UDC 004.9; 535.6; 681.5

V. Timchenko, Yu. Kondratenko, V. Kreinovich

EFFICIENT OPTICAL APPROACH TO FUZZY DATA PROCESSING BASED ON COLORS AND LIGHT FILTERS

Victor Timchenko

Admiral Makarov National University of Shipbuilding, Mykolaiv,

vl.timchenko58@gmail.com

Yuriy Kondratenko

Petro Mohyla Black Sea National University, Mykolaiv,

y_kondrat2002@yahoo.com

Vladik Kreinovich

University of Texas at El Paso, USA,

vladik@utep.edu

This work is devoted to the creation of effective optical logic systems based on the use of light emitter of a certain color directly as a fuzzy variable — the carrier of logical information and the basis for building logical solutions by transforming light emitter with appropriate light filters. Optical processing of color information, which reflects different values of input data (considered on the example of expert evaluations), is carried out by the proposed structural construction of fuzzy logic gates (logic coloroid) and is significantly simplified, in relation to existing systems, due to the implementation on the properties of additive and subtractive color processing using fairly simple light filters. A fuzzy database was formed based on the definition of the quantum of information as the corresponding color, and the components of the optical logic system based on the additive and subtractive processing of the light emitter of the corresponding colors; the basics of the synthesis of logical inference and decision-making systems have been developed. The paper synthesizes a generalized structural diagram of an optical logic coloroid as a basis for creating a multi-level decision-making system for further application in artificial intelligence systems. Schemes of optical logic coloroids can be combined into series-parallel hierarchically organized schemes, the color of the used light filters can also, in addition to expert assessments, reflect the tactile information of sensor systems about the environment, which is necessary for the formation of appropriate logical assessments or decisions. The use of color as a carrier of logical information allows you to create fast-acting technical devices with performance based on the calculations of which the speed of light is used to form a certain array of logical solutions.

Keywords: quantum of information, fuzzy logical gates, optical coloroid, light color filters.

Introduction. While modern computers are very fast, many practical problems still require even faster computations. In modern computers, there are usually several processors working in parallel. So, to speed up computations, we can speed up each processor and/or increase the number of processors. Both ideas are used when designing high-performance computing.

To speed up each processor, a natural idea is to transmit the information between different components of this processor. According to modern physics, all speeds are limited by the speed of light, and the only particles that can travel with the speed of light are photons (and other more exotic particles of zero rest mass). Thus, a natural idea is to use photons — in particular, the usual light — or process information. The use of light can also help with parallelization — it is easy to have a large number of light beams sending information in parallel.

The idea of optical computing — i.e., using optical signals to speed up data processing — dates back to the mid-20th century. Since then, a lot of progress has been achieved in implementing this idea. At present, there are two trends in optical computing.

The first trend is optoelectronic computational devices, in which optical components are used to transmit information (and even to perform some data processing tasks), while the more traditional semiconductor-based components transform optical signals into the usual electronic form and perform the remaining computational tasks on the resulting electric signals. The second — more ambitious — trend is to design all-optical computational devices in which all logic gates — the basis of modern computers — process optical information [1–11]. Optical switching gates are based on the properties of interference (for example, based on the displacement and combination of optical interference fringes, the use of a Mach-Zehnder interferometer, etc.), on the polarization and coherence of a light beam, and on using the properties of diffraction gratings and photonic crystals. The effectiveness of such all-optical gates is growing exponentially: while in 2009, the best all-optical gates could process 250 Megabytes per second [9], in 2020, the best all-optical gates are 4,000 times faster — they can process 1 Terabyte per second [10], much faster than electronic-based devices that only operate in the Gigabyte range. From the theoretical viewpoint, these gates can be made even faster, the only factor that limits further increase in processing speed is the imperfection of production technologies.

Because of this imperfection, at present, optical computing devices are more difficult to manufacture and thus, more expensive than electronics-based computers and are, therefore mostly used in problems where traditional computers cannot process the information sufficiently fast.

To analyze how we can better utilize the advantages of optical computing when processing data, let us recall where the original data comes from. A large amount of data comes from measurements, and processing the numbers coming from measurements was the main task for which the computers were designed in the first place. Researchers and engineers have been designing better and better algorithms for processing these numbers and better and better devices for implementing these algorithms. However, by the early 1960s, they encountered a problem. This problem was first explicitly formulated by Professor Lotfi A. Zadeh, who at that time was one of the world's leading specialists in optimal control and a co-author of the most popular textbook on this topic. He noticed that in many practical applications, e.g., in chemical plants, in manufacturing, and even in control of cars and airplanes, the control optimal with respect to all available numerical information was not as efficient as the control of the human experts. Zadeh naturally concluded that experts possess additional knowledge and skills not captured by the existing numerical models [12–14].

Experts were usually willing to share their knowledge, but the problem was that this knowledge was often formulated not in numerical terms, but by using imprecise («fuzzy») words from a natural language such as «small». For example, an expert driver driving on a freeway knows what to do when the car in front of him/her suddenly slows down a little bit: he/she needs to slow down his/her car similarly. But to implement this recommendation in an automatic system, we need to know with what pressure we need to hit the brakes and for how many milliseconds — and this information a driver cannot supply.

To translate the experts' imprecise information into computer-understandable terms, Zadeh designed a new methodology which he called fuzzy. In this methodology, to describe each imprecise property like «small», we ask the expert, for each possible value x of the corresponding quantity, to mark, on a scale from 0 to 1, to what extent this value satisfies the given property. The computer then processes the resulting degrees $m(x)$ corresponding to different values x. The function that maps a value x into the corresponding degree $m(x)$ is called a membership function.

Often, expert's decisions are based on several conditions. For example, the pressure with the driver need to hit the brakes depends also on the road condition, whether it is raining or not, whether there is ice on the road, etc. To represent correctly corresponding rules, we need to know not only to what extent one quantity is small, but to what extent the quantity x is small and some other quantity y is medium, etc. It would be nice to be able to extract the degrees corresponding to all possible tuples $(x, y, \ldots)$ from the experts, but this is not practically possible: the number of combinations becomes astronomic, and it is not possible to ask experts millions (or even billions) questions. In such situations, when we need to estimate the degree of expert's confidence in a composite statement like «A AND B» $(A \wedge B)$ and «A OR B» $(A \vee B)$, the only information that we have are the expert's degrees of confidence a and b in statements A and B. The algorithms that transform these degrees a and b into a degree for $A \wedge B$ and $A \vee B$ are known as, correspondingly, «AND» — operations (t — norms) and «OR» — operations (t — conorms). The simplest such operations are $\min(a, b)$ for «AND» and $\max(a, b)$ for «OR». These simplest operations are among the most frequently used.

Fuzzy methodology had a lot of successful applications, it led to efficient control of trains, elevators, cars, as well as rice cookers and video cameras. Interestingly, fuzzy logic was also successfully used to control light emission [15].

Due to these practical successes, fuzzy data processing has become an important part of data processing in general. So, not surprisingly, researchers have shown interest in the use of optical computing for processing fuzzy data [16–27]. The first optical computing devices for processing fuzzy data were based on several different ideas: on the representation of logical operations using the optical effect of anisotropic scattering [16], on a programmable array of prisms [17], and on the calibration of shadow diagrams, for example, an optical system based on a zone coding scheme and a shadow casting method [18].

Such devices also helped with parallelization: e.g., an optoelectronic fuzzy inference system for parallel processing of many fuzzy rules based on a spatial light modulator with the implementation of various membership functions was described in [19]. The authors of [20] showed how to use, for this purpose, a spatial modulator of a Gaussian laser light source and a system of micro prisms. Interestingly, similar parallelization ideas were proposed for chemical-based [21, 22] and biological-based [23, 24] processing of fuzzy data.

Systems in which optical computing is used for processing fuzzy data have been successfully used in technical applications; see, e.g., [25, 26]. However, such systems are not yet widely spread, and one the reasons for this is that these systems, in effect, copy the traditional computers when processing data. In the processing units of the traditional computers, all the numbers are processed with very high accuracy, 64 or even 128 bits, irrespective of whether these numbers come (a) from super-precise measurements where all these bits are important, (b) from a measurement with 10 % accuracy, where only the first decimal digit is reliable, or (c) from an expert estimate, where an expert can meaningfully distinguish between 7 plus minus 2 different degrees. The more bits we use, the more logical gates we need to process this data, and, as a result, the more complex is the resulting construction.

As a result, the currently proposed fuzzy optimal computing devices are very complex, requiring arrays of lenses, and prisms, complex diffraction gratings, additional devices in the form of piezoelectric crystal elements and optoelectronic phase shifters, coding systems, holograms, shadow images, etc. — as well as high accuracy requirements on the quality of light emitters — requirements that can only be satisfied by expensive laser light sources. A large number of processing elements leads to high costs and high energy consumption.

A known way to simplify the resulting computations in problems related to Artificial Intelligence — where a large amount of data comes from experts, and many other data points come from low-accuracy measurements — is to take this low accuracy into account, i.e., in effect, to «blur» and «fuzzify» the given numbers; see, e.g., [14, 27]. For optical data processing of fuzzy data, this will allow us to utilize fully the main advantages of optical computing: processing speed, compactness, and a practically unlimited possibility of parallelization.

How can we do it? To process a single fuzzy degree on a traditional computer, we need to store (and process) a real number from the interval [0, 1]. As we have mentioned, in practice, we only need to distinguish between about 7 degrees. So, a natural idea is to use an optical phenomenon where we humans also distinguish about 7 different main valued — the phenomenon of color, where we distinguish between 7 or so basic colors in the spectrum.

Based on this idea, we propose to use color to describe possible fuzzy degrees. The use of color to describe different degrees is not just a technical idea, it is perfectly in line with common sense. For example, we usually use red to describe a threat situation, green as an all-clear situation, and yellow as an intermediate degree of danger. Many linguistic expressions use white to indicate good and black to indicate bad; see, e,g., [28, 29].

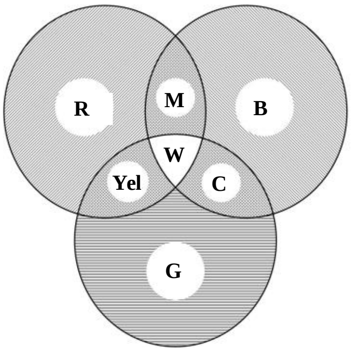
To process the corresponding optical filters, we propose to use color filters. This way, the optical processing of fuzzy information is greatly simplified. The main objective of this paper is to show how color can indeed be used to process fuzzy information.

General information about colors. It is well known that all visible colors can be obtained by an appropriate combination of three basic colors: red **R**, green **G**, and blue **B**. When we do not have any colors, we perceive it as the color black **Blc**. When we combine all three colors in equal proportion (Fig. 1, *a*, *b*), we get white color **W**; when we combine red and blue, we get magenta **M**; when we combine red and green, we get yellow **Yel**, and when we combine green and blue, we get cyan **C**:

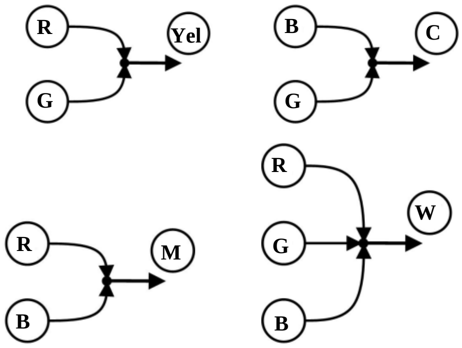
$$\mathbf{R} + \mathbf{G} + \mathbf{B} = \mathbf{W};\ \mathbf{R} + \mathbf{G} = \mathbf{Yel};\ \mathbf{R} + \mathbf{B} = \mathbf{M};\ \mathbf{G} + \mathbf{B} = \mathbf{C}. \quad (1)$$

These are the colors that we will use in our proposal. We assume that we have ideal filters [30] corresponding to all three basic colors (red, green, and blue) and all three combined colors (yellow, magenta and cyan). Of course, combining the two lights of the same color does not change this color:

$$\mathbf{R} + \mathbf{R} = \mathbf{R};\ \mathbf{G} + \mathbf{G} = \mathbf{G};\ \mathbf{B} + \mathbf{B} = \mathbf{B}.$$



a



b

Fig. 1

We can block some basic colors if we apply filters (Fig. 2). For example, the red filter blocks green and blue components, learning only the red color; we can describe it as **R** = **W** – **G** – **B**; we can write similar expressions describing the blue filter **B** = **W** – **R** – **G** and the green filter **G** = **W** – **R** – **B**.

We can also have a yellow filter that blocks the blue components of the white light and keeps only the red and green components, which form the yellow light filter (light filter F_1) **W** – **B** = **R** + **G** = **Yel**; we can similarly have a cyan filter (F_2) for which **W** – – **R** = **G** + **B** = **C** and a magenta filter (F_3) for which **W** – **G** = **R** + **B** = **M**; see Fig. 3.

If we block all three color components, we end up with black color (Fig. 4, *a*): **W** – – **R** – **G** – **B** = **Blc**.

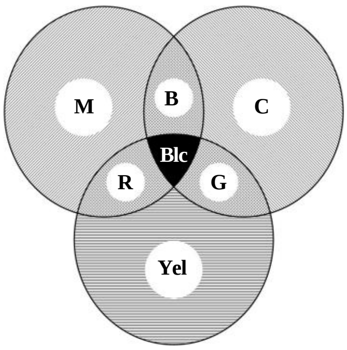


Fig. 2

When a yellow light emitter passes through a red filter, the green color is blocked, and the output is red

$$\mathbf{Yel} - \mathbf{G} = \mathbf{R}, \quad (2)$$

through the green filter, the red color is blocked, and the output is green

$$\mathbf{Yel} - \mathbf{R} = \mathbf{G}, \quad (3)$$

through the blue filter, red and green are blocked, and the output is black (i.e., the absence of light emitter) color

$$\mathbf{Yel} - \mathbf{R} - \mathbf{G} = \mathbf{Blc}. \quad (4)$$

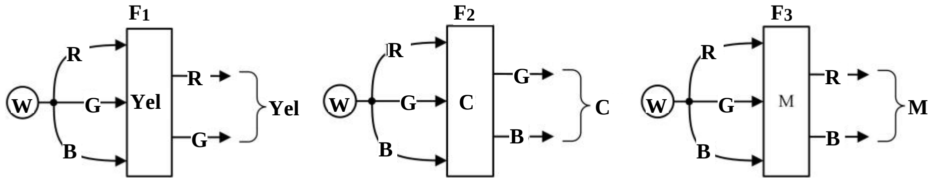


Fig. 3

Similar dependencies can be obtained for other combinations of the color of the light emitter and the light filter. In particular, by combining **Y**, **C** and **M** filters, it is possible to separate the main colors (Fig. 4, *b*–*d*):

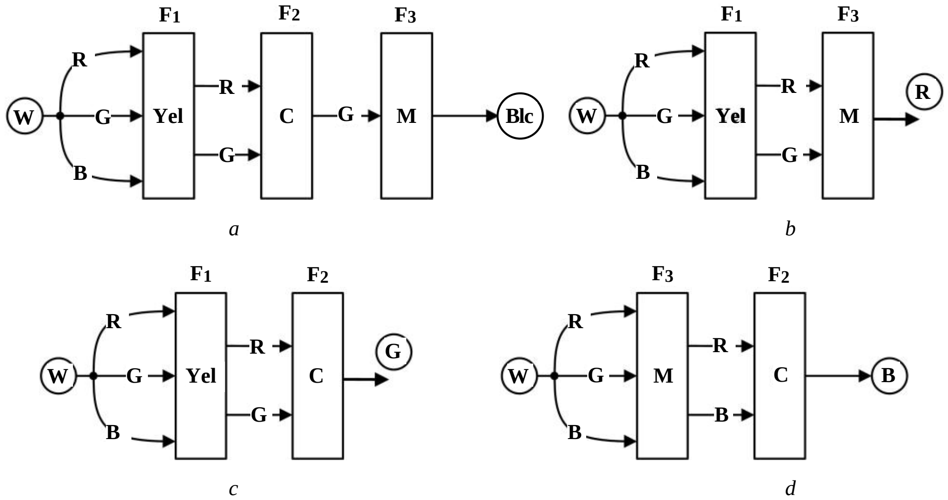


Fig. 4

Logic of colors. In line with the above idea, let us associate colors with fuzzy degrees of confidence. As we have mentioned, it is natural to associate «**no**» (**N**) — i.e., no confidence — with the red color **R**. The absolute confidence — described by «yes» — is the absolute opposite of «**no**». So, it is reasonable to associate «**yes**» with the color which is as far away from the «**no**» — color as possible, i.e., with the color blue which is on the other side of the spectrum from red. Thus, we take **B** = **Y** «**yes**». The remaining basic color green is natural to associate with the intermediate degree of confidence: **G** = **YN** «**probably yes**».

Interpretations of combinations of basic colors can be naturally associated with the combinations of the corresponding degrees of confidence:

W = **R** + **G** + **B** = **YYNN** «**positive decision**»; (5)

C = **G** + **B** = **YYN** «**very probably yes**»;

M = **R** + **B** = **NY** «**probably no**», negative evaluation (for additional color);

Yel = **R** + **G** = **NNY** «**very probably no**»;

Blc = **W** – **R** – **G** – **B** = 0 «**no decision**».

Then, for example, equations (2)–(4) will look like

NNY – **YN** = **N**; **NNY** – **N** = **YN**; **NNY** – **N** – **YN**= 0. (6)

ISSN 2786-6491

The operations of addition and subtraction of color, in view of (5), can be naturally interpreted as the operations of union (disjunction) and intersection (conjunction) of sets (logical statements, operations).

Tables 1 (disjunction) and 2 (conjunction) describe the correspondence between the usual Boolean logic and our logic color gates («coloroid») for red **R** (**false**, 0), blue **B** (**true**, 1), and magenta **M** (**false/true**, 0/1) colors:

Table 1

R(0)	**R**(0)	**R** ∨ **R** = **R**(0)
R(0)	**B**(1)	**R** ∨ **B** = **M**(0/1)
B(1)	**R**(0)	**B** ∨ **R** = **M**(0/1)
B(1)	**B**(1)	**B** ∨ **B** = **B**(1)

Table 2

R(0)	**R**(0)	**R** ∧ **R** = **R**(0)
R(0)	**B**(1)	**R** ∧ **B** = 0(0)
B(1)	**R**(0)	**B** ∧ **R** = 0(0)
B(1)	**B**(1)	**B** ∧ **B** = **B**(1)

The color subtraction operation corresponds to the operation of the intersection of the set determined by the color of the input light emitter entering the light filter and the set determined by the color of the light filter. In this case, set **R** and set **B** do not intersect, since the red filter blocks the blue color and the blue filter blocks the red color. For sets defined by the color **R** and **M**, the operations of disjunction and conjunction form Tables 3 (disjunction) and 4 (conjunction) (when added, a secondary purple-red color **MR** is formed).

Table 3

R(0)	**R**(0)	**R** ∨ **R** = **R**(0)
R(0)	**M**(0/1)	**R** ∨ **M** = **MR**(0/0/1)
M(0/1)	**R**(0)	**M** ∨ **R** = **MR**(0/0/1)
M(0/1)	**M**(0/1)	**M** ∨ **M** = **M**(0/1)

Table 4

R(0)	**R**(0)	**R** ∧ **R** = **R**(0)
R(0)	**M** (0/1)	**R** ∧ **M** = **R**(0)
M(0/1)	**R**(0)	**M** ∧ **R** = **R**(0)
M(0/1)	**M**(0/1)	**M** ∧ **M** = **M**(0/1)

In this case, for logical equations (5), when passing through the filters of the corresponding color, the corresponding blocking logical equations (6) will be subtracted (for convenience, we will continue to use the operations of summation and subtraction of colors adopted in colorimetry [28–30]) or the logical equations determined by the color of the used filters will be summed, and then reduced to the form of a logical equation corresponding to the primary or additional color

$$\mathbf{NNY} - \mathbf{YN} = \mathbf{N} \text{ or } \mathbf{NNY} + \mathbf{N} = \mathbf{NNNY} = \mathrm{N};$$
$$\mathbf{NNY} - \mathbf{N} = \mathbf{YN} \text{ or } \mathbf{NNY} + \mathbf{YN} = \mathbf{NNY}; \quad (7)$$
$$\mathbf{NNY} - \mathbf{N} - \mathbf{YN} = 0 \text{ or } \mathbf{NNY} + \mathbf{Y} = \mathbf{NNYY} = 0.$$

At the same time, for the additional equation of system (7), it follows that the operations of subtraction and summation lead to different results. This corresponds to the provisions of the theory of light filters [30] that it is more accurate to use the operation of subtracting the color blocked by the light filter than summing the color of the input light emitter and the color of the light filter.

Nevertheless, for a general illustration of the approach, in the future, both forms of operations with logical expressions for light filters will be used implying that if the results obtained differ, the exact solution is the logical operation of the intersection.

The remaining colors can be represented as follows: **G** (**true/false**, 1/0), **C** (**true/true/false**, 1/1/0), **Yel** (**true/false/false**, 1/0/0). To get a better relation with the usual fuzzy logic, in which degrees of confidence take values from the interval [0, 1], each color can be assigned an appropriate numerical value from this interval. For example, **R**(0); **Yel**(0,25); **G**(0,5); **C**(0,75); **B**(1); **M**(0,5); **R**(0), which corresponds to the location of the color on the inner hexagon of the circular spectrum when listed counterclockwise. We can also consider a secondary purple-red color (Table 3) **MR** (**false/false/true**, 0/0/1) with a value of 0,25.

Disjunction is carried out by adding light emissions based on equations (1), and conjunction using equations (2)–(4) by subtracting a certain color from the light flux using filters of the corresponding color (filter **R** passes **R**, blocks **B** and **G**; filter **B** passes **B**, blocks **R** and **G**; filter **G** passes **G**, blocks **B** and **R**).

Logical negation corresponds to the accepted (according to the information quantization system (5)), opposite color pairs $\mathbf{B} = \neg\mathbf{R}, \mathbf{C} = \neg\mathbf{Yel}, \mathbf{G} = \neg\mathbf{M}$ (or, in logical terms $\mathbf{Y} \neg \mathbf{N}$, $\mathbf{YYN} \neg \mathbf{NNY}$, $\mathbf{YN} \neg \mathbf{NY}$), as well as $\mathbf{W} = \neg\mathbf{Blc}$. The implementation of the negation operation is most difficult because it requires not just the transformation of the light emitter, as in disjunction and conjunction, but a change to the opposite color, which requires a more elaborate constructive solution — and we hope that such a solution will be designed. For example, this solution may be based on the separation of the primary and additional colors into separate light beams due to different refractive indices and, further, the production of light emitter opposite in color through the corresponding induced channel.

A more general approach to describing light emitters of different colors as sets can be considered, which is based on a matrix representation. Let's imagine a light emitter and a light filter of a certain color in the form of a 3×3 diagonal matrix:

$$\mathbf{R} = \begin{pmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \mathbf{G} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{pmatrix}; \mathbf{B} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B \end{pmatrix}; \mathbf{Yel} = \begin{pmatrix} R & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$\mathbf{M} = \begin{pmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B \end{pmatrix}; \mathbf{C} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & B \end{pmatrix}; \mathbf{W} = \begin{pmatrix} R & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & B \end{pmatrix}; \mathbf{Blc} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The disjunction operation will be defined as the union (addition) of sets, for example,

$$\mathbf{W} = \begin{pmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \vee \begin{pmatrix} 0 & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{pmatrix} \vee \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B \end{pmatrix} = \begin{pmatrix} R & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & B \end{pmatrix}.$$

We can also describe the idempotency property in these terms; for example, for red

$$\mathbf{R} = \begin{pmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \vee \begin{pmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (8)$$

The operation of conjunction (intersection of sets) will be determined by multiplying the corresponding sets (and taking into account the idempotency). For example, for yellow:

$$\mathbf{Yel} = \begin{pmatrix} R & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{pmatrix} \wedge \begin{pmatrix} R & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} R & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (9)$$

Then we get the following representation of white light emitter passing through the **Yel** filter, considering (9),

$$\begin{pmatrix} R & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & B \end{pmatrix} \wedge \begin{pmatrix} R & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} R & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The resulting yellow light then passes through a magenta light filter **M**

$$\begin{pmatrix} R & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{pmatrix} \wedge \begin{pmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B \end{pmatrix} = \begin{pmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and cyan light filter **C**

$$\begin{pmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \wedge \begin{pmatrix} 0 & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & B \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

At the output, we get an empty set corresponding to the black color **Blc**.

The corresponding matrix expressions for equations (2)–(4) will be written in the form

$$\mathbf{R} = \begin{pmatrix} R & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{pmatrix} \wedge \begin{pmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$\mathbf{G} = \begin{pmatrix} R & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{pmatrix} \wedge \begin{pmatrix} 0 & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$\mathbf{Blc} = \begin{pmatrix} R & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{pmatrix} \wedge \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The above operations can be written by using an abbreviated description of the corresponding diagonal matrices; for example, for the equations (7), we get:

$$\mathrm{diag}(R, G, 0) \wedge \mathrm{diag}(R, 0, 0) = \mathrm{diag}(R, 0, 0);$$

$$\mathrm{diag}(R, G, 0) \wedge \mathrm{diag}(0, G, 0) = \mathrm{diag}(0, G, 0);$$

$$\mathrm{diag}(R, G, 0) \wedge \mathrm{diag}(0, 0, B) = \mathrm{diag}(0, 0, 0).$$

With a matrix representation of light emitter and light filters, the advantage is that the operations of subtracting and adding colors accepted in colorimetry are correctly described, and only primary colors are used in this description. Similarly, the transformations of the light emitter of different colors for different filters are correctly described.

Let's continue to consider the system of equations (5), which specifies the appropriate gradation of the qualitative assessments of experts (for the considered example of input information in the form of expert assessments). On its basis, by considering the optical transformation of light emissions of different colors, we will form a certain logical conclusion.

For example, suppose that three expert ratings are given (let's call them a «**positive**» group, because, under conditions of opposite ratings, the key decision is defined as **YN** «**probably yes**»): **N**, which corresponds to red; **Y** (blue) and **YN** (green). The addition (combining) of these light emissions (optical schemes as shown in Fig. 1, *b*) forms the white color **W** and the estimate **YYNN**, which corresponds to a positive decision based on these conflicting estimates.

For three ratings (of the same positive group of experts) **N**, **N**, **N** the general decision **N** follows (color **R**, «**no**»); for **Y**, **Y**, **Y** — general solution Y (color **B**, «**yes**»); for **YN**, **YN**, **YN** — the general solution **YN** (color **G**, «**probably yes**»).

In general, with three estimates, if we consider the light emitters of the above three basic color and three combined colors, and take into account idempotency (8), we arrive at the following decisions:

for estimates

$$\mathbf{N}, \mathbf{N}, \mathbf{Y} = \mathbf{N}, \mathbf{Y}, \mathbf{Y} = \mathbf{NY} \qquad (10)$$

— the common decision **NY** (color **M**, «**probably no**»);

for estimates

$$\mathbf{N}, \mathbf{N}, \mathbf{YN} = \mathbf{N}, \mathbf{YN}, \mathbf{YN} = \mathbf{NNY} \qquad (11)$$

— the common decision **NNY** (color **Yel**, «**very probably no**»);

for

$$\mathbf{Y}, \mathbf{Y}, \mathbf{YN} = \mathbf{Y}, \mathbf{YN}, \mathbf{YN} = \mathbf{YYN} \qquad (12)$$

— the common decision **YYN** (color **C**, «**very probably yes**»).

If only primary and additional colors (discarding tones or secondary colors) are considered as well as the white light emitter as in expressions (10)–(12), then we get 7 solutions.

Let's form the second group of three experts and define it as «**negative**» (under conditions of opposite assessments, the key decision is defined as **NY** «**probably no**») with possible assessments of **YYN**, which corresponds to the blue color; **NY** (magenta) and **NNY** (yellow).

The solution when assessing the positive group **R**, **G**, **B** and obtaining white light, which then passes through a system of three light filters with expert estimates **C**, **M**, **Yel**, results in black light (Fig. 4, *a*), i.e. its absence, which corresponds to the absence of a decision

$$\mathbf{R} + \mathbf{G} + \mathbf{B} = \mathbf{W} - \mathbf{R} - \mathbf{G} - \mathbf{B} = 0 \text{ or } \mathbf{Y} + \mathbf{N} + \mathbf{YN} = \mathbf{YNYN} - \mathbf{Y} - \mathbf{N} - \mathbf{YN} = 0.$$

It is logical that with the total assessments of experts: «**no, yes, probably yes, very probably yes, probably no, very probably no**», and even with an intermediate positive decision, the final negative decision follows, taking into account completely contradictory expert assessments.

The disadvantage of the optical scheme shown in Fig. 4, *a* is the blocking by the system of light filters (negative group of experts with estimates of form **C**, **M**, **Yel**) also other estimates at the output of the positive decision-making block. For example, in the primary assessment of the positive group of experts **Y**, **Y**, **Y**, the blue color will be blocked by a yellow light filter, which does not correspond to the general primary decision **Y** in the secondary assessment by the negative group of experts

$$\mathbf{Y} + \mathbf{YYN} + \mathbf{NY} + \mathbf{NNY} = \mathbf{YYN},$$

those, output estimate **YYN**, which can be interpreted as **very probably yes**.

To consider this contradiction, optical schemes are proposed, shown in Fig. 4, *b*–*d*. The optical scheme that implements solutions for two groups of experts includes a system of light filters corresponding to the generated color estimates. Thus, there will be six expert assessments in total (Fig. 5). Then, for example, the light emission of blue in the initial evaluation of **Y** will pass through a group of two light filters cyan and magenta; red light emitter will pass through a group of two light filters yellow and magenta; green light emitter will pass through a group of two light filters yellow and cyan.

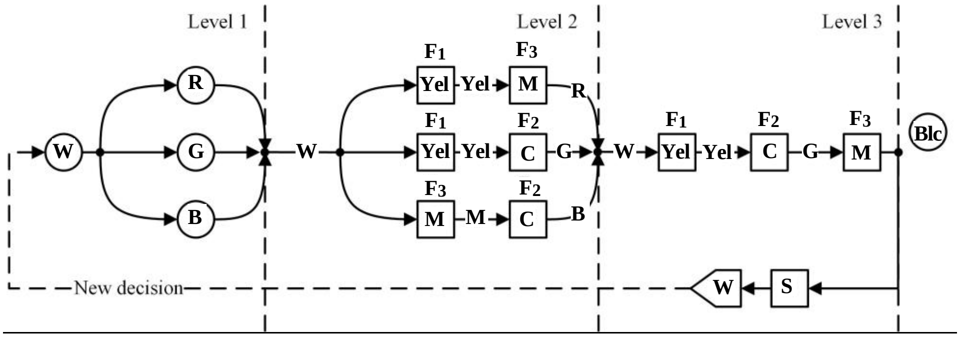


Fig. 5

In the case of white light emission at the output of each of the group of two light filters, with additional estimates of form **C**, **M**, **Yel** will give white **RGB**; color filter **C**, **M** will give blue color **YNYN** – **YN** – **N** = **Y**; color filter **M**,**Yel** will give red color **YNYN** – **YN** – **Y** = **N**; will give green color filter **C**,**Yel** will give green color **YNYN** – – **N** – **Y** = **YN**, and in the sum of three output emitters according to the optical scheme of Fig. 1 we obtain white emitter **YNYN**.

When white light passes through a yellow filter (this corresponds to the **NNY** expert's estimate), the blue color is subtracted and the remaining red and green give yellow light and the output solution «**very probably no**»

YNYN – **Y** = **NNY**.

When white light passes through a cyan filter (this corresponds to the **YYN** expert's estimate), the red color is subtracted and the remaining blue and green give cyan light, and the output solution is «**very probably yes**»

YNYN – **N** = **YYN**.

When white light passes through a magenta filter (this corresponds to the **NY** expert's estimate), the green color is subtracted and the remaining blue and red give magenta light and the output «**probably no**»

YNYN – **YN** = **NY**.

Fundamentals of synthesis of decision-making systems and logical conclusions. Let us consider an expanded optical scheme of a logical coloroid (Fig. 5, **Level** — evaluation; **S** — a white light emitter) with three levels of evaluation of the decision-making process. After the secondary evaluation (by Level 2) by the system of light filters, it is proposed, upon receipt of the white light emitter, to introduce a third group of experts who control the third level of the system of light filters, which, for example, with a tertiary evaluation Level 3 of form **C**, **M**, **Yel** will give **Blc** at the output, i.e. «**no decision**».

For example, for the primary evaluation **R**, **R**, **R** at the output of the Level 1, red light emitter **R** is formed, which passes through the filters **M**, **Yel**, and provides the secondary evaluation as a solution

N – 0 – 0 = **N** or $\operatorname{diag}(R, 0, 0) \wedge \operatorname{diag}(R, 0, B) \wedge \operatorname{diag}(R, G, 0) = \operatorname{diag}(R, 0, 0)$.

For the primary evaluation, for example, **R**, **R**, **B** magenta light **M** is produced at the output of the optical gates of Level 1. This light will pass through the filters **M**, **Yel** of Level 2, where the magenta light emitter will be blocked by the yellow filter **B** (remains **R**), and through the filters **M**, **C** of the secondary evaluation Level 2, where magenta light emission is blocked by **R** with a cyan filter (remains **B**)

$$\mathbf{M} - \mathbf{B} = \mathbf{R}; \mathbf{M} - \mathbf{R} = \mathbf{B}$$

or, correspondently,

$$\mathrm{diag}(R, 0, 0) \vee \mathrm{diag}(0, 0, B) \wedge \mathrm{diag}(R, 0, B) \wedge \mathrm{diag}(R, G, 0) = \mathrm{diag}(R, 0, 0);$$

$$\mathrm{diag}(R, 0, 0) \vee \mathrm{diag}(0, 0, B) \wedge \mathrm{diag}(R, 0, B) \wedge \mathrm{diag}(0, G, B) = \mathrm{diag}(0, 0, B).$$

When passing through a **Yel**, **C** filter, magenta will be blocked

$$\mathbf{M} - \mathbf{B} - \mathbf{R} = 0;$$

$$\mathrm{diag}(R, 0, B) \wedge \mathrm{diag}(R, G, 0) \wedge \mathrm{diag}(0, G, B) = \mathrm{diag}(0, 0, 0).$$

At the output of optical devices of Level 2, the sum of red and blue light **R** + **B** = **M** is formed, i.e. we get magenta light as the final score for this case

$$\mathbf{NY} - \mathbf{N} + \mathbf{NY} - \mathbf{Y} = \mathbf{NY} \text{ or}$$

$$\mathrm{diag}(R, 0, 0) \vee \mathrm{diag}(0, 0, B) = \mathrm{diag}(R, 0, B)..$$

Similarly, solutions are formed for various options for expert assessments. A logical coloroid can be an integral part of a system (network) of series-parallel, hierarchically organized elements, where the optical signal at the output of a certain coloroid will be one of the input signals for the next coloroid, and so on.

Operations by the logical coloroid can be described in the following matrix form:

1. $\mathbf{W} = \mathrm{diag}(R, 0, 0) \vee \mathrm{diag}(0, G, 0) \vee \mathrm{diag}(0, 0, B) = \mathrm{diag}(R, G, B).$

2. Light filter **Yel**, **M** $\mathrm{diag}(R, G, B) \wedge \mathrm{diag}(R, G, 0) \wedge \mathrm{diag}(R, 0, B) = \mathrm{diag}(R, 0, 0);$

light filter **Yel**, **C** $\mathrm{diag}(R, G, B) \wedge \mathrm{diag}(R, G, 0) \wedge \mathrm{diag}(0, G, B) = \mathrm{diag}(0, G, 0);$

light filter **M**, **C** $\mathrm{diag}(R, G, B) \wedge \mathrm{diag}(R, 0, B) \wedge \mathrm{diag}(0, G, B) = \mathrm{diag}(0, 0, B).$

3. $\mathbf{W} = \mathrm{diag}(R, 0, 0) \vee \mathrm{diag}(0, G, 0) \vee \mathrm{diag}(0, 0, B) = \mathrm{diag}(R, G, B).$

4. Light filter **Yel**, **M**, **C**

$$\mathrm{diag}(R, G, B) \wedge \mathrm{diag}(R, G, 0) \wedge \mathrm{diag}(R, 0, B) \wedge \mathrm{diag}(0, G, B) = \mathrm{diag}(0, 0, 0) = \mathbf{Blc}.$$

The above equations do not cover all possible combinations of filters but allow us to consider the basics of the formation of color logic (logical coloroid).

Continuing further, the main prerequisites for the concept of **decisio**n (in contrast to estimates) can be formulated for the method of information processing under consideration. Let’s formulate primary (**ordinary preliminary decision**) decisions (less important, less significant, based on the formed purposes and objectives of the final decision-making). For example, the question «**Is John tall?**» can be not essential for decision-making. On the other hand, if «**John's height**» plays a role in the process of making a decision, then this will be a **complicated decision** with a corresponding branched chain of estimates and logical conclusion.

The preliminary ordinary decision is based on the primary assessment and color (coloroid) summation of these assessments (it can be assumed that three are enough). On Fig. 5, this part of the decision-making system is implemented by Level 1 and is described as follows:

1 decision — three experts give the same estimates:

— «**John is not tall**», final estimate «**John is not tall**», color **R**;

— «**John is probably tall**», — «**John is probably tall**», color **G**;

— «**John is tall**», — «**John is tall**», color **B**.

2 decision — two experts give the same estimates (block diagram of the algorithm is shown in Fig. 6, *a*):

— «**John is not tall**» and «**John is probably tall**», final estimate «**John is very probably not tall**», color **Yel**;

— «**John is not tall**» and «**John is tall**», — (negative estimate) «**John is probably not tall**», color **M**;

— «**John is tall**» and «**John is probably tall**», — «**John is very probably tall**», color **C**.

3 decision — three experts give different estimates (block diagram of the algorithm is shown in Fig. 6, *b*) — «**John is not tall**», «**John is probably tall**», «**John is tall**», the main (basic) solution in terms of conflicting «**Truth, John is tall**», color **W** = **RGB**.

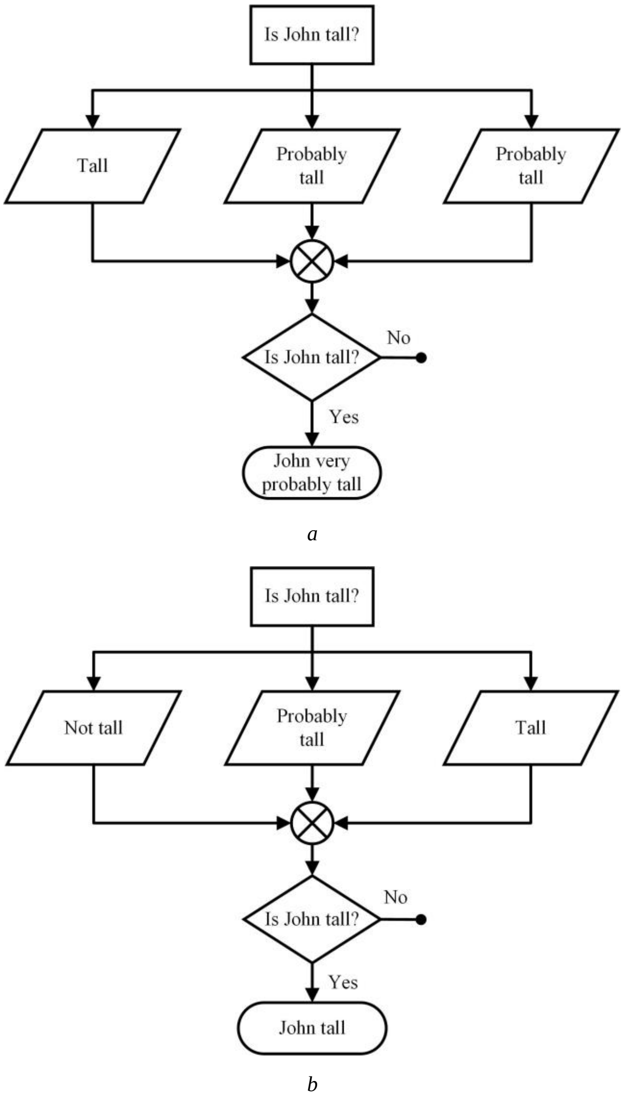


Fig. 6

A difficult decision problem involves an important (complicated) decision, for example, consider the assessment of the situation «**Is the bridge safe?**». According to the proposed scheme (Fig. 5), the decision-making process includes a group of three main experts and six auxiliary experts (e.g., less qualified ones).

When evaluating the decision Level 1 (the block diagram of the decision-making algorithm is shown in Fig. 7) of the main experts in the form of **RGB** the decision «**The bridge is safe**» is sent to a group of secondary experts with estimates in three grouped lines Level 2: **Yel** «**Very probably not safe**», **M** «**Probably not safe**» at the output gives an overall intermediate rating in the first line **R** — «**Not safe**»; **Yel**, **C** «**Very probably safe**» at the output gives in the second line **G** «**Probably safe**»; **M**, **C** in the third line gives **B** «**Safe**», which, with coloroid summation of the results of assessments in all three branches, gives a solution in terms of conflicting **RGB**, «**Safe**».

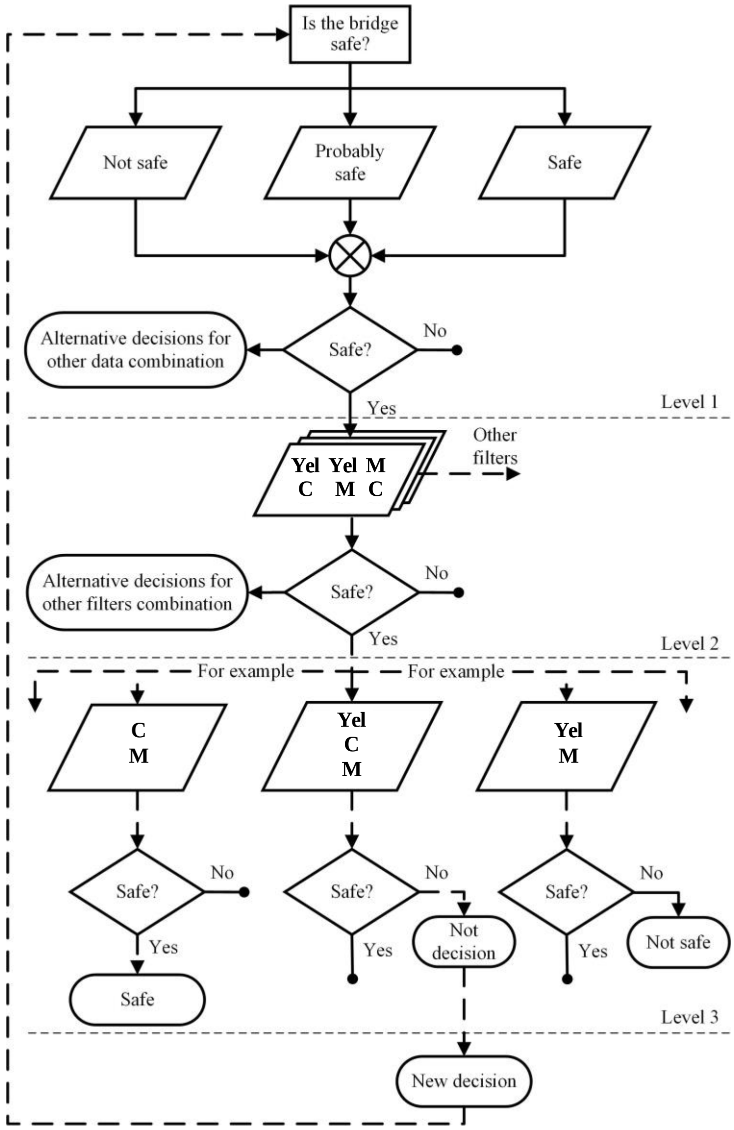


Fig. 7

The third group of experts, for example, with estimates **Yel**, **M**, **C** — gives the final decision **Blc** «**False. Not decision**». It should be noted that according to the proposed scheme for constructing the decision-making process, the experts gave 5 generally positive estimates and 7 generally negative ones. Such a result obviously requires a re-evaluation.

The optical schemes (shown in Fig. 4) give possible overall estimates for different (2 identical) estimates of the third group of experts. In the case of **Yel**, **C** the overall rating would be **G** «**Probably safe**»; for **Yel**, **M** – **R** «**Not safe**»; at **M**, **C** follows **B**

ISSN 2786-6491

«**Safe**». Three identical expert estimates give overall ratings for **Yel** — «**Very probably not safe**»; for three **M** — «**Probably not safe**»; for three **C** — «**Very probably safe**».

Conclusion

The proposed schemes for the optical implementation of logical coloroids using light filters and color information quantization make it possible to expand the possibilities of using logical gates for processing high-quality information and synthesis of high-speed decision support systems. The proposed approach can be supplemented, for example, by using secondary colors and thus increasing their total number to 12, thereby reducing the step and increasing the accuracy of information quantization. It is possible also to consider the use of the concept of color saturation or desaturation of color, for example, $\mathbf{R}^s$ — is the red color of saturation s, where s > 1 corresponds to the degree of saturation, and s < 1 — of desaturation. Color saturation carries additional information, which can be formulated for **R** as «more or less probably with the power s than *N*».

Optical logic coloroid gates can be combined into series-parallel hierarchically organized circuits, the color of the used filters can also reflect the tactile information of the environmental sensing systems necessary to form the corresponding logical conclusions or decisions. Optical schemes can be constructed using, for example, the properties of photonic crystals [10] or by using phenomena from other areas of optical physics.

The presented system of quantization of light emitter of different colors determines the information load of color as a carrier of qualitative information about the environment and/or expert estimates. Optical gates that form chains of logical coloroids allow the construction of logical conclusions for decision-making systems in intelligent control. For nanosize coloroids, in which the time during which light passes through the logical elements is very small, we can get high-performance logical gates and thus, very fast information processing.

В.Л. Тимченко, Ю.П. Кондратенко, В. Крейнович

ЕФЕКТИВНИЙ ОПТИЧНИЙ ПІДХІД ДО ОБРОБКИ НЕЧІТКИХ ДАНИХ НА ОСНОВІ КОЛЬОРІВ ТА СВІТЛОФІЛЬТРІВ

Тимченко Віктор Леонідович

Національний університет кораблебудування імені адмірала Макарова, м. Миколаїв,

vl.timchenko58@gmail.com

Кондратенко Юрій Пантелійович

Чорноморський національний університет імені Петра Могили, м. Миколаїв,

y_kondrat2002@yahoo.com

Крейнович Владик

Університет штату Техас в Ель Пасо, США,

vladik@utep.edu

Ця робота присвячена створенню ефективних оптичних логічних систем на основі використання світлового випромінювання визначеного кольору безпосередньо як нечіткої змінної — носія логічної інформації та основи побудови логічних рішень шляхом перетворення світлового випромінювання відповідними світловими фільтрами. Оптична обробка кольорової інформації, яка відображає різні значення вхідних даних (розглядається на прикладі експертних оцінок), здійснюється запропонованою структурною побудовою нечітких логічних вентилів (логічного колороїда) та значно спрощується, по відношенню до існуючих систем, за рахунок реалізації на властивостях адитивної та субтрактивної обробки кольорів з використанням достатньо простих світлових фільтрів. Сформовано нечітку базу даних на основі визначення кванта інформації, як відповідний колір, та компоненти оптичної логічної системи за допомогою адитивної та субтрактивної обробки світлового випромінювання відповідних кольорів; розроблено основи синтезу систем логічного висновку та прийняття рішень. В роботі синтезовано узагальнену структурну схему оптичного логічного колороїда як основу створення багаторівневої системи прийняття рішень для подальшого застосування в системах штучного інтелекту. Схеми оптичних логічних колороїдів можуть поєднуватися в послідовно-паралельні ієрархічно організовані схеми, колір використовуваних світлофільтрів може також, крім експертних оцінок, відображати тактильну інформацію систем сенсорів про навколишнє середовище, що необхідно для формування відповідних логічних оцінок або рішень. Використання кольору як носія логічної інформації дозволяє створити швидкодіючі технічні пристрої з продуктивністю, в основі розрахунків якої використовується швидкість світла для формування певного масиву логічних рішень.

Ключові слова: квант інформації, нечіткі логічні вентилі, оптичний колороїд, кольорові світлові фільтри.

REFERENCES

1. Tanida J., Ichioka Y. Optical logic array processor using shadowgrams. *Journal of the Optical Society of America*. 1983. Vol. 73, N 6. P. 200–209. https://doi.org/10.1364/JOSA.73.000800
2. Corletti C., Toroba R. Color coded logic. *Optics Communications*. 1996. Vol. 126, N 4–6. P. 197–204. https://doi.org/10.1016/0030-4018(96)00028-4
3. Jung Y.J., Yu S., Koo S. and others. Reconfigurable all-optical logic AND, NAND, OR, NOR, XOR and XNOR by photonic crystal nonlinear cavities. *Conference on Lasers and Electro-Optics*. Pacific Rim, 2009. paper TuB4_3. DOI: 10.1109/CLEOPR.2009.5292059
4. Zaghloul Y.A., Zaghloul A.R.M. Complete all-optical processiry polarization based binary logic gates and optical processors. *Optical Express*. 2006. Vol. 14, N 21. P. 9879–9895. https://doi.org/10.1364/OE.14.009879
5. Qian C., Lin X., Xu J. and others. Performing optical operators by a diffractive neural work. *Light Sciences*. www.nature.com/Lsa 2020/ Open Access DOI:10.1038/s41377-020-0303-2
6. Araujo A., Oliveira A. and others. Two all-optical logic gates in a single photonic interferometer. *Optics Communications*. 2005. Vol. 355. P. 485–491. http://www.repositorio.ufc.br/handle/riufc/46454
7. Wang L.A., Chang S.H., Lin Y.F. Novel implementation method to realize all-optical logic gates. *Optical Eng*. 1998. 37 (3). P. 1011–1018. https://doi.org/10.1117/1.601935
8. Azhigulov D., Nakami B., Ukaegbu I.A. High-speed thermally tuned electro-optical logic gates based on micro-ring resonators. *Optical and Quantum Electronics 52*. Article 412. 2020. https://doi.org/10.1007/s11082-020-02526-y
9. Ma S., Chen Z., Dutta N.K. All-optical logic gates based on two-photon absorption in semiconductor optical amplifiers. *Optics Communications*. 2009. Vol. 282, N 23. P. 4508–4512. 10.1016/j.optcom.2009.08.039
10. Amer Kotb. Simulation of high quality factor all-optical logic gates based on quantum-dot semiconductor optical amplifier at 1 Tb/s. *Optik*. 2016. Vol. 127, N 1. P. 320–325. https://doi.org/10.1016/j.ijleo.2015.10.093
11. Zhu Z., Yuan J., Jiang L. Multifunctional and multichannels all-optical logic gates based on the in-plane coherent control of localized surface plasmons. *Optics letters*. 2020. Vol. 45, N 23. P. 6362–6365. https://doi.org/10.1364/OL.402085
12. Zadeh L.A. Fuzzy Sets. *Information & Control*. 1965. Vol. 8. P. 338–353.
13. Zadeh L.A. The role of fuzzy logic in modeling, identification and control. *Modeling Identification and Control*. 1994. 15 (3). P. 191–203. doi:10.4173/mic.

14. Kondratenko Y.P., Simon D. Structural and parametric optimization of fuzzy control and decision making systems. In: Zadeh L., Yager R., Shahbazova S., Reformat M., Kreinovich V. (eds). Recent developments and the new direction in soft-computing foundations and applications. *Studies in Fuzziness and Soft Computing*. 2018. Vol. 361. P. 273–289. Springer, Cham.https://doi.org/10.1007/978-3-319-75408-6_22
15. Olenych I.B. Fuzzy logic controller for smart home lighting control. *Information and Telecommunication Sciences*. 2017. Vol. 9, N 2. P. 50–54. Google Scholar
16. Lohmann A.W., Weigelt J. Optical logic by anisotropic scattering. *Optics Communications*. 1985. Vol. 54, N 2. P. 81–86. https://doi.org/10.1016/0030-4018(85)90005-7
17. Liren Liu. Optical implementation of parallel fuzzy logic. *Optics Communications*. 1989. Vol. 73, N 3. P. 183–187. https://doi.org/10.1016/0030-4018(89)90080-1
18. Lin S., Kumazava I., Zhang S. Optical fuzzy image processing based on shadow-casting. *Optics Communications*. 1992. Vol. 94, N 5. P. 397–405. https://doi.org/10.1016/0030-4018(92)90582-C
19. Zhang S., Chen C. Parallel optical fuzzy logic gates based on spatial area-encoding technique. *Optics Communications*. 1994. Vol. 107, N 1-2. P. 11–16. https://doi.org/10.1016/0030-4018(94)90095-7
20. Gur E., Mendlovic D., Zalevsky Z. Optical implementation of fuzzy-logic controllers. Part I. *Applied Optics*. 1998. Vol. 37, N 29. P. 6937–6945. https://doi.org/10.1364/AO.37.006937
21. Tomonori Kawano. Printable optical logic gates with cielab color coding system for boolean, operation-mediated handling of colors genetic and Evolutionary Computing (ICGEC). *IEEE*. 2012. P. 270–275. DOI:10.1109/ICGEC.2012.121
22. Gentili P.L. The fundamental fuzzy logic operators and some complex Boolean logic circuits implemented by the chromogenism of a spirooxazine'. *Phys. Chem.* 2011. Vol. 13, N 45. P. 20335–20344. DOI https://doi.org/10.1039/C1CP21782H
23. Gentili P.L., Horvath V., Vanag V., Epstein I.R. Belousov-Zhabotinsky «chemical neuron» as a binary and fuzzy logic processor. *Int. Journ. of Unconventional Computing*. 2012. Vol. 8. P. 177–192. Google Scholar
24. Aby K. G., Harpreet S. Three-input majority gate using spatially localised DNA hairpins. *Micro & Nano Letters*. 2017. Vol. 12, N 3. P. 143–146. doi:10.1049/mnl.2016.0535
25. Golden E., Manic M. Optical fuzzy logic for low-power satellite controller. *34 Annual Conference.* 2008. P. 2647–2654. DOI:10.1109/IECON.2008.4758375
26. Wang Y., Wu X. and others. Fuzzy logic based feedback control systems for the frequency stabilization of external-cavity semiconductors lasers. *International Journal of Optomechatronics*. 2020. Vol. 14, N 1. P. 44–51. DOI:10.1080/15599612.2020.1828516
27. Gentili P.L. Establishing a new link between fuzzy logic, neuroscience and quantum mechanics through bayesian probability: perspectives in artificial intelligence and unconventional computing. *Molecules*. 2021. Vol. 26, N 5987. 18 p. https://doi.org/10.3390/molecules26195987
28. Hemcsics A. Coloroid color system. *Color Research and Application*. 1980. N 5. P. 113–120. https://doi.org/10.1002/col.5080050214
29. Medvedev V.Yu. Color science and coloring. St. Petersburg, 2005. 114 p.
30. Hayman R. Light filters. M. : Mir, 1988. 216 p.

Submitted 30.09.2022