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E. Aliyev, F. Salmanov

FUZZY APPROACH TO FORECASTING THE DYNAMICS OF VEGETATION INDICES

Elchin Aliyev

Institute of Control Systems of Azerbaijan National Academy of Sciences, *elchin.aliyev@sinam.net*

Fuad Salmanov

Institute of Control Systems of Azerbaijan National Academy of Sciences, *fuad.salmanli@sinam.net*

Modern technologies for satellite monitoring of the Earth's surface provide agricultural producers with useful information about the health status of crops. The remote sensor's ability to detect subtle differences in vegetation makes it a useful tool for quantifying variability within a given field, estimating crop growth, and managing land based on current conditions. Remote sensing data, collected on a regular basis, allows producers and agronomists to draw up a current vegetation map that reflects the condition and strength of crops, analyze the dynamics of changes in plant condition, and predict yields in a particular area under crops. To interpret these data, the most effective means are various vegetation indices calculated empirically, that is, by operations with different spectral ranges of satellite monitoring multispectral data. Based on the time series of one of these vegetation indices, the paper considers the annual dynamics of the development of a plant culture in a particular field. The possibility of predicting the yield of the given crop is considered based on fuzzy modeling of time series for the corresponding spectral ranges of vegetation reflection obtained from satellite monitoring images. The proposed fuzzy models of time series are investigated for adequacy and suitability in terms of analyzing the features of the intra-annual of average long-term dynamics of the vegetation index, typical for the given area under crop.

Keywords: crop, multispectral reflection of plants, vegetation index, fuzzy set, fuzzy time series.

Introduction

Most agricultural crops are characterized by changes in the phases of development, which is reflected in the dynamics of the spectral-reflective properties of plants. The study of seasonal and long–term changes in the spectral-brightness characteristics of crops is possible through the analysis and modeling of the dynamic series of vegetation indices, which makes it possible to quantify the features of the vegetation cover and the regularity of its temporal dynamics. At the same time, standard algorithms for solving problems of predicting the dynamics of the spectral–reflective properties of plants work, as a rule, with «crisp» or structured data from satellite sensing of the Earth, that is, with data presented in the form of averaged numbers. Therefore, averaging the results of measurements of spectral ranges for calculating vegetation indices is one of the most common empirical operations in data collection systems for accurate agriculture. In particular, the achievement of the required accuracy in the process of averaging the values of vegetation indices is achieved by multiple measurements, where the results of individual measurements are partially compensated for by positive and negative deviations from the exact value. At the same time, the accuracy of their mutual compensation improves with an increase in the number of measurements, since the average value of negative deviations in modulus verge towards the average value of positive deviations. Nevertheless, multispectral satellite monitoring data, for example, the values of spectral ranges should be considered as weakly structured, that is, those that are known to belong to a certain interval [1]. For example, the region of maximum reflection of plant cell structures is in the wavelength range from 750 nm to 900 nm, which is the near infrared region of the electromagnetic spectrum.

More adequate reflections of weakly structured spectral ranges can be evaluative concepts of the type «HIGH», «LOW», etc., which can be formally described by the corresponding fuzzy sets as the terms (values) of the linguistic variable «spectral reflectivity of plants» [2, 3]. Based on this premise, it becomes obvious the importance and relevance of studying methods for predicting seasonal and long–term changes in agricultural crops in spectral–brightness characteristics using fuzzy time series (FTS) of satellite monitoring indicators relative to spectral ranges.

Problem definition

Existing approaches to the calculation of vegetation indices are usually based on two independent parts of the electromagnetic spectrum of vegetation reflectivity [4]: on the reflection in the red region of the spectrum in the range from 620 nm to 750 nm, which accounts for the maximum absorption of solar radiation by chlorophyll of higher vascular plants, and on the reflection in the near infrared region of the spectrum in the range from 750 nm to 900 nm, where the region of maximum reflection of the cellular structures of the leaf is concentrated. One of the wide-spread indicators for solving problems regarding the assessment of vegetation cover is the NDVI (Normalized Difference Vegetation Index), which is calculated by the formula

$$\mathrm{NDVI} = \frac{\mathrm{NIR} - \mathrm{RED}}{\mathrm{NIR} + \mathrm{RED}}, \tag{1}$$

where NIR and RED are the reflection coefficients in the near infrared and red regions of the electromagnetic spectrum, respectively. Both coefficients are calculated by mapping the red and infrared regions of the spectrum onto a unit segment using trivial equalities:

$$\mathrm{RED} = \frac{\lambda_1 - 620}{750 - 620},\ \lambda_1 \in (620, 750), \quad \mathrm{NIR} = \frac{\lambda_2 - 750}{900 - 750},\ \lambda_2 \in (750, 900).$$

The NDVI value varies from 0 to 1: the higher its value is the higher the vegetation intensity, and vice versa, the lower the index value, the sparser is the vegetation, and the tendency to zero generally indicates open ground. So, it is necessary to adapt a fuzzy method for forecasting seasonal and long-term changes in agricultural crops in the spectral–brightness characteristics using FTS of spectral ranges of remote sensing data and reflecting certain vegetation parameters in a particular pixel of a satellite image. As an example of testing fuzzy models, time series were selected that reflect the annual dynamics of the coefficients of the spectral ranges RED and NIR (see Table 1 and Fig. 1), obtained from images of a fixed pixel in the corresponding MODIS images (LPDAAC — Land Processes Distributed Active Archive Center) (see Fig. 2) of crop area in Jonesboro (USA, Arkansas) with geographic coordinates (– 90,1614583252562, 35.8135416634583) [4]. Table 1 also shows the corresponding NDVI calculated using formula (1).

Table 1

N	Date	NIR	RED	NDVI	N	Date	NIR	RED	NDVI
1	18.02.2000	0,2036	0,0958	0,3599	16	25.06.2000	0,8565	0,1452	0,7101
2	26.02.2000	0,3175	0,1445	0,3745	17	02.07.2000	0,8651	0,1453	0,7124
3	05.03.2000	0,3639	0,1523	0,4099	18	11.07.2000	0,8702	0,1455	0,7135
4	15.03.2000	0,3623	0,1623	0,3812	19	20.07.2000	0,3357	0,1256	0,4554
5	21.03.2000	0,2219	0,1025	0,3680	20	27.07.2000	0,1125	0,0678	0,2479
6	29.03.2000	0,1717	0,0835	0,3457	21	02.08.2000	0,3666	0,1348	0,4623
7	06.04.2000	0,1676	0,0845	0,3296	22	12.08.2000	0,6051	0,1245	0,6587
8	15.04.2000	0,1407	0,0765	0,2957	23	20.08.2000	0,5828	0,1463	0,5987
9	22.04.2000	0,1106	0,0659	0,2535	24	28.08.2000	0,4628	0,1354	0,5473
10	29.04.2000	0,1214	0,0689	0,2759	25	03.09.2000	0,3492	0,1158	0,5019
11	08.05.2000	0,1502	0,0815	0,2966	26	13.09.2000	0,3523	0,1233	0,4815
12	15.05.2000	0,1529	0,0813	0,3058	27	20.09.2000	0,3450	0,1389	0,4259
13	24.05.2000	0,1664	0,0855	0,3211	28	29.09.2000	0,2457	0,1125	0,3719
14	09.06.2000	0,4084	0,1324	0,5104	29	07.10.2000	0,2173	0,1045	0,3505
15	15.06.2000	0,5890	0,1356	0,6257	30	15.10.2000	0,2058	0,1056	0,3217

Time series of the NDVI index against the background of the dynamics of the RED and NIR coefficients obtained for a fixed pixel of MODIS images (LPDAAC) are in Fig. 1.

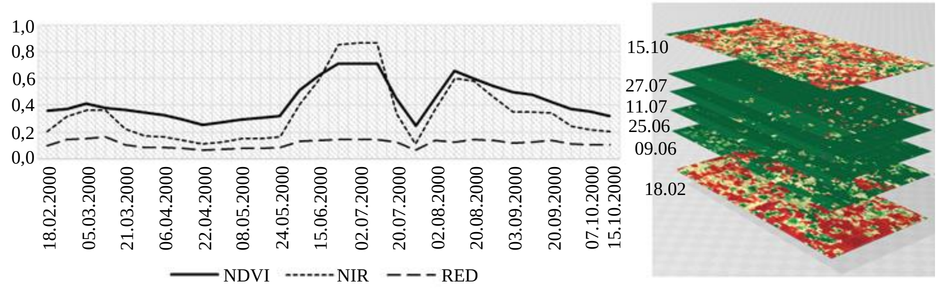

Fig. 1

FTS: Main Stages of Predictive Modeling

The existing approaches to fuzzy modeling of time series involve the sequential implementation of the following main stages (procedures): 1) establishing the coverage of the entire set of historical data in the form of a universal set (universe); 2) fuzzification of weakly structured historical data of time series; 3) establishing internal relationships in the form of fuzzy relations and dividing them into groups; 4) finding fuzzy outputs (predicts) of the applied model and their defuzzification.

One of the ways to establish the universe and calculate the optimal number of evaluative concepts for fuzzy evaluation of the historical data of the time series was proposed in [6], the essence of which is to perform sequentially the following steps.

Step 1. Assorting the time series data $\{x_t\}_{t=1}^{n}$ into an ascending sequence $\{x_{p(i)}\}$, where p is a permutation that sorts the data values in ascending order: $x_{p(i)} \le x_{p(i+1)}$.

Step 2. Calculation of the average value for all pairwise distances $d_i = |x_{p(i)} - x_{p(i+1)}|$ between any two consecutive values $x_{p(i)}$ and $x_{p(i+1)}$ and standard deviation by formulas:

$$AD(d_1, d_2, ..., d_n) = \frac{1}{n-1}\sum_{i=1}^{n-1}\left|x_{p(i)} - x_{p(i+1)}\right|, \qquad (2)$$

$$\sigma_{AD} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n-1}(d_i - AD)^2}. \quad (3)$$

Step 3. Detection and elimination of anomalies — outliers that need to be reset. For this, both the mean distance AD and the standard deviation σ_{AD}, established in the previous step, are used. In this case, the values of pairwise distances that do not satisfy the following condition are subject to outlier

$$AD - \sigma_{AD} \le d_i \le AD + \sigma_{AD}. \quad (4)$$

Step 4. Recalculation of the mean distance AD for the set of remaining values d_i.

Step 5. Establishing the universe U in the form $U = [D_{\min} - AD, D_{\max} + AD] = = [D_1, D_2]$, where $D_{\min}$ and $D_{\max}$ are the minimum and maximum values, respectively, on the entire data set.

Step 6. Finding the optimal number of evaluative concepts as criteria for evaluating the historical data of the time series. It is carried out based on the formula

$$m = \frac{D_2 - D_1 - AD}{2 \cdot AD}. \quad (5)$$

Considering the above step-by-step data fuzzification procedure FTS models of both the NDVI and the corresponding reflection coefficients NIR and RED are proposed below.

Forecasting the fuzzy time series of the NDVI: method №1

There are various ways to describe the qualitative criteria for assessing the values of historical data using fuzzy sets. One of them is quite trivial, initially implying a set of qualitative criteria, for example, of the form:

too low (value): $A_1 = \frac{1}{u_1} + \frac{0,5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7} + \frac{0}{u_8}$;

very low: $A_2 = \frac{0,5}{u_1} + \frac{1}{u_2} + \frac{0,5}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7} + \frac{0}{u_8}$;

more than low: $A_3 = \frac{0}{u_1} + \frac{0,5}{u_2} + \frac{1}{u_3} + \frac{0,5}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7} + \frac{0}{u_8}$;

low: $A_4 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0,5}{u_3} + \frac{1}{u_4} + \frac{0,5}{u_5} + \frac{0}{u_6} + \frac{0}{u_7} + \frac{0}{u_8}$;

high: $A_5 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0,5}{u_4} + \frac{1}{u_5} + \frac{0,5}{u_6} + \frac{0}{u_7} + \frac{0}{u_8}$;

more than high: $A_6 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0,5}{u_5} + \frac{1}{u_6} + \frac{0,5}{u_7} + \frac{0}{u_8}$;

very high: $A_7 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0,5}{u_6} + \frac{1}{u_7} + \frac{0,5}{u_8}$;

too high: $A_8 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0,5}{u_7} + \frac{1}{u_8}$,

where each historical data of the time series is interpreted considering the belonging of the interval of its localization u_j $(j = 1 \div 8)$ to one or another fuzzy set A_j with a rather trivial membership function.

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In [5], to describe the qualitative evaluation criteria by appropriate fuzzy sets, the following trapezoidal membership functions (TMF) are used

$$\mu_{A_k}(x) = \begin{cases} 0, & x < a_{k1} \\ \dfrac{x - a_{k1}}{a_{k2} - a_{k1}}, & a_{k1} \le x \le a_{k2}, \\ 1, & a_{k2} \le x \le a_{k3}, \\ \dfrac{a_{k4} - x}{a_{k4} - a_{k3}}, & a_{k3} \le x \le a_{k4}, \\ 0, & x > a_{k4} \end{cases} \quad (6)$$

whose parameters satisfy the conditions: $a_{k_2} - a_{k_1} = a_{k_3} - a_{k_2} = a_{k_4} - a_{k_3}$ $(k = 1 \div m)$.

So, on the entire data set of the NDVI time series (see Table 1), using formulas (2) and (3) the mean value $AD = 0,0161$ and the standard deviation $\sigma_{AD} = 0,0140$ are established, respectively. After resetting the pairwise distances d_i that do not satisfy condition (4) or, more specifically, the condition $0,0161 - 0,0140 \le d_i \le 0,0161 + 0,0140$, based on the remaining set of pairwise distances the final value of the mean value was obtained as $AD = 0,0132$. In this case, the desired universe is constructed as a segment $U = [0,2479 - 0,0132; 0,2479 + 0,0132] = [0,2347; 0,7267]$, where 0,2479 and 0,7135 are the minimum and maximum values of the NDVI index, respectively. At the same time, the number of fuzzy subsets of this universe, describing the qualitative criteria for evaluating NDVI indices, is calculated by equality (5) as follows:

$$m = \frac{0,7267 - 0,2347 - 0,0132}{2 \cdot 0,0132} = 18,1424 \approx 18.$$

Based on the use of the TMF (6) with parameters a_{k_1} summarized in Table 2, where $(i = 1 \div 4)$ and $(k = 1 \div 18)$, the corresponding fuzzy sets A_k are established (Fig. 2).

Table 2

Fuzzy set	Parameters of the TMF				Fuzzy set	Parameters of the TMF			
	a_{k_1}	a_{k_2}	a_{k_3}	a_{k_4}		a_{k_1}	a_{k_2}	a_{k_3}	a_{k_4}
A1	0,2347	0,2479	0,2611	0,2743	A10	0,4722	0,4854	0,4986	0,5118
A2	0,2611	0,2743	0,2875	0,3007	A11	0,4986	0,5118	0,5250	0,5382
A3	0,2875	0,3007	0,3139	0,3271	A12	0,5250	0,5382	0,5514	0,5646
A4	0,3139	0,3271	0,3403	0,3535	A13	0,5514	0,5646	0,5778	0,5910
A5	0,3403	0,3535	0,3667	0,3799	A14	0,5778	0,5910	0,6042	0,6174
A6	0,3667	0,3799	0,3931	0,4062	A15	0,6042	0,6174	0,6306	0,6438
A7	0,3931	0,4062	0,4194	0,4326	A16	0,6306	0,6438	0,6570	0,6702
A8	0,4194	0,4326	0,4458	0,4590	A17	0,6570	0,6702	0,6834	0,6965
A9	0,4458	0,4590	0,4722	0,4854	A18	0,6834	0,6965	0,7097	0,7267

Fuzzification of NDVI indices by the presented trapezoidal membership functions is carried out according to the principle: NDVI is described by the fuzzy set to which its value belongs with the highest degree. When the NDVI value belongs to the interval $[a_{k_2}, a_{k_3}]$, the appropriate fuzzy analog is easily determined. In other cases, clarifications are needed. According to (6) for $\mathrm{NDVI} = 0,5019$ we have: $\mu_{A_{11}}(0,5019) = 0,2491$ and $\mu_{A_{10}}(0,5019) = 0,7509$ (see Fig. 3). Then the fuzzy set A_{10} is the analog of NDVI, since the value of the corresponding membership function at the point 0,5019 is greater. The fuzzy analogs of all NDVIs are summarized in Table 3.

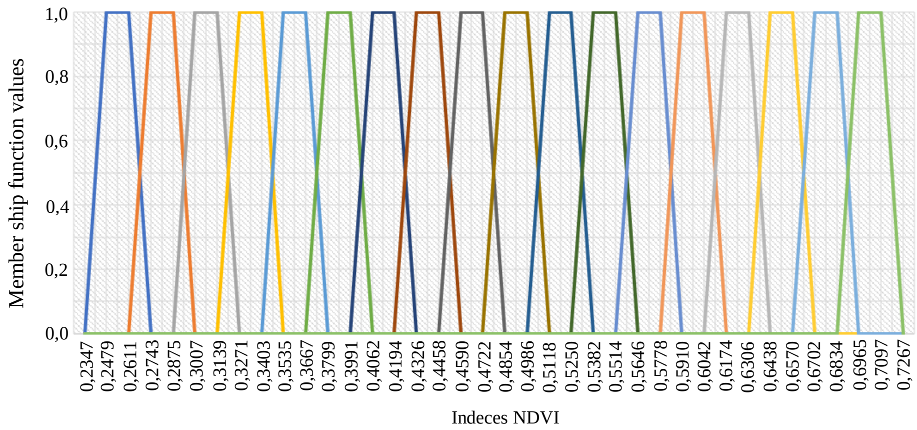

Fig. 2

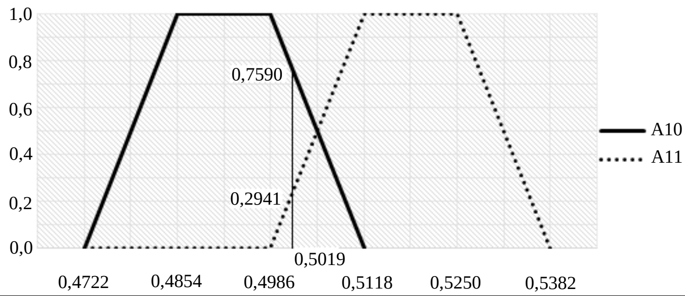

Fig. 3

Table 3

N	Date	NDVI	Fuzzy set	N	Date	NDVI	Fuzzy set
1	18.02.2000	0,3599	A5	16	25.06.2000	0,7101	A18
2	26.02.2000	0,3745	A6	17	02.07.2000	0,7124	A18
3	05.03.2000	0,4099	A7	18	11.07.2000	0,7135	A18
4	15.03.2000	0,3812	A6	19	20.07.2000	0,4554	A9
5	21.03.2000	0,3680	A5	20	27.07.2000	0,2479	A1
6	29.03.2000	0,3457	A4	21	02.08.2000	0,4623	A9
7	06.04.2000	0,3296	A4	22	12.08.2000	0,6587	A16
8	15.04.2000	0,2957	A3	23	20.08.2000	0,5987	A14
9	22.04.2000	0,2535	A1	24	28.08.2000	0,5473	A12
10	29.04.2000	0,2759	A2	25	03.09.2000	0,5019	A10
11	08.05.2000	0,2966	A3	26	13.09.2000	0,4815	A10
12	15.05.2000	0,3058	A7	27	20.09.2000	0,4259	A7
13	24.05.2000	0,3211	A4	28	29.09.2000	0,3719	A5
14	09.06.2000	0,5104	A11	29	07.10.2000	0,3505	A5
15	15.06.2000	0,6257	A15	30	15.10.2000	0,3217	A4

As is known, FTS modeling is based on the analysis of internal cause and effect relations, which are presented in the form of implication «If <…>, then <…>». Identified internal relations are grouped according to the principle: if the fuzzy set A_k or the bunch of sets A_i, A_{i+1} relates to one or several sets, then a group of the 1st order is localized relative to A_k or the group of the 2nd order is localized relative to A_i, A_{i+1} (see Table 4 and Table 5).

Table 4

A1 ⇒ A2	A3 ⇒ A1	A4 ⇒ A3	A5 ⇒ A4	A6 ⇒ A5	A7 ⇒ A5	A10 ⇒ A10	A12 ⇒ A10	A16⇒A14
A1 ⇒ A9	A3 ⇒ A7	A4 ⇒ A11	A5 ⇒ A5	A7 ⇒ A6	A9 ⇒ A1	A10 ⇒ A7	A14 ⇒ A12	A18 ⇒ A18
A2 ⇒ A3	A4 ⇒ A4	A5 ⇒ A6	A6 ⇒ A7	A7 ⇒ A4	A9 ⇒ A16	A11 ⇒ A15	A15 ⇒ A18	A18 ⇒ A9

Table 5

A5, A6 ⇒ A7	A5, A4 ⇒ A4	A1, A2 ⇒ A3	A4, A11 ⇒ A15	A18, A9 ⇒ A1	A16, A14 ⇒ A12	A10, A7 ⇒ A5
A6, A7 ⇒ A6	A4, A4 ⇒ A3	A2, A3 ⇒ A7	A11, A15 ⇒ A18	A9, A1 ⇒ A9	A14, A12 ⇒ A10	A7, A5 ⇒ A5
A7, A6 ⇒ A5	A4, A3 ⇒ A1	A3, A7 ⇒ A4	A15, A18 ⇒ A18	A1, A9 ⇒ A16	A12, A10 ⇒ A10	A5, A5 ⇒ A4
A6, A5 ⇒ A4	A3, A1 ⇒ A2	A7, A4 ⇒ A11	A18, A18 ⇒ A18, A9	A9, A16 ⇒ A14	A10, A10 ⇒ A7	

Within the fuzzy time series of the NDVI index, internal relationships are grouped according to the principle: if a fuzzy set A_k or a bunch of sets of the form A_i, A_{i+1} are connected to one or several sets at once, then a group of the 1st order is localized relative to A_k or, respectively, a group of 2nd order is localized relative to A_i, A_{i+1}. The 2nd order relationships for 27 groups are already presented in Table 5, and the 1st order relationships are divided into 15 groups, which are summarized in Table 6.

Table 6

G1: A1 ⇒ A2, A9	G4: A4 ⇒ A3, A4, A11	G7: A7 ⇒ A4, A5, A6	G10: A11 ⇒ A15	G13: A15 ⇒ A18
G2: A2 ⇒ A3	G5: A5 ⇒ A4, A5, A6	G8: A9 ⇒ A1, A16	G11: A12 ⇒ A10	G14: A16 ⇒ A14
G3: A3 ⇒ A1, A7	G6: A6 ⇒ A5, A7	G9: A10 ⇒ A7, A10	G12: A14 ⇒ A12	G15: A18 ⇒ A9, A18

Denoting by x_i the value of the NDVI index on the *i*-th day, and by x_{i+1} the value of the NDVI index on the next $(i+1)$-th day, a fuzzy relation of the 1st order, for example, $A_2 \Rightarrow A_3$ (from the group G2) can be interpreted as a fuzzy implicative rule «If $x_i = A_2$, then $x_{i+1} = A_3$». Or, say, a fuzzy relation of the 1st order of the form $A_4 \Rightarrow A_3, A_4, A_{11}$ (from the group G4) can be interpreted as a fuzzy implicative rule: «If $x_i = A_4$, then $x_{i+1} = A_3$ or $x_{i+1} = A_4$ or $x_{i+1} = A_{11}$». Accordingly, the fuzzy relation of the 2nd order, for example, $A_5, A_6 \Rightarrow A_7$ can be interpreted as «If $x_i = A_5$ and $x_{i+1} = A_6$, then $x_{i+2} = A_7$», or relationship $A_{18}, A_{18} \Rightarrow A_{18}, A_9$ can be interpreted as «If $x_i = A_{18}$ and $x_{i+1} = A_{18}$, then $x_{i+2} = A_{18}$ or $x_{i+2} = A_9$».

Various approaches are used to determine «crisp» (defuzzified) predicts. The essence of one of them is as follows [7]. If the value of the NDVI index for the *i*-th day is described as a fuzzy set A_j, which forms only one relationship within the fuzzy time series, say $A_j \Rightarrow A_k$, then the fuzzy predict for the next $(i+1)$-th day will be the set A_k. If there is a group of fuzzy relations, for example, of the form $A_j \Rightarrow A_{k_1}, A_{k_2}, ..., A_{k_p}$, then the union of fuzzy sets $A_{k_1} \cup A_{k_2} \cup ... \cup A_{k_p}$ is the fuzzy predict for the $(i+1)$-th day.

In our case, the numerical interpretation of fuzzy predicts is based on the application of the S. Chen rule (see [7]). As a numerical estimate of the fuzzy predict A_j (the output of the fuzzy model of the NDVI time series), the abscissa of the middle of the upper base of the *j*-th trapezoid is considered (see Fig. 2). For example, for the fuzzy predict A_4, described by the trapezoidal membership function with the parameters indicated in Table 2, the numerical analog is the abscissa of the middle of the upper base: $(0,3271+0,3403)/2=0,3337$. Really, according to the fuzzy set point estimate rule (see [3]), the following formula is used to defuzzify a fuzzy set A:

$$F(A)=\frac{1}{\alpha_{\max}}\int_0^{\alpha_{\max}} M(A_\alpha)\,d\alpha, \qquad (7)$$

where $A_\alpha=\{u \mid \mu_A(u)\ge\alpha, u\in U\}$ is the α-level set ($\alpha\in[0;1]$); $M(A_\alpha)=\frac{1}{n}\sum_{k=1}^{n}u_k$ ($u_k\in A_\alpha$) is the cardinal number of the corresponding α-level set. For a fuzzy predict $A_4=\frac{0}{0,3139}+\frac{1}{0,3271}+\frac{1}{0,3403}+\frac{0}{0,3535}$ (see Table 2) we have:

$$0<\alpha<1,\ \ \Delta\alpha=1,\ \ A_{4\alpha}=\{0,3139;\,0,3535\},$$

$$M(A_{4\alpha})=(0,3139+0,3535)/2=0,3337.$$

Then, according to (7), the point estimate of A_4 or the defuzzified output of the 1st order model is the following number:

$$F(A_4)=\frac{1}{1}\int_0^1 M(A_{4\alpha})\,d\alpha=M(A_{4\alpha})\cdot\Delta\alpha=0,3337.$$

For relation $A_i\Rightarrow A_j, A_t, A_p$, where A_i is the fuzzy analog of the NDVI for the i-th day, the numerical predict for the next $(i+1)$-th day is calculated as the arithmetic mean of the abscissas of the midpoints of the upper bases of trapezoids corresponding to the fuzzy sets A_j, A_t and A_p. In particular, the fuzzy predict for the date 29.09.2000 is the union $A_4\cup A_5\cup A_6$ with the numerical estimate obtained as following:

$$F=\frac{\frac{0,3271+0,3403}{2}+\frac{0,3535+0,3667}{2}+\frac{0,3799+0,3931}{2}}{3}=0,3601.$$

Thus, considering the internal relationships of the 1st and 2nd orders for the fuzzy time series of the NDVI index, the corresponding predictive models were obtained, which are summarized in Table 7. Their geometric interpretations are shown in Fig. 4.

Table 7

N	Date	NDVI	1st order model		2nd order model	
			Fuzzy output	Predict	Fuzzy output	Predict
1	18.02.2000	0,3599				
2	26.02.2000	0,3745	A4, A5, A6	0,3601		
3	05.03.2000	0,4099	A5, A7	0,3865	A7	0,4128
4	15.03.2000	0,3812	A4, A5, A6	0,3601	A6	0,3865
5	21.03.2000	0,3680	A5, A7	0,3865	A5	0,3601
6	29.03.2000	0,3457	A4, A5, A6	0,3601	A4	0,3337

7	06.04.2000	0,3296	A3, A4, A11	0,3865	A4	0,3337
8	15.04.2000	0,2957	A3, A4, A11	0,3865	A3	0,3073
9	22.04.2000	0,2535	A1, A7	0,3337	A1	0,2545
10	29.04.2000	0,2759	A2, A9	0,3733	A2	0,2809
11	08.05.2000	0,2966	A3	0,3073	A3	0,3073
12	15.05.2000	0,3058	A1, A7	0,3337	A7	0,4128
13	24.05.2000	0,3211	A4, A5, A6	0,3601	A4	0,3337
14	09.06.2000	0,5104	A3, A4, A11	0,3865	A11	0,5184
15	15.06.2000	0,6257	A15	0,6240	A15	0,6240
16	25.06.2000	0,7101	A18	0,7031	A18	0,7031
17	02.07.2000	0,7124	A9, A18	0,5844	A18	0,7031
18	11.07.2000	0,7135	A9, A18	0,5844	A18, A9	0,5844
19	20.07.2000	0,4554	A9, A18	0,5844	A18, A9	0,5844
20	27.07.2000	0,2479	A1, A16	0,4524	A1	0,2545
21	02.08.2000	0,4623	A2, A9	0,3733	A9	0,4656
22	12.08.2000	0,6587	A1, A16	0,4524	A16	0,6504
23	20.08.2000	0,5987	A14	0,5976	A14	0,5976
24	28.08.2000	0,5473	A12	0,5448	A12	0,5448
25	03.09.2000	0,5019	A10	0,4920	A10	0,4920
26	13.09.2000	0,4815	A7, A10	0,4524	A10	0,4920
27	20.09.2000	0,4259	A7, A10	0,4524	A7	0,4128
28	29.09.2000	0,3719	A4, A5, A6	0,3601	A5	0,3601
29	07.10.2000	0,3505	A4, A5, A6	0,3601	A5	0,3601
30	15.10.2000	0,3217	A4, A5, A6	0,3601	A4	0,3337
MSE			**0,0066**		**0,0017**	
MAPE (%)			**14,4730**		**4,6533**	
MPE (%)			**– 0,0488**		**– 0,0205**	

At the end of Table 5, the values of the MSE (Mean Squared Error), the MAPE (Mean Absolute Percentage Error) and the MPE (Mean Percentage Error) are presented, which reflect the quality of the constructed predictive models, their adequacy and accuracy. Errors according to these criteria are calculated by the formulas [7]:

$$\text{MSE}=\frac{1}{m}\sum_{j=1}^{m}(F_t-A_t)^2,\quad \text{MAPE}=\frac{1}{m}\sum_{t=1}^{m}\frac{|F_t-A_t|}{A_t}\times 100\,\%,\quad \text{MPE}=\frac{1}{m}\sum_{t=1}^{m}\frac{F_t-A_t}{A_t}\times 100\,\%,$$

where m is the length of the time series; A_t is the value of the NDVI index at time t; F_t is the predict of A_t.

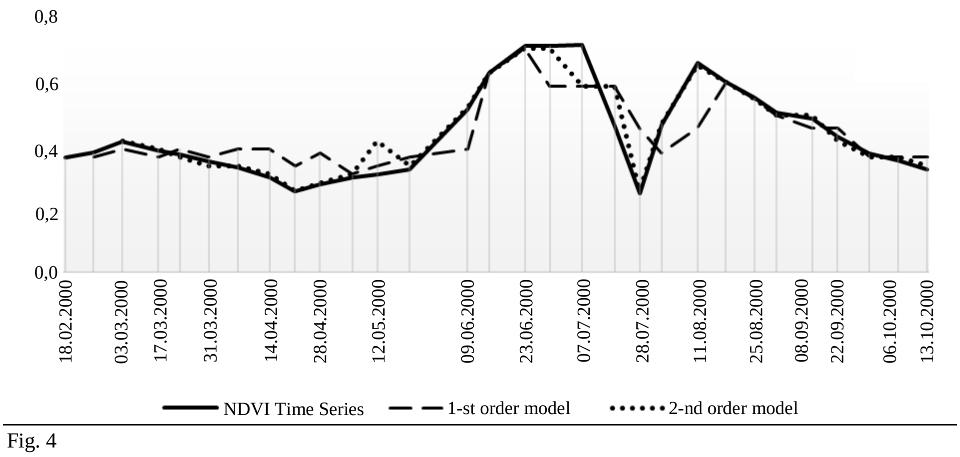

Fig. 4

The MSE criterion is most often used in choosing the optimal predictive model and emphasizes large forecast errors. As can be seen from the forecasting results, the error value for this criterion is quite low. The MAPE indicator shows how large the forecast errors are compared to the real values of the time series. MPE is a more informative criterion for assessing the adequacy of the predictive model, which determines the «bias» of the constructed predict, that is, its permanent underestimation or overestimation. MPE indicators for predictive models of the 1-st and 2-nd orders are $(-0,0488)$ and $(-0,0205)$, respectively, which reflects a slight bias, that is, having percentage values close to zero, not exceeding 5 %. Otherwise, if there were a large negative MPE percentage, then the predictive models would be consistently overestimating. If the MPE indicator showed a large positive percentage value, then the constructed predictive models would be consistently underestimating.

Forecasting the fuzzy time series of the NDVI: method №2

Based on the above procedures (Step 1–Step 5) for the time series relative to NIR and RED reflectance (see Table 1), the final average distance $AD_{\text{NIR}} = 0,0144$ and $AD_{\text{RED}} = 0,0022$ were established. In accordance with these values, the required universes for each time series are obtained in the form:

$$U_{\text{NIR}} = [0,1106 - 0,0144; 0,8702 + 0,0144] = [0,0962; 0,8846],$$

where 0,1106 and 0,8702 are the minimum and maximum values of the NIR reflection coefficient, respectively.

$$U_{\text{RED}} = [0,0659 - 0,0022; 0,1623 + 0,0022] = [0,0637; 0,1645],$$

where 0,0659 and 0,1623 are the minimum and maximum values of the RED reflection coefficient, respectively.

Based on the application of formula (5), the optimal values of the number of evaluation criteria for estimation the NIR and RED reflection coefficient were obtained as follows:

$$m_{\text{NIR}} = \frac{0,8846 - 0,0962 - 0,0144}{2 \cdot 0,0144} = 26,9158 \approx 27,$$

$$m_{\text{RED}} = \frac{0,1645 - 0,0637 - 0,0022}{2 \cdot 0,0022} = 22,2444 \approx 22.$$

To describe the qualitative evaluation criteria by fuzzy sets, trapezoidal membership functions in the form of (6) are also used. As a result, to evaluate NIR time series data (see Table 8 and Fig. 5) the appropriate fuzzy sets A_i $(i = 1 \div 27)$ were formed, and to evaluate RED time series data (see table 9 and Fig. 6) the appropriate fuzzy sets B_j $(j = 1 \div 22)$ were formed.

Table 8

Fuzzy set	Membership function parameters				Fuzzy set	Membership function parameters			
	a_{i_1}	a_{i_2}	a_{i_3}	a_{i_4}		a_{i_1}	a_{i_2}	a_{i_3}	a_{i_4}
A1	0,0962	0,1106	0,1250	0,1394	A15	0,4988	0,5132	0,5276	0,5419
A2	0,1250	0,1394	0,1537	0,1681	A16	0,5276	0,5419	0,5563	0,5707
A3	0,1537	0,1681	0,1825	0,1969	A17	0,5563	0,5707	0,5851	0,5994
A4	0,1825	0,1969	0,2112	0,2256	A18	0,5851	0,5994	0,6138	0,6282
A5	0,2112	0,2256	0,2400	0,2544	A19	0,6138	0,6282	0,6426	0,6570
A6	0,2400	0,2544	0,2688	0,2831	A20	0,6426	0,6570	0,6713	0,6857

Continuation Table 8

A7	0,2688	0,2831	0,2975	0,3119	A21	0,6713	0,6857	0,7001	0,7145
A8	0,2975	0,3119	0,3263	0,3406	A22	0,7001	0,7145	0,7288	0,7432
A9	0,3263	0,3406	0,3550	0,3694	A23	0,7288	0,7432	0,7576	0,7720
A10	0,3550	0,3694	0,3838	0,3982	A24	0,7576	0,7720	0,7864	0,8007
A11	0,3838	0,3982	0,4125	0,4269	A25	0,7864	0,8007	0,8151	0,8295
A12	0,4125	0,4269	0,4413	0,4557	A26	0,8151	0,8295	0,8439	0,8582
A13	0,4413	0,4557	0,4700	0,4844	A27	0,8439	0,8582	0,8726	0,8846
A14	0,4700	0,4844	0,4988	0,5132					

Trapezoidal membership functions of fuzzy sets A_i $(i = 1 \div 27)$.

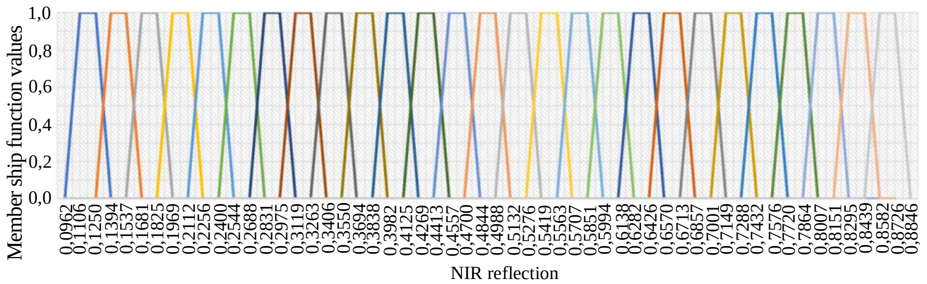

Fig. 5

In Table 9 fuzzy sets as qualitative criteria for RED reflection evaluation are presented.

Table 9

Fuzzy set	Membership function parameters				Fuzzy set	Membership function parameters			
	a_{i_1}	a_{i_2}	a_{i_3}	a_{i_4}		a_{i_1}	a_{i_2}	a_{i_3}	a_{i_4}
B1	0,0637	0,0659	0,0681	0,0703	B12	0,1125	0,1147	0,1169	0,1191
B2	0,0681	0,0703	0,0726	0,0748	B13	0,1169	0,1191	0,1213	0,1235
B3	0,0726	0,0748	0,0770	0,0792	B14	0,1213	0,1235	0,1258	0,1280
B4	0,0770	0,0792	0,0814	0,0836	B15	0,1258	0,1280	0,1302	0,1324
B5	0,0814	0,0836	0,0859	0,0881	B16	0,1302	0,1324	0,1346	0,1368
B6	0,0859	0,0881	0,0903	0,0925	B17	0,1346	0,1368	0,1391	0,1413
B7	0,0903	0,0925	0,0947	0,0969	B18	0,1391	0,1413	0,1435	0,1457
B8	0,0947	0,0969	0,0992	0,1014	B19	0,1435	0,1457	0,1479	0,1501
B9	0,0992	0,1014	0,1036	0,1058	B20	0,1479	0,1501	0,1524	0,1546
B10	0,1036	0,1058	0,1080	0,1102	B21	0,1524	0,1546	0,1568	0,1590
B11	0,1080	0,1102	0,1125	0,1147	B22	0,1568	0,1590	0,1612	0,1645

Trapezoidal membership functions of fuzzy sets B_j $(j = 1 \div 22)$.

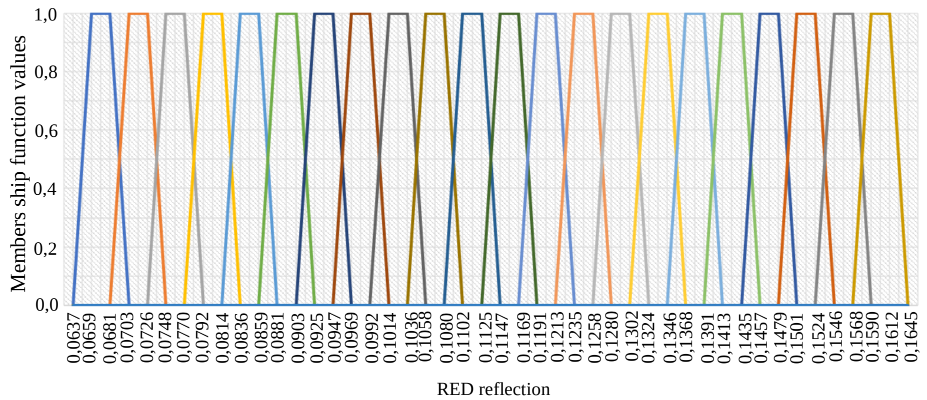

Fig. 6

According to the above principle, fuzzification of NIR and RED reflections is carried out by trapezoidal membership functions, the corresponding parameters of which are presented in Table 8 and Table 9. Obtained fuzzy analogs for all NIR and RED data are summarized in Table 10.

In Table 10 Fuzzy time series of reflection coefficients NIR and RED are shown.

Table 10

N	NIR time series		RED time series		N	NIR time series		RED time series	
	NIR	Fuzzy set	RED	Fuzzy set		NIR	Fuzzy set	RED	Fuzzy set
1	0,2036	A4	0,0958	B7	16	0,8565	A27	0,1452	B19
2	0,3175	A8	0,1445	B18	17	0,8651	A27	0,1453	B19
3	0,3639	A10	0,1523	B20	18	0,8702	A27	0,1455	B19
4	0,3623	A10	0,1623	B22	19	0,3357	A9	0,1256	B14
5	0,2219	A5	0,1025	B9	20	0,1125	A1	0,0678	B1
6	0,1717	A3	0,0835	B5	21	0,3666	A10	0,1348	B16
7	0,1676	A3	0,0845	B5	22	0,6051	A18	0,1245	B14
8	0,1407	A2	0,0765	B3	23	0,5828	A17	0,1463	B19
9	0,1106	A1	0,0659	B1	24	0,4628	A13	0,1354	B16
10	0,1214	A1	0,0689	B1	25	0,3492	A9	0,1158	B12
11	0,1502	A2	0,0815	B4	26	0,3523	A9	0,1233	B14
12	0,1529	A2	0,0813	B4	27	0,3450	A9	0,1389	B17
13	0,1664	A3	0,0855	B5	28	0,2457	A5	0,1125	B11
14	0,4084	A11	0,1324	B16	29	0,2173	A4	0,1045	B9
15	0,5890	A17	0,1356	B16	30	0,2058	A4	0,1056	B10

In the framework of the fuzzy time series NIR and RED, the identified internal relationships of the 1st and 2nd orders are summarized in Table 11 and Table 12, respectively.

Table 11

NIR time series		RED time series	
G1: A1 ⇒ A2, A10, A11	G9: A11 ⇒ A17	G1: B1 ⇒ B1, B4, B16	G9: B14 ⇒ B1, B17, B19
G2: A2 ⇒ A1, A2, A3	G10: A13 ⇒ A9	G2: B3 ⇒ B1	G10: B16 ⇒ B12, B14, B16, B19
G3: A3 ⇒ A2, A3, A11	G11: A17 ⇒ A13, A27	G3: B4 ⇒ B4, B5	G11: B17 ⇒ B11
G4: A4 ⇒ A4, A8	G12: A18 ⇒ A17	G4: B5 ⇒ B3, B5, B16	G12: B18 ⇒ B20
G5: A5 ⇒ A3, A4	G13: A27 ⇒ A9, A27	G5: B7 ⇒ B18	G13: B19 ⇒ B14, B16, B19
G6: A8 ⇒ A10		G6: B9 ⇒ B5, B10	G14: B20 ⇒ B22
G7: A9 ⇒ A1, A5, A9		G7: B11 ⇒ B9	G15: B22 ⇒ B9
G8: A10 ⇒ A5, A10, A18		G8: B12 ⇒ B14	

Table 12

NIR time series		RED time series	
G1: A1, A1 ⇒ A2	G15: A9, A5 ⇒ A4	G1: B1, B1 ⇒ B4	G15: B14, B17 ⇒ B11
G2: A1, A2 ⇒ A2	G16: A9, A9 ⇒ A9, A5	G2: B1, B4 ⇒ B4	G16: B14, B19 ⇒ B16
G3: A1, A10 ⇒ A18	G17: A10, A5 ⇒ A3	G3: B1, B16 ⇒ B14	G17: B16, B12 ⇒ B14
G4: A2, A1 ⇒ A1	G18: A10, A10 ⇒ A5	G4: B3, B1 ⇒ B1	G18: B16, B14 ⇒ B19
G5: A2, A2 ⇒ A3	G19: A10, A18 ⇒ A17	G5: B4, B4 ⇒ B5	G19: B16, B16 ⇒ B19
G6: A2, A3 ⇒ A11	G20: A11, A17 ⇒ A27	G6: B4, B5 ⇒ B16	G20: B16, B19 ⇒ B19
G7: A3, A2 ⇒ A1	G21: A13, A9 ⇒ A9	G7: B5, B3 ⇒ B1	G21: B17, B11 ⇒ B9
G8: A3, A3 ⇒ A2	G22: A17, A13 ⇒ A9	G8: B5, B5 ⇒ B3	G22: B18, B20 ⇒ B22
G9: A3, A11 ⇒ A17	G23: A17, A27 ⇒ A27	G9: B5, B16 ⇒ B16	G23: B19, B14 ⇒ B1
G10: A4, A8 ⇒ A10	G24: A18, A17 ⇒ A13	G10: B7, B18 ⇒ B20	G24: B19, B16 ⇒ B12
G11: A5, A3 ⇒ A3	G25: A27, A9 ⇒ A1	G11: B9, B5 ⇒ B5	G25: B19, B19 ⇒ B19, B14
G12: A5, A4 ⇒ A4	G26: A27, A27 ⇒ A27, A9	G12: B11, B9 ⇒ B10	G26: B20, B22 ⇒ B9
G13: A8, A10 ⇒ A10		G13: B12, B14 ⇒ B17	G27: B22, B9 ⇒ B5
G14: A9, A1 ⇒ A10		G14: B14, B1 ⇒ B16	

As a result, for each value of the NIR and RED reflections within the time series the corresponding crisp predicts were obtained by applications of obtained predictive models and the rule of defuzzification. Desired results are summarized in Table 13 and Table14, respectively.

Table 13

N	Date	NIR	Fuzzy analog	1st order model		2nd order model	
				Fuzzy output	Predict	Fuzzy output	Predict
1	18.02.2000	0,3599	A4				
2	26.02.2000	0,3745	A8	A4, A8	0,2616		
3	05.03.2000	0,4099	A10	A10	0,3766	A10	0,3766
4	15.03.2000	0,3812	A10	A5, A10, A18	0,4053	A10	0,3766
5	21.03.2000	0,3680	A5	A5, A10, A18	0,4053	A5	0,2328
6	29.03.2000	0,3457	A3	A3, A4	0,1897	A3	0,1753
7	06.04.2000	0,3296	A3	A2, A3, A11	0,2424	A3	0,1753
8	15.04.2000	0,2957	A2	A2, A3, A11	0,2424	A2	0,1465
9	22.04.2000	0,2535	A1	A1, A2, A3	0,1465	A1	0,1178
10	29.04.2000	0,2759	A1	A2, A10, A11	0,3095	A1	0,1178
11	08.05.2000	0,2966	A2	A2, A10, A11	0,3095	A2	0,1465
12	15.05.2000	0,3058	A2	A1, A2, A3	0,1465	A2	0,1465
13	24.05.2000	0,3211	A3	A1, A2, A3	0,1465	A3	0,1753
14	09.06.2000	0,5104	A11	A2, A3, A11	0,2424	A11	0,4053
15	15.06.2000	0,6257	A17	A17	0,5779	A17	0,5779
16	25.06.2000	0,7101	A27	A13, A27	0,6641	A27	0,8654
17	02.07.2000	0,7124	A27	A9, A27	0,6066	A27	0,8654
18	11.07.2000	0,7135	A27	A9, A27	0,6066	A9, A27	0,6066
19	20.07.2000	0,4554	A9	A9, A27	0,6066	A9, A27	0,6066
20	27.07.2000	0,2479	A1	A1, A5, A9	0,2328	A1	0,1178
21	02.08.2000	0,4623	A10	A2, A10, A11	0,3095	A10	0,3766
22	12.08.2000	0,6587	A18	A5, A10, A18	0,4053	A18	0,6066
23	20.08.2000	0,5987	A17	A17	0,5779	A17	0,5779
24	28.08.2000	0,5473	A13	A13, A27	0,6641	A13	0,4629
25	03.09.2000	0,5019	A9	A9	0,3478	A9	0,3478
26	13.09.2000	0,4815	A9	A1, A5, A9	0,2328	A9	0,3478
27	20.09.2000	0,4259	A9	A1, A5, A9	0,2328	A3, A9	0,2616
28	29.09.2000	0,3719	A5	A1, A5, A9	0,2328	A3, A9	0,2616
29	07.10.2000	0,3505	A4	A3, A4	0,1897	A4	0,2041
30	15.10.2000	0,3217	A4	A4, A8	0,2616	A4	0,2041

Table 14

N	Date	NIR	Fuzzy analog	1st order model		2nd order model	
				Fuzzy output	Predict	Fuzzy output	Predict
1	18.02.2000	0,0958	B7				
2	26.02.2000	0,1445	B18	B18	0,1424		
3	05.03.2000	0,1523	B20	B20	0,1512	B20	0,1512
4	15.03.2000	0,1623	B22	B22	0,1601	B22	0,1601
5	21.03.2000	0,1025	B9	B9	0,1025	B9	0,1025
6	29.03.2000	0,0835	B5	B5, B10	0,0958	B5	0,0847
7	06.04.2000	0,0845	B5	B3, B5, B16	0,0980	B5	0,0847
8	15.04.2000	0,0765	B3	B3, B5, B16	0,0980	B3	0,0759
9	22.04.2000	0,0659	B1	B1	0,0670	B1	0,0670
10	29.04.2000	0,0689	B1	B1, B4, B16	0,0936	B1	0,0670
11	08.05.2000	0,0815	B4	B1, B4, B16	0,0936	B4	0,0803
12	15.05.2000	0,0813	B4	B4, B5	0,0825	B4	0,0803
13	24.05.2000	0,0855	B5	B4, B5	0,0825	B5	0,0847
14	09.06.2000	0,1324	B16	B3, B5, B16	0,0980	B16	0,1335
15	15.06.2000	0,1356	B16	B12, B14, B16, B19	0,1302	B16	0,1335
16	25.06.2000	0,1452	B19	B12, B14, B16, B19	0,1302	B19	0,1468
17	02.07.2000	0,1453	B19	B14, B16, B19	0,1350	B19	0,1468
18	11.07.2000	0,1455	B19	B14, B16, B19	0,1350	B19, B14	0,1357
19	20.07.2000	0,1256	B14	B14, B16, B19	0,1350	B19, B14	0,1357
20	27.07.2000	0,0678	B1	B1, B17, B19	0,1173	B1	0,0670
21	02.08.2000	0,1348	B16	B1, B4, B16	0,0936	B16	0,1335
22	12.08.2000	0,1245	B14	B12, B14, B16, B19	0,1302	B14	0,1246
23	20.08.2000	0,1463	B19	B1, B17, B19	0,1173	B19	0,1468
24	28.08.2000	0,1354	B16	B14, B16, B19	0,1350	B16	0,1335
25	03.09.2000	0,1158	B12	B12, B14, B16, B19	0,1302	B12	0,1158
26	13.09.2000	0,1233	B14	B14	0,1246	B14	0,1246
27	20.09.2000	0,1389	B17	B1, B17, B19	0,1173	B17	0,1379
28	29.09.2000	0,1125	B11	B11	0,1113	B11	0,1113
29	07.10.2000	0,1045	B9	B9	0,1025	B9	0,1025
30	15.10.2000	0,1056	B10	B5, B10	0,0958	B10	0,1069

Thus, predictive model for NDVI time series is restored by application of the empirical formula (1) for each day according to the corresponding forecast data NIR and RED. The desired models are interpreted in the form of Table 15 and Fig. 7.

Table 15

N	Date	NDVI	Model N 1	Model N 2	N	Date	NDVI	Model N 1	Model N 2
1	18.02.2000	0,3599			16	25.06.2000	0,7101	0,6722	0,7099
2	26.02.2000	0,3745	0,2951		17	02.07.2000	0,7124	0,6360	0,7099

ISSN 1028-0979

Continuation Table 15

3	05.03.2000	0,4099	0,4269	0,4269	18	11.07.2000	0,7135	0,6360	0,6343
4	15.03.2000	0,3812	0,4337	0,4034	19	20.07.2000	0,4554	0,6360	0,6343
5	21.03.2000	0,3680	0,5964	0,3887	20	27.07.2000	0,2479	0,3301	0,2748
6	29.03.2000	0,3457	0,3287	0,3482	21	02.08.2000	0,4623	0,5356	0,4765
7	06.04.2000	0,3296	0,4240	0,3482	22	12.08.2000	0,6587	0,5138	0,6591
8	15.04.2000	0,2957	0,4240	0,3177	23	20.08.2000	0,5987	0,6626	0,5948
9	22.04.2000	0,2535	0,3724	0,2748	24	28.08.2000	0,5473	0,6622	0,5523
10	29.04.2000	0,2759	0,5356	0,2748	25	03.09.2000	0,5019	0,4553	0,5005
11	08.05.2000	0,2966	0,5356	0,2920	26	13.09.2000	0,4815	0,3026	0,4724
12	15.05.2000	0,3058	0,2795	0,2920	27	20.09.2000	0,4259	0,3301	0,3094
13	24.05.2000	0,3211	0,2795	0,3482	28	29.09.2000	0,3719	0,3530	0,4028
14	09.06.2000	0,5104	0,4240	0,5045	29	07.10.2000	0,3505	0,2985	0,3314
15	15.06.2000	0,6257	0,6323	0,6247	30	15.10.2000	0,3217	0,4638	0,3124
MSE								0,0137	0,0021
MAPE (%)								25,5716	5,9490
MPE (%)								– 0,1161	– 0,0182

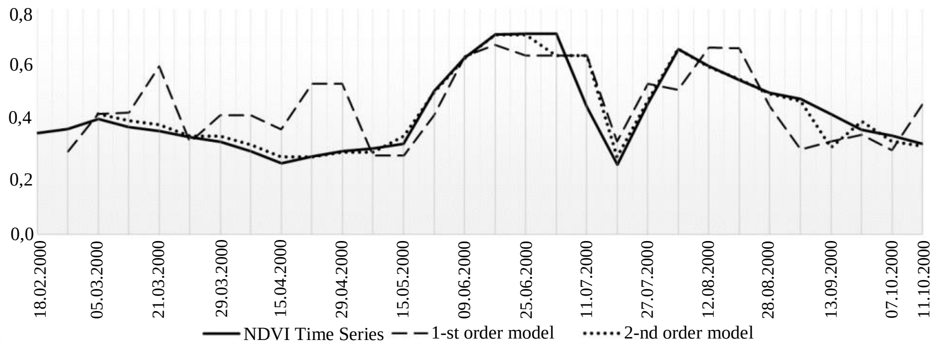

Fig. 7

As can be seen from the results of NDVI forecasting by the 2nd method, the error value according to the MSE criterion is quite low ($0,0137$ for Model N 1, and $0,0021$ for Model N 2), which cannot be said about the error value according to the MAPE criterion ($25,5716$ for Model N 1 and $5,9490$ for Model N 2). The MPEs for the 1st and 2nd order predictive models are $(-0,1161)$ and $(-0,0182)$, respectively, reflecting a slight bias below the 5 % threshold.

Conclusion

The article uses one of the methods of precision farming, associated with the applying of the NDVI vegetation index, which allows to predict crop volumes and most accurately assess the real state of growing plants. The last statement is relative since the index itself does not reflect the absolute values of plant volumes. Nevertheless, according to the obtained multispectral data, it is possible to evaluate the development of crops and predict its future yield. It should be considered that the value of the NDVI index changes throughout the entire growing season, that is, during the initial growth, the period of flowering and maturation its indicators differ significantly, in fact, as this is demonstrated by the dynamics of NDVI using the example of one pixel (see Fig. 1). The practice of precision farming has shown that the most active increase in the NDVI occurs during the growing season, during the flowering period, the growth of the crop slows down and halts, and in the process of crop maturation, the index gradually decreases.

The fuzzy approaches to predicting the annual dynamics of the NDVI proposed in the article can be easily projected to process multispectral data obtained from all pixels of the corresponding vegetation maps. If vegetation maps used in precision farming allow only visually determining differences in the state of plants, then owing to digitalization it becomes possible to interpret the color range of vegetation — from light tones with a low index to dark color with high NDVI index.

Е.Р. Алієв, Ф.М. Салманов

НЕЧІТКИЙ ПІДХІД ДО ПРОГНОЗУВАННЯ ДИНАМІКИ ВЕГЕТАЦІЙНИХ ІНДЕКСІВ

Алієв Ельчин Рашид огли

Інститут систем керування Національної Академії Наук Азербайджана,
elchin.aliyev@sinam.net

Салманов Фуад Мухтар огли

Інститут систем керування Національної Академії Наук Азербайджана,
fuad.salmanli@sinam.net

Сучасні технології супутникового моніторингу поверхні Землі надають сільськогосподарським виробникам корисну інформацію — стан здоров'я посівних культур. Здатність віддаленого датчика виявляти незначні відмінності у рослинності робить його корисним інструментом для кількісної оцінки мінливості в межах заданого поля, оцінки зростання сільськогосподарських культур та управління угіддями на основі поточних умов. Дані дистанційного зондування, що збираються на регулярній основі, дозволяють виробникам та агрономам складати поточну карту вегетації, що відображає стан та силу посівних культур, аналізувати динаміку зміни у стані рослин, а також прогнозувати врожайність на конкретній посівній площі. Для інтерпретації цих даних найефективнішими засобами є всілякі вегетаційні індекси, що розраховуються емпірично, тобто шляхом операцій із різними спектральними діапазонами мультиспектральних даних супутникового моніторингу. На основі часового ряду одного з таких вегетаційних індексів у статті розглядається річна динаміка розвитку посівної культури на конкретному полі. Можливість прогнозування врожайності цієї посівної культури розглядається на основі нечіткого моделювання часових рядів за відповідним спектральним діапазоном відображення вегетації, отриманим зі знімків супутникового моніторингу. Запропоновано нечіткі моделі часових рядів, вивчені на адекватність і придатність з погляду аналізу особливості внутрішньорічної середньорічної динаміки індексу, типової для цієї посівної площі.

Ключові слова: сільскогосподарська культура, мультиспектральне відображення рослин, вегетаційний індекс, нечітка множина, нечіткий часовий ряд.

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