UDC 681.5

L.S. Zhiteckii, K.Yu. Solovchuk

ADAPTIVE ROBUST MULTIVARIABLE CONTROL OF NONINVERTIBLE MEMORYLESS SYSTEMS WITH BOUNDED DISTURBANCES: A GENERALIZATION

Leonid Zhiteckii

International Research and Training Center for Information Technologies and Systems of the National Academy of Sciences of Ukraine and Ministry of Education and Science of Ukraine, Kyiv,
leonid_zhiteckii@i.ua

Klavdiia Solovchuk

Poltava Scientific Research Forensic Center of the MIA of Ukraine,
solovchuk_ok@ukr.net

The discrete-time robust adaptive control for some classes of the uncertain multivariable memoryless (static) systems in the presence of unmeasurable bounded disturbances, whose bounds are assumed to be known is addressed. The systems where the number of the control inputs do not exceed the number of their outputs are considered. The main feature of plants to be controlled is that their gain matrices are noninvertible. The assumption that the elements of these matrices are unknown a priori but there is information about possible bounds on these elements. The problem stated and solvesd here is to design a feedback controller to be capable to cope with the noninvertibility of the gain matrices and also with the parametric uncertainty in order to reject the external disturbances and to ensure the boundedness of all the control and output system signals. To solve the problem above mentioned, the robust adaptive approach together with the so-called pseudoinverse or inverse model-based concept is used. Three different cases are studied. In the first case, the robust adaptive controller applicable to the uncertain plant with the square singular gain matrix is designed. The robust method employing the pseudoinverse model-based controllers whose parameters are estimated via a standard recursive adaptation procedure is proposed in the second case to deal with the unknown nonsquare gain matrices having the full rank. The approach proposed in first case is extended to the third case dealing with the control of the unknown plants the gain matrices of which represent the nonsquare matrices of not full rank. Asymptotic properties of the robustly-adaptive controllers proposed in this paper are established. Results of numerical examples given to support the theoretic study.

Keywords: discrete time, multivariable memoryless plant, noninvertibility, pseudoinverse model-based concept, uncertainty, estimation algorithm, robust adaptive control.

Introduction

A long-standing in [1] problem of the optimal controller design for multivariable system in the presence of unmeasurable disturbances remains an important problem from both theoretical and practical points of view. Within the framework of this actual problem, new approaches have been proposed by many researches. The latest results in this scientific area have been reported in numerous papers including [2] and generalized in several recent books [3–6] dealing with advanced multivariable control systems.

Among other methods advanced in the modern control theory, the inverse model-based method that is an extension of the well-known internal model principle seems to be perspective in order to cope with arbitrary unmeasurable disturbances and to optimize some classes of multivariable control systems. It turned out that this method first intuitively devised in [7] makes it possible to optimize the closed-loop control system containing the multivariable static (memoryless) plants whose gain matrices are square and nonsingular. Since the beginning of the 21st century, a significant progress has been achieved utilizing the inverse model-based approach [8, 9]. However, this approach is quite unacceptable if the gain matrices are either square but singular or nonsquare because they are noninvertible.

To optimize the closed-loop control system containing an arbitrary multivariable static plant, the so-called pseudoinverse (generalized inverse) model-based concept has been proposed and substantiated in [10] dealing with the possible noninvertibility of gain matrices whose elements are assumed to be known. This fruitful concept was extended in [11, 12] to robust control of a noninvertible and uncertain plant with unmeasurable bounded disturbances. Unfortunately, the robust control theory may not be employed if the initial parametric uncertainty is «wide» enough. Meanwhile, the adaptive approach gives some universal tool to deal with such a type of uncertainty. Foundations of this approach have been extended and generalized in the books [13–19].

In the recent works [20, 21], the different adaptive control ideas are advanced to cope with the noninvertible multivariable memoryless system in the presence of parametric uncertainties. In particular, the adaptive pseudoinverse model-based control idea using the standard identification method is exploited in [20, 22] to reject unmeasurable bounded disturbances acting on the uncertain plants with the nonsquare gain matrices of full rank. Novel approaches to deal with the multivariable memoryless plants having unknown square and nonsquare gain matrices of not full rank are reported in [21–23].

The purpose of this paper is to generalize the results achieved in [20–23] and related to the case where there are no nonparametric uncertainties.

1. Problem formulation

Let

$$y_n = Bu_{n-1} + v_n \tag{1}$$

be the vector-valued difference equation of a static (memoryless) plant representing some linear multivariable discrete-time system to be stabilized. In this equation, $y_n \in R^m, u_n \in R^r$ and $v_n \in R^m$ are the m dimensional measured output, control input and unmeasured external disturbance vectors, respectively, at the n th time instant $(n = 1, 2, \ldots)$ defined by $y_n = [y_n^{(1)}, \ldots, y_n^{(m)}]^{\mathrm{T}}, u_n = [u_n^{(1)}, ..., u_n^{(r)}]^{\mathrm{T}}$ and $v_n = [v_n^{(1)}, ..., v_n^{(m)}]^{\mathrm{T}}$,

$$B = \begin{pmatrix} b^{(11)} & \ldots & b^{(1r)} \\ \cdots & \cdots & \cdots \\ b^{(m1)} & \ldots & b^{(mr)} \end{pmatrix} \tag{2}$$

is an arbitrary time-invariant $m \times r$ gain matrix.

Consider the case when the number of the control inputs $u_n^{(1)}, ..., u_n^{(r)}$ is not less than two but does not exceed the number of the outputs $y_n^{(1)}, \ldots, y_n^{(m)}$ meaning that

$$2 \le r \le m. \tag{3}$$

Next, suppose that the rank of B satisfies the inequality

$$\text{rank } B \le r \tag{4}$$

implying that B may be not a full rank matrix. Note that the rank of B satisfying (4) together with (3) give that B becomes a noninvertible matrix if either $r = m$ but $\text{rank } B < r$ or $r < m$ irrespective of $\text{rank } B$.

The following basic assumptions with respect to the gain matrix B and the sequences $\{v_n^{(i)}\} = v_0^{(i)}, v_1^{(i)}, \ldots$ $(i = 1, \ldots, m)$ are made.

A1) all the elements of B are all unknown. However, there are some interval estimates defined as

$$\underline{b}^{(ij)} \le b^{(ij)} \le \bar{b}^{(ij)}, \ \ i = 1, \ldots, m, \ \ j = 1, \ldots, r, \tag{5}$$

where the upper and lower bounds $\underline{b}^{(ij)}$ and $\bar{b}^{(ij)}$, respectively, on $b^{(ij)}$ are assumed to be known.

A2) $v_n^{(i)}$s $(i = 1, ..., m)$ are all the arbitrary scalar sequences bounded in modulus according to

$$\left|v_n^{(i)}\right| \le \varepsilon^{(i)} < \infty \ \forall \ n = 0, 1, 2, ...,$$

where $\varepsilon^{(i)}s$ are constant.

A3) The upper bounds, $\varepsilon^{(i)}s$ are known a priori.

Let $y^0 = [y^{0(1)}, \ldots, y^{0(m)}]^{\mathrm{T}}$ be a desired output vector $(y^{0(i)} \equiv \text{const} \ \forall \ i = = 1, \ldots, m$. Suppose that $\left|y^{0(1)}\right| + \ldots + \left|y^{0(m)}\right| \ne 0$ implying that, at least, one $y^{0(i)}$ of $y^{0(1)}, ..., y^{0(m)}$ is nonzero.

Define the output error vector

$$e_n = y^0 - y_n, \tag{6}$$

with the components $e_n^{(i)} = y^{0(i)} - y_n^{(i)}$, i.e., $e_n = [e_n^{(1)}, ..., e_n^{(m)}]^{\mathrm{T}}$.

The problem is to design the feedback controller guaranteeing the ultimate boundedness of the sequence $\{e_n\} = e_1, e_2, \ldots,$ in the form

$$\limsup_{n\to\infty} \left\|e_n\right\| < \infty \tag{7}$$

provided

$$\limsup_{n\to\infty} \left\|u_n\right\| < \infty. \tag{8}$$

Remark. The requirement (8) is here introduced additionally since it may not be satisfied even if (7) takes place.

2. Preliminaries

Assume, for the time being, that B is a known noninvertible matrix. In this case, the so-called pseudoinverse control

$$u_n = u_{n-1} + B^+ e_n \tag{9}$$

proposed in several works (see, e.g., [24]) ensures the minimum of the upper bound on the Euclidean norm $\| e_n \|_2$ of the output error vector of the closed-loop control system (1), (6), (9) with any bounded sequence $\{v_n\} = v_1, v_2, \ldots$ Here the notation P^+ of any pseudoinverse matrix introduced in [24, Theorem 3.4] and defined as

$$P^{+} = \lim_{\delta \to 0} (P^{\mathrm{T}} P + \delta I_r)^{-1} P^{\mathrm{T}},$$

where I_r denotes the identity $r \times r$ matrix is used. Recall that if $\operatorname{rank} P = r$, for some $P \in R^{(m \times r)}$, then the expression of P^{+} is simplified [25, item 7.46]; we have

$$P^{+} = (P^{\mathrm{T}} P)^{-1} P^{\mathrm{T}}. \tag{10}$$

The closed-loop control system (1), (6), (9) is designed as shown in Fig. 1. In this control system, the variable $\nabla u_n := u_n - u_{n-1}$ produced by the pseudoinverse model represents the increment of the control action during one step determined as

$$\nabla u_n := B^{+} e_n$$

whereas the signal u_n is formed as the sum

$$u_n = \sum_{k=1}^{n} \nabla u_k .$$

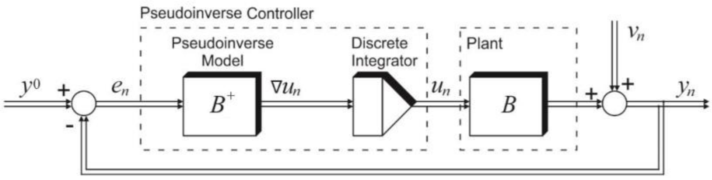


Fig. 1

3. Robustly-adaptive control of plants with square singular gain matrices

Let B be an unknown square singular $r \times r$ matrix, i.e.,

$$\det B = 0. \tag{11}$$

Basic idea to deal with a matrix B satisfying the requirement (11) is to replace adaptive identification of the true plant having the singular gain matrix B to the adaptive identification of a so-called fictitious plant with the nonsingular gain matrix $\tilde{B}$ of the form

$$\tilde{B} = B + \delta_0 I_r, \tag{12}$$

where I_r denotes the identity $r \times r$ matrix and δ_0 is a fixed quantity [26].

Although $\tilde{B}$ as well as B remain unknown, the requirement

$$\det \tilde{B} \neq 0 \tag{13}$$

can always be satisfied by the suitable choice of δ_0 in the expression (12). In fact, each ith eigenvalue $\lambda_i(B)$ of B lies in one of the r closed regions of the complex z-plane consisting of all the Gerŝgorin discs [27, p. 146]:

$$\left| z - b^{(ii)} \right| \le \sum_{\substack{j=1 \\ j \neq i}}^{r} \left| b^{(ij)} \right|, \; i = 1, \dots, r. \tag{14}$$

Since, at least, one of the eigenvalues $\lambda_i(B)$ is equal to zero (due to the singularity of B), by virtue of (12) there are an integer i $(1 \le i \le r)$ and the numbers

$$\underline{\beta}^{(i)} := b^{(ii)} - \sum_{\substack{j=1 \\ j \neq i}}^{r} \left| b^{(ij)} \right|, \; \overline{\beta}^{(i)} := b^{(ii)} - \sum_{\substack{j=1 \\ j \neq i}}^{r} \left| b^{(ij)} \right| \tag{15}$$

such that if

$$\underline{\beta}^{(i)} \neq \bar{\beta}^{(i)} \tag{16}$$

then either $\underline{\beta}^{(i)} \leq 0$ but $\bar{\beta}^{(i)} > 0$ or $\underline{\beta}^{(i)} < 0$ but $\bar{\beta}^{(i)} \geq 0$. These numbers are defined as the intersection of the *i* th Gerŝgorin disc with the real axis of the complex *z*-plane as shown in Fig. 2, *a* and 3, *a*, respectively. In both cases, $\underline{\beta}^{(i)}\bar{\beta}^{(i)} \leq 0$ if the inequality (16) is satisfied because $\underline{\beta}^{(i)}$ and $\bar{\beta}^{(i)}$ cannot have the same sign.

Denoting

$$\underline{\beta} := \min\{\underline{\beta}^{(1)}, ..., \underline{\beta}^{(r)}\}, \ \bar{\beta} := \max\{\bar{\beta}^{(1)}, ..., \bar{\beta}^{(r)}\}, \tag{17}$$

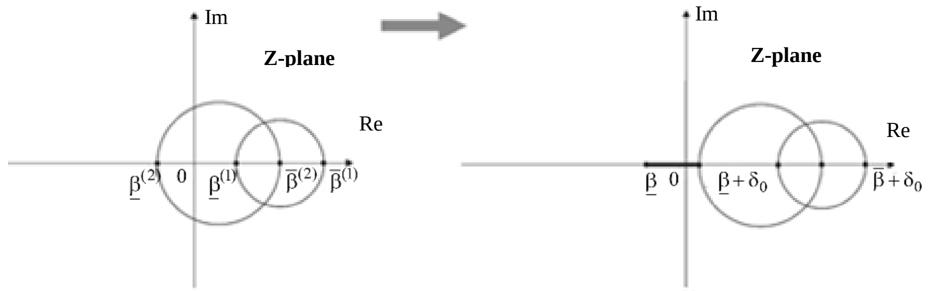
consider the following two cases: (i) $|\underline{\beta}| < |\bar{\beta}|$; (ii) $|\underline{\beta}| > |\bar{\beta}|$ (The case when $|\underline{\beta}| = |\bar{\beta}|$ can be combined with any of two cases.) In order to go to the gain matrix $\tilde{B}$ of the fictitious plant having the form (12) in the case (i), it is sufficient to shift the Gerŝgorin discs (14) right taking

$$\delta_0 > |\underline{\beta}| \tag{18}$$

as shown in Fig. 2b. In the case (ii), the discs (14) need to be shifted left according to

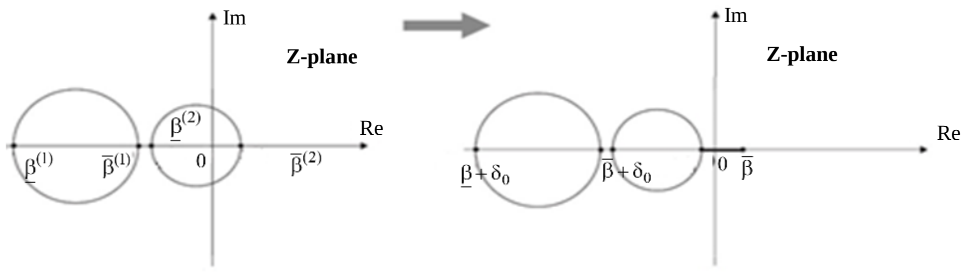
$$\delta_0 < -|\bar{\beta}|; \tag{19}$$

see Fig. 3, *b*. In both cases, the nonsingularity of $\tilde{B}$ is guaranteed. Nevertheless, the conditions (18) and (19) cannot be satisfied, as yet. In fact, the numbers $\underline{\beta}$ and $\bar{\beta}$ given by the expressions (17) depend on $\underline{\beta}^{(i)}s$ and $\bar{\beta}^{(i)}s$ defined by (15). But they are unknown because $b^{(ij)}s$ are all unknown.



a *б*

Fig. 2



a *б*

Fig. 3

The following operations are proposed to choose a number δ_0 satisfying the reqiurement (13). Introduce

$$\underline{\beta}_{\min}^{(i)} := \underline{b}^{(ii)} - \sum_{j=1,\ j\neq i}^{r} \max\{|\underline{b}^{(ij)}|, |\overline{b}^{(ij)}|\},$$
$$\overline{\beta}_{\max}^{(i)} := \overline{b}^{(ii)} + \sum_{j=1,\ j\neq i}^{r} \max\{|\underline{b}^{(ij)}|, |\overline{b}^{(ij)}|\} \qquad (20)$$

minimizing and maximizing in $b^{(ij)} \in [\underline{b}^{(ij)}, \overline{b}^{(ij)}]$ the right-hand sides of (15) for $\underline{\beta}^{(i)}$ and $\overline{\beta}^{(i)}$ respectively.

Further, the number δ_0 is found to satisfy the conditions

$$\delta_0 > -\underline{\beta}_{\min} \ \text{ if } \ |\underline{\beta}_{\min}| < |\overline{\beta}_{\max}|, \ \delta_0 > \overline{\beta}_{\max} \ \text{ if } \ |\underline{\beta}_{\min}| > |\overline{\beta}_{\max}| \qquad (21)$$

where $\underline{\beta}_{\min}$, $\overline{\beta}_{\max}$ represent some quantities defined as follows:

$$\underline{\beta}_{\min} := \min\{\underline{\beta}_{\min}^{(1)}, ..., \underline{\beta}_{\min}^{(r)}\}, \ \overline{\beta}_{\max} := \max\{\overline{\beta}_{\min}^{(1)}, ..., \overline{\beta}_{\min}^{(r)}\}. \qquad (22)$$

It can be clarified that if the conditions (21) together with (20) and (22) are satisfied then the condition (13) will without fail be ensured.

After determining the quantity δ_0 we can proceed to the consideration of the fictitious plant. Since the input variables $u_n^{(1)}, ..., u_n^{(r)}$ and the disturbances $v_n^{(1)}, ..., v_n^{(r)}$ of both true plant and fictitious plant are the same, this feature makes it possible to describe our fictitious plant by the equation

$$\tilde{y}_n = \tilde{B}u_{n-1} + v_n, \qquad (23)$$

similar to the equation (1), where $\tilde{y}_n = [\tilde{y}_n^{(1)}, ..., \tilde{y}_n^{(m)}]^{\mathrm{T}}$, denotes the output vector of the fictitious plant.

It is interesting that the components of $\tilde{y}_n$ can be measured while the components of v_n in the equation (23) remain unmeasurable. In fact, substituting the expression (12) into (23), due to (1) we produce

$$\tilde{y}_n = y_n + \delta_0 u_{n-1}. \qquad (24)$$

It is seen from the equation (24) that $\tilde{y}_n$ can always be found indirectly having u_n and y_n to be measured.

Now, our problem reduces to the known problem of adaptive control applicable to the fictitious plant (23) with the unknown gain matrix $\tilde{B}$ in the presence of arbitrary bounded disturbances $v_n^{(1)}, ..., v_n^{(r)}$. Its solving follows the steps of the section above. Namely, the adaptive control law is designed in the form

$$u_n = u_{n-1} + \tilde{B}_n^{-1} e_n, \qquad (25)$$

in which, instead of the current estimate B_n of B, another $\tilde{B}_n$ is exploited, and the error vector e_n defined in (6) is replaced by

$$\tilde{e}_n = y^0 - \tilde{y}_n \tag{26}$$

with $\tilde{y}_n$ given by the expression (24).

The adaptive identification algorithm used to determine the estimates $\tilde{B}_n$ may be taken as

$$\tilde{b}_n^{(i)} = \begin{cases} \tilde{b}_{n-1}^{(i)} & \text{if } |\tilde{e}_n^{*(i)}| \le \varepsilon_i^0, \\ \tilde{b}_{n-1}^{(i)} + \gamma_n^{(i)} \dfrac{\tilde{e}_n^{*(i)} - \bar{\varepsilon}_i \operatorname{sign} \tilde{e}_n^{*(i)}}{\| \nabla u_{n-1} \|_2^2} \nabla u_{n-1} & \text{otherwise}, \quad i = 1, \ldots, r \end{cases} \tag{27}$$

which is similar to that in [22]. In this algorithm, ε_i^0 and $\bar{\varepsilon}_i$ are given by

$$\varepsilon_i^0 > \bar{\varepsilon}_i = 2\varepsilon_i, \;\; i = 1, \ldots, r \tag{28}$$

$$\tilde{e}_n^{*(i)} = \nabla \tilde{y}_n^{(i)} - \tilde{b}_{n-1}^{(i)\mathrm{T}} \nabla u_{n-1} \tag{29}$$

represent the i th component of the identification error $\tilde{e}_n^*$ given as

$$\tilde{e}_n^* = \nabla \tilde{y}_n - \tilde{B}_{n-1} \nabla u_{n-1}, \tag{30}$$

where $\nabla \tilde{y}_n^{(i)} := \tilde{y}_n^{(i)} - \tilde{y}_{n-1}^{(i)}$, and the notation $\tilde{b}_n^{(i)\mathrm{T}} := [\tilde{b}_n^{(i1)}, ..., \tilde{b}_n^{(ir)}]$ of the i th row of $\tilde{B}_n$ is introduced. The coefficients $\gamma_n^{(i)} s$ are chosen from the intervals

$$0 < \gamma' \le \gamma_n^{(i)} \le \gamma'' < 2 \tag{31}$$

to satisfy

$$\det \tilde{B}_n \ne 0. \tag{32}$$

The feedback adaptive robust control system described in the (1), (25), (27) is designed as depicted in Fig. 4. In this figure, the notation $\nabla \tilde{y}_n^* := \tilde{B}_{n-1} \nabla u_{n-1}$ is introduced.

The asymptotic properties of the adaptive control system are established in the theorem below.

Theorem 1. Determine δ_0 using the formula (21) together with the expressions (20) and (22), and choose an arbitrary initial $\tilde{B}_0 = B_0 + \delta_0 I_r$ with $B_0 = (b_0^{(ij)})$ whose elements satisfy the conditions $\underline{b}^{(ij)} \le b_0^{(ij)} \le \bar{b}^{(ij)}$. Subject to assumptions A1–A3, the adaptive controller described in the equations (25) and (27) together with (24) and (26) when applied to the plant (1) leads to (7) and (8).

Proof. See [26]. □

ISSN 1028-0979

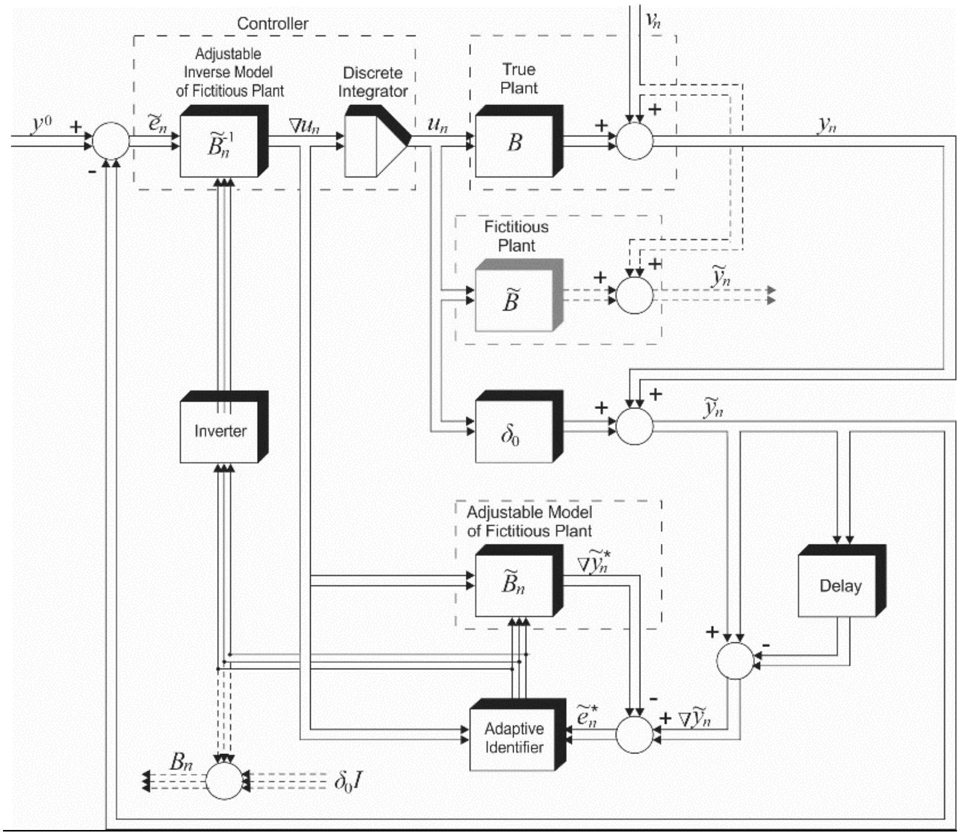


Fig. 4

4. Robustly-adaptive control of plants with nonsquare gain matrices having full rank

Let (1) be the equation of a nonsquare multivariable memoryless system ($2 \le r < m$) whose gain matrix, *B*, as full rank:

$$\text{rank } B = r \tag{33}$$

Suppose that $\varepsilon^{(i)}s$ are known. For controling this system under condition of the parametric uncertainty given by (5), the adaptive controller will be designed here as the adaptive pseudoinverse model-based controller described by

$$u_n = u_{n-1} + B_n^{+} e_n \tag{34}$$

with B_n^{+} defined by (10). Noting that the requirement

$$\text{rank } B_n = r \tag{35}$$

will be satisfied to calculate B_n^{+} by (10), we will update the elements of $B_n = (b_n^{(ij)})$, exploiting the following standard adaptive estimation algorithm proposed in [28, sect. 4.2]:

$$b_n^{(i)} = \begin{cases} b_{n-1}^{(i)} & \text{if } \ |e_n^{(i)}| \le \bar{\varepsilon}^{0(i)}, \\ P_{\Xi^{(i)}}\{b_{n-1}^{(i)} - \gamma_n^{(i)} \dfrac{e_n^{(i)} - \bar{\varepsilon}^{(i)} \operatorname{sign} e_n^{(i)}}{\| \nabla u_{n-1} \|_2^2} \nabla u_{n-1}\} & \text{otherwise} \quad (i = 1, \ldots, m) \end{cases} \tag{36}$$

In this algorithm, $b_n^{(i)} := [b_n^{(i1)}, ..., b_n^{(ir)}]^{\mathrm{T}}$ is the i th row of B_n, $P_{\Xi^{(i)}}\{w\}$ represents the projection of w onto the i th set

$$\Xi^{(i)} = [\underline{b}^{(i1)}, \overline{b}^{(i1)}] \times \ldots \times [\underline{b}^{(ir)}, \overline{b}^{(ir)}]$$

and $\overline{\varepsilon}^{(i)} = 2\varepsilon^{(i)}$, $\overline{\varepsilon}^{0(i)} \ge \overline{\varepsilon}^{(i)}$. The coefficients $\gamma_n^{(i)}s$ in (36) are chosen from (31) to satisfy (35).

The recursive procedure (36) is the on-line identification algorithm need to implement the adaptive pseudoinverse model-based controller (34). The adaptive closed-loop control system containing this controller is shown in Fig. 5.

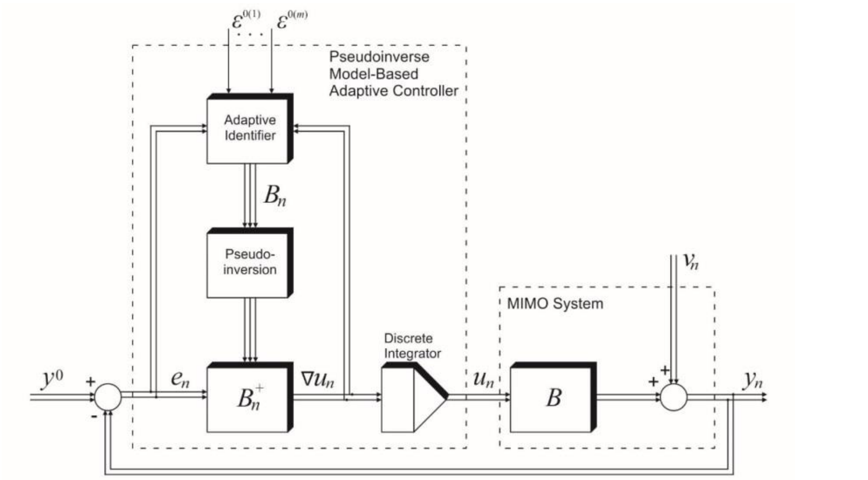


Fig. 5

The convergence and robustness properties of the adaptive pseudoinverse model-based controller (34), (36) are given below.

Proposition. Provided that Assumptions A1) to A3) are valid, and the condition (33) is satisfied, the adaptive control algorithm defined in (34), (36) and applied to the plant (1) gives:

(i) the estimate sequence $\{B_n\}$ converges in the sense of

$$B_n \xrightarrow[n\to\infty]{} B_\infty;$$

(ii) the ultimate boundedness given in the expressions (7), (8) is achieved.

Proof. Due to space limitation, the proof of Proposition is omitted. □

5. Robustly-adaptive control of plants with nonsquare gain matrices having not full rank

Let B be a nonsquare $m \times r$ matrix of the form (2) with unknown rank satisfying (4). Define the so-called submatrices $B[i_1[k], \ldots, i_r[k] | 1, \ldots, r] \in R^{r\times r}$ [43, part I, subsect. 2.2] whose rows represent the rows of B with the numbers $i_1[k], \ldots, i_r[k]$ $(1 \le i_1[k] < \ldots < i_r[k] \le m)$. The quantity of these matrices is equal to $N = \begin{pmatrix} m \\ r \end{pmatrix}$. Denoting by $B[k]$ the submatrix which corresponds to a k th subset $\{i_1[k], \ldots, i_r[k]\}$, write the equations of some k plants as:

$$y_n[k] = B[k]u_{n-1} + v_n[k], \quad k = 1, \dots, N, \tag{37}$$

where $y_n[k] = [y_n^{(i_1[k])}, \dots, y_n^{(i_r[k])}] \in R^r$ and $v_n[k] = [v_n^{(i_1[k])}, \dots, v_n^{(i_r[k])}] \in R^r$.

In accordance with the approach proposed in the previous section, pass from the equation (37) to the equations of the fictitious plants described by

$$\tilde{y}_n[k] = \tilde{B}[k]u_{n-1} + v_n[k], \quad k = 1, \dots, N, \tag{38}$$

with the same u_{n-1} and $v_n[k]$. In these equations, $\tilde{y}_n[k]$ denotes the r-dimensional output vector related to the k th fictitious plant whose gain matrix $\tilde{B}[k]$ is defined as follows:

$$\tilde{B}[k] = B[k] + \delta_0[k]I_r, \tag{39}$$

where $\delta_0[k]$ is a fixed quantity depending on k. This quantity is calculated for each $k = 1, \dots, N$ using the technique described in the previous section. Namely, with the constraints (5) in mind, $\delta_0[k]$ can always be found to satisfy the conditions

$$\det \tilde{B}[k] \neq 0 \quad \forall k = 1, \dots, N \tag{40}$$

similar to (32).

It follows from the equations (37) to (39) that

$$\tilde{y}_n[k] = y_n[k] + \delta_0[k]u_{n-1}. \tag{41}$$

This expression shows that although $\tilde{B}[k]$ as $B[k]$ remain unknown, however, the components of all N the vectors $\tilde{y}_n[k]$ can indirectly be «measured» after measuring the components of y_n and u_{n-1}, and it is essential.

If the conditions (40) are satisfied, then the problem of the adaptive stabilization of the true plant (1) can be reduced to the problem of simultaneous adaptive stabilization of all N fictitious plants (38) with unknown but nonsingular $r \times r$ gain matrices $\tilde{B}[k]$ ($k = 1, \dots, N$) via forming at each n th time instant a set of N different «potentially» possible controls $u_n[1], \dots, u_n[N]$ and selecting one of them in accordance with certain choice rule [23] given below.

Following [23], the adaptive control law to be applicable to any fictitious plant is designed in the form

$$u_n[k] = u_{n-1} + \tilde{B}_n^{-1}[k]\tilde{e}_n[k], \quad k = 1, \dots, N, \tag{42}$$

where $\tilde{e}_n[k] = y^0[k] - \tilde{y}_n[k]$ with $y^0[k] = [y^{0(i_1[k])}, ..., y^{0(i_r[k])}]^{\mathrm{T}}$ defines the output error vector related to the k th fictitious plant at the n th time instant, and $\tilde{B}_n[k] \in R^{r \times r}$ is the current estimate of unknown $r \times r$ matrix $\tilde{B}[k]$ at the same time instant satisfying

$$\det \tilde{B}_n[k] \neq 0 \quad \forall k = 1, \dots, N. \tag{43}$$

As the adaptation algorithms, the standard recursive procedures for the adaptive identification of each k th fictitious plant (37) described by

$$\tilde{b}_n^{(i)}[k] = \begin{cases} \tilde{b}_{n-1}^{(i)}[k] & \text{if } |\tilde{e}_n^{*(i)}[k]| \le \varepsilon_i^0, \\ \tilde{b}_{n-1}^{(i)}[k] + \gamma_n^{(i)} \dfrac{\tilde{e}_n^{*(i)}[k] - \bar{\varepsilon}_i \operatorname{sign} \tilde{e}_n^{*(i)}[k]}{\| \nabla u_{n-1} \|_2^2} \nabla u_{n-1} & \text{otherwise}, \\ i = 1, \ldots, r, \quad k = 1, \ldots, N \end{cases} \tag{44}$$

are proposed. In these algorithms, $\tilde{b}_n^{(i)}[k]$ denotes the r-dimensional estimate vector obtained by transposing the i th row of $\tilde{B}_n[k]$, and

$$\tilde{e}_n^{*(i)}[k] = \tilde{y}_n^{(i)}[k] - \tilde{y}_{n-1}^{(i)}[k] - \tilde{b}_{n-1}^{(i)\mathrm{T}}[k] \nabla u_{n-1} \tag{45}$$

represents the scalar variable making sense of the i th component of $\tilde{e}_n^{*(i)}[k] \in R^r$ that is the identification error vector related to the k th fictitious plant. The coefficients $\gamma_n^{(i)}s$ are chosen from the ranges (31) to satisfy the requirement (43).

Next, add the adaptation algorithms described in the formulas (44) together with (45) by an algorithm for estimating unknown B defined as follows:

$$b_n^{(i)} = \begin{cases} b_{n-1}^{(i)} & \text{if } |e_n^{*(i)}| \le \varepsilon_i^0, \\ b_{n-1}^{(i)} + \gamma_n^{(i)} \dfrac{e_n^{*(i)} - \bar{\varepsilon}_i \operatorname{sign} e_n^{*(i)}}{\| \nabla u_{n-1} \|_2^2} \nabla u_{n-1} & \text{otherwise}, \quad i = 1, \ldots, m, \end{cases} \tag{46}$$

where $b_n^{(i)\mathrm{T}}$ represents the i th row of the estimate matrix B_n and

$$e_n^{*(i)} = y_n^{(i)} - y_{n-1}^{(i)} - b_{n-1}^{(i)\mathrm{T}} \nabla u_{n-1} \tag{47}$$

is the i th component of the identification error vector $e_n^* = y_n - y_{n-1} - B_{n-1} \nabla u_{n-1}$ ($\bar{\varepsilon}_i$ and ε_i^0 are given by the conditions (28)).

The estimation procedure defined in the algorithm (46) together with the equation (47) makes it possible to estimate the m predicted output errors $\vec{e}_{n+1}^{(i)}[k]$ ($i = 1, \ldots, m$) for each i th output of true plant (1) at any n using the formula

$$|\vec{e}_n^{(i)}[k]| = |y^{0(i)} - b_n^{(i)\mathrm{T}} u_n[k]| + \varepsilon^{(i)}, \quad i = 1, \ldots, m. \tag{48}$$

The synthesis of the adaptive controller is finished by the choice of the control u_n from the set $\{u_n[1], \ldots, u_n[N]\}$ with $u_n[k]$ given by (42). This choice is implemented by the rule giving the minimum of the 1-norm of $\vec{e}_{n+1}[k] = [\vec{e}_{n+1}^{(1)}[k], \ldots, \vec{e}_{n+1}^{(m)}[k]]^{\mathrm{T}}$ according to

$$u_n = \arg \min_{u_n[k]} \sum_{i=1}^{m} |\vec{e}_n^{(i)}[k]|, \tag{49}$$

where $\vec{e}_{n+1}^{(i)}[k]s$ are specified by (48).

The asymptotic properties of the adaptive controller described in this section are given in theorem below.

Theorem 2. Consider the feedback control system containing the plant (1) in which $r < m$, and the adaptive controller defined in the equations (44), (49) together with (41), (48) and (43). Using the constraints (5), determine $\delta_0[1],\ldots$ $\ldots,\delta_0[N]$ to satisfy the requirement (40). Let assumptions A1)–A3) be valid. Then, this controller applied to the plant (1) guarantees that the control objectives (7) and (8) will be achieved.

Proof. See [39]. □

Note that Theorem 2 does not guarantee that the ultimate error $\lim_{n\to\infty}\sup\|e_n\|$ will become as in the nonadaptive case when there is no parametric uncertainty and the pseudoinverse model-based controller proposed in [24] can by applied.

6. Simulation

To demonstrate the behavior of the robust adaptive closed-loop control system designed in Section 3–5, several simulation experiments were conducted.

Simulation experiment 1. In this experiment, the adaptive closed-loop control system (1), (6), (25) to (32) was simulated. The elements of B were given as: $b^{(11)}=4$, $b^{(12)}=2$, $b^{(21)}=2$, $b^{(22)}=1$ $(\det B=0)$. The interval estimates of these elements were chosen as follows: $1\le b^{(11)}\le 5$, $0\le b^{(12)}\le 2$, $0\le b^{(21)}\le 2$, $1\le b^{(22)}\le 2$. By the formulas (20) and (22), it was found: $\underline{\beta}^{(1)}_{\min}=-1$, $\underline{\beta}^{(2)}_{\min}=-1$, $\overline{\beta}^{(1)}_{\max}=7$, $\overline{\beta}^{(2)}_{\max}=4$, $\underline{\beta}_{\min}=-1$, $\overline{\beta}_{\max}=7$. It turned out that $|\underline{\beta}_{\min}|<|\overline{\beta}_{\max}|$. Therefore it is required that $\delta_0>1$ to satisfy the inequalities (21). Namely, $\delta_0=1{,}1$ was put. From the conditions $b_0^{(11)}\in[1,5]$, $b_0^{(12)}\in[0,2]$, $b_0^{(21)}\in[0,2]$, $b_0^{(22)}\in[1,2]$ the following initial estimates of B_n were taken: $b_0^{(11)}=1$, $b_0^{(12)}=1$, $b_0^{(21)}=0$, $b_0^{(22)}=1{,}9$. Then $\tilde b_0^{(11)}=2{,}1$, $\tilde b_0^{(12)}=1$, $\tilde b_0^{(21)}=0$, $\tilde b_0^{(22)}=3$.

In this simulation experiment, the sequences $\{v_n^{(i)}\}\,(i=1,2)$ were generated as i.i.d. random variables [29, p.40] belonging to $[-1,1]$. It was put: $y^0=[1,3]^{\mathrm T}$.

The performance of the adaptive estimation algorithm is shown in Fig. 6.

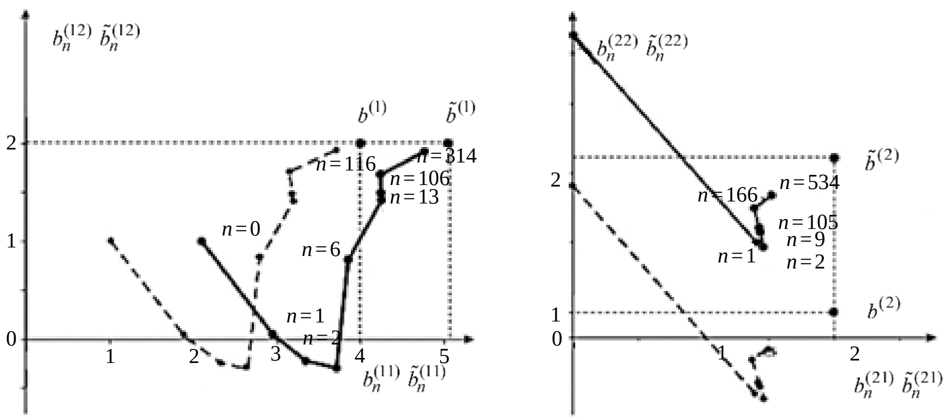


Fig. 6

Fig. 7 shows simulation results illustrating a successful behavior of the robustly-adaptive control system when this experiment was conducted.

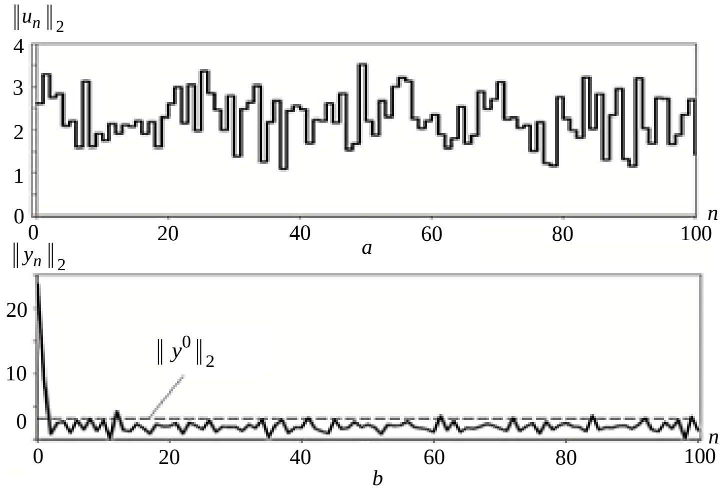


Fig. 7

Simulation experiment 2. To verify how the adaptive controller proposed in Section 4 performs, we give an illustrative example. In this example, the true B and the initial B_0 were chosen as:

$$B = \begin{pmatrix} 0,2 & 1,4 \\ 0,8 & 2,4 \\ 1,1 & 0.5 \end{pmatrix}, \quad B_0 = \begin{pmatrix} 50 & 20 \\ 30 & 40 \\ 10 & 10 \end{pmatrix}$$

to ensure $\operatorname{rank} B_0 = \operatorname{rank} B = 2.$ The desired output vector was taken as $y^0 = [2, 7, 3]^{\mathrm{T}}$. $\{v_n^{(i)}\}$ were generated as pseudorandom independently identically distributed (i.i.d.) sequences satisfying to $v_n^{(1)} \in [-0,1; 0,1]$, $v_n^{(2)} \in [-0,2; 0,2]$ and $v_n^{(3)} \in [-0,08; 0,08]$.

Computer simulations have been carried out to evaluate the behavior of the adaptive control system (1), (34), (36). This behavior is presented in Fig. 8. It demonstrates that the closed-loop control systems containing the adaptive pseudoinverse model-based controller (34), (36) is successful enough.

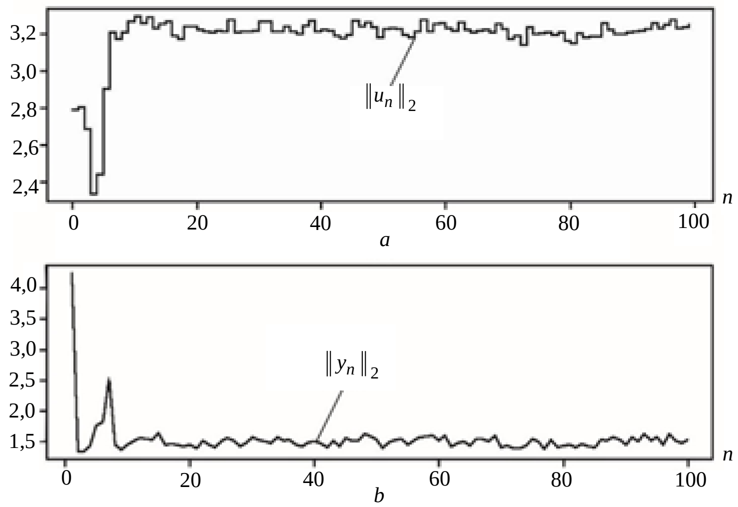


Fig. 8

ISSN 1028-0979

Simulation experiment 3. This simulation experiment was conducted to illustrate the performance of the adaptive control proposed in Section 5 for the case when $r=2$, $m=3$. As the gain matrix,

$$B=\begin{pmatrix}4 & 2\\ 2 & 1\\ 3 & 1,5\end{pmatrix}$$

of not full rank ($\operatorname{rank} B=1$) was taken. Since $N=3$, it produces the following three submatrices:

$$B[1]=\begin{pmatrix}4 & 2\\ 2 & 1\end{pmatrix},\quad B[2]=\begin{pmatrix}4 & 2\\ 3 & 1,5\end{pmatrix}\quad \text{and}\quad B[3]=\begin{pmatrix}2 & 1\\ 3 & 1,5\end{pmatrix}$$

Further, the three vectors $y_n[1]=[y_n^{(1)}, y_n^{(2)}]^{\mathrm{T}}$, $y_n[2]=[y_n^{(1)}, y_n^{(3)}]^{\mathrm{T}}$ and $y_n[3]=[y_n^{(2)}, y_n^{(3)}]^{\mathrm{T}}$ were introduced to describe the plants (37) having the gain matrices $B[1], B[2]$ and $B[3]$ respectively. The quantities $\delta_0[1]=1{,}1$, $\delta_0[2]=1{,}2$ and $\delta_0[3]=1{,}3$ were taken to satisfy the conditions (40) guaranteeing $\tilde{B}[k]$ to be nonsingular were derived from the equation (5). The initial $\tilde{B}_0[1], \tilde{B}_0[2]$ and $\tilde{B}_0[3]$ were chosen as $\tilde{B}_0[k]=B_0[k]+\delta_0[k]I_r$ with the initial elements of $B_0[k]$ which were selected from B inside the corresponding ranges $[\underline{b}^{(ij)}, \bar{b}^{(ij)}]$ specified as follows: $b^{(11)}\in[1, 5]$, $b^{(12)}\in[0, 2]$, $b^{(21)}\in[0, 2]$, $b^{(22)}\in[1, 2]$, $b^{(31)}\in[1, 4]$, $b^{(32)}\in[0, 5]$. Namely, we set $b_0^{(11)}=1$, $b_0^{(12)}=1$, $b_0^{(21)}=0$, $b_0^{(22)}=1{,}9$, $b_0^{(31)}=2$, $b_0^{(32)}=2{,}1$. The desired output vector was given as $y^0=[1, 3, 7]^{\mathrm{T}}$.

The performance of the simulated adaptive closed-loop control system with the disturbance sequences $\{v_n^{(i)}\}=v_0^{(i)}, v_1^{(i)}, \ldots$ generated as some pseudorandom i.i.d. variables taken from $-0{,}1\le v_n^{(1)}\le 0{,}1; -0{,}2\le v_n^{(2)}\le 0{,}2; -0{,}08\le v_n^{(3)}\le 0{,}08$ is presented in Fig. 9.

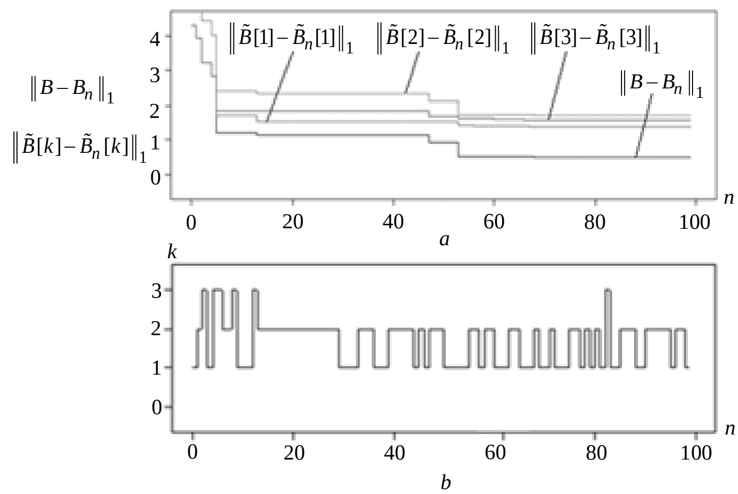


Fig. 9

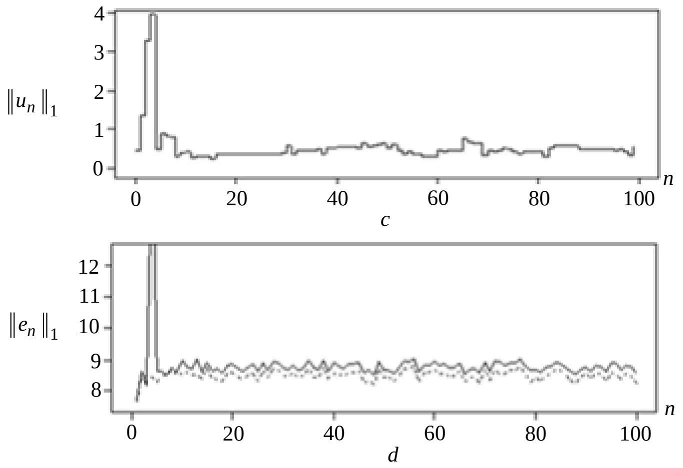


Fig. 9, *a–d* demonstrate that the performance of the proposed adaptive controller applied to the static multivariable plant having some nonsquare gain matrix with not full rank is successful enough.

Conclusion

The adaptive control concept together with the pseudoinverse (generalized inverse) model-based approach is the suitable tool to deal with discrete-time uncertain multivariable systems whose gain matrices are noninvertible. It has been shown that a successful behavior of the robustly-adaptive closed-loop control system containing the square singular or nonsquare plant can be achieved regardless of the rank of its gain matrix.

Л.С. Житецький, К.Ю. Соловчук

АДАПТИВНЕ РОБАСТНЕ КЕРУВАННЯ БАГАТОЗВ’ЯЗНИМИ СИСТЕМАМИ БЕЗ ПАМ’ЯТІ, ЩО НЕ ЗДАТНІ БУТИ ОБЕРНЕНИМИ, З ОБМЕЖЕНИМИ ЗБУРЕННЯМИ: УЗАГАЛЬНЕННЯ

Житецький Леонід Сергійович

Міжнародний науково-навчальний центр інформаційних технологій та систем НАН України і МОН України, м. Київ,
leonid_zhiteckii@i.ua

Соловчук Клавдія Юріївна

Полтавський науково-дослідний експертно-криміналістичний центр МВС України,
solovchuk_ok@ukr.net

Розглядається жорстке адаптивне керування у дискретному часі для деяких класів невизначених багатозмінних безпам’ятних (статичних) систем за наявності невимірних обмежених збурень, межі яких вважаються відомими. Розглядаються системи, де кількість керуючих входів не перевищує кількість їх виходів. Основною особливістю об’єктів, які підлягають контролю, є те, що їх матриці підсилення є незворотними. Припущення, що елементи цих матриць апріорі невідомі, але є інформація про можливі межі цих еле-

 ISSN 1028-0979

ментів. Поставлена та вирішувана тут проблема полягає в тому, щоб розробити контролер зворотного зв'язку, здатний впоратися з необоротністю матриць підсилення, а також з параметричною невизначеністю, щоб відкинути зовнішні збурення та забезпечити обмеженість усіх сигналів системи керування та виведення. Для вирішення вищезгаданої проблеми використовується надійний адаптивний підхід разом із так званою концепцією на основі псевдоінверсної або зворотної моделі. Вивчаються три різні випадки. У першому випадку розроблено надійний адаптивний контролер, застосовний до невизначеної установки з квадратною матрицею сингулярного підсилення. У другому випадку для роботи з невідомими неквадратичними матрицями підсилення, що мають повний ранг, пропонується надійний метод із застосуванням контролерів на основі псевдоінверсної моделі, параметри яких оцінюються за допомогою стандартної рекурсивної процедури адаптації. Підхід, запропонований у першому випадку, поширюється на третій випадок, що стосується керування невідомими об'єктами, матриці підсилення яких представляють неквадратні матриці неповного рангу. Встановлено асимптотичні властивості запропонованих у цій роботі робастно-адаптивних регуляторів. Наведено результати чисельних прикладів на підтвердження теоретичного дослідження.

Ключові слова: дискретний час, багатозмінна установка без пам'яті, незворотність, концепція на основі псевдозворотної моделі, невизначеність, алгоритм оцінки, надійне адаптивне керування.

1. Davison E. The output control of linear time-invariant multivariable systems with unmeasurable arbitrary disturbances'. *IEEE Trans. Autom. Contr. Oct.* 1972. **AC-17**, N 5. P. 621–631.
2. Gubarev V.F., Mishchenko M.D., Snizhko B.M. Model predictive control for discrete MIMO linear systems. In: Y.P. Kondratenko, A.A. Chikrii, V.F. Gubarev, J. Kacprzyk (eds.). Advanced control techniques in complex engineering systems: Theory and applications. Dedicated to Prof. V.M. Kuntsevich. Studies in systems, decision and control, Springer Nature Switzerland AG, Cham, 2019. **203**. P. 63–81. DOI: https://doi.org/10.1007/978-3-030-21927-7_10.
3. Skogestad S., Postlethwaite I. Multivariable feedback control. Wiley, Chichester, 1996.
4. Albertos P., Sala A. Multivariable control systems: an engineering approach. Springer, London, 2006.
5. Glad T., Ljung L. Control theory: multivariable and nonlinear methods. Taylor & Francis, New York, 2000.
6. Tan L. A generalized framework of linear multivariable control. Elsevier, Oxford, 2017.
7. Pukhov G.E., Zhuk K.D. Synthesis of interconnected control systems via inverse operator method. Nauk. dumka, Kiev, 1966 (in Russian).
8. Lyubchyk L.M. Disturbance rejection in linear discrete multivariable systems: inverse model approach. *In: Proc. 18th IFAC World Congress*. Milano, Italy, 2011. P. 7921–7926.
9. Lyubchyk L.M. Inverse model approach to disturbance rejection problem, In: Kondratenko Y.P., Kuntsevich V.M., Chikrii A.A., Gubarev V.F. (Eds), Advanced control systems: theory and applications. River Publishers, Gistrup. 2021. P. 129–166.
10. Skurikhin V.I., Zhiteckii L.S., Solovchuk K.Yu. Control of interconnected plants with singular and ill-conditioned transfer matrices based on pseudo-inverse operator method. *Upravlyayushchye sistemy i mashiny*. 2013. **29**, N 3. P. 14−20.
11. Zhiteckii L.S., Solovchuk K.Yu. Pseudoinversion in the problems of robust stabilizing multivariable discrete-time control systems of linear and nonlinear static objects under bounded disturbances. *Journal of Automation and Information Sciences*. 2017. **49**, N 5. P. 35–48.
12. Skurikhin V.I., Zhiteckii L.S., Solovchuk K.Yu. Stabilization of a nonlinear multivariable discrete-time time-invariant plant with uncertainty on a linear pseudoinverse model. *Journal of Computer and Systems Sciences International*. 2017. **56**, N 5. P. 759–773.
13. Kuntsevich V.M. Control under uncertainty: guaranteed results in control and identification problems. Nauk. dumka, Kiev, 2006 (in Russian).

14. Fomin V.N., Fradkov A.L., Yakubovich V.A. Adaptive control of dynamic plants. Nauka, Moscow, 1981 (in Russian).
15. Goodwin G.C., Sin K.S. Adaptive filtering, prediction and control. Prentice-Hall, Engewood Cliffs, 1984.
16. Tao G. Adaptive control design and analysis. John Wiley and Sons, New York, 2003.
17. Narendra K.S., Annaswamy A.M. Stable adaptive systems. Dover Publications, New York, 2012.
18. Ioannou P., Sun J. Robust adaptive control. Dover Publications, New York, 2013.
19. Ǻström K.J. and B. Wittenmark. Adaptive control: 2nd Edition, Dover Publications, New York, 2014.
20. Zhiteckii L.S., Solovchuk K.Yu. Adaptive stabilization of some multivariable systems with nonsquare gain matrices of full rank. *Cybernetics and Computer Engineering*. 2018. **192**, N 2. P. 44–60.
21. Zhiteckii L.S., Solovchuk K.Yu. Solving a problem of adaptive stabilization for some static mimo systems. *Cybernetics and Computer Engineering*. 2019. **197**, N 3. P. 33–50.
22. Zhiteckii L.S., Solovchuk K.Yu. Robust adaptive controls for a class of nonsquare memoryless systems. In: Kondratenko, Y.P., Kuntsevich, V.M., Chikrii, A.A., Gubarev, V.F. (Eds), Advanced Control Systems: Theory and Applications. River Publishers, Gistrup, 2021. P. 203–226.
23. Zhiteckii L.S., Azarskov V.N., Solovchuk K.Yu. Adaptive robust control of inter-connected static plants with nonsquare gain matrixes. *In: Proc. 13th All-Russian Control Problems Council (VSPU-2019)*. IPU, Moscow, 2019. P. 713–718.
24. Albert A. Regression and the Moore-Penrose pseudoinverse. Academic Press, New York, 1972.
25. Voevodin V.V., Kuznetsov Yu.A. Matrices and calculations. Nauka, Moscow, 1984 (in Russian).
26. Azarskov V.N., Zhiteckii L.S., Solovchuk K.Yu. Parametric identification of the interconnected static closed-loop system: a special case. *In: Proc. 12th All-Russian Control Problems Council (VSPU-2014)*. IPU, Moscow, 2014. P. 2764–2776.
27. Marcus M., Minc H. A survey of matrix theory and matrix inequalities. Aliyn and Bacon, Boston, 1964.
28. Zhiteckii L.S., Skurikhin V.I. Adaptive control systems with parametric and nonparametric uncertainties. Nauk. dumka, Kiev, 2010 (in Russian).
29. Walter E., Pronzato L. Identification of parametric models. Springer, Paris/Milan/Barcelona, 1997.

Submitted 01.02.2022