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STAGES AND MAIN TASKS OF THE CENTURY-LONG CONTROL THEORY AND SYSTEM IDENTIFICATION DEVELOPMENT. PART II. METHODS FOR DESIGNING LINEAR CONTROL SYSTEMS BASED ON MATHEMATICAL MODELS OF INPUT-OUTPUT PROCESSES

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The article provides a review of the mathematical description of the dynamics of continuous and discrete linear stationary systems and objects, used at the development stage of the classical theory of automatic control in the form of mathematical models of the «input-output» type. The time and frequency characteristics of continuous and discrete control systems are described, typical links of stationary systems are considered, parametric discrete models of objects as part of typical digital control loops are presented. Stochastic discrete autoregressive models of stationary time series used to describe the dynamic objects in the synthesis of digital control systems are considered. A review of standard control laws for the implementation of continuous and discrete controllers has been completed. A method for synthesizing discrete controllers for multidimensional controlled objects with different, unknown and changing delays is considered, through which variable delays are compensated in the characteristic equation of a closed-loop control system. A common technique for synthesizing one-dimensional and multidimensional controllers for stochastic objects with delays based on ARMAX models is considered. An analysis of approaches to identifying delays in controlled objects is carried out and a method for identifying delays when using input-output models is considered, based on the calculation and comparison of impulse responses for extended and non-extended models of the controlled object. An analysis of the advantages and disadvantages of «input-output» type models is given, as well as the possibilities of their application for solving various classes of control theory problems.

Keywords: transfer function, control laws, discrete controllers, identification, objects with delay.

Introduction

In the first part [1] of this series of articles, the main directions in the development of control theory based on the state space method were analyzed. It was noted that one of the main limitations of this method when designing automatic control systems is the fact that state space method models are very difficult to apply to systems with delay, especially when delays in control objects change over time. Therefore, the authors considered it appropriate, to describe as well other available control methods developed over several decades for the design of control systems based on input-output process models, and to focus attention on methods for control synthesis in the presence of delays in control objects with various features of the delays influence on the dynamics objects, and also analyze the problem of estimating delays in input-output models that describe digital control systems.

Mathematical description of continuous linear stationary systems

In classical control theory, the main mathematical model of the dynamics of linear systems and control objects of the «input-output» type is the transfer function

$$W(s)=\frac{y(s)}{u(s)}=\frac{b_m s^m+b_{m-1}s^{m-1}+\ldots+b_0}{a_n s^n+a_{n-1}s^{n-1}+\ldots+a_0}, \tag{1}$$

which is the ratio of the Laplace transform $y(s)$ of the coordinate $y(t)$ at the output of the system to the Laplace transform $u(s)$ of the input signal $u(t)$ under zero initial conditions. Moreover, $m \le n$ and the Laplace operator $s=\sigma+j\omega$.

For absolutely integrable functions, we can put $\sigma=0$ in the operator s, then the transfer function (1) can be written for $s=j\omega$ as

$$W(j\omega)=\frac{y(j\omega)}{u(j\omega)}=\frac{b_m(j\omega)^m+b_{m-1}(j\omega)^{m-1}+\ldots+b_0}{a_n(j\omega)^n+a_{n-1}(j\omega)^{n-1}+\ldots+a_0}=P(\omega)+jQ(\omega), \tag{2}$$

where $W(j\omega)$ is called the frequency transfer function, which in control theory is used in exponential notation and is called the amplitude-phase frequency characteristic

$$W(j\omega)=A(\omega)e^{j\phi(\omega)},$$

where $A(\omega)=\sqrt{P^2(\omega)+Q^2(\omega)}$ is amplitude-frequency characteristic, and $\phi(\omega)=\mathrm{arctg}\frac{Q(\omega)}{P(\omega)}$ — phase-frequency characteristic.

As standard input in control theory, a step signal $u(t)=1(t)$ and signal in the form of a δ-function (delta function) are used.

The response of the system's output signal $y(t)$ to a single step disturbance $u(t)=1(t)$ under zero initial conditions is called the system's transient response $h(t)$.

The system's response to a single impulse input disturbance, that is to $\delta(t)$, under zero initial conditions, was called the impulse transition or weighting function of the system $k(t)$.

In this case, the transfer function of the system is equal to the Laplace image of the impulse transition function

$$L\{k(t)\} = W(s) . \tag{3}$$

Based on the transfer function model (1), the following typical links of stationary systems have been developed and are widely used in control theory, which are shown in Table 1.

Table 1

Link type	Transition function $h(t)$	Weight weighting function $w(t)$	Transmission function $W(s)$
1	2	3	4
Inertia-free link	$h(t) = k \cdot 1(t)$	$w(t) = k \cdot \delta(t)$	$W(s) = k$
Aperiodic link of 1st order	$h(t) = k \cdot (1 - e^{-\frac{t}{T}})1(t)$	$w(t) = \frac{k}{T} e^{-\frac{t}{T}} \cdot 1(t)$	$W(s) = \frac{k}{1+Ts}$
The ideal integrating link	$h(t) = k \cdot t \cdot 1(t)$	$w(t) = k \cdot 1(t)$	$W(s) = \frac{k}{s}$
Integrating link with deceleration	$h(t) = k \cdot [t - T(1 - e^{-\frac{t}{T}})]1(t)$	$w(t) = k \cdot (1 - e^{-\frac{t}{T}})1(t)$	$W(s) = \frac{k}{s(1+Ts)}$
The ideal differentiator	$h(t) = k \cdot \delta(t)$	$w(t) = k \frac{d\delta(t)}{dt}$	$W(s) = ks$
Differentiating link with deceleration	$h(t) = \frac{k}{T} e^{-\frac{t}{T}} \cdot 1(t)$	$w(t) = \frac{k}{T}\delta(t) -$ $-\frac{k}{T^2} e^{-\frac{t}{T}} \cdot 1(t)$	$W(s) = \frac{ks}{1+Ts}$
Oscillatory link	$h(t) = k \cdot [1 - e^{-\gamma t}(\cos\lambda t +$ $+\frac{\gamma}{\lambda}\sin\lambda t)] \cdot 1(t);$ $\gamma = \frac{\lambda}{\pi} \ln \frac{A_1}{A_2}$; $\lambda = \frac{1}{T}\sqrt{1-\xi^2}$	$w(t) =$ $= \frac{k}{\lambda T^2} \cdot e^{-\gamma t} \sin \lambda t \cdot 1(t)$	$W(s) =$ $= \frac{k}{1 + 2\xi Ts + T^2 s^2}$
Delayed link	$1(t - \tau)$	$\delta(t - \tau)$	$W(s) = 1 \cdot e^{-\tau s}$

The frequency characteristics of the links shown in the table are described in detail in [2].

2. Mathematical description of discrete linear stationary systems

Discrete automatic control systems (DACS) are characterized by the fact that at least one of the coordinates that determine the state of the system is discretized. Depending on the type of quantization (by level, by time, by level and by time), discrete systems are divided accordingly into three types: relay, pulse and digital.

Unlike continuous systems, the dynamics of which are described by differential equations, pulsed systems, as was established in the classical work [3], are described by difference equations, which have received limited use in the design of control systems because of cumbersome calculations.

As a result of the search for new approaches, a new mathematical apparatus was developed using the discrete Laplace transform

$$D\{x(nT_0)\} = \sum_{n=0}^{\infty} x(nT_0)\, e^{-nsT_0} = X^*(s)\,, \tag{4}$$

where $x(nT_0)$ is the lattice function, and T_0 is the sampling period. From (4) it follows that $X^*(s)$ is a function of e^{sT_0}. This led to the fact that the description of the dynamics of control systems was obtained in the form of transcendental equations for the operator s. . To eliminate this complexity, the operator was used in [4–8]

$$z = e^{sT_0}\ , \tag{5}$$

on the basis of which the discrete Laplace transform (4) was presented in the form of *z*-transform

$$X(z) = \sum_{n=0}^{\infty} x(nT_0) z^{-n}\ . \tag{6}$$

The use of *z*-transformation for the coordinates of the control system made it possible to describe the dynamics of systems in the form of algebraic equations with respect to the operator *z*.

For a continuous system $W_0(s)$, the input of which is a pulse signal (Fig. 1), the formation of a discrete transfer function is implemented as follows:

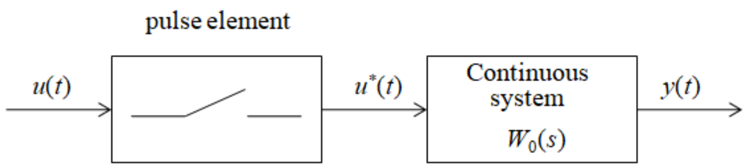

Fig. 1

The signal at the output of the pulse element will be equal to

$$u^*(t) = \sum_{n=-\infty}^{\infty} u(nT_0)\delta(t - nT_0)$$

or in the form of discrete Laplace transform

$$u^*(s) = \sum_{n=-\infty}^{\infty} u(nT_0) e^{-nT_0 s}\ .$$

Then the signal at the output of the continuous system

$$y(t) = \sum_{k=-\infty}^{\infty} w_0(t - kT_0) u(kT_0),$$

where $w_0(t)$ is the impulse transition (weighting) function of a continuous system.

The discrete signals $y(nT_0)$ at the output of a continuous system, previously at rest, will have the form

$$y(nT_0) = \sum_{k=-\infty}^{\infty} \Gamma[(n-k)T_0] u(kT_0)\,,$$

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where $\Gamma_0(nT_0)$ is the pulsed lattice-frequent transition function of a continuous system. For a lattice function of the output signal, the z-transform will be equal to

$$z\{y(nT_0)\} = y(z) = \sum_{n=-\infty}^{\infty} y(nT_0)z^{-n} = \sum_{n=-\infty}^{\infty}\left[\sum_{k=-\infty}^{\infty}\Gamma_0\left[(n-k)T_0\right]u(kT_0)z^{-n}\right] =$$

$$= \sum_{k=-\infty}^{\infty} u(kT_0) \sum_{n=-\infty}^{\infty}\Gamma_0\left[(n-k)T_0\right]z^{-n}.$$

By performing the replacement $m = n - k$ we obtain

$$y(z) = \sum_{k=-\infty}^{\infty} u(kT_0)z^{-k} \cdot \sum_{m=-\infty}^{\infty}\Gamma_0(mT_0)z^{-m} = u(z)W_0(s),$$

from here the expression for the discrete transfer function of the system is obtaine

$$W_0(z) = \frac{y(z)}{u(z)} = \sum_{m=-\infty}^{\infty}\Gamma_0(mT_0)z^{-m}. \quad (7)$$

2.1. Parametric discrete models of controlled objects of the «input-output» type. To implement control systems based on microprocessor systems, a mathematical description of controlled objects was presented in the form of parametric discrete models that describe the dynamics of a series-connected digital-to-analog converter and a controlled object. For the convenience of analysis and synthesis of digital control systems, the mathematical model of the DACS in the form of a zero-order extrapolator $W_E(s)$ and the model of the controlled object $W_0(s)$ were combined together. Such a union was called the reduced continuous part (RCP) of the object, the transfer function of which has the form

$$W_R(s) = W_E(s) \cdot W_0(s).$$

The discrete transfer function of the RCP is defined as follows:

$$W_R(z) = Z\{W_E(s) \cdot W_0(s)\} = Z\left\{\frac{1-e^{-T_0 s}}{s}W_0(s)\right\} = \left(\frac{z-1}{z}\right)Z\left\{\frac{W_0(s)}{s}\right\}. \quad (8)$$

For widely used models of typical DACS links, Table 2 shows discrete transfer functions of the RCP, developed on the basis of (8).

When using the reverse shift operator $z^{-1}y_n = y_{n-1}$; $z^{-1}u_n = u_{n-1}$ it is possible to obtain difference equations of the RCP for aperiodic links of the first and second order, respectively

$$y_n + a_1 y_{n-1} = b_1 u_{n-1},$$

where

$$a_1 = -e^{-T_0/T_1}; \; b_1 = k(1 - e^{-T_0/T_1}),$$

and also

$$y_n + a_1 y_{n-1} + a_2 y_{n-2} = b_1 u_{n-1} + b_2 u_{n-2},$$

where

$$a_1 = -\left(e^{-T_0/T_1} + e^{-T_0/T_2}\right), \; a_2 = e^{-T_0/T_1} \times \times e^{-T_0/T_2}, \; b_1 = kC_3, \; b_2 = kC_4.$$

Table 2

Link type	Transfer function $W_0(s)$	Discrete transfer function RCP $W_R(z)$
Aperiodic link of 1st order	$\frac{k}{1+T_1 s}$	$\frac{k\cdot\left(1-e^{-T_0/T_1}\right)z^{-1}}{1-\left(e^{-T_0/T_1}\right)z^{-1}}$
Aperiodic 1st order link with delay	$\frac{k\cdot e^{-\tau s}}{1+T_1 s}$	$\frac{k\cdot(C_1+C_2 z^{-1})z^{-d-1}}{1-\left(e^{-T_0/T_1}\right)z^{-1}}$; $d=\left[\frac{\tau}{T_0}\right]$; $C_1=1-e^{-\frac{\mu T_0}{T_1}}$; $C_2=e^{-\frac{\mu T_0}{T_1}}-e^{-\frac{T_0}{T_1}}$; $\mu=1-\frac{(\tau-dT_0)}{T_0}$
Aperiodic link of 2nd order	$\frac{k}{(1+T_1 s)(1+T_2 s)}$	$\frac{k\cdot(C_3+C_4 z^{-1})z^{-1}}{\left[1-\left(e^{-T_0/T_1}\right)z^{-1}\right]\cdot\left[1-\left(e^{-T_0/T_2}\right)z^{-1}\right]}$; $C_3=1+\frac{T_2 e^{-\frac{T_0}{T_2}}-T_1 e^{-\frac{T_0}{T_1}}}{T_1-T_2}$; $C_4=e^{-\frac{T_0}{T_1}}\cdot e^{-\frac{T_0}{T_2}}+\frac{T_2 e^{-\frac{T_0}{T_1}}-T_1 e^{-\frac{T_0}{T_2}}}{T_1-T_2}$;

2.2. Stochastic discrete models of stationary time series. In [9], stochastic discrete models of stationary time series were proposed to describe the dynamics of economic processes. In the theory of automatic control, the ARMAX model (autoregressive and moving average with an additional input signal) began to be widely used.

$$A(z^{-1})y_t = B(z^{-1})u_t + C(z^{-1})\xi_t\,, \tag{9}$$

where y_t are discrete samples of the output coordinate of the control object, presented in the form of deviations relative to the zero average; u_t — discrete samples of the control signal in the form of deviations relative to the zero average; ξ_t is disturbance in the form of discrete white noise.

Polynomial expressions for the reverse shift operator z^{-1} have the form:

$$A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_{n_a} z^{-n_a}\,;$$

$$B(z^{-1}) = b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{n_b} z^{-n_b}\,;$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + \ldots + c_{n_c} z^{-n_c}\,.$$

Subsequently, the ARMAX model began to take into account control delay and displacement v as follows:

$$A(z^{-1})y_t = z^{-d} B(z^{-1})u_t + C(z^{-1})\xi_t + v,$$

where $d = \left[\frac{\tau}{T_0}\right]$ is an integer dividing the delay time τ by the quantization period T_0, and v is the offset, which is set if the mathematical expectation is $E\{y_t\} \neq 0.$

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The ARMAX model is also used to describe multidimensional controlled objects in a stochastic environment

$$\bar{A}(z^{-1})\bar{y}_t = z^{-d}\bar{B}(z^{-1})\bar{u}_t + \bar{C}(z^{-1})\bar{\xi}_t + \bar{v}\,, \tag{10}$$

where $\bar{y}_t$, $\bar{u}_t$, $\bar{\xi}_t$ are the vectors of output coordinates, control actions and disturbances respectively, and the matrix polynomials have the form

$$\bar{A}(z^{-1}) = I + \bar{A}_1 z^{-1} + \ldots + \bar{A}_{n_a} z^{-n_a}\,;$$

$$\bar{B}(z^{-1}) = \bar{B}_1 z^{-1} + \bar{B}_2 z^{-2} + \ldots + \bar{B}_{n_b} z^{-n_b}\,;$$

$$\bar{C}(z^{-1}) = I + \bar{C}_1 z^{-1} + \ldots + \bar{C}_{n_c} z^{-n_c}\,.$$

3. Methods for design of linear automatic control systems

3.1. Model control laws. The control law is the function of the control action synthesized in the controller from the control error $u(t) = \phi\{e(t)\}$. For continuous controllers, the typical control law is the proportional-integral-differential (PID) control law

$$u(t) = K_p \left[e(t) + \frac{1}{T_\text{p}} \int e(t)dt + T_\text{d} \frac{de(t)}{dt} \right]. \tag{11}$$

which is widely implemented to control various technological processes.

In discrete form, the PID control law is implemented through a positional and high-speed algorithm [10].

The positional algorithm assumes the formation of the full value of the control action at each sampling period

$$u(n) = K_p \left[e(n) + \frac{T_0}{T_\text{p}} \sum_{i=1}^{n} \frac{e(i) + e(i-1)}{2} + T_\text{d} \frac{e(n) + e(n-1)}{T_0} \right]. \tag{12}$$

In a unified form, law (12) is implemented in a microprocessor system as follows:

$$u(n) = u(n-1) + A_0 e(n) + A_1 e(n-1) + A_2 e(n-2)\,, \tag{13}$$

where

$$A_0 = K_p \left[1 + \frac{T_0}{2T_\text{p}} + \frac{T_\text{d}}{T_0} \right],$$

$$A_1 = -K_p \left(1 - \frac{T_0}{2T_\text{p}} + \frac{2T_\text{d}}{T_0} \right),$$

$$A_2 = K_p \frac{T_\text{d}}{T_0}.$$

In distributed microprocessor PID control systems, the control law is implemented in the form of a high-speed algorithm, in which at each sampling period an increment of the control action is formed in the discrete controller

$$\Delta u(n) = u(n) - u(n-1) =$$

$$= K_p \left\{ [e(n) - e(n-1)] + \frac{T_0}{2T_\text{p}} [e(n) + e(n-1)] + \frac{T_\text{d}}{T_0} [e(n) - 2e(n-1) + e(n-2)] \right\}. \tag{14}$$

The discrete transfer function of the PID controller (13) has the form

$$W_p(z) = \frac{u(z)}{e(z)} = \frac{A_0 + A_1 z^{-1} + A_2 z^{-2}}{1 - z^{-1}} . \tag{15}$$

3.2. A method for designing discrete controllers for multidimensional objects with various unknowns and varying delays. The discrete model of these control objects has the form [11]:

$$\overline{A}(z^{-1})\overline{y}(z) = \overline{B}(z^{-1})\overline{u}(z) , \tag{16}$$

where $\overline{y}(z)$ is the vector of controlled output variables with dimension $(m \times 1)$; $\overline{u}(z)$ is the vector of control actions $(m \times 1)$; $\overline{A}(z^{-1})$, $\overline{B}(z^{-1})$ are matrix polynomials of dimension $(m \times m)$ with elements

$$A_{ij}(z^{-1}) = \sigma_{ij} - a_{1_{ij}} z^{-1} - \ldots - a_{r_{ij}} z^{-r_{ij}} ; \tag{17}$$

$$B_{ij}(z^{-1}) = (b_{1_{ij}} z^{-1} + b_{2_{ij}} z^{-2} \ldots + b_{p_{ij}} z^{-p_{ij}}) z^{-d_{\min_{ij}}} ; \tag{18}$$

$$(i = 1, \ldots, m; \; j = 1, \ldots, m) ,$$

where σ_{ij} is the Kronecker function, and z^{-1} is the inverse shift operator by T_0, and $d_{\min_{ij}}$ is the known minimum delay in discrete form along the *ij* control channel. In this case, the order p_{ij} in (18) is selected taking into account the maximum interval of change of the discrete delay along the *ij* channel

$$p_{ij} = r_{ij} + (d_{\max_{ij}} - d_{\min_{ij}}). \tag{19}$$

In this method, delays d_{ij} change at intervals $(d_{\max_{ij}} - d_{\min_{ij}})$ and are unknown during the operation of the controlled object.

In [11, 12], a method for designing a discrete controller with compensation of variable delays in the characteristic equation of a multidimensional closed-loop control system is implemented, the block diagram of which is shown in Fig. 2.

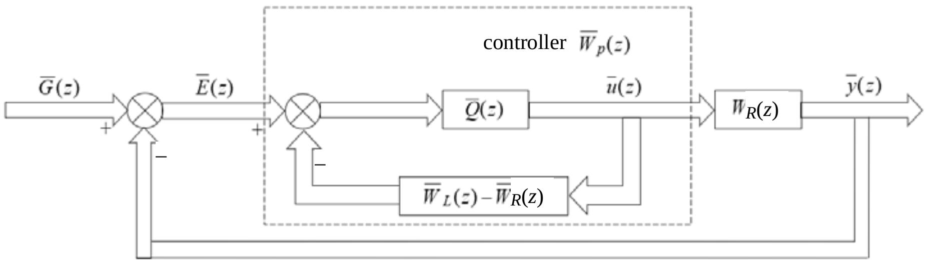

Fig. 2

According to model (16), the discrete transfer function of the controlled object will be equal to

$$\overline{W}_R(z) = \overline{A}^{-1}(z^{-1})\overline{B}(z^{-1}) , \tag{20}$$

and the discrete transfer function of the delay compensator is formed as follows:

$$\bar{W}_k(z) = \bar{W}_L(z) - \bar{W}_R(z), \tag{21}$$

where $\bar{W}_L(z)$ is the discrete transfer function of the proposed object without delay, which equals

$$\bar{W}_L(z) = [\bar{A}(z^{-1})]^{-1} \Sigma \bar{B}(z), \tag{22}$$

where the matrix $\Sigma\bar{B}(z)$ elements are defined as follows:

$$\Sigma\bar{B}(z) = \left(\sum_{k=1}^{p_{ij}} b_{k_{ij}} \right) z^{-1}.$$

Designing of the optimal structure of a multidimensional digital controller with variable delays compensation with known dynamics of the controlled object begins with the formation of a matrix discrete transfer function of a closed-loop system, which includes the proposed controlled object without delays (22)

$$\bar{H}(z) = [I + \bar{W}_L(z)\bar{Q}(z)]^{-1} \bar{W}_L(z)\bar{Q}(z), \tag{23}$$

where $\bar{Q}(z)$ is the discrete transfer function of the controller designed for the intended object without delay $\bar{W}_L(z)$. If the desired function $H(z)$ is given, then from (23) we can determine

$$\bar{Q}_{\text{opt}}(z) = \bar{W}_L^{-1}(z)\bar{H}(z)\left[I - \bar{H}(z)\right]^{-1}. \tag{24}$$

In this case, the equation of the closed system will be closed-loop system

$$\tilde{\bar{y}}(z) = \bar{H}(z)\bar{G}(z),$$

where $\tilde{\bar{y}}(z)$ is the vector of undelayed controlled variables.

As the desired discrete transfer function $\bar{H}(z)$, a diagonal matrix with elements is selected

$$H_{ij}(z) = \frac{\tilde{y}_i(z)}{G_i(z)} = \frac{\left(1 - e^{-T_0/T_i}\right) z^{-1}}{1 - e^{-T_0/T_i} z^{-1}}, \quad i = 1, \ldots, m,$$

which provides an aperiodic transient process via the «$\bar{G}_i(z) - \tilde{y}_i(z)$» channel. With the selected diagonal matrix $\bar{H}(z)$, there will be no cross connections between the elements of the vectors $\bar{G}(z)$ and $\tilde{\bar{y}}(z)$. To regulate the speed of transient processes in a closed-loop system, a time constant T_i is used, which is selected when designing the control system.

According to Fig. 2, the discrete transfer function of a multidimensional controller with a delay compensator has the form

$$\bar{W}_p(z) = \left[I + \bar{Q}(z)[\bar{W}_L(z) - \bar{W}_R(z)]\right]^{-1} \bar{Q}(z). \tag{25}$$

To obtain the optimal value $W_{\text{popt}}(z)$, its optimal value (24) is substituted into this expression instead of $Q(z)$. Then after transformations there is

$$\bar{W}_{\text{popt}}(z) = \left[\bar{W}_L(z) - \bar{H}(z)\bar{W}_{\text{п}}(z)\right]^{-1} \bar{H}(z). \tag{26}$$

Statement. If for the considered object without delay (22) a controller $\bar{Q}(z)$ is used, then the introduction of a multidimensional compensator $\bar{W}_k(z)$ according to (21) in the feedback circuit with this controller excludes the discrete transfer function $\bar{W}_R(z)$ with delays from the characteristic equation of the closed-loop system Fig. 2, the dynamics of which will be presented in the form

$$\bar{y}(z) = \bar{W}_R(z)\left[I + \bar{Q}(z)\bar{W}_L(z)\right]^{-1} \bar{Q}(z)\bar{G}(z) . \tag{27}$$

The proof of this statement is given in [13].

The stability of the closed-loop system (27) is determined by placing the roots of the characteristic equation $\det[I + \bar{Q}(z)\bar{W}_L(z)] = 0$, which contains only the discrete transfer function $\bar{W}_L(z)$ of the considered object without delays. Thus, the delay variables have no effect in the system of Fig. 2 for its stability.

The coefficients of polynomials $\bar{A}(z^{-1})$, $\bar{B}(z^{-1})$ model (16) are estimated using the recurrent least squares method.

3.3. Design method of discrete controllers for stochastic objects with delays. This method is described in [14] for one-dimensional object with a delay and has found wide application for control various technological processes.

The mathematical model of the control object is presented in the form of the ARMAX model

$$A(z^{-1})y_t = z^{-d} B(z^{-1})u_t + C(z^{-1})\xi_t + \eta , \tag{28}$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_m z^{-m} , \tag{29}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{m-1} z^{-(m-1)} , \tag{30}$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + \ldots + c_m z^{-m} . \tag{31}$$

The discrete delay time for the control action in model (28) is determined according to $d = \left[\frac{\tau}{T_0}\right] + 1$.

The implementation of the optimal digital controller is performed based on the minimization of the quadratic optimality criterion

$$J = E\{(Py_{t+d} - RG_t)^2 + \lambda(u_t - u_{t-1})^2\} . \tag{32}$$

where E is the mathematical expectation operator, G_t is the setting action of the digital controller at $[nT_0] \le t < [(n+1)T_0]$; P and R — weighting coefficients; λ — coefficient of amplification, with the help of which the quality of the transient process is regulated. From a mathematical point of view, criterion (32) represents the generalized variance of the control error and the increment of the control action.

Criterion (32) uses the predicted value of the output coordinate y_{t+d} for d sampling periods ahead. For this purpose, the work [14] developed a forecasting function

$$y^*_{t+d/t} = \frac{1}{C(z^{-1})}[L(z^{-1})B(z^{-1})u_t + F(z^{-1})y_t + L(z^{-1})\eta] , \tag{33}$$

where polynomials $L(z^{-1})$, $F(z^{-1})$ are formed based on the application of Diophantine equations

$$C(z^{-1}) = L(z^{-1})A(z^{-1}) + z^{-d}F(z^{-1}). \tag{34}$$

In this case, the future coordinate

$$y_{t+d} = y^*_{t+d/t} + e_{t+d}, \tag{35}$$

where e_{t+d} is the prediction error.

The synthesis of the optimal structure of the digital controller is implemented based on criterion (32) and (35)

$$J = E\{(Py^*_{t+d/t} + Pe_{t+d} - RG_t)^2 + \lambda(u_t - u_{t-1})^2\}. \tag{36}$$

The random error e_{t+d} is not correlated with the discrete values of u_{t-1}, y_{t-1}, G_{t-1} for $i \geq 0$, included in the forecasting function (33). Based on this, criterion (36) is divided into deterministic and stochastic components according to

$$J = (Py^*_{t+d/t} - RG_t)^2 + \lambda(u_t - u_{t-1})^2 + \sigma^2, \tag{37}$$

where $\sigma^2 = E\{(Pe_{t+d})^2\}$, since the mathematical expectation $E\{2(Py^*_{t+d/t} - RG_t) \times$ $\times Pe_{t+d}\} = 0$ due to the lack of correlation.

Differentiating criterion (37) with respect to the control action u_t and then from equation $\frac{\partial J}{\partial u_t} = 0$, we obtain the equation of the optimal controller

$$J = \psi^*_{t+d/t} = Py^*_{t+d/t} - RG_t + \lambda'(u_t - u_{t-1}) = 0. \tag{38}$$

When substituting the prediction function (33), the controller equation (38) takes the form

$$C(z^{-1})\psi^*_{t+d/t} = F(z^{-1})y_t + D(z^{-1})u_t + H(z^{-1})G_t + \delta = 0. \tag{39}$$

where

$$F(z^{-1}) = \sum_{j \geq 0} PF'_{d-j}; \; D(z^{-1}) = \sum_{j \geq 0} Pz^{-1}G'_{d-j} + C(z^{-1})\lambda'(1 - z^{-1});$$

$$H(z^{-1}) = -RC(z^{-1}).$$

A block diagram of control system with synthesized in such a way closed-loop control system, including a controlled object (28) and a digital controller (39), is shown in Fig. 3.

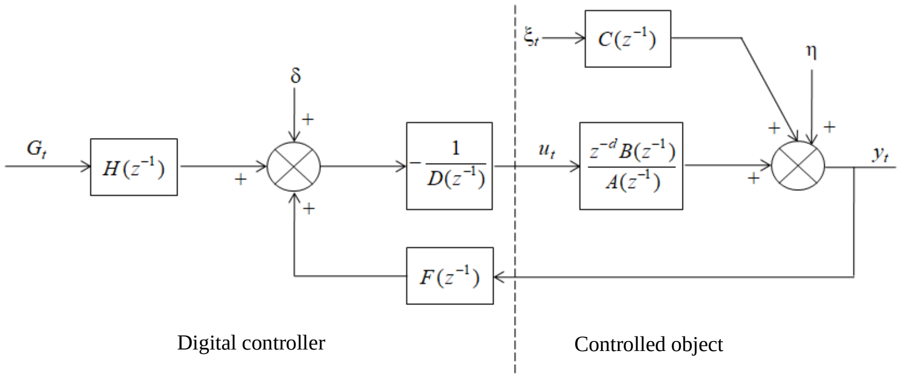

Fig. 3

In [15, 16], the controller synthesis method (39), performed for a one-dimensional object (28), was implemented for a multidimensional controlled object, the model of which has the form ARMAX:

$$\bar{A}(z^{-1})\bar{y}_t = \bar{B}(z^{-1})\bar{u}_{t-d} + \bar{C}(z^{-1})\bar{\xi}_t + \bar{v}\,, \tag{40}$$

where $\bar{y}$ is the vector of deviations of the output (measured) controlled variables from their mathematical expectations with dimension $(m \times 1)$; $\bar{u}$ is the m-dimensional vector of deviations of control signals from zero average values; $\bar{\xi}_t$ is the m-dimensional vector of sequences of independent, identically distributed random signals with zero mean; $\bar{v}$ is the m-dimensional vector consisting of constant elements characterizing the steady-state values of the output signals $\bar{y}$ at a zero value of the input signals $\bar{u}$; d is discrete delay time $d = \left[\frac{\tau}{T_0}\right] + 1$; z^{-1} is delay operator (reverse shift) $z^{-1}y_t = y_{t-1}$. Matrix polynomials $\bar{A}$, $\bar{B}$, $\bar{C}$ of dimensions $(m \times m)$ are determined by the equations

$$\bar{A}(z^{-1}) = I + \bar{A}_1 z^{-1} + \ldots + \bar{A}_k z^{-k}\,;$$

$$\bar{B}(z^{-1}) = \bar{B}_0 + \bar{B}_1 z^{-1} + \ldots + \bar{B}_k z^{-k}\,;$$

$$\bar{C}(z^{-1}) = I + \bar{C}_1 z^{-1} + \ldots + \bar{C}_k z^{-k}\,,$$

where $\bar{B}_0$ is a non-singular matrix, where I is the identity matrix.

The quality of control is determined by the quadratic optimality criterion

$$J = E\left\{ \left\| \bar{P}(z^{-1})\bar{y}_{t+d} - \bar{R}(z^{-1})\bar{G}_t \right\|^2 + \left\| \bar{L}(z^{-1})\bar{u}_t \right\|^2 \right\}. \tag{41}$$

where $\bar{P}$, $\bar{R}$, $\bar{L}$ are matrix polynomials, and $\bar{G}$ is a m-dimensional vector of setting actions. The most common special cases of this criterion are

$$J_1 = E\{ \left\| \bar{y}_{t+d} - \bar{G}_t \right\|^2 + [\bar{u}_t - \bar{u}_{t-1}]^{\mathrm{T}} \operatorname{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\}[\bar{u}_t - \bar{u}_{t-1}]\}\,. \tag{42}$$

To apply optimality criteria (41), (42), a forecasting function has been developed [15, 16]

$$\bar{y}^*_{t+d/t} = \left[\tilde{\tilde{C}}(z^{-1})\right]^{-1}\left[\tilde{\tilde{F}}(z^{-1})\bar{y}_t + \tilde{\tilde{L}}(z^{-1})B(z^{-1})u_t + \bar{\delta}\right], \tag{43}$$

which is obtained based on the modified Diophantine equation

$$\tilde{\tilde{C}}(z^{-1}) = \tilde{\tilde{L}}(z^{-1})\bar{A}(z^{-1}) + z^{-d}\tilde{\tilde{F}}(z^{-1})\,, \tag{44}$$

where matrix polynomials $\tilde{\tilde{C}}(z^{-1})$, $\tilde{\tilde{L}}(z^{-1})$, $\tilde{\tilde{F}}(z^{-1})$ are formed based on the identities

$$\tilde{\tilde{L}}(z^{-1})F(z^{-1}) = \tilde{\tilde{F}}(z^{-1})L(z^{-1})\,;$$

$$\tilde{\tilde{C}}(z^{-1})L(z^{-1}) = \tilde{\tilde{L}}(z^{-1})\bar{C}(z^{-1})\,;$$

$$\det \tilde{\tilde{L}}(z^{-1}) = \det L(z^{-1})\,;\ \tilde{\tilde{L}}(0) = I\,,$$

and the shift vector $\bar{\delta} = \tilde{\tilde{L}}(1)\bar{v}$.

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The error in predicting the future vector of output coordinates $\bar{y}_{t+d}$ is equal to

$$\bar{e}_{t+d} = \bar{y}_{t+d} - y^*_{t+d/t}, \tag{45}$$

where $\bar{e}_{t+d} = \tilde{\bar{L}}(z^{-1})\bar{\xi}_{t+d}$.

When substituting into criterion (42) $\bar{y}_{t+d} = \bar{y}^*_{t+d/t} + \bar{e}_{t+d}$, it can be divided into deterministic and stochastic components

$$J_1 = E\{\left\|\bar{y}_{t+d/t} - \bar{G}_t\right\|^2 + (\bar{u}_t - \bar{u}_{t-1})^{\mathrm{T}} \operatorname{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\}(\bar{u}_t - \bar{u}_{t-1})\} + E\{\left\|\bar{e}_{t+d}\right\|^2\}. \tag{46}$$

The optimal vector of control actions is determined based on minimizing criterion (46)

$$\frac{\partial J}{\partial u_t} = (\bar{B}_0)^{\mathrm{T}}[\bar{y}^*_{t+d/t} - \bar{G}_t] + \operatorname{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\}(\bar{u}_t - \bar{u}_{t-1}) = 0. \tag{47}$$

where $(\bar{B}_0)^{\mathrm{T}} = \left[\dfrac{\partial \bar{y}^*_{t+d/t}}{\partial u_t}\right]^{\mathrm{T}}$.

Based on (43), (47), the equation of the optimal multidimensional controller is presented in the form

$$\tilde{\bar{C}}(z^{-1})\bar{\psi}^*_{t+d/t} = \tilde{\bar{F}}(z^{-1})\bar{y}_t - \tilde{\bar{R}}(z^{-1})u_t + \tilde{\bar{H}}(z^{-1})\bar{G}_t + \bar{\delta} = 0, \tag{48}$$

where

$$\tilde{\bar{R}}(z^{-1}) = \tilde{\bar{L}}(z^{-1})\bar{B}(z^{-1}) + \tilde{\bar{C}}(z^{-1})(\bar{B}_0^T)^{-1} \cdot \operatorname{diag}\{\lambda_1, \ \ldots, \lambda_m\}(1 - z^{-1});$$

$$\tilde{\bar{H}}(z^{-1}) = -\tilde{\bar{C}}(z^{-1}).$$

The controller (48) minimizes the generalized dispersion (42) along the direct and cross channels of the controlled object (40).

In [13], the controller (48) was modernized for a multidimensional controlled plant with various delays in the control channels

$$\bar{A}(z^{-1})\bar{y}_t = \bar{B}(z^{-1}) \operatorname{diag}\{z^{-d_i}\}\bar{u}_t + \bar{C}(z^{-1})\bar{\xi}_t + \bar{v},$$

where $d_i = \left[\dfrac{\tau_i}{T_0}\right] + 1$ is the known delay at $i = 1, 2, \ldots, m$.

3.4. «Pure» delay identification of the controlled object. The time delay along the control channels of technological objects varies depending on the operating modes and state of the object and connecting paths.

Many works have considered the issues of delay estimation for controlled objects represented by a discrete transfer function

$$W(z) = \frac{y(z)}{u(z)} = z^{-d}\,\frac{B(z^{-1})}{A(z^{-1})}, \tag{49}$$

where the polynomials with respect to the inverse shift operator have the form

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_m z^{-m};$$

$$B(z^{-1}) = b_1 z^{-1} + b_2 z^{-2} + \ldots + b_m z^{-m}.$$

In this case, the discrete delay is equal to $d = \left[\dfrac{\tau}{T_0}\right]$.

To derive the relations necessary to estimate the delay, model (49) was presented in an expanded form

$$y(z) = \frac{B'(z^{-1})}{A(z^{-1})} u(z) , \tag{50}$$

where

$$B'(z^{-1}) = b_1' z^{-1} + b_2' z^{-2} + \ldots + b_m' z^{-m} + \ldots + b_m' z^{-m} + b_{m+d_{\max}}' z^{-(m+d_{\max})} . \tag{51}$$

In this case, $d_{\max}$ is the maximum possible delay, which is considered, to be known for each controlled object.

For the extended polynomial (51), which takes into account the delay, the following relations hold:

$$b_i' = 0 \text{ at } i = 1, 2, \ldots, d ;$$

$$b_j' = b_{j-d} \text{ at } j = (1+d), (2+d), \ldots, (m+d) ;$$

$$b_k' = 0 \text{ at } k = (m+d+1), \ldots, (m+d_{\max}).$$

The algorithm for estimating the parameters of the object model (50) using the recurrent least squares method (RLSM) was described as follows:

$$\hat{\bar{\theta}}_n = \hat{\bar{\theta}}_{n-1} + K_{n-1}(y_n - \bar{X}_{n-1}^T \hat{\bar{\theta}}_{n-1}) , \tag{52}$$

$$K_{n-1} = \mu_n P_{n-1} \bar{X}_{n-1} ,$$

where

$$\hat{\bar{\theta}} = \{\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_m, \hat{b}_1', \hat{b}_2', \ldots, \hat{b}_{m+d_{\max}}' \}^{\mathrm{T}} , \tag{53}$$

$$\bar{X}_{n-1}^{\mathrm{T}} = \{-y_{n-1}, \ldots, y_{n-m}, u_{n-1}, \ldots, u_{n-m-d_{\max}}\} . \tag{54}$$

$$\mu_n = \{\beta + \bar{X}_{n-1}^{\mathrm{T}} P_{n-1} \bar{X}_{n-1}\}^{-1} , \ 0 < \beta \le 1 ,$$

$$P_{n-1} = [I - K_{n-1} \bar{X}_{n-1}^{\mathrm{T}}] P_{n-2} / \beta , \ P_0 >> 0 , \tag{55}$$

If the condition for obtaining consistent estimates according to the RLSM is met, then the parameters $\hat{b}_i'$ ($i = 1, \ldots, \hat{d}$) and $\hat{b}_k'$ at $k = (m + \hat{d} + 1), \ldots, (m + d_{\max})$ converge to zero at $n \to \infty$. For a finite number of steps when estimating using RLSM, the values of these parameters will be significantly smaller compared with the parameters $\hat{b}_j'$ for $j = (\hat{d} + 1), \ldots, (\hat{d} + m)$, where m is the order of the polynomial $B(z^{-1})$ without delay. In this case, the estimate of the delay time $\hat{d}$ was determined by founding the maximum number i of the first parameters $\hat{b}_i'$ of the polynomial $\hat{B}'(z)$, which will be small compared with the subsequent parameters of this polynomial.

However, repeated simulations of such algorithm showed that the resulting delay time estimates $\hat{d}$ very often turned erroneous.

The error of estimates $\hat{d}$ ranged from 0÷100 %.

To obtain reliable estimates $\hat{d}$, a method based on calculating the impulse characteristics of the controlled object was developed in [17]. In Fig. 4 it is wown shown a block diagram of the algorithm for estimating the delay. According to the algorithm, the estimation of delay $\hat{d}$ and coefficients of the polynomial $\hat{B}(z^{-1})$, that is $\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_m$, is performed in the following sequence.

1. In block 1, to identify the parameters of the high-order model (50), the coefficients $a_1, a_2, \ldots, a_m$, $b_1, b_2, \ldots, b_{m+d_{\max}}$ are estimated at each sampling period in accordance with the measurements of the output and control action included in the vector $\overline{X}_{n-1}$ according to (54) using the extended RLSM (52)–(55) .

2. To estimate the true delay time of an object $\hat{d}$, a method is used to calculate and compare the weighting functions (impulse characteristics) of the object using models (49) and (50), in which the polynomials in the denominators are equal. Using the extended model (50), in block 2 the weighting function $H'(n)$ is calculated as an inverse transformation from

$$W'(z) = \frac{\hat{B}'(z^{-1})}{A(z^{-1})}. \tag{56}$$

As a result of dividing the numerator polynomial of this discrete function by the denominator polynomial, we obtain recurrence relations for calculating samples $\hat{h}'_k$ of the weighting function $H'(n)$

$$\begin{aligned} &\hat{h}'_1 = \hat{b}'_1; \\ &\hat{h}'_2 = \hat{b}'_2 - \hat{a}_1\hat{b}'_1 = \hat{b}'_2 - \hat{a}_1\hat{h}'_1; \\ &\vdots \\ &\hat{h}'_{m+d_{\max}} = \hat{b}'_{m+d_{\max}} - \hat{a}_1\hat{h}'_{m+d_{\max}-1} - \ldots - \hat{a}_m\hat{h}'_{d_{\max}}. \end{aligned} \tag{57}$$

In this case, the estimated coefficients $\hat{a}_i$, $\hat{b}'_i$ of parameters of the object model (56) from the output of block (1) are supplied to the inputs of blocks 2, 3, 6.

3. From the output of block 2 for calculating the weighting function according to the high-order model $\hat{W}'(z)$, that is, samples (57), go to the second inputs of block 3 to calculate the weighting function $H(n)$ according to the low-order model $\hat{W}(z)$, according to (49). In this case, estimates of the coefficients $\hat{a}_i$, $\hat{b}'_i$ from identification block 1 are supplied to the first inputs of block 3.

In block 3, the inverse z-transformation procedure for model (49) is implemented, as a result of which the weighting function samples $\hat{h}_k$ for the low-order model are determined. Since the transfer functions $\hat{W}'(z)$ and $\hat{W}(z)$ have the same polynomials in the denominator, and the numerator $\hat{W}(z)$ polynomial $\hat{B}(z)$ affects only those values of the weighting function samples $\hat{h}_k$ that have indices $k = (d+1), \ldots, (d+m)$, then the assumption $\hat{h}_k = \hat{h}'_k$ is permissible at $k = (d+1), \ldots, (d+m)$. Given this assumption, we can write

$$\begin{aligned} &\hat{h}_1 = \hat{h}_2 = \ldots = \hat{h}_d \triangleq 0; \\ &\hat{h}_{d+1} = \hat{h}'_{d+1} = \hat{b}_1; \\ &\hat{h}_{d+2} = \hat{h}'_{d+2} = \hat{b}_2 - \hat{a}_1\hat{h}'_{d+1}; \\ &\vdots \\ &\hat{h}_{d+m} = \hat{h}'_{d+m}; \\ &\vdots \\ &\hat{h}_{m+d_{\max}} = -\hat{a}_1\hat{h}_{m+d_{\max}-1} - \ldots - \hat{a}_m\hat{h}_{d_{\max}}. \end{aligned} \tag{58}$$

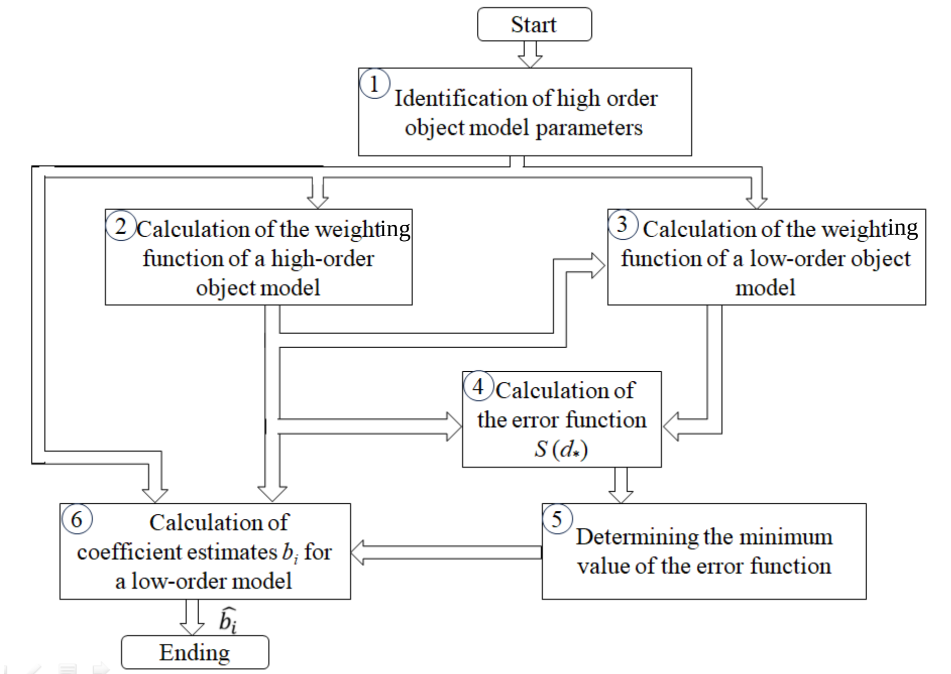

Fig. 4

The weighting function (58) samples are calculated for the entire values range of lag times $0, \ldots, d_{\max} - 1$. In this case, samples of the weighting function (57) of the extended object model, which were calculated in block 2, are used.

4. Samples of the weighting functions calculated in blocks 2 and 3 are fed into block 4 to find the error function $S(d_*)$ defined by the relation

$$S(d_*) = \begin{cases} \sum_{k=1}^{d_*} (\hat{h}'_k)^2; \\ 0 \quad \text{at } k = d_* + 1, \ldots, d_* + m; \\ \sum_{k=m+d_*+1}^{L} (\hat{h}'_k - \hat{h}_k)^2, \ \forall d_* \in [0, \ldots, d_{\max} - 1], \end{cases} \quad (59)$$

where $L = t_{95} / T_0$; t_{95} — time for the transient response to reach 95 % of the steady-state value; T_0 is the sampling period.

5. The error function values $S(d_*)$ from block 4 go to block 5, where the minimum value is determined

$$S_{\min}(d_*) = \min\{S(d_*)\}; \ \forall d_* \in [0, \ldots, \hat{d}_{\max} - 1]. \quad (60)$$

After the values $S(d_i)$ are calculated, according to (60), the minimum value of this function is found for all $d_* \in [0, \ldots, \hat{d}_{\max} - 1]$.

The value d_* at $\min\{S(d_i)\}$ is the desired estimate of the delay time of the object.

Thus, from the set of weighting functions (58) of the discrete model (49), one $\hat{h}_k$ is determined that best agrees with the weighting function $\hat{h}'_k$ of the extended model of the object $\hat{W}'(z)$.

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6. The value d found in block 5 is transferred to the first input of block 6 to calculate estimates of the coefficients $\hat{h}'_k$ of the low-order model, the second input of which receives coefficients $\hat{a}_i$ from block 1, and the third input receives weighting function samples $\hat{h}'_k$ from block 2.

Based on the assumption that $\hat{h}_k = \hat{h}'_k$ for $k = (d+1), \ldots, (d+m)$, in block 6, estimates of the coefficients $\hat{b}_i$ of model (49) are calculated according to (58)

$$\begin{aligned}
&\hat{b}_1 = \hat{h}'_{1+\hat{d}};\\
&\hat{b}_2 = \hat{h}'_{2+\hat{d}} + a_1\hat{h}'_{1+\hat{d}};\\
&\vdots\\
&\hat{b}_m = \hat{h}'_{m+\hat{d}} + a_1\hat{h}'_{m+\hat{d}-1} + \ldots + \hat{a}_{m-1}\hat{h}'_{1+\hat{d}},
\end{aligned} \tag{61}$$

which, together with the delay time estimate $\hat{d}$ and coefficients $\hat{a}_i$, will be used in the synthesis of optimal digital controllers.

To identify the parameters a_i, b_i and delay time d, it is necessary to apply a signal containing a wide range of harmonic components to the input, for example, a sequence of rectangular pulses.

Conclusion

The theory of automatic control based on input-output models (transfer functions) began to develop in the late 20s of the twentieth century. In the early 1970s, input-output models received significant development with the advent of stochastic discrete time series regression models. These models have become widely used to describe the dynamics of economic and financial processes due to their simplicity, since it has become possible to describe the dynamics of complex systems using algebraic equations with respect to the inverse shift operator. In this case, it turned out to be very simple to take into account the delay of the input signal of the controlled object. This made it possible to take into account the delay in the control actioned when designing regulators in automatic control systems.

Also, when using input-output models in discrete time, it is easy to use the recurrent least squares method to estimate model coefficients, which can change due to changes in the dynamics of the control object during operation.

It is necessary to note the following significant limitations for the use of input-output models for solving various classes of control theory problems.

In time series regression models, «white» noise signals are used as input stochastic disturbances, the mathematical expectation of which is zero. However, in real control processes over limited periods of time, the average value of disturbances, as a rule, does not equal zero.

When describing the dynamics of controlled objects based on transfer functions during the design and study of control systems in transient modes, it is necessary to assume the presence of zero initial conditions.

When designing a multidimensional control system using a matrix discrete transfer function or an ARMAX matrix model, it is necessary to satisfy the condition of equality of dimensions of the vectors of output controlled coordinates and control actions. This sharply narrows the class of controlled objects when using these models.

When using input-output models, the dynamics of unmeasured internal movements in multidimensional controlled objects, which are not taken into account in the vector of output measured coordinates, are not taken into account.

And finally, when using input-output models, it is almost impossible to automate the operation of individual control loops in multidimensional automatic control systems in transient modes.

Due to these limitations, the use of input-output models has found wide application in automatic control systems to stabilize the operation of technological processes.

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ЕТАПИ ТА ОСНОВНІ ЗАДАЧІ СТОЛІТНЬОГО РОЗВИТКУ ТЕОРІЇ СИСТЕМ КЕРУВАННЯ ТА ІДЕНТИФІКАЦІЇ. Частина 2. МЕТОДИ ПРОЄКТУВАННЯ ЛІНІЙНИХ СИСТЕМ КЕРУВАННЯ НА ОСНОВІ МАТЕМАТИЧНИХ МОДЕЛЕЙ ПРОЦЕСІВ ТИПУ «ВХІД–ВИХІД»

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У статті виконано огляд математичного опису динаміки неперервних та дискретних лінійних стаціонарних систем та об'єктів, що застосовується на етапі розвитку класичної теорії автоматичного керування у формі математичних моделей типу «вхід–вихід». Описано часові та частотні характеристики неперервних та дискретних систем керування, розглянуто типові ланки стаціонарних систем, наведено параметричні дискретні моделі об'єктів у складі типових контурів цифрового керування. Розглянуто стохастичні дискретні авторегресійні моделі стаціонарних часових рядів, які застосовуються для опису динаміки об'єктів при синтезі цифрових систем керування. Здійснено огляд типових законів керування для реалізації неперервних та дискретних регуляторів. Розглянуто метод синтезу дискретних регуляторів для багатовимірних об'єктів керування з різними запізненнями, невідомими та змінними, за допомогою якого здійснюється компенсація змінних запізнювань в характеристичному рівнянні замкнутої системи керування. Розглянуто поширену методику синтезу одно- та багатовимірних регуляторів для стохастичних об'єктів із запізнюваннями на основі моделей ARMAX. Проведено аналіз підходів до ідентифікації запізнювань в об'єктах керування та розглянуто метод ідентифікації запізнення при застосуванні моделей типу «вхід–вихід», заснований на обчисленні та порівнянні імпульсних характеристик для розширеної та нерозширеної моделей об'єкта керування. Наведено аналіз переваг та недоліків моделей типу «вхід–вихід», а також можливостей їх застосування для вирішення різних класів задач теорії керування.

Ключові слова: передатна функція, закони керування, дискретні регулятори, ідентифікація, об'єкти із запізненням.

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