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ADAPTIVE SUBOPTIMAL CONTROL
OF SOME DISCRETE-TIME PLANTS
WITH NONSTOCHASTIC BOUNDED DISTURBANCES

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This paper deals with the adaptive suboptimal control of linear, discrete-time, time-invariant, minimum phase, scalar plants in the presence of nonstochastic bounded unmeasurable disturbances whose upper and lower bounds, which may be asymmetric, are assumed to be unknown *a priori*. Additional assumption is that an order of the difference equation describing the plant is known *a priori*. The distinguishing feature of the problem stated in this paper is that neither bounds on the unmeasured disturbances, nor bounds on an allowable region to which the unknown plant parameters belong are assumed to be known *a priori*. To solve this problem, adaptation procedures for the point and membership set estimation are utilized. The standard recursive procedure with adjustable dead zone is employed in order to derive the point estimates of unknown plant parameters together with the point estimate of time-invariant disturbance component. The size of this dead zone depends on the previous point estimate of the bounds on the time-varying disturbance component and also on a fixed suboptimality index chosen by the designer. The estimates generated by the point estimation procedure are directly exploited to derive the adaptive control law. The main idea advanced in this paper is that, instead of unknown *a priori* membership set of these parameters, their peculiar hypothetical *a posteriori* membership sets are designed via the use of the meas-

ured system's signals together with the current point estimate of bounds on the time-varying disturbance component. Contrary to the usual membership set estimation approach, this set is updated if only it is discovered that the unknown parameter vector does not belong in reality to this set. To this end, a remarkable property of the point estimation procedure is utilized. Such an approach makes it possible to reconstruct this set and to update the previous estimate of the bounds on the time-varying disturbance component. The finite convergence of the adaptation procedures and also the ultimate boundedness of system's signals are established. To demonstrate an efficiency of the adaptive controller and support the theoretical study, simulation results are presented.

Keywords: adaptive control, point estimation, membership set estimation, discrete-time, nonstochastic disturbance.

Introduction

The problem of a perfect performance of modern control systems with parametric and nonparametric uncertainties remains an actual problem from both theoretical and practical points of view. Within the framework of this scientific problem, novel ideas have been advanced in recent work [1] dealing with the model predictive control extended last time in the modern control theory [2]. An essential progress has been achieved by many researches using adaptive control approaches presented in [3–5]. Important theoretical results have been reported in numerous papers and generalized in several books including, in particular, [6–9]. Some practical application of these adaptive control approaches for a manufacturing systems with uncertainties has been presented in [10].

There are exist stochastic and nonstochastic approaches to the adaptive control design in the presence of unmeasurable external disturbances. The nonstochastic approach has been advanced by V.M. Kuntsevich who studied methods for the so-called membership set estimation of unknown plant parameters in his book [11] and also by V.A. Yakubovich dealing with the traditional point estimation of these parameters in the book [12, chap. 4]. Their works have been extended for over last two decades in [13, 14]. Recent results achieved in this scientific field can be found in the papers [4, 15].

A distinguishing feature of the disturbances considered in the works above mentioned is that they have no probabilistic nature [12, p. 137]. Namely, their sequences do not have any stochastic characteristics such as the expected value, variance and others. Nevertheless, it is known [12, item 4.2.1] that, in contrast to stochastic case, the exact identification of a plant subjected to unmeasured nonstochastic disturbances is impossible, in principle, even when the asymptotic behavior of a plant to be controlled is considered. One of the areas which attracts an attention of the adaptive control community up-to-date remains the rejection of arbitrary bounded nonstochastic disturbances. Meanwhile, traditional techniques for rejecting random disturbances based on stochastic approaches become here unacceptable.

The adaptive optimal control of the linear discrete-time systems without a pure delay in the presence of arbitrary bounded disturbances with known bounds has originally been proposed in the paper [16]. To establish the asymptotic properties of the adaptive closed-loop control system, the Key Technical Lemma reported in [17, p. 181] is utilized. The adaptive suboptimal controller for rejecting nonstochastic bounded disturbance has been proposed before by V.A. Yakubovich together with his disciple V.A. Bondarko utilizing their Frequency Theorem [12, sec. 4.2]. It turns out that adaptive optimal and suboptimal controls based on the point estimation without the membership set estimation are possible if only the time delay is absent even when bounds on the disturbances are known.

The method for adaptive robust control of the systems with bounded disturbances whose bounds are unknown has been developed by G. Feng in [18] who has established the ultimate boundedness of all signals in the closed-loop control system, assuming that a lower bound on the high frequency gain of the open-loop system and also its sign are known *a priori*. However, no suboptimality of his adaptive controller has been achieved. To cope with the absence of knowledge concerning the bounds on these disturbances, V.A. Bondarko devised the adaptive suboptimal control algorithm using a remarkable property of the finitely convergent estimation procedure examined in [12, chap. 2]; see [19]. In order to implement this algorithm, certain knowledge of *a priori* bounded membership set of unknown plant parameters was needed. The adaptive suboptimal feedback controller for the first-order nonlinear plant in the presence of bounded disturbances with unknown bounds is designed in [13]. Fruitful ideas of this paper have been extended in [14], where the problem of adaptive suboptimal control of the first-order plant with two unknown parameters and unmodelled dynamics is studied. One of the basic assumptions introduced in [13, 14] is that some constraints on the allowable values of unknown parameters are known *a priori*. A disadvantage of the works [13, 14, 18, 19] is that the bounds on the disturbance are assumed to be symmetric. This disadvantage has been overcome in [20]. Unfortunately, some *a priori* knowledge of the membership set is also required to implement this algorithm.

A common feature of the works above mentioned is that they require certain *a priori* information about a bounded membership set of unknown plant parameters. Difficulties that may arise in practice are how to get suitable estimates of the bounds on admissible parameters which will not be conservative and simultaneously to guarantee that each unknown parameter will lie within corresponding bounds. To the best of our knowledge, the existing works do not provide an answer to this question.

This paper proposes a new adaptive suboptimal control method to deal with arbitrary bounded disturbances having unknown bounds in the absence of any information about *a priori* membership set of unknown parameters. Its original version was presented in [21] at 22nd IFAC World Congress which was held in Yokohama (Japan) in July 2023.

1. Problem statement

Let the system to be controlled be a linear discrete-time, time-invariant single-input single-output system with no pure delay described by

$$y_t + a_1 y_{t-1} + \dots + a_n y_{t-n} = b_1 u_{t-1} + b_n u_{t-n} + v_t \quad (b_1 \neq 0), \quad (1)$$

where y_t , u_t and v_t denote the scalar output, input (control) and unmeasurable disturbance, respectively, at time instant t . Using the backward shift operator q^{-1} defined by $q^{-1}x_t = x_{t-1}$, rewrite (1) in the form

$$A(q^{-1})y_t = B(q^{-1})u_t + v_t, \quad (2)$$

where $A(q^{-1})$ and $B(q^{-1})$ are the polynomials of the order n given as

$$A(q^{-1}) = 1 + a_1 q^{-1} + \dots + a_n q^{-n}, \quad B(q^{-1}) = b_1 q^{-1} + \dots + b_n q^{-n}.$$

The following assumptions about the system (2) are made:

- A1) the integer n is known *a priori*;
- A2) the system (1) is the minimum phase plant;
- A3) the coefficients of $A(q^{-1})$ and $B(q^{-1})$ are unknown;

A4) the disturbance v_t represents the so-called nonstochastic but bounded variable [11, 12] defined by

$$-\infty < \varepsilon_{\min} \leq v_t \leq \varepsilon_{\max} < \infty \quad \forall t \quad (3)$$

with possibly asymmetric lower and upper bounds, i.e., $-\varepsilon_{\min} \neq \varepsilon_{\max}$;

A5) ε_{\min} and ε_{\max} are unknown *a priori*.

An illustrative example of nonstochastic bounded disturbance v_t with asymmetric bounds is depicted in Fig. 1 below.

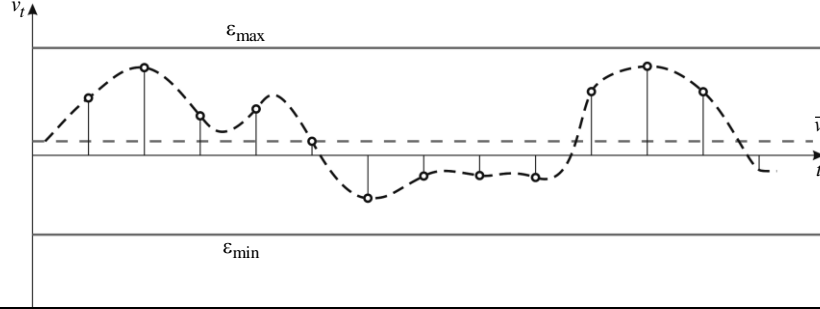


Fig. 1

Remark 1. Note that the open-loop minimum phase system (2) may be unstable, in principle. Besides, $A(q^{-1})$ and $B(q^{-1})$ may not be relatively prime, i.e., the pole-zero cancellation is valid. See [12, item. 4.2.1].

Denoting by y^0 a desired output of the system (1), the regulation problem when $y^0 \equiv \text{const}$ will be considered. To this end, introduce the ultimate performance index

$$J(u, v) = \limsup_{t \rightarrow \infty} |y^0 - y_t|$$

depending on the control sequence $u := \{u_t\} = u_1, u_2, \dots$ generated by some controller and on the disturbance sequence $v := \{v_t\} = v_1, v_2, \dots$. It is clear that

$$J^0(v) = \inf_{u \in \ell_\infty} J(u, v) = \inf_{u \in \ell_\infty} \limsup_{t \rightarrow \infty} |e_t|$$

where ℓ_∞ is the standard notation of the space of any bounded sequence, characterizes the optimal asymptotic behavior of the output error

$$e_t = y^0 - y_t \quad (4)$$

for a given disturbance sequence v .

Definition [19, 20]. The closed-loop control system is said to be suboptimal with a suboptimality index $\delta > 0$ if the requirement

$$\limsup_{t \rightarrow \infty} |e_t| \leq \sup_{v: v_t \in [\varepsilon_{\min}, \varepsilon_{\max}]} J^0(v) + \delta \quad (5)$$

is satisfied for any positive δ .

Remark 2. In contrast with [12], the expression (5) defines the suboptimality index in the additive but not in multiplicative form, and it is here essential.

With assumptions A1) to A5) in mind, the problem is to design the adaptive suboptimal controller guaranteeing the achievement of the control goal (5) for any positive δ chosen by the designer.

2. Optimal nonadaptive control (ideal case)

Suppose, for the time being, that the coefficients of $A(q^{-1})$ and $B(q^{-1})$ are known. To derive the optimal control satisfying the requirement

$$\limsup_{t \rightarrow \infty} |e_t| \leq \sup_{v: v_t \in [\varepsilon_{\min}, \varepsilon_{\max}]} J^0(v), \quad (6)$$

write the variable v_t in the following form:

$$v_t = \bar{v} + \tilde{v}_t, \quad (7)$$

where \bar{v} and \tilde{v}_t are the time-invariant and the time-varying components of v_t , respectively. Due to (3) they are given as

$$\begin{aligned} \bar{v} &= (\varepsilon_{\min} + \varepsilon_{\max}) / 2. \\ |\tilde{v}_t| &\leq \varepsilon, t = 0, 1, 2, \dots \end{aligned} \quad (8)$$

with

$$\varepsilon = (\varepsilon_{\max} - \varepsilon_{\min}) / 2. \quad (9)$$

Next, substituting (7) into (1) and shifting the indices of y_t and u_t , one obtains

$$y_{t+1} + a_1 y_t + \dots + a_n y_{t-n+1} = b_1 u_t + b_n u_{t-n+1} + \bar{v} + \tilde{v}_{t+1}. \quad (10)$$

Following the standard steps in deriving the optimal control of the discrete-time systems in the presence of bounded disturbances taken, e.g., from [12], one puts

$$-a_1 y_t - \dots - a_n y_{t-n+1} + b_1 u_t + b_n u_{t-n+1} + \bar{v} = y^0$$

to get the control law

$$u_t = b_1^{-1} [y^0 + a_1 y_t + \dots + a_n y_{t-n+1} - b_2 u_{t-1} - b_n u_{t-n+1} - \bar{v}]. \quad (11)$$

By virtue of (10) and the constraints (8), this control law yields

$$|y^0 - y_t| \leq \varepsilon \quad \forall t$$

implying that the optimality requirement (6) rewritten as

$$\limsup_{t \rightarrow \infty} |e_t| \leq \sup_{\tilde{v}: \tilde{v}_t \in [-\varepsilon, \varepsilon]} J^0(\tilde{v}),$$

will be achieved with $\sup_{\tilde{v}: |\tilde{v}_t| \leq \varepsilon} J^0(\tilde{v}) \leq \varepsilon$, where (4) has been utilized.

3. Main result

Basic ideas. Return to the true situation when the components of the $2n$ -dimensional vector $\theta^T = [a_1, \dots, a_n, b_1, \dots, b_n]$ and also the $(2n+1)$ -dimensional extended vector $\theta^T = [\theta^T, \bar{v}]$ are unknown. According to the so-called certainty equivalence adaptive control principle [17], replace unknown θ by some updated estimate $\theta_t^T = [a_1(t), \dots, a_n(t), b_1(t), \dots, b_n(t), \bar{v}(t)]$ which will be found later. Then from (11), the adaptive control law in the form

$$u_t = b_1^{-1}(t) [y^0 + a_1(t) y_t + \dots + a_n(t) y_{t-n+1} - b_2(t) u_{t-1} - \dots - b_n(t) u_{t-n+1} - \bar{v}(t)] \quad (12)$$

can directly be written.

Further, recalling that the upper bound, ε on $\{\tilde{v}_t\}$ defined in (9) remains unknown (since ε_{\min} and ε_{\max} are unknown), a current estimate, ε_t of ε will be exploited to devise the adaptive algorithm for updating both the parameter estimate vector $\theta_t \in \mathbb{R}^{2n+1}$ and scalar ε_t .

Define some strip S_t in the $(2n+1)$ th Euclidean space of extended vectors $\theta^T = [a_1, \dots, a_n, \hat{b}_1, \dots, \hat{b}_n, \bar{v}]$ as

$$S_t := \{\theta : |y_t - \theta^T \phi_{t-1}| \leq \varepsilon_t\} \subset \mathbb{R}^{2n+1}, \quad (13)$$

where $\tilde{\phi}_{t-1}^T = [\phi_{t-1}^T, 1] = [-y_{t-1}, \dots, -y_{t-n}, u_{t-1}, \dots, u_{t-n}, 1]$ denotes the extended regression vector. It follows from (10) together with (8) that S_t will contain the unknown vector $\theta^T = [\theta^T, \bar{v}]$ if $\sup_{t \in [0, \infty)} |\tilde{v}_t| \leq \varepsilon_t$.

It turns out that two different cases shown in Fig. 2 are possible. In the case depicted in Fig. 2 left, a current estimate ε_t of unknown upper bound ε on the absolute value of the disturbance \tilde{v} is not less than its true value ε . Then the unknown extended parameter vector $\tilde{\theta}$ will belong to the strip defined by the expression (13). However, in the other case depicted in this figure right when $\varepsilon_t < \varepsilon$, the unknown extended parameter vector θ may not belong to this strip.

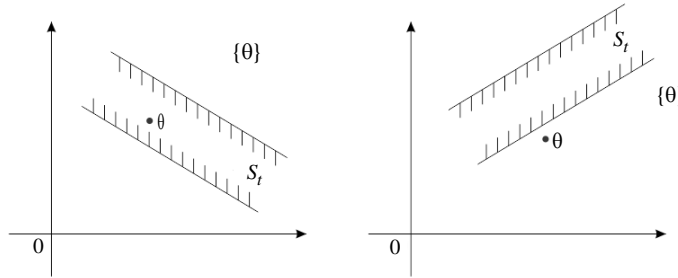


Fig. 2

Now, consider some intersection written in the recursive form

$$\Omega_t = \Omega_{t-1} \cap S_t \quad (14)$$

as depicted in Fig. 3, a, b.

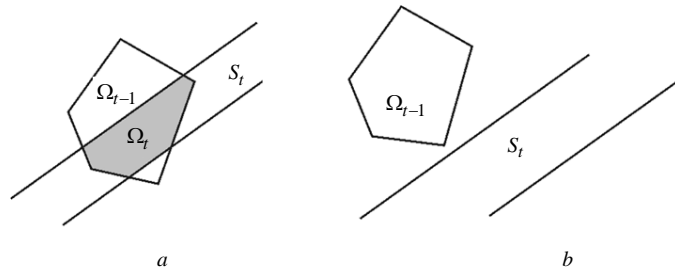


Fig. 3

It can be understood that if $\tilde{\theta} \in \Omega_{t-1}$ and also $\tilde{\theta} \in S_t$ then $\Omega_t \neq \emptyset$ (see Fig. 3, a) whereas if $\tilde{\theta} \in \Omega_{t-1}$ but $\tilde{\theta} \notin S_t$ then Ω_t becomes empty (see Fig. 3, b) meaning that the set of the inequalities

$$|y_k - \hat{\theta}^T \tilde{\varphi}_{k-1}| \leq \varepsilon_k \quad (k = k_0, k_0 + 1, \dots, t)$$

with respect to unknown θ will be incompatible. With this fact in mind, put

$$\varepsilon_t = \varepsilon_{t-1} + \delta/2 \quad (15)$$

if it is discovered that $\Omega_t = \emptyset$, and

$$\varepsilon_t = \varepsilon_{t-1} \quad (16)$$

otherwise. It is clear that the increase of the estimates ε_t s of the upper bound on $|\tilde{v}_t|$ cannot be infinite, since Ω_t becomes nonempty if $\varepsilon_t \geq \sup_{t \in [0, \infty)} |\tilde{v}_t|$.

To avoid the case $\Omega_t = \emptyset$, fix a $t_k \geq 0$, put $\varepsilon_{t_k} = \varepsilon_{t_k+1} = \dots = \varepsilon_{t_k+2n}$ ($k = 0, 1, \dots$) and consider the intersection $\Omega[k]$ of the $2n+1$ strips S_t defined in (13) at the time instants $t = t_k, t_k + 1, \dots, t_k + 2n$ as

$$\Omega[k] = S_{t_k} \cap S_{t_k+1} \cap \dots \cap S_{t_k+2n}. \quad (17)$$

Obviously, the set $\Omega[k]$ defined in (17) represents a convex bounded polytope iff the vectors $\tilde{\varphi}_{t_k}, \dots, \tilde{\varphi}_{t_k+2n}$ are linearly independent as illustrated in Fig. 4. Its Chebyshev center $\tilde{\theta}^c$ and radius $\text{rad } \Omega$ are specified as follows:

$$\tilde{\theta}^c = \arg \min_{\theta' \in \Omega} \max_{\theta'' \in \Omega} \|\theta' - \theta''\|_2,$$

$$\text{rad } \Omega = \max_{i=1, \dots, N} \|\tilde{\theta}^c - h^{(i)}\|_2,$$

where $h^{(i)}$ is the notation of i th vertex of Ω and N is the number of these vertices, and $\text{rad } \Omega$ denotes the radius of Ω , $\|\cdot\|_2$ is the Euclidean vector norm.

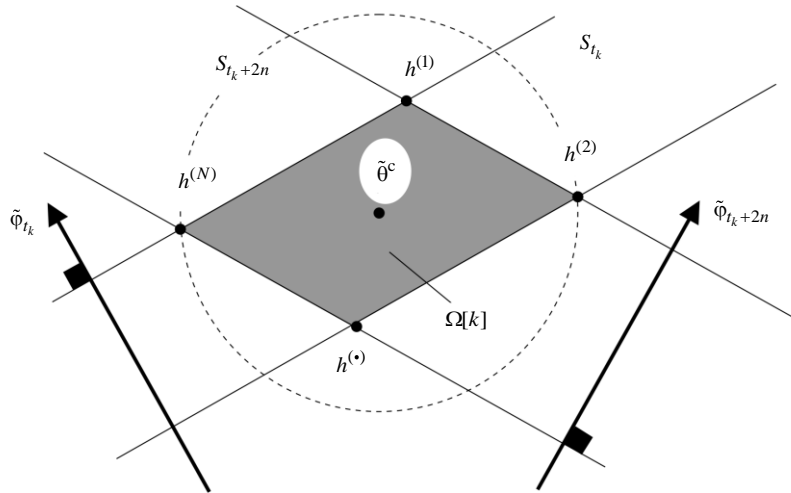


Fig. 4

One of the basic ideas advanced in this paper is to utilize the set $\Omega[k]$ given by (17) as some hypothetical *a posteriori* membership set of unknown θ since $\Omega[k]$ may not contain θ in reality if ε_{t_k} is sufficiently small. On the other hand, the condition $\theta \in \Omega[k]$ will be guaranteed via a sequential increase of ε_{t_k} from a time t_k to some time t_{k+1} for $k = 0, 1, \dots$ (Such an approach will be described in the subsection below.)

Adaptive law. The adaptation algorithm is implemented via the following subsequent steps.

Step 1. Starting from $k = 0$, fix some initial $t = t_0$, put $\varepsilon_{t_0} = \delta/2$, and find the vertices $h^{(1)}, \dots, h^{(N)}$ of $\Omega[0]$ using the measured vectors φ_t at $t = t_0, \dots, t_0 + 2n$. These vectors must be linearly independent to ensure the boundedness of $\Omega[k]$. To do it, choose arbitrarily the initial control sequence $u_{t_0}, u_{t_0+1}, \dots, u_{t_0+2n}$ so that the Gram's determinants Γ_t satisfy

$$\Gamma_t \neq 0 \text{ for } t \in [t_0, t_0 + 2n], \quad (18)$$

where

$$\Gamma_{t_0+j} = \det \begin{pmatrix} \tilde{\varphi}_{t_0}^T \tilde{\varphi}_{t_0} & \dots & \tilde{\varphi}_{t_0}^T \tilde{\varphi}_{t_0+j} \\ \dots & \dots & \dots \\ \tilde{\varphi}_{t_0}^T \tilde{\varphi}_{t_0+j} & \dots & \tilde{\varphi}_{t_0+j}^T \tilde{\varphi}_{t_0+j} \end{pmatrix}, \quad j = 1, \dots, 2n.$$

(Note that the requirement (18) defines the necessary and sufficient condition for the linear independence of these vectors [22, chap. 9, item 3].)

Step 2. Compute the Chebyshev center $\theta_{t_k+2n}^c$ using, e.g., [23] and the radius of $\Omega[k]$.

Step 3. Choose the initial estimate θ_{t_k+2n} moving it to the Chebyshev center $\theta_{t_k+2n}^c$, i.e., set $\theta_{t_k+2n} = \theta_{t_k+2n}^c$.

Step 4. Set $\chi_t = 0$ at $t = t_k + 2n$, where χ_t denotes an auxiliary variable determined below.

Step 5. Update θ_t s using the adaptive estimation procedure with the adjustable dead zone as

$$\theta_t = \theta_{t-1} - \gamma_t \frac{f(e_t, \varepsilon_{t-1}, \varepsilon_{t-1}^0)}{\|\tilde{\varphi}_{t-1}\|_2^2} \tilde{\varphi}_{t-1}, \quad (19)$$

at $t \geq t_k + 2n + 1$, where

$$\varepsilon_t^0 = \varepsilon_t + \delta/2, \quad (20)$$

$$f(e, \bar{\varepsilon}, \bar{\varepsilon}^0) = \begin{cases} e - \bar{\varepsilon} & \text{if } e > \bar{\varepsilon}^0, \\ 0 & \text{if } |e| \leq \bar{\varepsilon}^0 \quad (0 < \bar{\varepsilon} < \bar{\varepsilon}^0), \\ e + \bar{\varepsilon} & \text{if } e < -\bar{\varepsilon}^0 \end{cases}$$

denotes the dead-zone function caused by the identification algorithm of [12, chap. 2] and γ_t is the coefficient chosen free from

$$0 < \gamma' \leq \gamma_t \leq \gamma'' < 2 \quad (21)$$

to ensure $b_1(t) \neq 0$.

Step 6. If $t \neq t_k$ then calculate χ_t as follows:

$$\chi_t = \begin{cases} \chi_{t-1} & \text{if } f(e_t, \varepsilon_{t-1}, \varepsilon_{t-1}^0) = 0, \\ \chi_{t-1} + (2\gamma_t - \gamma_t^2)(\|e_t - \varepsilon_{t-1}\| \|\varphi_{t-1}\|_2)^{-2} & \text{otherwise.} \end{cases} \quad (22)$$

Step 7. Verify the condition

$$\chi_t \leq (\text{rad } \Omega[k])^2. \quad (23)$$

If (23) is violated then put $\chi_t = 0$ and increase k by 1 and instead of ε_{t-1} the updated ε_t specified in (15) exploit, reconstruct $\Omega[k]$ using the same initial vectors $\tilde{\varphi}_t$ given in Step 1 with updated ε_t and return to Step 2.

Remark 3. It can be proved that the violation of (23) indicates that $\tilde{\theta} \notin \Omega[k]$ for given ε_{t_k} . This remarkable fact makes it possible to discover indirectly that the intersection (17) is indeed empty without performing complex computations.

Convergence and ultimate boundedness analysis. A remarkable property of the procedure (19)–(21) leading to (23) is given in the following lemma:

Lemma. *If $\sup_{t \in [0, \infty)} |v_t| \leq \varepsilon_t$ and $\tilde{\theta}_0 = \theta^c[k-1]$ then*

$$\sum_{t \geq t_{k-1}} (2\gamma_t - \gamma_t^2) f^2(e_t, \varepsilon_{t-1}, \varepsilon_{t-1}^0) \|\tilde{\varphi}_{t-1}\|_2^{-2} \leq (\text{rad } \Omega[k-1])^2.$$

Proof. Follows from the features of adaptive estimation procedure established in [12, chap. 2]. \square

With this lemma, the ultimate properties of the adaptive controller proposed here is summarized in next theorem.

Theorem. *Let assumptions A1) to A5) be satisfied. Then the resulting closed-loop control system including the plant (1), the control law (12) and the adaptation algorithm described in Steps 1 to 7 has the following properties:*

- i) the adaptive estimation algorithm determined by (15), (16), (19)–(23) together with (13) and (17) converges at a finite time;*
- ii) the system signals are ultimately bounded;*
- iii) the suboptimal control objective (5) is achieved.*

Proof. Due to space limitation, details are omitted. \square

4. A numerical example and simulations

To demonstrate the features of the proposed adaptive controller, the simplest first-order system described by

$$y_t - 2,4y_{t-1} = u_{t-1} + v_t$$

was considered. (Although the instability is not inherent for many industrial plants to be controlled, firstly, for open process control systems, $A(q^{-1})$ was purposely chosen to be unstable (as in [14]), in order to show that even in such a «bad» case, the adaptive suboptimal performance can be achieved.) In this example, the suboptimality index was chosen as $\delta = 0,1$.

Before going to simulation experiments, the parameters of sets $\Omega[k]$ of vectors $\hat{\theta}^T = [\hat{a}_1, \hat{b}_1]$, $k = 0, 1, 2, 3$ for $\varepsilon_t = 0,05; 0,1; 0,15; 0,2$ were calculated and summarized in the Table. These sets are depicted in Fig. 5, showing that $\Omega[3]$ which contains unknown θ is an appropriate *a posteriori* membership set of this vector while $\Omega[0], \Omega[1]$ and $\Omega[2]$ are not.

Table

Ordinal number k	Chebyshev center	Radius of $\Omega[k]$
0	$[-2,28, 1,26]$	0,092
1	the same	0,126
2	the same	0,181
3	the same	0,320

Results of the simulation experiment conducted for $v_t \in [-0,2, 0,2]$ and $y^0 = 5$ are given in Fig. 6. They show the behavior of the adaptive control system in this example. It can be seen from Fig. 6, *a* that if the variable χ_t exceeds the square of the radius of $\Omega[k]$ designed at the time instant $t = t_k$ ($k = 0, 1, 2, 3$), then by virtue of the algorithm determined by (15), (16), (19)–(23) together with (13) and (17) the variable χ_t is reset to zero, whereas the current estimate of the bound on disturbance increases without exceeding a certain threshold $\varepsilon + \delta/2 = 0,25$ (Fig. 6, *b*). One observes that the components of $\theta_t^T = [a_1(t), b_1(t)]$ jump moving toward the Chebyshev center θ^c at the same time instants $t = t_k$ as shown in Fig. 6, *c* and *d*.

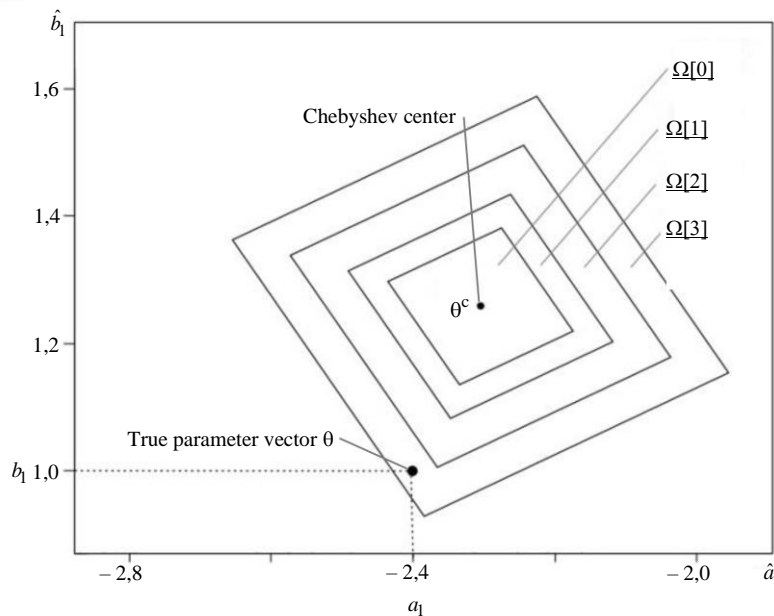


Fig. 5

For comparison, the behavior of nonadaptive control system with the same plant and fixed parameters of the controller specified by θ^c (instead of unknown θ) is presented in Fig. 7. It turned out that this control system was also stable and its output error remained bounded. However, one can observe that the performance of the adaptive control system is essentially better than in the nonadaptive case. Moreover, it is seen that the suboptimality property of the proposed adaptive controller is ensured (Fig. 6, *e*).

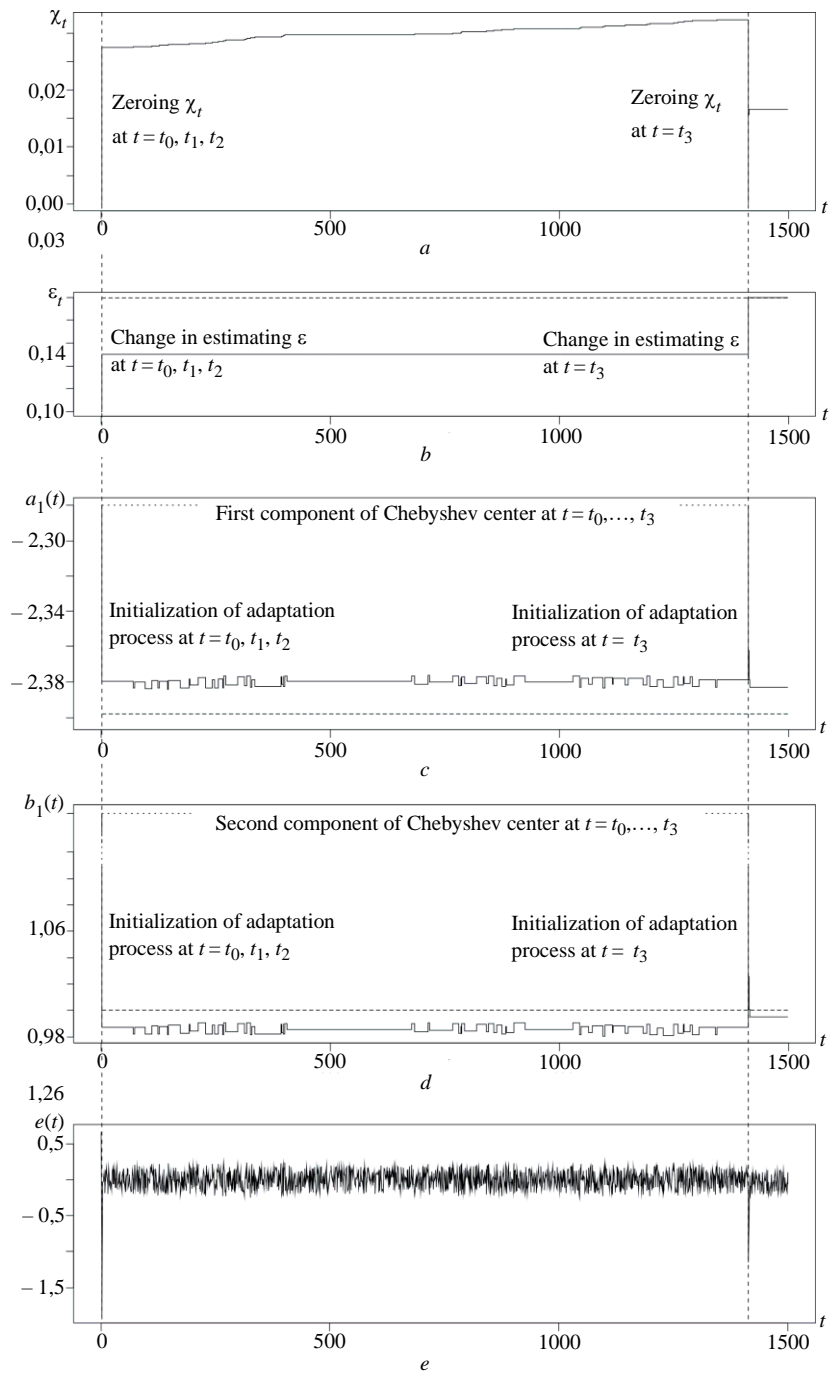


Fig. 6

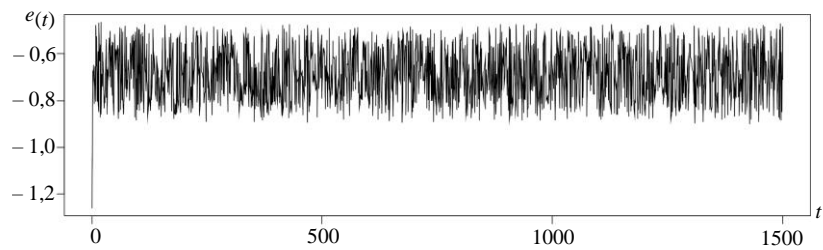


Fig. 7

Conclusion

The adaptive suboptimal discrete-time closed-loop system containing the linear uncertain scalar minimum phase plant subjected to nonstochastic bounded disturbances with unknown bounds in the absence of any *a priori* information about the membership set of its parameters is addressed in this paper. The asymptotical features of the adaptive controller above proposed are established. They shed some light on the possibility of achieving any suboptimality index utilizing the adaptive controller. In future, this controller can be improved via the use of additional ellipsoid estimation procedure in order to accelerate the adaptation process. The MIMO case will also be studied.

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АДАПТИВНЕ СУБООПТИМАЛЬНЕ КЕРУВАННЯ ДЕЯКИМИ ДИСКРЕТНИМИ ОБ'ЄКТАМИ З НЕСТОХАСТИЧНИМИ ОБМЕЖЕНИМИ ЗБУРЕННЯМИ

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Розглядається адаптивне субоптимальне керування лінійними, дискретними, стаціонарними, мінімально-фазовими, скалярними об'єктами за наявності нестохастичних обмежених невимірюваних збурень, верхня та нижня межі яких можуть бути асиметричними та вважаються априорі невідомими. Додаткове припущення полягає в тому, що порядок різницевого рівняння, яке описує об'єкт, априорі відомий. Відмінною особливістю поставленої в статті задачі є те, що межі ані невимірюваних збурень, ані допустимої області, до якої належать невідомі параметри об'єкта, не вважаються априорі відомими. Для вирішення цієї задачі використовуються адаптивні процедури точкового і множинного оцінювання. Стандартна рекурентна процедура з регульованою зоною нечутливості застосовується для отримання точкових оцінок невідомих параметрів об'єкта разом з точковою оцінкою постійної компоненти збурення. Розмір цієї зони нечутливості залежить від попередньої точкової оцінки меж змінної у часі компоненти збурення, а також від фіксованого показника субоптимальності, вибраного конструктором системи. Оцінки, отримані за допомогою процедури точкового оцінювання, безпосередньо використовуються для отримання закону адаптивного керування. Головна ідея, висунута в статті, полягає в тому, що замість невідомої априорної множини належності цих параметрів використовуються їхні своєрідні гіпотетичні апостеріорні множини належності, розроблені

за допомогою вимірюваних сигналів системи разом з поточною точковою оцінкою меж змінної у часі компоненти збурення. На відміну від звичайного підходу до оцінки множини належності цей набір оновлюється лише тоді, коли виявляється, що невідомий вектор параметрів насправді не належить цій множині. Для цього застосовується певна властивість процедури точкового оцінювання. Такий підхід дає змогу реконструювати цю множину і оновити попередню оцінку меж компоненти збурення, змінної в часі. Встановлено скінченну збіжність процедур адаптації, а також граничну обмеженість сигналів системи. Для демонстрації ефективності адаптивного регулятора та підтримки теоретичного дослідження репрезентовані результати моделювання.

Ключові слова: адаптивне керування, точкове оцінювання, множинне оцінювання, дискретний час, нестохастичне збурення.

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