}_{+}(\mathfrak{H}, [\vartheta]), \mathcal{B}; \mathfrak{H}\right)$ is certainly time irreversible. ♦

Remark 13. In the begin of the article we have also introduced the kinematics $\mathfrak{U}H_0(\mathfrak{H},\mathcal{B},\vartheta)$ (together with $\mathfrak{U}H(\mathfrak{H},\mathcal{B},\vartheta)$). Using sufficient condition of conditionally time reversibility (see [13, Theorem 1], see also [12, Theorem 3]) it can be proven that the kinematics $\mathfrak{U}H_0(\mathfrak{H},\mathcal{B},\vartheta)$ is conditionally time reversible (for any function $\vartheta \in \Upsilon$). Detailed proof of the latter fact may be set forth in future publications.

1. *O.-M. P. Bilaniuk, V. K. Deshpande, E. C. G. Sudarshan.* "Meta" Relativity // American Journal of Physics. − 1962. − **30**, No. 10. − P. 718−723. − http://doi.org/10.1119/1.1941773.
2. *O.-M. P. Bilaniuk, E. C. G. Sudarshan.* Particles beyond the light barrier // Physics Today. − 1969. − **22**, No. 5. − P. 43−51. − http://doi.org/10.1063/1.3035574.
3. *Garrett Birkhoff.* Lattice theory // Third edition. American Mathematical Society Colloquium Publications, Vol. XXV. American Mathematical Society, Providence, R.I., New York, 1967.
4. *R. Goldoni.* Faster-than-light inertial frames, interacting tachyons and tadpoles // Lettere al Nuovo Cimento. − 1972. − **5**, No. 6. − P. 495−502. − http://doi.org/10.1007/BF02785903.
5. *Ya. I. Grushka.* Base changeable sets and mathematical simulation of the evolution of systems // Ukr. Math. J. − 2014. − **65**, No. 9. − P. 1332−1353. − http://doi.org/10.1007/s11253-014-0862-6.
6. *Ya. I. Grushka.* Changeable sets and their application for the construction of tachyon kinematics // Zb. Pr. Inst. Mat. NAN Ukr. − 2014. − **11**, No. 1. − P. 192−227.
7. *Ya. I. Grushka.* Draft introduction to abstract kinematics // Preprint (2017). − 208 p. − https://doi.org/10.13140/RG.2.2.28964.27521.
8. *Ya. I. Grushka.* Evolutional extensions and analogues of the union operation for base changeable sets // Zb. Pr. Inst. Mat. NAN Ukr. − 2014. − **11**, No. 2. − P. 66−99.
9. *Ya. I. Grushka.* Evolutionary extensions of kinematic sets and universal kinematics // Zb. Pr. Inst. Mat. NAN Ukr. − 2015. − **12**, No. 2. − P. 139−204.
10. *Ya. I. Grushka.* Kinematic changeable sets with given universal coordinate transforms // Zb. Pr. Inst. Mat. NAN Ukr. − 2015. − **12**, No. 1. − P. 74−118.
11. *Ya. I. Grushka.* On some properties of Hassani transforms // Mat. Studii. − 2022. − **57**, № 1. − P. 79−91. − https://doi.org/10.30970/ms.57.1.79-91.
12. *Ya. I. Grushka.* On time irreversibility of universal kinematics // Reports of the National Academy of Sciences of Ukraine. − 2016. − **7**. − P. 14−21. − http://doi.org/10.15407/dopovidi2016.07.014.
13. *Ya. I. Grushka.* On time reversibility of tachyon kinematics // Zb. Pr. Inst. Mat. NAN Ukr. − 2016. − **13**, No. 2. − P. 125−174.
14. *Ya. I. Grushka.* Tachyon generalization for Lorentz transforms // Meth. Funct. Anal. Topology. − 2013. − **19**, No. 2. − P. 127−145.
15. *Ya. I. Grushka.* Theorem of Non-Returning and Time Irreversibility of Tachyon Kinematics // Progress in Physics. − 2017. − **13**, No. 4. − P. 218−228.
16. *Mohamed Elmansour Hassani.* Foundations of Superluminal Relativistic Mechanics // Communications in Physics. − 2014. − **24**, No. 4. − P. 313−332. − https://doi.org/10.15625/0868-3166/24/4/4850.
17. *James M. Hill, Barry J. Cox.* Einstein's special relativity beyond the speed of light // Proc. Roy. Soc. London A: Mathematical, Physical and Engineering Sciences. − 2012. − **468**. − P. 4174−4192. − http://doi.org/10.1098/rspa.2012.0340.
18. *S. Yu. Medvedev.* On the possibility of broadening special relativity theory beyond light barrier // Nauk. Visn. Uzhhorod. Univ. Ser. Phys. − 2005. − **18**. − P. 7−15.
19. *C. Møller.* The theory of relativity. − Oxford: Clarendon Press, 1957.
20. *M. A. Naimark.* Linear Representations of the Lorentz Group // International Series of Monographs in Pure and Applied Mathematics, Vol. **63.** − Oxford: Pergamon Press, 1964.
21. *E. Recami.* Classical Tachyons and Possible Applications // Riv. Nuovo Cim. − 1986. − **9**, No. 6. − P. 1−178. − http://doi.org/10.1007/BF02724327.
22. *E. Recami, V. S. Olkhovsky.* About Lorentz transformations and tachyons // Lettere al Nuovo Cimento. − 1971. − **1**, No. 4. − P. 165−168. − http://doi.org/10.1007/BF02799345.

ПРО ЧАСОНЕЗВОРОТНІСТЬ УЗАГАЛЬНЕНИХ КІНЕМАТИК ХАССАНІ

Оригінальні перетворення Хассані були отримані в роботах алжирського фізика М. Е. Хассані. Узагальнені (надсвітлові) кінематики Хассані з'явилися в контексті узагальнення і розвитку ідей Хассані. У цій статті за допомогою теореми про неповернення для універсальних кінематик доведено, що довільна узагальнена кінематика Хассані з додатним напрямком часу є безумовно часонезворотною. З фізичної точки зору цей результат означає, що в будь-якій часопозитивній узагальненій кінематиці Хассані в принципі відсутні часові парадокси, пов'язані з можливістю впливати на власне минуле за допомогою «подорожей» і «перестрибувань» між системами відліку.

Ключові слова: *універсальні кінематики, мінливі множини, інерціальні системи відліку, тахіони, часові парадокси, часонезворотність.*

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