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CLASSIFICATION OF MINIMAL AND MAXIMAL NON-SERIAL POSITIVE POSETS

The finite posets with positive Tits quadratic form, which are called positive, are analogs of Dynkin diagrams. They were first described in 2005 by the authors. In particular, according to this result such a poset can be serial if it belongs to an infinite strictly increasing sequence of positive posets, or non-serial otherwise. In the following years the authors studied various classes of posets that are related to the Tits quadratic form. In this paper, positive posets are studied in more detail, namely in relation to their ordering. The main theorems classify all non-serial positive posets that are minimal or maximal. The case of serial posets are trivial: there are no maximal posets and all minimal posets are one-element. The number of non-serial minimal posets up to isomorphism and duality is 10, and the number of maximal ones is 66 (out of a total 108).

Keywords: Tits quadratic form, positive poset, serial and non-serial poset, minimal and maximal non-serial poset, Dynkin diagram.

Introduction. In [9] P. Gabriel introduced the notion of representation of a finite quiver $Q=(Q_0,Q_1)$ (with the set of vertices Q_0 and the set of arrows Q_1) over a field k and defined a quadratic form $q_Q : \mathbb{Z}^n \to \mathbb{Z}$, $n = |Q_0|$, called by him the *Tits quadratic form of the quiver Q*:

$$q_Q(z) = q_Q(z_1,\dots,z_n) := \sum_{i\in Q_0} z_i^2 - \sum_{i\to j} z_i z_j ,$$

where $i\to j$ runs through the set Q_1. He proved that the quiver Q has finite representation type if and only if its Tits form is positive. He also received a criterion of finiteness representation type in terms of the quivers themselves. Namely in the main case, when Q is connected, it has finite type if and only if it is a Dynkin diagram (if one does not take into account the orientation of arrows).

In [8], Yu. A. Drozd showed that a poset $0 \notin S$ has finite representation type if and only if its *Tits quadratic form*

$$q_S(z) = z_0^2 + \sum_{i\in S} z_i^2 + \sum_{i<j,\, i,j\in S} z_i z_j - z_0 \sum_{i\in S} z_i .$$

is weakly positive, i.e. takes positive value on any nonzero vector with non-negative coordinates (representations of posets were introduced by L. A. Nazarova and A. V. Roiter [11]). A criterion of finiteness type in terms of the posets themselves was obtained by M. M. Kleiner [10]. Namely, a poset S is of finite representation type if and only if it does not contain subsets of the form K_1=(1, 1, 1, 1), K_2=(2, 2, 2), K_3=(1, 3, 3), K_4=(1, 2, 5); and K_5=(N, 4).

For posets, in contrast to quivers the sets of posets with weakly positive and with positive Tits quadratic form do not coincide. Since the posets with positive Tits form, which are called positive, are analogs of the Dynkin diagrams, their study is an important problem. Such posets were studied by the authors (from different points of view) in many papers (see e.g. [2–7]). Note that the paper [3] is practically inaccessible, but its main ideas and many results are outlined in the first part of [1].

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In this paper, positive posets are studied in more detail. The main theorems classified all non-serial positive posets that are minimal or maximal in the set of all non-serial positive ones. The case of serial posets are trivial (there are no maximal posets and all minimal posets are one-element).

1. The main result. Throughout the paper all posets are finite of order $n>0$ (without an element 0). We consider only complete subposets (i.e. with partial orders induced by those on the posets). One-element subsets are identified with the elements themselves. The dual poset for S is denoted by S^{op}) i.e. S^{op} and S are equal as usual sets, and $x<y$ in S^{op} if and only if $x>y$ in S). Posets T and S are called *anti-isomorphic* if T and S^{op} are isomorphic.

A finite poset S is called *positive* if so is its Tits quadratic form $q_S(z)$ (see Introduction). They were classified in [3]. According to this result such a poset can be both *serial*, when it belongs to an infinite strictly increasing sequence of positive posets, or *non-serial*, when otherwise.

The following two theorems are classified all non-serial positive posets S that are minimal (i.e. do not contain proper non-serial complete subposets) or maxinal (i.e. are not contained in other non-serial positive posets as proper complete subposets).

Theorem 1. *Let S be a non-serial positive poset. Then the following conditions are equivalent*:

(1) *S is minimal*;

(2) *S is of order* 5;

(3) *S is isomorphic or anti-isomorphic to a poset from Theorem* 6 (see below).

Theorem 2. *Let S be a non-serial positive poset. Then the following conditions are equivalent*:

(1) *S is maximal*;

(2) *S is of order 7*;

(3) *S is isomorphic or anti-isomorphic to a poset from Theorem* 8 (see below).

For a better understanding these theorems see below Theorem 5.

A subposet X of a poset S is called *lower* (respectively, upper) if $x \in X$ whenever $x < y$ (resp. $x > y$) and $y \in X$. We call a subposet *extremal* if it is lower or upper.

From the proof of Theorem 2 it will be followed the next theorem.

Theorem 3. *Let $n, m \in \{5,6,7\}$ and $n < m$. Then for any non-serial positive poset S of order n, there exists a non-serial positive poset T of order m such that S is isomorphic to a subposet of T. One can assumed that the image of isomorphism is an exstremal subposet*.

Finally, we have the following theorem which follows from Theorem 3 and the classifying the serial positive posets [3, 5].

Theorem 4. *Let m be a natural number. For a non-maximal positive poset S of order m, there exists a positive poset T of order $m+1$ such that S is isomorphic to a subposet of T.*

2. Non-serial positive posets. The positive posets were classified by the authors in [3]. Such a poset can be serial or non-serial (see Section 1).

Theorem 5. *Any positive poset of order $n < 5$ or $n > 7$ is serial.*

In the case of serial posets the answer was obtained on the set-theoretic language (more precisely, in the terms of various sums of chains and almost chains (see also [1] and [5]). In the case of non-serial posets the answer was obtained in the terms of Hasse diagrams. In this section we indicate the classification of non-serial positive posets on the set-theoretic language (in terms of sums of chains) following the paper [6].

For subposets X, Y of a poset S, let us denote by $X \coprod Y$ their direct sum (i.e. such union that elements $x \in X$ and $y \in Y$ are incomparable). From

Dilworth's theorem it follows that any poset can be represented in the form $\coprod_{i=1}^{m} X_i$ with X_i being chains and additional relations $y<z$ for y and z belonging to different components. Note that additional relations are indicated up to transitivity. By A_s, B_s, C_s are denoted, respectively, the chains $a_1 < ... < a_s$, $b_1 < ... < b_s$, $c_1 < ... < c_s$.

Now we formulate three theorems which classify the non-serial positive posets and which are a set-theoretic reformulation of our classification of non-serial positive posets in [3].

By *m* in parentheses is denoted the corresponding number from [3], and m^{op} means that we must take the poset dual to that with number *m*. Partial order relations are denoted by $\preceq$.

Theorem 6. *The non-serial posets of order 5 are exhausted, up to isomorphism and duality,by the following 10 posets:*

$NSP5.1(3)$ $A_2 \coprod B_3$, $a_1 \prec b_2$;

$NSP5.2(4)$ $A_2 \coprod B_3$, $a_2 \prec b_3$;

$NSP5.3(5)$ $A_2 \coprod B_3$, $a_1 \prec b_2$, $a_2 \prec b_3$;

$NSP5.4(1)$ $A_1 \coprod B_4$, $a_1 \prec b_3$;

$NSP5.5(2)$ $A_2 \coprod B_3$, $a_1 \prec b_1$, $a_2 \prec b_3$;

$NSP5.6(46)$ $A_1 \coprod B_2 \coprod C_2$;

$NSP5.7(48)$ $A_1 \coprod B_2 \coprod C_2$, $b_1 \prec c_2$;

$NSP5.8(49)$ $A_1 \coprod B_2 \coprod C_2$, $a_1 \prec b_2$, $b_1 \prec c_2$;

$NSP5.9(47)$ $A_1 \coprod B_1 \coprod C_3$, $b_1 \prec c_3$;

$NSP5.10(50)$ $A_1 \coprod B_3 \coprod C_1$, $a_1 \prec b_3$, $b_1 \prec c_1$.

Theorem 7. *The non-serial posets of order 6 are exhausted, up to isomorphism and duality,by the following 32 posets:*

$NSP6.1(12)$ $A_3 \coprod B_3$, $a_1 \prec b_2$;

$NSP6.2(20)$ $A_3 \coprod B_3$, $a_1 \prec b_2$, $a_2 \prec b_3$;

$NSP6.3(10)$ $A_2 \coprod B_4$, $a_1 \prec b_2$;

$NSP6.4(11)$ $A_2 \coprod B_4$, $a_1 \prec b_3$;

$NSP6.5(13)$ $A_2 \coprod B_4$, $a_2 \prec b_4$;

$NSP6.6(14)$ $A_2 \coprod B_4$, $a_1 \prec b_2$, $a_2 \prec b_3$;

$NSP6.7(16)$ $A_2 \coprod B_4$, $a_1 \prec b_2$, $a_2 \prec b_4$;

$NSP6.8(18)$ $A_2 \coprod B_4$, $a_1 \prec b_3$, $a_2 \prec b_4$;

$NSP6.9(19^{op})$ $A_3 \coprod B_3$, $a_1 \prec b_2$, $a_3 \prec b_3$;

$NSP6.10(14^{op})$ $A_3 \coprod B_3$, $a_2 \prec b_2$, $a_3 \prec b_3$;

$NSP6.11(15)$ $A_3 \coprod B_3$, $a_1 \prec b_1$, $a_2 \prec b_2$, $a_3 \prec b_3$;

$NSP6.12(6)$ $A_1 \coprod B_5$, $a_1 \prec b_3$;

$NSP6.13(8)$ $A_1 \coprod B_5$, $a_1 \prec b_4$;

$NSP6.14(7)$ $A_2 \coprod B_4$, $a_1 \prec b_1$, $a_2 \prec b_3$;

$NSP6.15(9)$ $A_2 \coprod B_4$, $a_1 \prec b_1$, $a_2 \prec b_4$;

$NSP6.16(58)$ $A_2 \coprod B_2 \coprod C_2$, $b_1 \prec c_2$;

$NSP6.17(66)$ $A_2 \coprod B_2 \coprod C_2$, $a_1 \prec b_2$, $b_1 \prec c_2$;

$NSP6.18(51)$ $A_1 \coprod B_2 \coprod C_3$;

$NSP6.19(55)$ $A_1 \coprod B_2 \coprod C_3$, $b_1 \prec c_2$;

$NSP6.20(56)$ $A_1 \coprod B_2 \coprod C_3$, $b_1 \prec c_3$;

$NSP6.21(57)$ $A_2 \coprod B_1 \coprod C_3$, $b_1 \prec c_3$;

$NSP6.22(60)$ $A_1 \coprod B_2 \coprod C_3$, $a_1 \prec b_2$, $b_1 \prec c_2$;

$NSP6.23(61)$ $A_1 \coprod B_2 \coprod C_3$, $a_1 \prec b_2$, $b_1 \prec c_3$;

$NSP6.24(62)$ $A_1 \coprod B_3 \coprod C_2$, $a_1 \prec b_3$, $b_1 \prec c_1$;

$NSP6.25(63)$ $A_1 \coprod B_3 \coprod C_2$, $a_1 \prec b_3$, $b_1 \prec c_2$;

$NSP6.26(59)$ $A_1 \coprod B_2 \coprod C_3$, $b_1 \prec c_2$, $b_2 \prec c_3$;

$NSP6.27(67)$ $A_1 \coprod B_3 \coprod C_2$, $a_1 \prec b_3$, $b_1 \prec c_1$, $b_2 \prec c_2$;

$NSP6.28(52)$ $A_1 \coprod B_1 \coprod C_4$, $b_1 \prec c_3$;

$NSP6.29(54)$ $A_1 \coprod B_1 \coprod C_4$, $b_1 \prec c_4$;

$NSP6.30(64)$ $A_1 \coprod B_4 \coprod C_1$, $a_1 \prec b_3$, $b_1 \prec c_1$;

$NSP6.31(64)$ $A_1 \coprod B_4 \coprod C_1$, $a_1 \prec b_4$, $b_1 \prec c_1$;

$NSP6.32(53)$ $A_1 \coprod B_2 \coprod C_3$, $b_1 \prec c_1$, $b_2 \prec c_3$.

Theorem 8. *The non-serial posets of order 7 are exhausted, up to isomorphism and duality,by the following 66 posets:*

$NSP7.1(29)$ $A_3 \coprod B_4$, $a_1 \prec b_3$;

$NSP7.2(30)$ $A_3 \coprod B_4$, $a_2 \prec b_4$;

$NSP7.3(42)$ $A_3 \coprod B_4$, $a_1 \prec b_2$, $a_2 \prec b_4$;

$NSP7.4(43)$ $A_3 \coprod B_4$, $a_1 \prec b_3$, $a_2 \prec b_4$;

$NSP7.5(44)$ $A_3 \coprod B_4$, $a_1 \prec b_3$, $a_2 \prec b_4$;

$NSP7.6(45)$ $A_3 \coprod B_4$, $a_1 \prec b_2$, $a_2 \prec b_3$, $a_3 \prec b_4$;

$NSP7.7(26)$ $A_2 \coprod B_5$, $a_1 \prec b_2$;

$NSP7.8(27)$ $A_2 \coprod B_5$, $a_1 \prec b_4$;

$NSP7.9(28)$ $A_2 \coprod B_5$, $a_2 \prec b_5$;

$NSP7.10(31)$ $A_2 \coprod B_5$, $a_1 \prec b_2$, $a_2 \prec b_3$;

$NSP7.11(33)$ $A_2 \coprod B_5$, $a_1 \prec b_2$, $a_2 \prec b_4$;

$NSP7.12(36)$ $A_2 \coprod B_5$, $a_1 \prec b_2$, $a_2 \prec b_5$;

$NSP7.13(38)$ $A_2 \coprod B_5$, $a_1 \prec b_3$, $a_2 \prec b_5$;

$NSP7.14(40)$ $A_2 \coprod B_5$, $a_1 \prec b_4$, $a_2 \prec b_5$;

$NSP7.15(35^{op})$ $A_3 \coprod B_4$, $a_2 \prec b_2$, $a_3 \prec b_3$;

$NSP7.16(41^{op})$ $A_4 \coprod B_3$, $a_1 \prec b_2$, $a_4 \prec b_3$;

$NSP7.17(39^{op})$ $A_4 \coprod B_3$, $a_2 \prec b_2$, $a_4 \prec b_3$;

$NSP7.18\left(37^{op}\right)$ $A_4 \coprod B_3$, $a_3 \prec b_2$, $a_4 \prec b_3$;

$NSP7.19(32)$ $A_3 \coprod B_4$, $a_1 \prec b_1$, $a_2 \prec b_2$, $a_3 \prec b_3$;

$NSP7.20(34)$ $A_3 \coprod B_4$, $a_1 \prec b_1$, $a_2 \prec b_2$, $a_3 \prec b_4$;

$NSP7.21(21)$ $A_1 \coprod B_6$, $a_1 \prec b_3$;

$NSP7.22(24)$ $A_1 \coprod B_6$, $a_1 \prec b_5$;

$NSP7.23(22)$ $A_2 \coprod B_5$, $a_1 \prec b_1$, $a_2 \prec b_3$;

$NSP7.24(25)$ $A_2 \coprod B_5$, $a_1 \prec b_1$, $a_2 \prec b_5$;

$NSP7.25(23)$ $A_3 \coprod B_4$, $a_2 \prec b_1$, $a_3 \prec b_3$;

$NSP7.26(75)$ $A_1 \coprod B_3 \coprod C_3$, $b_1 \prec c_3$;

$NSP7.27(78)$ $A_2 \coprod B_2 \coprod C_3$, $b_1 \prec c_2$;

$NSP7.28(79)$ $A_3 \coprod B_1 \coprod C_3$, $b_1 \prec c_3$;

$NSP7.29(80)$ $A_3 \coprod B_2 \coprod C_2$, $b_1 \prec c_2$;

$NSP7.30(89)$ $A_1 \coprod B_3 \coprod C_3$, $a_1 \prec b_2$, $b_1 \prec c_3$;

$NSP7.31(91)$ $A_1 \coprod B_3 \coprod C_3$, $a_1 \prec b_3$, $b_1 \prec c_2$;

$NSP7.32(91)$ $A_1 \coprod B_3 \coprod C_3$, $a_1 \prec b_3$, $b_1 \prec c_3$;

$NSP7.33(99)$ $A_2 \coprod B_2 \coprod C_3$, $a_1 \prec b_2$, $b_1 \prec c_2$;

$NSP7.34(100)$ $A_2 \coprod B_2 \coprod C_3$, $a_1 \prec b_2$, $b_1 \prec c_3$;

$NSP7.35(101)$ $A_2 \coprod B_3 \coprod C_2$, $a_1 \prec b_3$, $b_1 \prec c_2$;

$NSP7.36(102)$ $A_2 \coprod B_3 \coprod C_2$, $a_2 \prec b_3$, $b_1 \prec c_1$;

$NSP7.37(85)$ $A_1 \coprod B_3 \coprod C_3$, $b_1 \prec c_2$, $b_2 \prec c_3$;

$NSP7.38(86)$ $A_2 \coprod B_2 \coprod C_3$, $b_1 \prec c_2$, $b_2 \prec c_3$;

$NSP7.39(108)$ $A_2 \coprod B_3 \coprod C_2$, $a_2 \prec b_3$, $b_1 \prec c_1$, $b_2 \prec c_2$;

$NSP7.40\left(107^{op}\right)$ $A_2 \coprod B_3 \coprod C_2$, $a_1 \prec b_2$, $a_2 \prec b_3$, $b_1 \prec c_2$;

$NSP7.41(68)$ $A_1 \coprod B_2 \coprod C_4$;

$NSP7.42(72)$ $A_1 \coprod B_2 \coprod C_4$, $b_1 \prec c_2$;

$NSP7.43(73)$ $A_1 \coprod B_2 \coprod C_4$, $b_1 \prec c_3$;

$NSP7.44(74)$ $A_1 \coprod B_2 \coprod C_4$, $b_1 \prec c_4$;

$NSP7.45(76)$ $A_2 \coprod B_1 \coprod C_4$, $b_1 \prec c_3$;

$NSP7.46(87)$ $A_1 \coprod B_2 \coprod C_4$, $a_1 \prec b_2$, $b_1 \prec c_2$;

$NSP7.47(88)$ $A_1 \coprod B_2 \coprod C_4$, $a_1 \prec b_2$, $b_1 \prec c_4$;

$NSP7.48(90)$ $A_1 \coprod B_3 \coprod C_3$, $a_1 \prec b_3$, $b_1 \prec c_1$;

$NSP7.49(93)$ $A_1 \coprod B_4 \coprod C_2$, $a_1 \prec b_3$, $b_1 \prec c_1$;

$NSP7.50(94)$ $A_1 \coprod B_4 \coprod C_2$, $a_1 \prec b_3$, $b_1 \prec c_2$;

$NSP7.51(95)$ $A_1 \coprod B_4 \coprod C_2$, $a_1 \prec b_4$, $b_1 \prec c_2$;

$NSP7.52(81)$ $A_1 \coprod B_2 \coprod C_4$, $b_1 \prec c_2$, $b_2 \prec c_3$;

$NSP7.53(83)$ $A_1 \coprod B_2 \coprod C_4$, $b_1 \prec c_2$, $b_2 \prec c_4$;

$NSP7.54(84^{op})$ $A_1 \coprod B_3 \coprod C_3$, $b_2 \prec c_2$, $b_2 \prec c_3$;

$NSP7.55(77)$ $A_2 \coprod B_2 \coprod C_3$, $b_1 \prec c_1$, $b_2 \prec c_3$;

$NSP7.56(103)$ $A_1 \coprod B_3 \coprod C_3$, $a_1 \prec b_3$, $b_1 \prec c_1$, $b_2 \prec c_2$;

$NSP7.57(104)$ $A_1 \coprod B_3 \coprod C_3$, $a_1 \prec b_3$, $b_1 \prec c_1$, $b_2 \prec c_3$;

$NSP7.58(105)$ $A_1 \coprod B_4 \coprod C_2$, $a_1 \prec b_4$, $b_1 \prec c_1$, $b_2 \prec c_2$;

$NSP7.59(106)$ $A_1 \coprod B_4 \coprod C_2$, $a_1 \prec b_4$, $b_2 \prec c_1$, $b_3 \prec c_2$;

$NSP7.60(82)$ $A_1 \coprod B_3 \coprod C_3$, $b_1 \prec c_1$, $b_2 \prec c_2$, $b_3 \prec c_3$;

$NSP7.61(69)$ $A_1 \coprod B_1 \coprod C_5$, $b_1 \prec c_3$;

$NSP7.62(71)$ $A_1 \coprod B_1 \coprod C_5$, $b_1 \prec c_5$;

$NSP7.63(96)$ $A_1 \coprod B_5 \coprod C_1$, $a_1 \prec b_3$, $b_1 \prec c_1$;

$NSP7.64(96)$ $A_1 \coprod B_5 \coprod C_1$, $a_1 \prec b_4$, $b_1 \prec c_1$;

$NSP7.65(96)$ $A_1 \coprod B_5 \coprod C_1$, $a_1 \prec b_5$, $b_1 \prec c_1$;

$NSP7.66(70)$ $A_1 \coprod B_2 \coprod C_4$, $b_1 \prec c_1$, $b_2 \prec c_3$.

3. Proof of Theorem 1. The implication $(3) \Rightarrow (2)$ is obvious and $(2) \Rightarrow (1)$ follows from Theorem 5. The implication $(2) \Rightarrow (3)$ follows from Theorem 6.

Obviously, to complete the proof of the theorem it is sufficient to prove the implication $(1) \Rightarrow (2)$. Since S and S^{op} are simultaneously minimal or not minimal all further reasoning can be carried out with precision up to duality.

So, in essence, we need to show that all posets indicated in Theorems 7 and 8 are not minimal. This follows from the following easily verifiable facts:

$(a.1)$ each of the posets 6.1–6.2; 6.4–6.9; 6.13; 6.15–6.16; 6.26–6.27; 6.31–6.32 of Theorem 7 without the element b_1 is isomorphic or anti-isomorphic to one of the posets from Theorem 6;

$(a.2)$ each of the posets 6.6; 6.10–6.11; 6.14; 6.21–6.22; 6.24; 6.28 of Theorem 7 without the element a_1 is isomorphic or anti-isomorphic to one of the posets from Theorem 6;

$(a.3)$ each of the posets 6.17–6.20; 6.23; 6.25; 6.29–6.30 of Theorem 7 without the element c_1 is isomorphic or anti-isomorphic to one of the posets from Theorem 6;

$(a.4)$ each of the posets 6.3; 6.12 of Theorem 7 without the element b_4 is isomorphic or anti-isomorphic to one of the posets from Theorem 6;

$(b.1)$ each of the posets 7.1–7.6; 7.8–7.9; 7.11–7.14; 7.16–7.17; 7.20; 7.22; 7.24; 7.26–7.27; 7.29; 7.35–7.39; 7.51–7.60; 7.64–7.66 of Theorem 8 without the element b_1 is isomorphic or anti-isomorphic to one of the posets from Theorem 7;

$(b.2)$ each of the posets 7.10; 7.15; 7.18–7.19; 7.23; 7.25; 7.28; 7.45–7.46; 7.48; 7.61 of Theorem 8 without the element a_1 is isomorphic or anti-isomorphic to one of the posets from Theorem 7;

$(b.3)$ each of the posets 7.30–7.24; 7.40–7.44; 7.47; 7.49–7.50; 7.62–7.63 of Theorem 8 without the element c_1 is isomorphic or anti-isomorphic to one of the posets from Theorem 7;

$(b.4)$ each of the posets 7.7; 7.21 of Theorem 8 without the element b_4 is isomorphic or anti-isomorphic to one of the posets from Theorem 7.

Theorem 1 is proved.

4. Proof of Theorem 2. The implication $(3)\Rightarrow(2)$ is obvious and $(2)\Rightarrow(1)$ follows from Theorem 5. The implication $(2)\Rightarrow(3)$ follows from Theorem 6.

Obviously, to complete the proof of the theorem, it is sufficient to prove the implication $(1)\Rightarrow(2)$. Since S and S^{op} are simultaneously minimal or not minimal, all further reasoning can be carried out with precision up to duality.

So in essence,we need to show that all posets indicated in Theorems 6 and 7 are not maximal. This follows from the following easily verifiable facts:

$(c.1)$ $5.1 \cong 7.7\setminus\{b_4,b_5\}$, $5.2\cong 7.36\setminus\{c_1,c_2\}$, $5.3\cong 7.10\setminus\{b_4,b_5\}$, $5.4\cong 7.21\setminus\{b_5,b_6\}$, $5.5\cong 7.23\setminus\{b_5,b_6\}$, $5.6\cong 7.41\setminus\{c_3,c_4\}$, $5.7\cong 7.29\setminus\{a_2,a_3\}$, $5.8\cong 7.46\setminus\{c_3,c_4\}$, $5.9\cong 7.61\setminus\{c_4,c_5\}$, $5.10\cong 7.63\setminus\{b_4,b_5\}$;

$(c.2)$ $6.1\cong 7.2^{op}\setminus\{b_1\}$, $6.2\cong 7.6\setminus\{b_4\}$, $6.3\cong 7.7\setminus\{b_5\}$, $6.4\cong 7.1\setminus\{a_3\}$, $6.5\cong 7.23\setminus\{a_3\}$, $6.6\cong 7.11\setminus\{b_5\}$, $6.7\cong 7.3\setminus\{b_3\}$, $6.8\cong 7.4\setminus\{a_3\}$, $6.9\cong 7.3^{op}\setminus\{b_1\}$, $6.10\cong 7.15\setminus\{b_4\}$, $6.11\cong 7.19\setminus\{b_4\}$, $6.12\cong 7.21\setminus\{b_6\}$, $6.13\cong 7.8\setminus\{a_2\}$, $6.14\cong 7.23\setminus\{b_6\}$, $6.15\cong 7.16^{op}\setminus\{b_1\}$, $6.16\cong 7.29\setminus\{a_3\}$, $6.17\cong 7.33\setminus\{c_3\}$, $6.18\cong 7.41\setminus\{c_4\}$, $6.19\cong 7.42\setminus\{c_4\}$, $6.20\cong 7.32\setminus\{b_3\}$, $6.21\cong 7.45\setminus\{c_4\}$, $6.22\cong 7.46\setminus\{c_4\}$, $6.23\cong 7.33\setminus\{a_2\}$, $6.24\cong 7.48\setminus\{c_3\}$, $6.25\cong 7.510\setminus\{b_4\}$, $6.26\cong 7.52\setminus\{c_4\}$, $6.27\cong 7.56\setminus\{c_3\}$, $6.28\cong 7.61\setminus\{c_5\}$, $6.29\cong 7.47\setminus\{b_2\}$, $6.30\cong 7.63\setminus\{b_5\}$, $6.31\cong 7.64\setminus\{b_5\}$, $6.32\cong 7.66\setminus\{c_4\}$.

Theorem 2 is proved.

5. Proof of Theorem 3. The case $n = 5$, $m = 7$ follows from (c.1) and the case $n = 6$, $m = 7$ from (c.2) (see the proof of Theorem 2).

The case $n = 5$, $m = 6$ follows from the isomorphisms $5.1\cong 6.3\setminus\{b_4\}$; $5.2\cong 6.1^{op}\setminus\{b_1\}$; $5.3\cong 6.6\setminus\{b_4\}$; $5.4\cong 6.12\setminus\{b_5\}$; $5.5\cong 6.14\setminus\{b_4\}$; $5.6\cong 6.18\setminus\{c_3\}$; $5.7\cong 6.16\setminus\{a_2\}$; $5.8\cong 6.22\setminus\{c_3\}$; $5.9\cong 6.28\setminus\{c_4\}$; $5.10\cong 6.30\setminus\{b_4\}$ which in turn follows from (c.1).

Obviously, in any case $n = 5$, $m = 7$, $n = 6$, $m = 7$, $n = 5$, $m = 6$ all right parts of isomorphisms are lower subposets in the corresponding positive posets of order 7, 7, 6, respectively.

Note that $S\cong T$ imply $S^{op}\cong T^{op}$. Hence if we will repeat previous reasoning for all posets dual to those from Theorems 6, 7 and 8, we obtain that the right parts of isomorphisms are upper subposets.

Theorem 3 is proved.

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1. *Bondarenko V. M.* Minimax equivalence method: initial ideas, first applications and new concepts // Algebra and Discrete Math. – 2024. – 38, No. 1. – P. 1–22.
2. *Bondarenko V. M., Styopochkina M. V.* On posets of width two with positive Tits form // Algebra and Discrete Math. – 2005. – 4, No. 2. – P. 20–35.
3. *Bondarenko V. M., Styopochkina M. V.* (Min, max)-equivalence of partially ordered sets and Tits quadratic form // Analysis and Algebra Problems. – 2005. – 2, No. 3. – P. 18–58 (in Russian).

4. *Bondarenko V. M., Styopochkina M. V.* On finite posets of inj-finite type and their Tits forms // Algebra and Discrete Math. – 2006. – 5, No. 2. – P. 17–21.
5. *Bondarenko V. M., Stepochkina M. V.* On serial posets with positive-definite quadratic Tits form // Nonlinear Oscillations. – 2006. – 9, No. 3. – P. 312–316.
6. *Bondarenko V. M., Styopochkina M. V.* Combinatorial properties of non-serial posets with positive Tits quadratic form // Algebra and Discrete Math. – 2023. – 36, No. 1. – P. 1–13.
7. *Bondarenko V. M., Styopochkina M. V.* On the transitivity coefficients for minimal posets with nonpositive quadratic Tits form // J. Math. Sci. – 2023. – 274, No. 5. – P. 583–593.
8. *Drozd Yu. A.* Coxeter transformations and representations of partially ordered sets // Funkts. Anal. i Prilozhen. – 1974. – 8, No. 3. – P. 34–42 (in Russian).
9. *Gabriel P.* Unzerlegbare Darstellungen I // Manuscripta Math. – 1972. – 6. – P. 71–103.
10. *Kleiner M. M.* Partially ordered sets of finite type // Zap. Nauch. Semin. LOMI. – 1972. – 28. – P. 32–41 (in Russian).
11. *Nazarova L. A., Roiter A. V.* Representations of posets // Zap. Nauch. Semin. LOMI. – 1972. – 28. – P. 5–31 (in Russian).

КЛАСИФІКАЦІЯ МІНІМАЛЬНИХ І МАКСИМАЛЬНИХ НЕСЕРІЙНИХ ДОДАТНИХ ЧАСТКОВО ВПОРЯДКОВАНИХ МНОЖИН

Скінченні частково впорядковані множини з додатною квадратичною формою Тітса, які називаються додатними, є аналогами діаграм Динкіна. Вони вперше описані в 2005 р. авторами. За цим результатом така множина може бути серійною, якщо вона належить нескінченній строго зростаючій послідовності додатних частково впорядкованих множин, або несерійною, якщо це не так. У наступні роки авторами були вивчені різні класи частково впорядкованих множин, які пов'язані з квадратичною формою Тітса. У цій статті додатні частково впорядковані множини вивчаються більш детально, а саме відносно їхньої впорядкованості. Основні теореми класифікують всі несерійні додатні частково впорядковані множини, які є максимальними або мінімальними. Випадок серійних частково впорядкованих множин є тривіальним: максимальних множин немає взагалі, а всі мінімальні є одноелементними. Кількість несерійних мінімальних частково впорядкованих множин з точністю до ізоморфізму та дуальності становить 10, а максимальних – 66 (із загальної кількості 108).

Ключові слова*:* *квадратна форма Тітса, додатна частково впорядкована множина, серійні і несерійні частково впорядковані множини, мінімальні і максимальні несерійні частково впорядковані множини, діаграма Динкіна.*

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