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THE COEFFICIENTS OF TRANSITIVENESS OF THE POSETS MINIMAX ISOMORPHIC TO THE NON-PRIMITIVE SUPERCRITICAL POSET

The coefficients of transitiveness for all posets that are minimax isomorphic to the poset $N_6 = (N, 5) = \{1, 2, \ldots, 9 \mid 1 \prec 2 \prec 3 \prec 4 \prec 5,\ 6 \prec 7,\ 8 \prec 9,\ 6 \prec 9\}$ *are calculated.*

Key words: supercritical poset, minimax isomorphism, coefficient of transitiveness, MM-type, nodal element, dence subposet.

Introduction. M. M. Kleiner [10] proved that a poset S is of finite representation type if and only if it does not contain subsets of the form $K_1 = (1, 1, 1, 1)$, $K_2 = (2, 2, 2)$, $K_3 = (1, 3, 3)$, $K_4 = (1, 2, 5)$ and $K_5 = (N, 4)$. These posets are called the critical posets or the Kleiner's posets. On the other hand, Yu. A. Drozd [9] proved that a poset is of finite representational type if and only if its Tits quadratic form is weakly positive, i.e. it is positive on the non-zero non-negative vectors. Hence the Kleiner's posets are also critical with respect to weak positiveness of the Tits form, and there are no other such posets. In [2] the authors proved that a poset is P-critical (i.e. critical with respect to the positiveness of the Tits form) if and only if it is minimax isomorphism to a Kleiner's poset.

A similar situation takes place in the case of tame posets. L. A. Nazarova [11] proved that a poset S is tame if and only if it does not contain subsets of the form $N_1 = (1, 1, 1, 1, 1)$, $N_2 = (1, 1, 1, 2)$, $N_3 = (2, 2, 3)$, $N_4 = (1, 3, 4)$, $N_5 = (1, 2, 6)$ and $N_6 = (N, 5)$; these conditions are equivalent to weak non-negativity of the quadratic Tits form. She called these posets supercritical. So the supercritical posets are critical with respect to weak non-negativity of the Tits form and there are no other such posets. The authors proved that a poset is critical with respect to non-negativity of the Tits form if and only if it is minimax isomorphism to a supercritical poset; all such critical posets were described by them in [3].

In many papers (see e.g. [4–8] combinatorial properties were studied for various classes of posers. The present paper is devoted to the investigation of combinatorial properties of supercritical posets.

1. The list of posets of MM-type $(N, 5)$. Let P be a fix poset. A poset S is called of MM-type P if S is minimax (in other words, (min, max)-) isomorphic to P (the notions of (min, max)-equivalence and (min, max)-isomorphism were introduced in [1]; see also [2]). From the results of [3] it follows that the table below contains all (up to isomorphism and duality) posets of MM-type $N_6 = (N, 5) = \{1, 2, \ldots, 9 \mid 1 \prec 2 \prec 3 \prec 4 \prec 5,\ 6 \prec 7,\ 8 \prec 9,\ 6 \prec 9\}$.

2. Main result. Let S be a finite poset and $S^2_< := \{(x, y) \mid x, y \in S, x < y\}$. If $(x, y) \in S^2_<$ and there is no z satisfying $x < z < y$, then x and y are called *neighboring*. Put $n_w = n_w(S) := |S^2_<|$ and denote by $n_e = n_e(S)$ the number of pairs of neighboring elements. On the language of the Hasse diagram $H(S)$ (that represents S in the plane), n_e is equal to the number of all its edges and n_w to the number of all its paths, up to parallelity, going bottom-up (two path is called parallel if they start and terminate at the same vertices).

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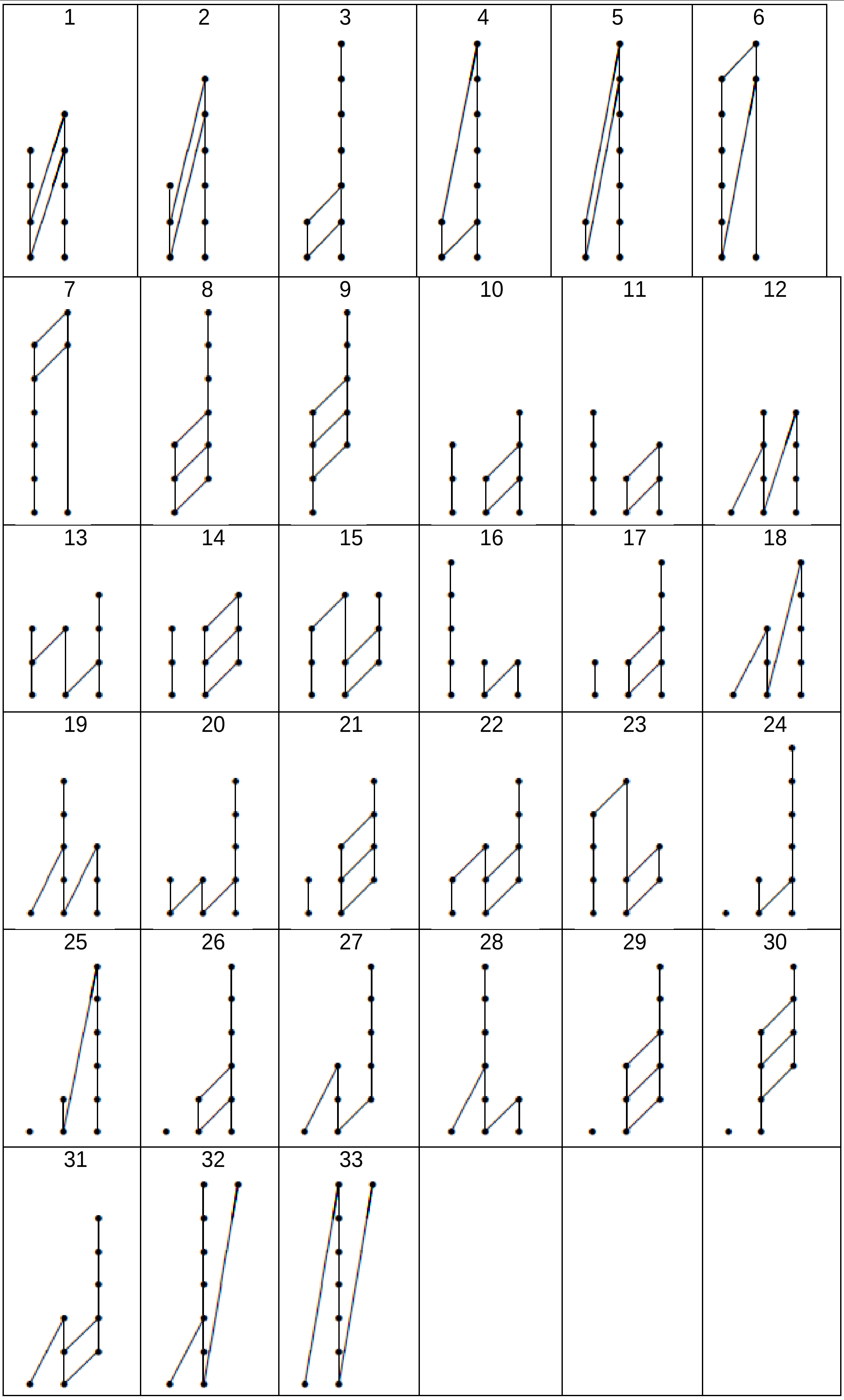
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The ratio $k_t = k_t(S)$ of the numbers $n_w - n_e$ and n_w is called *the coefficient of transitiveness of S*. If $n_w = 0$ (then $n_e = 0$), we assume $k_t = 0$ (see [5]). Obviously, dual poset have the same coefficient of transitiveness.

The aim of this paper is to calculate k_t for all posets of MM-type $N_6 = (N, 5) = \{1, 2, ..., 9 \mid 1 \prec 2 \prec 3 \prec 4 \prec 5, 6 \prec 7, 8 \prec 9, 6 \prec 9\}$. All the coefficients of transitiveness k_t are calculated up to the second decimal place.

Theorem. *The following holds for posets* 1–33:

N	n_e	n_w	k_t	N	n_e	n_w	k_t	N	n_e	n_w	k_t
1	9	19	0.52632	12	8	15	0.46667	23	9	17	0.47059
2	9	21	0.57143	13	8	15	0.46667	24	7	21	0.66667
3	9	33	0.72727	14	9	15	0.4	25	7	17	0.58824
4	9	29	0.68966	15	9	17	0.47059	26	8	25	0.68
5	9	25	0.64	16	7	13	0.46154	27	8	19	0.57895
6	9	25	0.64	17	8	19	0.57895	28	8	21	0.61905
7	9	29	0.68966	18	8	15	0.46667	29	9	25	0.64
8	10	33	0.69697	19	8	17	0.52941	30	9	25	0.64
9	10	33	0.69697	20	8	17	0.52941	31	9	23	0.60870
10	8	15	0.46667	21	9	19	0.52632	32	8	27	0.70370
11	8	13	0.38462	22	9	19	0.52632	33	8	23	0.65217

The proof is carried out by direct calculations.

Recall that an element of a poset T is called *nodal*, if it is comparable with all elements of T. A subposet X of T is said to be dense if there is not $x_1, x_2 \in X$, $y \in T \setminus X$ such that $x_1 < y < x_2$.

It is easy to see that the theorem emplies the following statement.

Corollary. A poset of MM-type N_6 has the largest coefficient of transitiveness if and only if it contains a dense subposet with five nodal element.

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КОЕФІЦІЄНТИ ТРАНЗИТИВНОСТІ Ч. В. МНОЖИН, МІНІМАКСНО ІЗОМОРФНИХ НЕПРИМІТИВНІЙ СУПЕРКРИТИЧНІЙ Ч. В. МНОЖИНІ

Обчислено коефіцієнти транзитивності для всіх ч. в. множин, мінімаксно ізоморфних ч. в. множині $N_6 = (N, 5) = \{1, 2, \ldots, 9 \mid 1 \prec 2 \prec 3 \prec 4 \prec 5,\ 6 \prec 7,\ 8 \prec 9,\ 6 \prec 9\}$.

Ключові слова*: суперкритична ч. в. множина, мінімаксний ізоморфізм, коефіцієнт транзитивності, MM-тип, вузловий елемент, щільна ч. в. підмножина.*

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