АДАПТИВНЕ КЕРУВАННЯ ТА МЕТОДИ ІДЕНТИФІКАЦІЇ

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STAGES AND MAIN TASKS OF THE CENTURY-LONG CONTROL THEORY AND SYSTEM IDENTIFICATION DEVELOPMENT.

Part IV. METHODS AND PROBLEMS OF DESIGNING ROBUST CONTROL SYSTEMS

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The article provides a review of the most important methods and problems in the design of robust discrete control systems. In this case, the main attention was focused on the problems of suppressing limited external disturbances, information about which is presented only in the form of a limitation on their maximum value. The use of invariant ellipsoids ε_x is considered as the first mathematical apparatus for describing the characteristics of the influence of external disturbances on the trajectory of motion of dynamic systems. Theorems on the representation of invariant ellipsoids ε_x in the form of linear matrix inequalities (LMI) are formulated, which are further used to synthesize discrete state controllers that suppress external disturbances. The solution to a more general problem of robust suppression of limited disturbances based on the use of LMI in the presence of system uncertainties in the parameters of the mathematical model of the control object is considered. The use of $H^{\propto}$-control theory is considered as a second mathematical apparatus for suppressing external l_2-limited external disturbances. In this case, the optimality criterion consists in minimizing the maximum ratio of the l_2-norm of the vector of output stabilized coordinates to the l_2-norm of the vector of input disturbances. The problem is solved by reducing it to the problem of robust control of a discrete dynamic system in space H^{∞} based on the Two-Ricatti approach. The standard $H^{\propto}$-optimization problem is also considered.

Keywords: state controller, invariant ellipsoid, linear matrix inequalities, robust control, $H^{\propto}$-theory, external disturbances.

Introduction

The methods for designing control systems based on state-space object models and input-output models discussed in previous articles [1, 2] were used to meet the requirement that these models accurately describe the dynamics of control objects. To do this, in the third part [3] of this series of articles, methods for identifying the parameters of mathematical models were considered. However, in modern and classical control theories, to describe the dynamics of controlled objects operating in a stochastic environment, it is assumed that white noise, the mathematical expectation of which is zero, is used as disturbances acting on the plant. In control systems, it is very rare that real disturbances over limited periods of time can be represented as white noise. For example, a gust of wind during take off or landing of an airplane in no way fits into the probabilistic characteristics of white noise. Thus, the uncertainty of the characteristics of external changing disturbances acting on the control object is the main problem in the design of controllers that guarantee stability of the control system in a stochastic environment. Also, in almost every engineering problem of designing a control system, there is uncertainty, which lies in the fact that the mathematical model of the object, obtained as a result of identifying its parameters, will be adequate to the true dynamics of the object only for a limited period of time, after which it will change in a certain area according to an unknown law. As a result, the problem of controller synthesis under conditions of uncertainty of the dynamic characteristics of the object and external disturbances required the development of new approaches to the design of control systems in comparison with the methods that were described in review articles [1–3].

At the end of the 70s of the last century, in the works [4–6], the problem of optimal suppression of arbitrary limited external disturbances acting on the control object was formulated. This problem is called l_1-optimization when describing control systems in discrete time. Later, in [7–9], the linear matrix inequalities (LMI) technique began to be used to suppress external disturbances with arbitrary characteristics.

The problem of optimal suppression of external disturbances is one of the main ones in control theory and besides remain one of the difficult problems of linear control theory [10]. To solve this problem, methods have currently been developed and applied that belong to two theoretical directions:

1) methods of invariant ellipsoids;

2) methods of H^∞-theory.

The development of these directions over several decades was mainly carried out for the original mathematical models of control objects in continuous time. In this article we will analyze this stage of development of the theory of control systems based on discrete-time models aimed at the practical implementation of synthesized systems in microprocessor execution.

In real problems of functioning control systems, there is inevitably system uncertainty and the control being formed must be operational under the most extreme conditions. This type of control is called robust. This control must ensure robust stability of the designed control system when changing external disturbances in given intervals and for a given family of state matrices of the mathematical model of the object

$$F = F_0 + \gamma\Delta,$$

where F_0 is the nominal matrix of the family, and Δ is the uncertainty. Coefficient $\gamma > 0$ determines the range of uncertainty.

1. The problem of suppressing limited external disturbances using the method of invariant ellipsoids

In [11, 12], invariant ellipsoids are considered as a characteristic of the influence of external disturbances on the trajectory of a dynamic system, which is represented by a mathematical model in state space in discrete time

$$\begin{aligned} &\bar{x}(k+1) = F\bar{x}(k) + \Phi\bar{\xi}(k), \\ &\bar{y}(k) = C\bar{x}(k), \end{aligned} \tag{1}$$

where $\bar{x}(k) \in \mathrm{R}^n$ are the phase variables of the system; $\bar{y}(k) \in \mathrm{R}^l$ are output coordinates of the system; $\bar{\xi}(k) \in \mathrm{R}^m$ are external disturbances, which are limited as follows:

$$\left\| \bar{\xi}(k) \right\| \le 1, k = 0, 1, 2, \dots, \tag{2}$$

where $\| \cdot \|$ is a vector norm in Euclidean space. In this way, l_∞-limited external disturbances are considered. No other restrictions are imposed on the disturbances, that is, it is assumed that the they are arbitrary and only bounded. It is also assumed that system (1) is stable and the pair (F, Φ) will be controllable. The matrix C has maximum row rank.

The family of invariant ellipsoids is defined as follows: an ellipsoid centered at the origin

$$\varepsilon_x = \{\bar{x}(k) \in \mathrm{R}^n : \bar{x}^{\mathrm{T}}(k) P^{-1} \bar{x}(k) \le 1\}, P > 0 \tag{3}$$

is called state $\bar{x}(k)$ invariant for a discrete dynamic system (1), (2), if the condition $\bar{x}(0) \in \varepsilon_x$ follows that the condition $\bar{x}(k) \in \varepsilon_x$ is satisfied for all discrete moments of time $k = 0, 1, 2, \dots$. The matrix P is called the ellipsoid matrix.

The following important theorem was formulated and proven in [12].

Theorem 1. *Ellipsoid* ε_x *(3) will be invariant for dynamical system (1) with* l_∞*-bounded external disturbance if and only if the matrix* P *satisfies linear matrix inequalities (LMI)*

$$\frac{1}{\alpha} FPF^{\mathrm{T}} - P + \frac{1}{(1-\alpha)} \Phi \Phi^{\mathrm{T}} \le 0, P \ge P(0) \tag{4}$$

at some coefficient $0 < \alpha < 1$.

The objective function is used as an optimality criterion

$$f(P) = \mathrm{tr}[CPC^{\mathrm{T}}], \tag{5}$$

which corresponds to the sum of the squares of the semiaxes of the invariant ellipsoid

$$\varepsilon_y = \{\bar{y}(k) \in \mathrm{R}^m : \bar{y}^{\mathrm{T}}(k)(CPC^{\mathrm{T}})^{-1} \bar{y}(k) \le 1\}.$$

Function (5) specifies the size of the invariant ellipsoid ε_y.

To synthesize a control system, a controlled discrete model of an object in state space is considered

$$\begin{aligned} &\bar{x}(k+1) = F\bar{x}(k) + G_1\bar{u}(k) + \Phi\bar{\xi}(k), \\ &\bar{y}(k) = C\bar{x}(k) + G_2\bar{u}(k), \end{aligned} \tag{6}$$

$\bar{x}(0) \in \varepsilon_0$, $\bar{u} \in \mathrm{R}^p$. At the same time $G_2^T C = 0$, the pair (F, G_1) is controllable.

To suppress disturbances, a state controller is implemented

$$\bar{u}(k) = K_p \bar{x}(k), \tag{7}$$

which ensures minimization of criterion (5), that is, the minimum size of the invariant ellipsoid with respect to the output. Based on (6), (7), the model of the closed-loop control system will have the form

$$\begin{aligned} \bar{x}(k+1) &= (F + G_1 K_p)\,\bar{x}(k) + \Phi\bar{\xi}(k), \\ \bar{y}(k) &= (C + G_2 K_p)\,\bar{x}(k). \end{aligned} \tag{8}$$

In [12] a theorem for the synthesis of state controller (7) was formulated and proven.

Theorem 2. *If for a discrete system (6) external disturbances are* l_∞*-limited and the pair* (F, G_1) *is controllable, then the problem of synthesizing an optimal state controller (7), which suppresses external disturbances, will be equivalent to the problem of minimizing a linear function*

$$\mathrm{tr}[CPC^{\mathrm{T}} + G_2 Z G_2^T] \to \min \tag{9}$$

under restrictions

$$\frac{1}{\alpha}[FPF^{\mathrm{T}} + G_1 Y F^{\mathrm{T}} + F Y^{\mathrm{T}} G_1^T + G_1 Z G_1^T] - P + \frac{\Phi\Phi^{\mathrm{T}}}{(1-\alpha)}, \alpha < 1, \tag{10}$$

$$\begin{bmatrix} Z & Y \\ Y^T & P \end{bmatrix} \geq 0, P \geq P(0), \tag{11}$$

where $Y = K_p P,$ *and minimization is performed over variables* $\alpha \in \mathbf{R},$ $P = P^{\mathrm{T}} \in \mathbf{R}^{n\times n},$ $Y \in \mathbf{R}^{p\times n}$ *and* $Z = Z^{\mathrm{T}} \in \mathbf{R}^{p\times p}.$

Minimization of criterion (5) is performed using the variables $\alpha,$ $P,$ $Y,$ $Z,$ using the semidefinite programming method, using SeDuMi Toolbox [13] based on Matlab. Let $\hat{\alpha},$ $\hat{P},$ $\hat{Y},$ $\hat{Z}$ provide the minimum of criterion (5) under restrictions (10), (11). Then the matrix $\hat{K}_p$ of the optimal controller (7) will be determined as follows:

$$\hat{K}_p = \hat{Y}\hat{P}^{-1}. \tag{12}$$

The requirement $G_2^T C = 0$ is not restrictive. If it is not satisfied, then all the results of Theorem 2 will be correct, only the objective function (9) will be presented in a different form

$$\mathrm{tr}[CPC^{\mathrm{T}} + G_2 Y C^{\mathrm{T}} + C Y^{\mathrm{T}} G_2^T + G_2 Z G_2^T] \to \min. \tag{13}$$

2. Robust suppression of disturbances

In the presence of system uncertainties, the mathematical model of the object (1) will have the form

$$\begin{aligned} \bar{x}(k+1) &= (F + \Delta F)\,\bar{x}(k) + (\Phi + \Delta\Phi)\,\bar{\xi}(k), \\ \bar{y}(k) &= C\bar{x}(k), \end{aligned} \tag{14}$$

where $F \in \mathbf{R}^{n\times n},$ $\Phi \in \mathbf{R}^{n\times m},$ $C \in \mathbf{R}^{l\times n}$ and $\bar{x}(k) \in \mathbf{R}^n$ is the phase state with the initial condition $\bar{x}(0)$ and output $\bar{y}(k) \in \mathbf{R}^l.$ External disturbances $\bar{\xi}(k) \in \mathbf{R}^m$ satisfy constraints (2).

System uncertainties ΔF and $\Delta\Phi$ have the following structure:

$$\begin{aligned} \Delta F &= A_F \Delta_F H_F\,, \\ \Delta\Phi &= A_\Phi \Delta_\Phi H_\Phi\,, \end{aligned} . \tag{15}$$

where $A_F \in \mathbf{R}^{n\times P_F}$, $A_\Phi \in \mathbf{R}^{n\times P_\Phi}$, $H_F \in \mathbf{R}^{q_F\times n}$, $H_\Phi \in \mathbf{R}^{q_\Phi\times m}$ are constant matrices, and the matrix uncertainties $\Delta_F \in \mathbf{R}^{P_F\times q_F}$ and $\Delta_\Phi \in \mathbf{R}^{P_\Phi\times q_\Phi}$ satisfy the following restrictions:

$$\|\Delta_F\| \le 1, \;\; \|\Delta_\Phi\| \le 1. \tag{16}$$

It is assumed that system (14) is stable, the pair (F, Φ) is controllable, and the matrix C has maximum row rank.

Definition. Ellipsoid centred at the origin

$$\varepsilon_x = \{\bar{x}(k) \in \mathrm{R}^n : \bar{x}^{\mathrm{T}}(k) P^{-1} \bar{x}(k) \le 1\},\; P > 0 \tag{17}$$

is called invariant for system (14), (15) if the condition $\bar{x}(0) \in \varepsilon_x$ implies the fulfilment $\bar{x}(k) \in \varepsilon_x$ for all discrete moments of time $k = 0, 1, \ldots$ for all admissible disturbances $\bar{\xi}(k)$ and all admissible uncertainties Δ_F and Δ_Φ.

The following theorem was formulated and proven in [14].

Theorem 3. *Ellipsoid (17) is invariant for system (14) for* $\bar{x}(0) = 0$, *if its matrix* P *satisfies linear matrix inequalities*

$$\begin{bmatrix} -\alpha P & PF^{\mathrm{T}} & 0 & PH_F^T & 0 \\ * & (-P + \varepsilon_1 A_F A_F^T + \varepsilon_2 A_\Phi A_\Phi^T & \Phi & 0 & 0 \\ * & * & -(1-\alpha)I & 0 & H_\Phi^T \\ * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & -\varepsilon_2 I \end{bmatrix} \le 0, \tag{18}$$

$$P > 0$$

at some coefficients $\alpha, \varepsilon_1, \varepsilon_2 \in \mathbf{R}$.

This theorem is a robust analogue of Theorem 1.

The problem of robust suppression of external arbitrary disturbances has been solved in the presence of system uncertainties in the controlled model of the object

$$\begin{aligned} \bar{x}(k+1) &= (F + \Delta F)\bar{x}(k) + (G_1 + \Delta G_1)\bar{u}(k) + (\Phi + \Delta\Phi)\bar{\xi}(k), \\ \bar{y}(k) &= C\bar{x}(k) + G_2 \bar{u}(k), \end{aligned} \tag{19}$$

where control $\bar{u}(k) \in \mathbf{R}^p$, and external disturbance satisfies constraint (2).

System uncertainties of matrices F and Φ have structure (15), and

$$\Delta G_1 = A_{G_1} \Delta_{G_1} H_{G_1}. \tag{20}$$

In this case, the matrix uncertainties $\Delta_F \in \mathbf{R}^{q_1^F \times q_2^F}$, $\Delta G_1 \in \mathbf{R}^{q_1^G \times q_2^G}$, $\Delta_\Phi \in \mathbf{R}^{q_1^\Phi \times q_2^\Phi}$ satisfy the constraint

$$\|\Delta_F\| \le 1, \;\; \|\Delta G_1\| \le 1, \;\; \|\Delta_\Phi\| \le 1, \tag{21}$$

the pair (F, G_1) is controllable and the pair (F, C) is observable. Constant matrices $A_F \in \mathbf{R}^{n\times q_1^F}$, $A_{G_1} \in \mathbf{R}^{n\times q_1^G}$, $A_\Phi \in \mathbf{R}^{n\times q_1^\Phi}$, $H_F \in \mathbf{R}^{q_2^F \times n}$, $H_{G_1} \in \mathbf{R}^{q_2^G \times p}$, $H_\Phi \in \mathbf{R}^{q_2^\Phi \times m}$ are given.

The task is to design a controller (7) in the form of a static linear state feedback $\bar{x}(k)$ that will stabilize a closed-loop control system and suppress the influence of external arbitrary limited disturbances $\bar{\xi}(k)$ in the presence of system uncertainties (15), (20) based on minimizing the trace of the limiting ellipsoid for the output $\bar{y}(k)$.

To solve this problem, the following theorem was formulated and proven in [14].

Theorem 4. *Let* $\hat{P}$, $\hat{Y}$, $\hat{Z}$ *be the solution to the problem for the optimality criterion (13)* $\mathrm{tr}[CPC^{\mathrm{T}} + CY^{\mathrm{T}}G_2^T + G_2YC^{\mathrm{T}} + G_2ZG_2^T] \to \min$ *under restrictions*

$$\begin{bmatrix} -\alpha P & PF^{\mathrm{T}} + Y^{\mathrm{T}}G_1^T & 0 & PH_F^T & Y^{\mathrm{T}}H_{G_1}^T & 0 \\ * & \Omega & \Phi & 0 & 0 & 0 \\ * & * & -(1-\alpha)I & 0 & 0 & H_\Phi^T \\ * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & -\varepsilon_3 I \end{bmatrix} \le 0, \qquad (22)$$

$$\begin{bmatrix} Z & Y \\ Y^T & P \end{bmatrix} > 0,, \qquad (23)$$

where $Y = K_pP$, $\Omega = -P + \varepsilon_1 A_F A_p^T + \varepsilon_2 A_{G_1} A_{G_2}^T + \varepsilon_3 A_\Phi A_\Phi^T$ *with respect to matrix variables* $P = P^{\mathrm{T}} \in \mathbf{R}^{n\times n}$, $Y \in \mathbf{R}^{p\times n}$, $Z = Z^{\mathrm{T}} \in \mathbf{R}^{p\times p}$ *and scalar variables* ε_1, ε_2, ε_3 *and scalar parameter* $0 < \alpha < 1$.

Finding the optimal values of $\hat{P}$, $\hat{Y}$, $\hat{Z}$ to minimize criterion (13) is performed using the semidefinite programming method in the Matlab system [13]. Then the matrix K_p of the robust state controller (7) is determined according to (12), that is

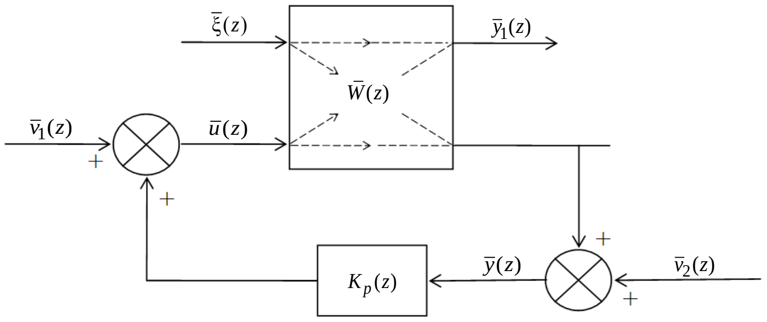
$$\hat{K}_p = \hat{Y}\hat{P}^{-1},$$

which stabilizes the system (14), and the matrix $C\hat{P}C^{\mathrm{T}} + C\hat{Y}^{\mathrm{T}}G_2^T + G_2\hat{Y}C^{\mathrm{T}} + G_2\hat{Z}G_2^T$ determines the size of the limiting ellipsoid for the vector of output variables for a closed-loop control system at $\bar{x}(0) = 0$.

3. Synthesis of discrete controllers using the control H^∞-theory method

3.1. Standard H^∞-optimization problem [15, 16]. When setting up a specific problem in control H^∞-theory, it is advisable to reduce it to the so-called standard problem. We consider the general diagram of a closed-loop control system with a discrete controller (Figure), where $\bar{y}(z)$ is the vector of output measured coordinates of a multidimensional controlled object; $\bar{y}_1(z)$ is vector of output adjustable coordinates; $\bar{\xi}(z)$ is vector of external disturbances; $\bar{u}(z)$ is vector of control actions. Signals

$\bar{v}_1(z)$, $\bar{v}_2(z)$ are introduced to define the concept of stability of a closed-loop system. For example, a vector $\bar{v}_1(z)$ arises due to the inaccuracy of actuators, and $\bar{v}_2(z)$ may be a vector of measurement noise.



The matrix discrete transfer function (MDTF) $\bar{W}(z)$ consists of limited rational MDFTs of the generalized controlled object, that is $\bar{W}(z)\in \mathbf{R}H^{\infty}$, $K_p(Z)$ — matrix discrete transfer function of the controller.

The matrix $\bar{W}(z)$ can be represented as

$$\bar{W}(z)=\begin{bmatrix}\bar{W}_{11}(z) & \bar{W}_{12}(z)\\ \bar{W}_{21}(z) & \bar{W}_{22}(z)\end{bmatrix}, \tag{24}$$

on the basis of which the dynamics of the controlled object can be represented in the form of an input-output type model with $\bar{v}_1=\bar{v}_2=0$

$$\bar{y}_1(z)=\bar{W}_{11}(z)\,\bar{\xi}(z)+\bar{W}_{12}(z)\bar{u}(z), \tag{25}$$

$$\bar{y}(z)=\bar{W}_{21}(z)\,\bar{\xi}(z)+\bar{W}_{22}(z)\bar{u}(z). \tag{26}$$

The dynamics of the controlled object can also be represented in the form of a state space model

$$\bar{x}(k+1)=F\bar{x}(k)+G\bar{u}(k)+\Phi\bar{\xi}(k), \tag{27}$$

$$\bar{y}(k)=C_1\bar{x}(k)+D\bar{\xi}(k). \tag{28}$$

The equation of stabilized coordinates is introduced separately in the form

$$\bar{y}_1(k)=C_2\bar{x}(k). \tag{29}$$

The correspondence between models (25), (26) and (27)–(29) is carried out as follows. Let us present the dynamics of the equation of state (27) and measurement (28) in the form of an input-output model

$$\bar{x}(z)=(zI-F)^{-1}G\bar{u}(z)+(zI-F)^{-1}\Phi\bar{\xi}(z), \tag{30}$$

which we substitute into expressions (28) and (29)

$$\bar{y}(z)=C_1(zI-F)^{-1}G\bar{u}(z)+C_1[(zI-F)^{-1}\Phi+D]\bar{\xi}(z), \tag{31}$$

$$\bar{y}_1(z)=C_2(zI-F)^{-1}G\bar{u}(z)+C_2(zI-F)^{-1}\Phi\bar{\xi}(z). \tag{32}$$

Comparing expressions (32), (31), respectively, with models (25), (26), we obtain the MDPF values

$$
\begin{aligned}
&\bar{W}_{11}(z) = C_2(zI - F)^{-1}\Phi,\\
&\bar{W}_{12}(z) = C_2(zI - F)^{-1}G,\\
&\bar{W}_{21}(z) = C_1(zI - F)^{-1} + D,\\
&\bar{W}_{22}(z) = C_1(zI - F)^{-1}G.
\end{aligned}
\tag{33}
$$

Let us represent the regulator control law in the form

$$\bar{u}(z) = K_p(z)\bar{y}(z). \tag{34}$$

Using expressions (31), (33), (34), we obtain the equation of the closed-loop system for the channel $\bar{\xi}(z) - \bar{y}(z)$

$$\bar{y}(z) = [I - \bar{W}_{22}(z)K_p(z)]^{-1}\bar{W}_{21}(z)\bar{\xi}(z). \tag{35}$$

Based on expression (25), control law (34) and equation (35), we find the equation of the closed-loop system along the channel $\bar{\xi}(z) - \bar{y}(z)$

$$\bar{y}_1(z) = \{\bar{W}_{11}(z) + \bar{W}_{12}(z)K_p(z)[I - \bar{W}_{22}(z)K_p(z)]^{-1} \cdot \bar{W}_{21}(z)\}\bar{\xi}(z). \tag{36}$$

Thus, the matrix discrete transfer function of the closed-loop system will be

$$\bar{W}_{cls}[\bar{W}(z), K_p(z)] = \bar{W}_{11}(z) + \bar{W}_{12}(z)K_p(z)[I - \bar{W}_{22}(z)K_p(z)]^{-1} \cdot \bar{W}_{21}(z). \tag{37}$$

For the closed-loop system equation (37), the standard H^∞-optimization problem is formulated as follows: it is necessary to design a controller $K_p(z)$ that will minimize the H^∞-norm of the matrix discrete function along the channel $\bar{\xi}(z) - \bar{y}_1(z)$ according to

$$\inf_{K_p} \left\| \bar{W}_{cls}[\bar{W}(z), K_p(z)] \right\|_\infty = \gamma_{\min}. \tag{38}$$

In this case, the influence of external disturbances $\bar{\xi}(z)$ on the vector of output regulative coordinates $\bar{y}_1(z)$ is maximally suppressed.

The dynamics of a closed-loop control system (Figure) can be described by a system of equations

$$
\begin{aligned}
&\bar{y}_1(z) = \bar{W}_{11}(z)\bar{\xi}(z) + \bar{W}_{12}(z)\bar{u}(z),\\
&\bar{y}(z) = \bar{W}_{21}(z)\bar{\xi}(z) + \bar{W}_{22}(z)\bar{u}(z) + \bar{v}_2(z),\\
&\bar{u}(z) = K_p(z)\bar{y}(z) + \bar{v}_1(z),
\end{aligned}
$$

which can be represented in the form

$$
\begin{aligned}
&\bar{y}_1(z) - \bar{W}_{12}(z)\bar{u}(z) = \bar{W}_{11}(z)\bar{\xi}(z),\\
&\bar{u}(z) - K_p(z)\bar{y}(z) = \bar{v}_1(z),\\
&\bar{y}(z) - \bar{W}_{22}(z)\bar{u}(z) = \bar{W}_{21}(z)\bar{\xi}(z) + \bar{v}_2(z).
\end{aligned}
$$

This system can be written in a generalized vector-matrix form:

$$\begin{bmatrix} I & -\bar{W}_{12}(z) & 0 \\ 0 & I & -K_p(z) \\ 0 & -\bar{W}_{22}(z) & I \end{bmatrix} \cdot \begin{bmatrix} \bar{y}_1(z) \\ \bar{u}(z) \\ \bar{y}(z) \end{bmatrix} = \begin{bmatrix} \bar{W}_{11}(z) & 0 & 0 \\ 0 & I & 0 \\ \bar{W}_{21}(z) & 0 & I \end{bmatrix} \cdot \begin{bmatrix} \bar{\xi}(z) \\ \bar{v}_1(z) \\ \bar{v}_2(z) \end{bmatrix}. \tag{39}$$

This system will be well conditioned if the matrix

$$\begin{bmatrix} I & -\bar{W}_{12}(z) & 0 \\ 0 & I & -K_p(z) \\ 0 & -\bar{W}_{22}(z) & I \end{bmatrix}$$

will be reversible [16]. This system will be internally stable if and only if the nine discrete transfer functions from vectors $\bar{\xi}(z)$, $\bar{v}_1(z)$, $\bar{v}_2(z)$ to vectors $\bar{y}_1(z)$, $\bar{u}(z)$, $\bar{y}(z)$ are asymptotically stable. In this case, the synthesized regulator $K_p(z)$, according to (38), belongs to the set of internally stabilizing regulators, that is, the regulator $K_p(z)$ will stabilize the object $\bar{W}(z)$.

3.2. The problem of suppressing limited external disturbances based on H^{∞}-theory. In the early 90s, a number of works appeared devoted to the application of the H^{∞}-theory in motion control problems. One of these types of problems is the problem of forming a control that minimizes the effect of an external disturbance on the controlled object. For example, a typical disturbance is a sudden gust of wind of high intensity, which is especially dangerous during take-off and landing of an aircraft and its flight at low altitudes.

Currently, a number of H^{∞}-optimization methods have been developed [17–21], based on which the so-called Two-Ricatti approach began to be used as the main tool for solving problems of synthesis of H^{∞}-optimal controllers. In this case, the synthesized system is presented in Figure. The standard object is specified as

$$\bar{W}(z) = \left[\begin{array}{c|cc} F & G & \Phi \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right], \tag{40}$$

and the system (Figure) is described by the following system of equations in the state space

$$\begin{aligned} \bar{x}(k+1) &= F\bar{x}(k) + G\bar{u}(k) + \Phi\bar{\xi}(k), \\ \bar{y}_1(k) &= C_1\bar{x}(k) + D_{11}\bar{u}(k) + D_{12}\bar{\xi}(k), \\ \bar{y}(k) &= C_2\bar{x}(k) + D_{21}\bar{\xi}(k), \end{aligned} \tag{41}$$

where $\bar{x}$ is the state vector, $\bar{u}$ is the control vector, $\bar{y}$ is the vector of output measured variables; $\bar{y}_1$ is vector of controlled variables; $\bar{\xi}$ is vector of external disturbances.

In this case, a generalized sequence of disturbance vectors $\bar{v} = (\bar{v}(0), \bar{v}(1), \ldots$ $\ldots, \bar{v}(k), \ldots)$ in the space l_2 is considered, the norm of which is determined according to [16, 17]

$$\|\bar{v}\| = \left(\sum_{k=0}^{\infty} \bar{v}^{\mathrm{T}}(k)\bar{v}(k) \right)^{1/2}. \tag{42}$$

By definition, a sequence $\bar{v}(k) \in l_2$ if this series converges, that is, $\|\bar{v}\| < \infty$.

We consider the problem of minimizing the H^{∞}-norm of a matrix discrete transfer function of a closed-loop system with a controller

$$\bar{u}(k) = K_p \bar{y}(k) \tag{43}$$

to suppress disturbances along the $\bar{\xi}(k) \to \bar{y}_1(k)$ channel, where $\bar{\xi}(k)$ is the vector of all input signals that contain external disturbances.

The solution to this problem will guarantee the robustness of the closed-loop control system, which consists in the fact that for all possible values of the sequence of disturbances $\bar{\xi}(k) \in l_2$, the maximum l_2-norm of the controlled output signal $\bar{y}_1(k)$ will be minimized.

The optimality criterion is formulated as follows:

$$J = \sup_{\bar{\xi}(k) \in l_2} \frac{\|\bar{y}_1\|}{\|\bar{\xi}\|} \to \min, \tag{44}$$

where $\bar{y}_1$ is a sequence of vectors $\{\bar{y}_1(k), k = 0, 1, \ldots\}$, $\bar{\xi}$ is a sequence $\{\bar{\xi}(k), k = 0, 1, \ldots\}$ that belong to the space l_2. Minimization of the criterion J is performed by forming an optimal sequence of control vectors $\bar{u} = \{\bar{u}(k), k = 0, 1, \ldots\}$ that influence the vector $\bar{y}_1$. In this way, the minimax problem (43) is solved under restrictions $\|\bar{\xi}\| < \infty$ on the norm l_2.

In works [17–21] it is shown that the value J of criterion (44) corresponds to the norm $\|\bar{W}_{cls}(z)\|_{\infty}$, where $\bar{W}_{cls}(z)$ is the MDFT of the closed-loop system along the « $\bar{\xi}(z) \to \bar{y}_1(z)$ » channel, that is $\bar{y}_1(z) = \bar{W}_{cls}(z)\bar{\xi}(z)$, and the norm $\|\bar{W}_{cls}(z)\|_{\infty}$ is calculated in space H^{∞} (Hardy space of complex matrix functions, analytic in the unit disk $|z| < 1$ and limited to circle $|z| = 1$ to which all MDFTs of stable discrete systems belong) and is equal to the singular number $\sigma(\bar{W}_{cls}(z))$ for all $|z| \le 1$. Indeed, according to Parseval's theorem, the norm l_2 (in the time domain) is equal to the norm L_2 (in the frequency domain), that is

$$\|\bar{v}\| = \left(\sum_{k=0}^{\infty} \bar{v}^*(k)\bar{v}(k) \right)^{1/2} = \left(\frac{1}{2\pi} \int_0^{2\pi} \bar{v}^{\mathrm{T}}(e^{j\omega})\bar{v}(e^{j\omega})\, d\omega \right)^{1/2} = \| \bar{v}(e^{j\omega}) \|_{L_2},$$

where $\bar{v}(e^{j\omega})$ is the discrete Fourier transform of an arbitrary discrete time sequence $\bar{v}$, $\bar{v}(e^{j\omega}) = \sum_{k=0}^{\infty} \bar{v}(k) e^{j\omega k} = \sum_{k=0}^{\infty} \bar{v}(k) z^k = \bar{v}(z)$, $z = e^{j\omega}$, $\bar{v}^*(e^{j\omega}) = \bar{v}^{\mathrm{T}}(e^{-j\omega})$ is complex conjugate vector. Taking account that $\bar{y}_1(z) = \bar{W}_{cls}(z)\bar{\xi}(z)$ or $\bar{y}_1(e^{j\omega}) = W_{cls}(e^{j\omega})\bar{\xi}(e^{j\omega})$, we will have

$$\|\bar{y}_1\| = \| \bar{y}_1(e^{j\omega}) \|_{L_2} = \left(\frac{1}{2\pi} \int_0^{2\pi} \bar{y}_1^*(e^{j\omega})\bar{y}_1(e^{j\omega})\, d\omega \right)^{1/2} =$$

$$= \left(\frac{1}{2\pi} \int_0^{2\pi} \bar{\xi}^*(e^{j\omega}) \bar{W}_{cls}^*(e^{j\omega}) W_{cls}(e^{j\omega})\, \bar{\xi}(e^{j\omega})\, d\omega \right)^{1/2} \le$$

$$\le \left(\frac{1}{2\pi} \int\limits_0^{2\pi} \sigma(\bar{W}_{cls}(e^{j\omega}))\, \bar{\xi}^*(e^{j\omega})\, \bar{\xi}(e^{j\omega})\, d\omega \right)^{1/2} \le$$

$$\le \sup_{\omega} \sigma(\bar{W}_{cls}(e^{j\omega})) \left(\frac{1}{2\pi} \int\limits_0^{2\pi} \bar{\xi}^*(e^{j\omega})\, \bar{\xi}(e^{j\omega})\, d\omega \right)^{1/2} =$$

$$= \| \bar{W}_{cls}(e^{j\omega}) \|_\infty \, \| \bar{\xi}(e^{j\omega}) \|_{L_2} = \| \bar{W}_{cls}(z) \|_\infty \, \| \bar{\xi} \|.$$

Then

$$J = \sup_{\bar{\xi}(k) \in l_2} \frac{\| \bar{y}_1 \|}{\| \bar{\xi} \|} = \sup_{\bar{\xi}(e^{j\omega}) \in L_2} \frac{\| \bar{y}_1(e^{j\omega}) \|_{L_2}}{\| \bar{\xi}(e^{j\omega}) \|_{L_2}} \le \left\| \bar{W}_{cls}(z) \right\|_\infty = \sup_{\omega} \sigma(\bar{W}_{cls}(e^{j\omega})) = \sup_{\|z\| \le 1} \sigma(\bar{W}_{cls}(z)).$$

In [17] it is shown that it is not even inequality that holds, but equality.

Thus, criterion (44) is equivalent to the following optimality criterion

$$J = \left\| \bar{W}_{cls}(z) \right\|_\infty = \sup_{\|z\| \le 1} \sigma(\bar{W}_{cls}(z)) \to \min_{\bar{u}}. \qquad (45)$$

Since $\bar{W}_{cls}(z)$ there is an MDPF of a closed-loop control system, it depends on the control law (43), which forms the control $\bar{u}$. In this case, the vector $\bar{u}$ should not depend on the state vector $\bar{x}$, which may be unmeasurable, but on the vector of measured variables $\bar{y}$. That is, the state vector $\bar{x}$ must be implicitly evaluated at the same time. The following theorem was formulated in [21].

Theorem 5. *Let system (41) be given and the following conditions be satisfied*

a) the pair (F, G) *must be stabilized;*

b) the pair (C_2, F) *must be detectable;*

c) matrices D_{12} *and* D_{21} *must have full rank, then the controller, which ensures the stability condition of the closed-loop system, and the optimality criterion* $J = \left\| \bar{W}_{cls}(z) \right\|_\infty < 1$ *is designed in the following form:*

$$\begin{aligned} &\bar{u}(k) = C_{\text{contr}} r(k+1) + D_{\text{contr}} \bar{y}(k), \\ &\bar{r}(k+1) = F_{\text{contr}} \bar{r}(k) + B_{\text{contr}} \bar{y}(k), \\ &\dim \bar{r} = \dim \bar{x} \end{aligned} \qquad (46)$$

if and only if there exist non-negative definite symmetric matrices $P \ge 0$, $R \ge 0$ *such that:*

— first Ricatti equation

$$P = F^{\mathrm{T}} P F + C_1^T C_1 - \begin{bmatrix} \Phi^{\mathrm{T}} P F & D_{11}^T C_1 \\ G P F & D_{12}^T C_1 \end{bmatrix}^{\mathrm{T}} \cdot T(P)^{-1} \cdot \begin{bmatrix} \Phi^{\mathrm{T}} P F & D_{11}^T C_1 \\ G P F & D_{12}^T C_1 \end{bmatrix},$$

$$T(P) = \begin{bmatrix} D_{11}^T D_{11} & D_{11}^T D_{12} \\ D_{12}^T D_{11} & D_{12}^T D_{12} - I \end{bmatrix} + \begin{bmatrix} \Phi^{\mathrm{T}} \\ G^{\mathrm{T}} \end{bmatrix} P \begin{bmatrix} \Phi & G \end{bmatrix};$$

— closed-loop control system matrix

$$W_{cp} = F - (\Phi \;\; G) \cdot T(P)^{-1} \cdot \begin{bmatrix} \Phi^{\mathrm{T}} P F & D_{11}^T C_1 \\ G^{\mathrm{T}} P F & D_{12}^T C_1 \end{bmatrix} \text{— asymptotically stable;}$$

— *matrix* $V = \Phi^{\mathrm{T}} P\Phi + D_{11}^T D_{11} > 0$,

$$M = I - D_{12}^T D_{12} - G^{\mathrm{T}} PG + (G^{\mathrm{T}} P\Phi + D_{12}^T D_{11})V^{-1}(\Phi^{\mathrm{T}} PG + D_{11}^T D_{12}) > 0.$$

If there is a matrix P that satisfies the specified conditions, then we define the following auxiliary matrices:

$$L = G^{\mathrm{T}} PF + D_{12}^T C_1 - (G^{\mathrm{T}} P\Phi + D_{12}^T D_{11})V^{-1}(\Phi^{\mathrm{T}} PF + D_{11}^T C_1),$$

$$A_p = F + GM^{-1}L, \;\; E_p = GM^{-1/2},$$

$$C_{1p} = V^{-1/2}(\Phi^{\mathrm{T}} PF + D_{11}^T C_1) + V^{-1/2}(\Phi^{\mathrm{T}} PG + D_{11}^T D_{12})M^{-1}L,$$

$$C_{2p} = C_2 + D_{21}M^{-1}L, \;\; D_{21p} = D_{21}M^{-1/2}, \;\; D_{12p} = V^{1/2},$$

$$D_{11p} = V^{-1/2}(\Phi^{\mathrm{T}} PG + D_{11}^T D_{12})M^{-1/2}.$$

Then the matrix R must satisfy the following conditions:

— second Ricatti equation

$$R = A_p RA_p^T + E_p E_p^T - \begin{bmatrix} C_{2p}RA_p^T + D_{21p}E_p^T \\ C_{1p}RA_p^T + D_{11p}E_p^T \end{bmatrix}^{\mathrm{T}} \cdot H(R)^{-1} \cdot \begin{bmatrix} C_{2p}RA_p^T + D_{21p}E_p^T \\ C_{1p}RA_p^T + D_{11p}E_p^T \end{bmatrix},$$

where

$$H(R) = \begin{bmatrix} D_{21p}D_{21p}^T & D_{21p}D_{11p}^T \\ D_{11p}D_{21p}^T & D_{11p}D_{11p}^T \end{bmatrix} + \begin{bmatrix} C_{2p} \\ C_{1p} \end{bmatrix} R[C_{2p}^T \;\; C_{1p}^T];$$

— matrix of a closed system based on observations

$$W_{cy} = A_p - \begin{bmatrix} C_{2p}RA_p^T + D_{21p}E_p^T \\ C_{1p}RA_p^T + D_{11p}E_p^T \end{bmatrix} \cdot H(R)^{-1} \cdot \begin{bmatrix} C_{2p} \\ C_{1p} \end{bmatrix}$$ must be asymptotically stable;

— matrix $Q = D_{21p}D_{21p}^T + C_{2p}RC_{2p}^T > 0$;

$$S = I - D_{11p}D_{11p}^T - C_{1p}RC_{1p}^T +$$

$$+ (C_{1p}RC_{2p}^T + D_{11p}D_{21p}^T)\,Q^{-1} \cdot (C_{2p}RC_{1p}^T + D_{21p}D_{11p}^T) > 0.$$

If such matrices P, R exist, then the designed controller in the form (46) will be defined as follows:

$$\begin{aligned} D_{\mathrm{contr}} &= -D_{12p}^{-1}(C_{1p}RC_{2p}^T + D_{11p}D_{21p}^T)\,Q^{-1}, \\ C_{\mathrm{contr}} &= -(D_{12p}^{-1}C_{1p} + D_{\mathrm{contr}}C_{2p}), \\ B_{\mathrm{contr}} &= \Phi D_{\mathrm{contr}} - (A_p RC_{2p}^T + E_p D_{21p}^T)\,Q^{-1}, \\ A_{\mathrm{contr}} &= W_{cp} - B_{\mathrm{contr}}C_{2p}. \end{aligned} \tag{47}$$

For system (41), the conditions of theorem (b), (c) are satisfied automatically. Thus, to implement the Two-Ricatti algorithm, one requirement remains — the stabilization of the pair (F, G).

To solve problem (41), (44), (46), (47), you can use the hinfsyn function from the Robust Control Toolbox package (Matlab R2017a) [22]. It is based on some modification of the result formulated in the theorem. In this modification, another requirement $J=\left\|\bar{W}_{cls}(z)\right\|_{\infty}<\gamma$ is required instead $J=\left\|\bar{W}_{cls}(z)\right\|_{\infty}<1$, and the parameter γ is selected iteratively [18–20]. This procedure is called γ-iterations. In this case, γ decreases as long as a solution to the problem exists. In this way, the minimum possible value of criterion (44) is achieved.

Conclusion

The article provides an overview of the main approaches to the design of controllers at the next stage of control theory development — robust control systems.

The first universal approach to solving the problem of suppressing arbitrary limited external disturbances acting on the controlled object is to design state controllers based on the method of invariant ellipsoids, which reduces the problem of synthesizing an optimal controller to searching for the smallest invariant ellipsoid of a closed-loop control system. The concept of using invariant ellipsoids makes it possible to represent the problem of minimizing the size of an ellipsoid in terms of linear matrix inequalities, which allow the synthesis of a state controller through the use of semidefinite programming tools implemented by numerical methods. This approach was extended to solve the more general problem of robust suppression of external limited disturbances in the presence of system uncertainties in the parameters of the model of the controlled object. This approach to the synthesis of robust control ensures the stability of a closed-loop system with one controller in the presence of system uncertainty of the object mathematical model and the influence of arbitrary limited disturbances.

The second approach to solving the problem of minimizing the effect of arbitrary external disturbances on the controlled object is to apply $H^{\propto}$-theory to solving the problem of minimizing the maximum ratio of the l_2-norm of the vector of output stabilized coordinates to the l_2-norm of the vector of external disturbances. The $H^{\propto}$-optimization problem was first solved in the form of a standard problem of suppressing external limited disturbances when describing the dynamics of a controlled object in the form of a matrix discrete transfer function. This $H^{\propto}$-controllers synthesis problem was later implemented for state-space plant models in the Two-Ricatti approach that is now accepted as a standard.

In practice, the problem of forming a control that minimizes the influence of external disturbances on the system was solved by synthesizing a robust aircraft control system, which ensures stability of motion in the event of a sudden gust of wind of high intensity during take off and landing of the aircraft and its flight at low altitudes. At the same time, the mathematical model of the aircraft, as a controlled object, was characterized by system uncertainty, which was caused by changes in the mass of the aircraft under different loads.

Robust along with their undeniable advantages, also have certain disadvantages. This is explained by the fact that a robust control system must remain operational under the maximum possible disturbances, without having information when this disturbance will occur. Therefore, robust controllers are configured for the worst case. As a result, the quality of operation of a robust control system with a small normally distributed disturbance is inferior to the quality of operation of a control system with linear-quadratic Gaussian controllers.

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ЕТАПИ ТА ОСНОВНІ ЗАДАЧІ СТОЛІТНЬОГО РОЗВИТКУ ТЕОРІЇ СИСТЕМ КЕРУВАННЯ ТА ІДЕНТИФІКАЦІЇ.

Частина 4. МЕТОДИ І ЗАДАЧІ ПРОЄКТУВАННЯ РОБАСТНИХ СИСТЕМ КЕРУВАННЯ

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У статті зроблено огляд найважливіших методів і задач проєктування робастних дискретних систем керування. При цьому основну увагу привернуто до задач приглушення обмежених зовнішніх збурень, інформація про які надана у формі обмеження на їх максимальну величину. Для опису характеристики впливу зовнішніх збурень на траєкторію руху динамічних систем як перший математичний апарат розглянуто застосування інваріантних еліпсоїдів. Розглянуто теореми про переформулювання проблеми інваріантності еліпсоїдів у терміни лінійних матричних нерівностей, які в подальшому використовуються для синтезу дискретних регуляторів стану для приглушення зовнішніх збурень. Розглянуто розв'язок більш загальної задачі робастного приглушення обмежених збурень на основі застосування лінійних матричних нерівностей за наявності системних невизначеностей параметрів математичної моделі об'єкта керування у просторі стану. Як другий математичний апарат для приглушення зовнішніх l_2-обмежених зовнішніх обурень розглянуто застосування H^{∞}-теорії керування. При цьому критерій оптимальності полягає в мінімізації максимального відношення l_2-норми вектора вихідних стабілізованих координат об'єкта до l_2-норми вектора вхідних збурень. Задачу розв'язано за допомогою приведення її до задачі робастного керування дискретною динамічною системою у просторі H^{∞} на основі застосування підходу Два-Рікатті. Розглянуто також стандартну задачу H^{∞}-оптимізації.

Ключові слова: регулятор стану, інваріантний еліпсоїд, лінійні матричні нерівності, робастне керування, теорія H^{∞}, зовнішні збурення.

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