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B. Panchenko, L. Bukata, D. Bahachuk, L. Martynovych, O. Zui

CLUSTER MODELING OF THE INTERACTION OF STATYONARY SH-WAVES WITH A SYSTEM OF CURVILINEAR CRACKS IN A HALF-SPACE

Borys Panchenko

Odesa I.I. Mechnykov National University,
orcid: 0000-0002-8085-4043

pr-bob@ukr.net

Liudmyla Bukata

State University Intellectual Technologies and Communication, Odesa,
orcid: 0000-0003-1707-1633

ygrikluda@gmail.com

Denys Bahachuk

State University Intellectual Technologies and Communication, Odesa,
orcid: 0000-0001-8798-891X

bagachukdg@gmail.com

Larysa Martynovych

Odesa I.I. Mechnykov National University,
orcid: 0000-0001-7351-1467

larysa.yaroslavna@onu.edu.ua

Oksana Zui

Odesa I.I. Mechnykov National University,
orcid: 0000-0001-9520-4441

oks.zuj@gmail.com

A method is proposed for solving problems of mathematical physics for semi-infinite media containing systems of curvilinear cuts. Based on singular integral equations (SIE), a unified approach to solving the problem has been developed. Numerical implementation was carried out using parallelisation. A graph of the duration of the initial cluster hour is given for calculating an array of searched functions in the context of a parabolic form as a function of the number of processes for one variant of implementation. It is shown that the entire algorithm scales well and has an efficient number of processes. The expected result was obtained, because the structure of the calculation process in models based on SIE is mostly unchanged and built on well-defined procedures. It turned out that 150–200 processes are effective. An accuracy of 10^{-12} was achieved with the number of collocation points of the contour of each section $N = 300$, because the algorithm for numerical solution of the problem uses interpolation by Chebyshev polynomials in accordance with the fact that the unknown function has a key feature at the ends of the section, which causes a higher speed of convergence of the algorithm. The study of the question of the further increase in the accuracy of the result was not conducted. The corresponding dynamic boundary value problems for a restrained and force-free half-plane are studied. The influence of the curvature of defects, their interaction and proximity of the boundary on the magnitude and nature of the change in the dynamic stress intensity coefficients was studied. To check the reliability of the algorithm, two tests were carried out: removing three inhomogeneities at a distance of 10^6 of their length from each other and saturating the system by increasing the number of geometrically equal reflectors to 13–15. When removed, the characteristics of each reflector tend to the characteristics of a single one, and when saturated, they tend to the results of the corresponding periodic problem. The agreement between the results showed good reliability of the algorithm. The proposed method can be used to assess the influence of various mechanical or geometric factors on the strength of bodies with defects.

Keywords: boundary value problem, shear waves, singular integral equations, parallel computing, system of cracks-cuts.

Introduction

Modern structures and structures operate under conditions of not only multiple static, but also dynamic loads. It is known that under dynamic loading on bodies with crack-like defects, the probability of their development can increase. Therefore, to estimate the limiting state of such bodies, it is important to find out the influence of the inertial effect on the propagation of defects.

The study of the problem of dynamic destruction of structures should be preceded by an analysis of wave fields in model problems. In this regard, it is topical to develop methods for solving dynamic problems of mathematical physics for infinite and semi-infinite media with cracks.

From a theoretical point of view, a crack is a mathematical cut, at the transition through which the displacements undergo discontinuities. If discontinuities in displacements are not known in advance, the exact mathematical description of the wave field that arises due to the presence of a crack turns out to be very difficult.

Most of the studies that are available in the literature relate to the problems of diffraction of elastic waves on straight and circular cracks-cuts. However, in reality, the crack, as a rule, does not have a straight or circular shape. And, as studies have shown, the curvature of the defect significantly affects the value of dynamic stress intensity factors. The value of this parameter also depends on the proximity of the defects to each other, since they always fall within the area of the reflected wave.

This work is devoted to the development of a method for solving dynamic problems of mathematical physics for an isotropic half-space weakened by curvilinear tunnel cuts. A stationary wave process is considered.

The stress-strain state of media with complicated properties can be highly efficiently simulated by computer systems in combination with software systems. Although the issue of automated synthesis of applications that can be reconfigured depending on the change in the configuration of mechanical systems has not been practically studied. Most of the research is devoted to the development of the finite element method [1]. There are other approaches that significantly save computational resources and improve the accuracy of calculations. In particular, software tools (CASE-tools) [2] make it possible to synthesize and maintain applications that simulate the dynamic behavior of complex mechanical systems.

It is important to study the diffraction of elastic waves on systems of arbitrary inhomogeneities. Efficient parallel algorithms [3] based on sound analytical methods are of particular importance. For solving anti-plane problems of diffraction theory [4, 5], the method of integral equations [6–14] is very effective. The advantage of the method is the reduction in the number of spatial variables, a fairly high rate of convergence, and the possibility of using various effective numerical methods of solution [6, 7]. The method also has the ability to build efficient parallel computational circuits [8, 9].

1. Formulation of the problem

Let us consider an elastic isotropic half-space weakened by a system of tunnels along the axis *OZ* curvilinear cuts $L_j \ \ (j=\overline{1,K})$ (Fig. 1), where x is a simple open Lyapunov arc with origin at a point a_j and end at a point b_j (we assume that $\cap L_j = 0$).

We consider that the edges of the cuts are free from forces, and the displacements during the transition through $L=\cup L_j$ suffer a break.

Let a monochromatic shear wave be emitted from infinity, the normal to the front of which makes an angle ψ with axle *OX* (dependence on time is expressed by the multiplier $e^{-i\omega t}$).

$$W_0=\tau e^{-i\gamma_2(x\cos\psi+y\sin\psi)},\ \tau=\text{const},\ \ \gamma_2=\frac{\omega}{c_2}. \tag{1}$$

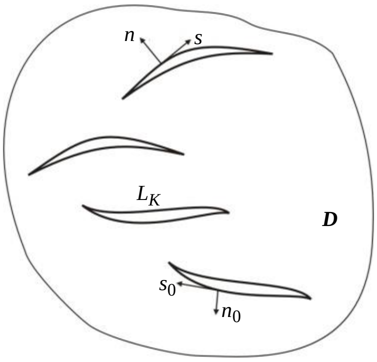

Fig. 1

Here ω is oscillation frequency, c_2 is the speed of propagation of the transverse wave.

As a result of the interaction of the incident W_0 waves with cuts, a scattered wave of displacements arises W_1 which satisfies the anti-plane deformation equation [7, 10].

$$\Delta W_1+\gamma_2^2 W_1=0,\ \ \Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}. \tag{2}$$

For fracture mechanics, the asymptotic distribution of stresses in the vicinity of defect tips is of decisive importance [7]. Nonzero Stress Tensor Components σ_{31},σ_{32} are shear stresses in the plane of the cross section of the cracks. They are connected with movement $W=W_0+W_1$ formulas (μ is a shear modulus)

$$\sigma_{31}=\mu\frac{\partial W}{\partial x},\sigma_{32}=\mu\frac{\partial W}{\partial y},\ \ \sigma_{31}-i\sigma_{32}=2\mu\frac{\partial W}{\partial z},\ \ z=x+iy. \tag{3}$$

The stress σ_n acting on L at the point $\zeta=\xi+i\eta\in L$ from the side of the positive normal (as for (3)), is equal to

$$\sigma_n=\sigma_{31}\sin\varphi-\sigma_{32}\cos\varphi=\operatorname{Im}\{e^{i\varphi}(\sigma_{31}-i\sigma_{32})\}. \tag{4}$$

Where φ is the angle between the positive tangent to L at the point ζ and axis *OX*.

When passing through L it provides a jump in displacements ($[W_1] = W_1^+ - W_1^- = f(s)$) and stress continuity ($[\sigma_n] = \sigma_n^+ - \sigma_n^- = 0$). For (4), it remains to fulfill the boundary condition on L:

$$\sigma_{n_0} = \mu \frac{\partial}{\partial n_0}(W_0 - W_1) = 0, \tag{5}$$

which we rewrite in the form

$$\left(e^{i\varphi_0} \frac{\partial W}{\partial z} - e^{-i\varphi_0} \frac{\partial W}{\partial \bar{z}} \right)^{\pm} = 0. \tag{6}$$

The solution of the anti-plane dynamical problem of mathematical physics is reduced to defining the function $W_1(x, y)$ from (5), (6) — solving the equation the Helmholtz equation (2) in a plane with a system of cracks-cuts under Sommerfeld type conditions of radiation at infinity [6].

Three variants of an infinite medium are considered: unlimited space $(A = 0)$; half space with boundary $y = 0$, free from forces $(A = -1)$; half-space with pinched boundary $y = 0$ $(A = 1)$.

$$A = -1: \quad \sigma_y \Big|_{y=0} = \mu \frac{\partial W}{\partial y}\Big|_{y=0} = 0;$$

$$A = 1: \quad W\Big|_{y=0} = 0.$$

2. Solution method

Following [10], we write the function $W_1(x, y)$ characterizing the wave of displacements scattered by the cuts in the domain D, in the following way:

$$W_1(x, y) = \frac{1}{4} \int_L f(s) \times \left[\left(\frac{\partial}{\partial z} H_0^{(1)}(\gamma_2 r)\, d\varsigma - A \frac{\partial}{\partial z} H_0^{(1)}(\gamma_1 r) \right] - \right.$$
$$\left. - \left[\frac{\partial}{\partial \bar{z}} H_0^{(1)}(\gamma_2 r)\, d\bar{\varsigma} - A \frac{\partial}{\partial \bar{z}} H_0^{(1)}(\gamma_1 r) \right] \right) + (\upsilon_0 - A\bar{\upsilon}_0), \;\; r = |z - \zeta|, \, r_1 = |z - \bar{\zeta}|. \tag{7}$$

Here $H_n^{(1)}(x)$ is Hankel function of the first kind n-th order; $f(s)$ is an unknown function that satisfies on L the Hölder condition.

The integral representation (7) automatically satisfies the Helmholtz equation (2) in the region D and radiation conditions at infinity.

We will assume that the displacement jumps at the ends of the cuts L_j are equal to zero, i.e.

$$f(a_j) = f(b_j) = 0, \quad j = \overline{1, k}. \tag{8}$$

In integral form (8) can be written as

$$\int_L df(s) = \int_L f'(s)\, ds = 0. \tag{9}$$

Let us use the known relations [10]:

$$\frac{\partial^2}{\partial z^2} H_0^{(1)}(\gamma r) = \frac{\gamma^2}{4} e^{-2i\alpha} H_2^{(1)}(\gamma r), \qquad \frac{\partial^2}{\partial \bar{z}^2} H_0^{(1)}(\gamma r) = \frac{\gamma^2}{4} e^{2i\alpha} H_2^{(1)}(\gamma r),$$
$$\frac{\partial^2}{\partial z \partial \bar{z}} H_0^{(1)}(\gamma r) = -\frac{\gamma^2}{4} H_2^{(1)}(\gamma r), \quad z - \zeta = re^{i\alpha}. \tag{10}$$

The behavior at zero ($x \to 0$) of the zero and second order Hankel functions is characterized by the asymptotic formulas:

$$H_0^{(1)}(x) = \frac{2i}{\pi}\ln x + H_0(x), \quad H_2^{(1)}(x) = \frac{4}{i\pi x^2} + H_2(x), \tag{11}$$

where $H_0(x)$ and $H_2(x)$ are continuous at a point $x = 0$.

Substituting the integral representation (7) into the expression on the left side of the boundary condition (6), taking into account (10), (11), gives:

$$\begin{aligned}
& e^{i\varphi}\frac{\partial \upsilon_*}{\partial z} - e^{-i\varphi}\frac{\partial \upsilon_*}{\partial \overline{z}} = \frac{1}{4}\int_L f(s)\Bigg\{\Bigg[\Bigg(e^{i(\varphi_0+\varphi)}\frac{\partial^2 H^{(1)}(\gamma r)}{\partial z^2} - e^{i(\varphi_0-\varphi)}\frac{\partial^2 H^{(1)}(\gamma r)}{\partial z \partial \overline{z}}\Bigg) - \\
& -\Bigg(e^{-i(\varphi_0-\varphi)}\frac{\partial^2 H^{(1)}(\gamma r)}{\partial z \partial \overline{z}} - e^{-i(\varphi_0+\varphi)}\frac{\partial^2 H^{(1)}(\gamma r)}{\partial \overline{z}^2}\Bigg]\Bigg) - \\
& -A\Bigg[\Bigg(e^{i(\varphi_0+\varphi)}\frac{\partial^2 H^{(1)}(\gamma r_1)}{\partial z \partial \overline{z}} - e^{i(\varphi_0-\varphi)}\frac{\partial^2 H^{(1)}(\gamma r_1)}{\partial z^2}\Bigg]\Bigg) - \\
& -\Bigg(e^{-i(\varphi_0-\varphi)}\frac{\partial^2 H^{(1)}(\gamma r_1)}{\partial \overline{z}^2} - e^{-i(\varphi_0+\varphi)}\frac{\partial^2 H^{(1)}(\gamma r_1)}{\partial z \partial \overline{z}}\Bigg\}\Bigg]\Bigg) ds = \\
& = \frac{\gamma^2}{16}\int_L f(s)\{[(e^{i(\varphi_0+\varphi-2\alpha)} + e^{i(\varphi_0+\varphi-2\alpha)})H_2^{(1)}(\gamma r) + \\
& + (e^{i(\varphi_0-\varphi)} + e^{-i(\varphi_0-\varphi)})H_0^{(1)}(\gamma r)] + A[(e^{i(\varphi_0-\varphi-2\alpha_1)} + e^{-i(\varphi_0-\varphi-2\alpha_1)})H_2^{(1)}(\gamma r_1) + \\
& + (e^{i(\varphi_0+\varphi)} + e^{-i(\varphi_0+\varphi)})H_0^{(1)}(\gamma r_1)\}\, ds.
\end{aligned} \tag{12}$$

Here we use formulas for integration by parts for hypersingular integrals [12] under additional conditions (9).

Using the Sokhotsky-Plemelj formulas [11] to calculate the limiting values of the Cauchy-type integrals that arise when the boundary condition (6) is satisfied, taking into account relations (1), (12), reduces the boundary value problem under consideration to a singular integro-differential equation for an unknown function $f(s)$ [10]:

$$\begin{aligned}
& \frac{1}{2i\pi}\int_L \operatorname{Re}\left\{\frac{e^{i\varphi_0}}{\xi - \xi_0}\right\} df - \frac{\gamma^2}{4i\pi}\int_L f(s)\cos(\varphi_0 - \varphi)\ln r_0 ds + \\
& + \frac{\gamma^2}{8}\int_L \Bigg\{ f(s)\left[\begin{aligned} & \cos(\varphi_0 + \varphi - 2\alpha_0)\left(H_2^{(1)}(\gamma r) - \frac{4}{i\pi\gamma^2 r_0^2}\right) \\ & + \cos(\varphi_0 - \varphi)\left(H_0^{(1)}(\gamma r_0) - \frac{2i}{\pi}\ln r_0\right)\end{aligned}\right] + \\
& + A[\cos(\varphi_0 - \varphi - 2\alpha_{10})H_2^{(1)}(\gamma r_{10}) + \cos(\varphi_0 + \varphi)H_0^{(1)}(\gamma r_{10})]\Bigg\} ds = \\
& = \tau\gamma_2 \sin(\varphi_0 - \psi)\, e^{-i\gamma_2(\xi_0 \cos\psi + \eta_0 \sin\psi)}.
\end{aligned} \tag{13}$$

For the unique solvability of the integro-differential equation (13), an additional condition (9) should be added to it.

Let us represent the contour L in parametric form. To do this, on each of the contours L_j $(j=\overline{1,K})$, we select a local coordinate system, taking into account the parallel translation and rotation of the coordinate axes. Considering this, we will assume L_j on the contour

$$\frac{\zeta_0'}{\zeta-\zeta_0}=\frac{1}{\beta-\beta_0}+\left(\frac{\zeta_0'}{\zeta-\zeta_0}-\frac{1}{\beta-\beta_0}\right),$$

$$s'\ln|\zeta-\zeta_0|=s_0'\ln|\beta-\beta_0|+(s'\ln|\zeta-\zeta_0|-s_0'\ln|\beta-\beta_0|).$$

$$\zeta=\zeta(\beta),\ \zeta_0=\zeta(\beta_0),\ -1\le\beta,\ \beta_0\le 1,\ \zeta(-1)=a_j,\ (+1)=b_j. \tag{14}$$

We multiply equation (13) by $s'(\beta_0)$ and select the Cauchy-type kernel and the logarithmic kernel (14) in it as follows:

$$\begin{gathered}\frac{\zeta_0'}{\zeta-\zeta_0}=\frac{1}{\beta-\beta_0}+\left(\frac{\zeta_0'}{\zeta-\zeta_0}-\frac{1}{\beta-\beta_0}\right),\\ s'\ln|\zeta-\zeta_0|=s_0'\ln|\beta-\beta_0|+(s'\ln|\zeta-\zeta_0|-s_0'\ln|\beta-\beta_0|).\end{gathered} \tag{15}$$

Here, using L'Hopital's rule in (15), we find

$$\lim_{\beta\to\beta_0}\left(\frac{\zeta_0'}{\zeta-\zeta_0}-\frac{1}{\beta-\beta_0}\right)=-\frac{\zeta''(\beta_0)}{2\zeta'(\beta_0)},$$

$$\lim_{\beta\to\beta_0}(s'\ln|\zeta-\zeta_0|-s_0'\ln|\beta-\beta_0|)=s_0'\ln|\zeta_0'|. \tag{16}$$

Taking into account the additional condition (9) and using (16), we integrate the integral with a logarithmic kernel by parts:

$$\int_{-1}^{1}f(\beta)\ln|\beta-\beta_0|\,d\beta=-\int_{-1}^{1}(\beta-\beta_0)(\ln|\beta-\beta_0|-1)\,df(\beta). \tag{17}$$

The parametric form of the integral equation taking into account (17) takes the form:

$$\frac{1}{2i\pi}\int_{-1}^{1}\frac{f'(\beta)}{\beta-\beta_0}d\beta+\frac{\gamma^2}{4i\pi}\operatorname{Re}(\zeta_0'\zeta')\int_{-1}^{1}(\beta-\beta_0)(\ln|\beta-\beta_0|-1)f'(\beta)d\beta+$$

$$+\frac{\gamma^2}{8}\int_{-1}^{1}f(\beta)\left\{\left[\begin{matrix}\operatorname{Re}\left(\zeta_0'\zeta'\dfrac{\overline{\zeta}_0-\overline{\zeta}}{\zeta_0-\zeta}\right)H_2^{(1)}(\gamma r_0)\\ -\dfrac{4}{i\pi\gamma^2(\beta_0-\beta)^2}\end{matrix}\right]+\left[\operatorname{Re}(\zeta_0'\overline{\zeta}')H_0^{(1)}(\gamma r_0)-\frac{2i}{\pi}\operatorname{Re}(\zeta_0'\overline{\zeta}_0')\ln|\beta-\beta_0|\right]+\right.$$

$$\left.+A\left[\begin{matrix}\operatorname{Re}\left(\zeta_0'\overline{\zeta'}\dfrac{\overline{\zeta}_0-\zeta}{\zeta_0-\overline{\zeta}}\right)H_2^{(1)}(\gamma r_{10})\\ -\operatorname{Re}(\zeta_0'\overline{\zeta}')H_0^{(1)}(\gamma r_{10})\end{matrix}\right]\right\}d\beta=\tau\gamma\zeta_0'(e^{-i\gamma\eta_0}+Ae^{i\gamma\eta_0}). \tag{18}$$

The proposed procedure for regularizing the integral with a logarithmic kernel allows us to reduce the problem to a singular integral equation (SIE) with respect to the function $f'(\beta)$. Kernels of integrals corresponding to the function $f(\beta)$ are continuous. The unique solution of the obtained singular integro-differential equation (18) in the presence of additional condition (9) should be sought in the class of functions that have a root singularity at the ends of the cuts [11]. Thus, we suppose

$$f'(\beta) = (\beta),\ f(\beta) = \int_{-1}^{\beta} f'(\beta)\,d\beta,\ \beta = \cos\theta.$$

3. Discretization of the problem

Imagine an unknown density $\Omega(\beta)$ integral equation (18) as a set of functions $\Omega_j(\beta^j)$ defined on the contours L_j, $j = \overline{1, K}$. The numerical implementation of the integral equation (18) is carried out by the mechanical quadrature method [6]. The equation corresponding to the contour Lj, is satisfied at Chebyshev nodes of the second kind $\theta_m = \dfrac{\pi m}{n_j}\ (m = \overline{1, n_j - 1})$ and reduces to a system of linear algebraic equations (SLAE) [15] with respect to the values of the function $\Omega_j(\beta)$ at Chebyshev knots of the first kind $\theta_k = \dfrac{2k-1}{n_j}\pi\ (k = \overline{1, n_j})$, where n_j is the number of points for splitting the contour L_j.

For the Cauchy-type integral, we use the quadrature formula

$$\int_{-1}^{1} \frac{\Omega_j(\beta)}{\sqrt{1-\beta^2}\,(\beta-\beta_m)}\,d\beta \approx \frac{\pi}{n_j}\sum_{k=1}^{n_j} \frac{\Omega_j(\beta_k)}{\beta_k - \beta_m},\ \beta_m = \cos\theta_m,\ \beta_k = \cos\theta_k. \tag{19}$$

To an integral containing a regular kernel $D(\beta_0, \beta)$ and having a root singularity, we apply the Gaussian quadrature formula

$$\int_{-1}^{1} \frac{\Omega_j(\beta)}{\sqrt{1-\beta^2}} D(\beta_m, \beta)\,d\beta \approx \frac{\pi}{n_j}\sum_{k=1}^{n_j} D(\beta_m, \beta_k)\,\Omega_j(\beta_k). \tag{20}$$

As applied to the additional condition (9), we have

$$\sum_{k=1}^{n_j} \Omega_j(\beta_k) = 0. \tag{21}$$

Then, taking into account (20), (21), the Lagrange interpolation polynomial for a function $f_j(\beta)$ has the form [6]:

$$f_j(\beta) = \int_{-1}^{\beta} f'(\beta)\,d\beta \approx -\frac{2}{n_j}\sum_{k=1`}^{n_j} \Omega_j(\beta_k) \sum_{l=1}^{n_j-1} \frac{\cos l\theta_k \sin l\theta}{l}. \tag{22}$$

SLAE regarding functions Ω_j, $j = \overline{1, K}$ (22) takes the form:

$$\sum_{k=1}^{n_j} A_{mk}^{j}\,\Omega_j(\beta_k) = N_m^j,\ \sum_{k=1}^{n_j} \Omega_j(\beta_k) = 0, \tag{23}$$

$$A_{mk}^{j} = \frac{\pi}{n_j}\left\{\frac{1}{2i\pi}\frac{1}{\beta_k - \beta_m} + \frac{{\gamma_2}^2}{4i\pi}\mathrm{Re}(\zeta'_m \overline{\zeta'_m})(\beta_k - \beta_m)\times(\ln|\beta_k - \beta_m| - 1) + \right.$$

$$+\frac{\gamma_2{}^2}{8}\sum_{v=1}^{n_l} B_{mv}\sin\theta_v\left(-\frac{2}{n_j}\sum_{j=1}^{n_j-1}\frac{\cos l\theta_k\sin l\theta_v}{l}\right)\Bigg\},$$

$$B_{mv}=\mathrm{Re}\left(\zeta'_m\zeta'_v\frac{\bar\zeta_m-\bar\zeta_v}{\zeta_m-\zeta_v}\right)H_2^{(1)}(\gamma|\zeta_m-\zeta_v|)-\frac{4}{i\pi\gamma^2(\beta_v-\beta_m)^2}+$$

$$+\mathrm{Re}(\zeta'_m\bar\zeta'_v)H_0^{(1)}(\gamma|\zeta_m-\zeta_v|)-\frac{2i}{\pi}\mathrm{Re}(\zeta'_m\bar\zeta'_m)\ln|\beta_v-\beta_m|+$$

$$+A\left[\begin{array}{l}\mathrm{Re}\left(\zeta'_m\bar\zeta'_v\dfrac{\bar\zeta_m-\zeta_v}{\zeta_m-\bar\zeta_v}\right)H_2^{(1)}(\gamma|\zeta_m-\bar\zeta_v|)\\ +\mathrm{Re}\left(\zeta'_m\zeta'_v\right)H_0^{(1)}(\gamma|\zeta_m-\bar\zeta_v|)\end{array}\right].$$

Thus, in the numerical implementation of SIE (18), (9), the problem is reduced to solving SLAE (23) with $N=n_1+n_2+\ldots+n_k$ unknowns.

4. Calculation scheme

A numerical study of the described problem has been carried out. In order to study the convergence of the constructed algorithm, the case of a normal incidence of a shear wave on a system [8] consisting of parabolic cracks alternately located in an elastic half-space at the same distance from one another and symmetrically oriented along the X axis (Fig. 2) was considered. However, the number of cracks on the right and left is not necessarily equal $(L_{-T}\neq L_K)$

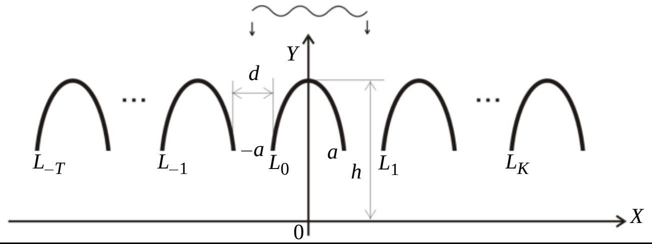

Fig. 2

The elements of the SLAE matrix, to which the SIE are ultimately reduced, are the result of discretization of the contours. Obviously, the size of the matrix is proportional to the number of cracks. We apply the parallelization of the algorithm, in which each element of the matrix is determined by the coordinates of the discretization nodes. As shown in [8], this method in the computational sense reduces to traversing each contour by collocation points of the non-integral variable ζ_{k0} and simultaneously bypassing each contour along similar or other nodes of the integration variable ζ_k.

The parallel-pipeline scheme of calculations is constructed similarly to [8]. The calculations have the following stages: synthesis of arrays of initial data, synthesis of the SLAE matrix, solution of SLAE by the Gauss method, synthesis of arrays of final solutions. The first, second and fourth stages of the macro pipeline do not require data transfers, which means that the calculations are independent. At the third stage, there is an optimal number of processes for solving the SLAE, which is determined by the specifics of the matrix. This means that without using parallelization of calculations for each value of the Green's functions [7], for steps 1, 2 and 4 of the algorithm, the optimal number of processes is strictly equal to the number of unknown SLAEs.

In this technique for solving the boundary value problem, the main operation is to determine the current distance between the collocation and integration points, given on the set of values of the parametric coordinates of inhomogeneities. The specified distance is an argument to the Green's function.

And since the combinations of Green's functions themselves and their coefficients are elements of the SLAE matrix, for its numerical solution the optimal number of processes is much smaller than for independent calculations — «proportional» minimization of the resulting time (ordering an increasingly large number of processes) is «impeded» by transfers data Such an asymmetric algorithm is supported, for example, by the MPI-2 operating system using the spawn procedure. But in these studies, such provision was not used. Therefore, the final optimal number of processes of the numerical implementation of the SIE (13) was obtained experimentally exclusively for the sequential parallelization of the Gaussian processing of the SLAE matrix.

This procedure can be basic when developing an application. As shown in [8, 9], the algorithm scales well across computing nodes.

Fig. 3 shows a graph of the dependence of the total time of cluster calculations of the array of intensity coefficients (10) on a section of a parabolic shape on the number of processes for one load variant. It follows from the graph that the entire algorithm scales well and has an effective number of processes, when the further increase in the number of cluster nodes no longer leads to a significant reduction in the algorithm implementation time. The graph shows the section of the asymptotic approximation of the dependence curve to the abscissa axis.

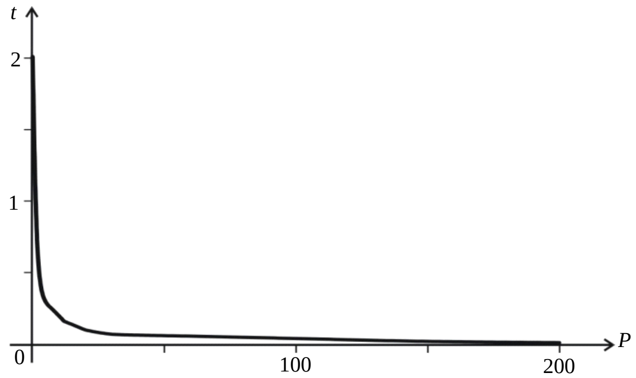

Fig. 3

Also, as in [8], 150–200 processes were effective for the parameter calculation algorithm (23). However, here the accuracy of 10^{-12} was achieved with the number of collocation points of the contour of each section $N = 300$, because the numerical solution algorithm of the problem uses interpolation by Chubyshev polynomials in accordance with the fact that the unknown function has its own root feature at the ends of the section, which causes a higher speed of convergence . The study of the question of the further increase in the accuracy of the result was not conducted.

The computational process of SLAE solution is parallelized according to [8, 15]. Parallel calculation of the final desired characteristics is carried out by substituting arrays of values of unknown functions $f_k(\beta_p)$ into integral representations of solutions similarly to the procedures for forming the SLAE matrix. To solve SLAE, it is more efficient to use line-by-line parallelization when transfers and calculations are in balance.

5. Results of numerical studies

Imagine a contour L in parametric form:

$$\begin{gathered} \zeta = \zeta(\beta),\ \zeta_0 = \zeta(\beta_0), \\ -1 \le \beta,\ \beta_0 \le 1,\ \zeta(-1) = a_j,\ \zeta(+1) = b_j. \end{gathered} \tag{24}$$

The parametric equation of the central parabola (24) was given as:

$$x = p_1\beta, \quad y = p_2\beta^2. \tag{25}$$

Thus, Fig. 2 shows the configuration, where the coefficient of curvature of the parabola (25) is $p_2 = -0,5$.

The dependences of the value of the stress intensity coefficients were determined in the work (maximum stresses is on the continuation *L* per vertex) on the parameter *q* [10, 13]:

$$q = 2l\gamma/\pi \tag{26}$$

$$K_3 = \lim_{\rho_0 \to 0} \sqrt{2\pi\rho_0}\,\sigma_n^c = \frac{\mu}{2}\sqrt{\frac{\pi}{s'(\mp 1)}}\,|\,f_0(\mp 1)\,| \,. \tag{27}$$

Calculations of the dimensionless quantity

$$\delta = \frac{\mu}{\sigma_{32}^{\max}} \frac{|\Omega(\mp 1)|}{2l}, \tag{28}$$

where $\sigma_{32}^{\max} = \mu\tau\gamma$ is the maximum voltage in the incident wave. Here τ is a load amplitude, γ is a wave number, *l* is a half-length of the contour, i.e., $L = 2l$.

To check the reliability of the algorithm, two tests were carried out: removal of three inhomogeneities at a distance of 10^6 of their lengths from each other and saturation of the system [8] with an increase in the number of geometrically equal reflectors in the system up to 13–15. When removing the characteristics of each reflector tend to the characteristics of a single one. The obtained results were compared with the results given in [13, 14, 16]. The coincidence of the results showed good reliability of the algorithm.

It should be noted that the expected result was obtained, which can be predictably observed on SIE both for more simplified problems of mathematical physics, such as diffraction of harmonic shear waves, and for three-dimensional static problems [9]. After all, the structure of the calculation process in models based on SIE is mostly unchanged and is built on well-defined procedures.

In the case of incrising in the number of defects located parallel to the oncoming wave front, the characteristics for most reflectors coincide and the problem is reduced to a periodic one [14]. Deviations of values are found only at the extreme reflectors.

The accuracy of the calculations was verified by comparing the results for different values of *N*. Stabilization of the values up to 10^{-10} is already achieved at 50 collocation points of each contour. The convergence of the algorithm, as well as in [8], does not depend on the number of cracks. The conditionality of the matrices was checked on the basis of the algorithm described in [15].

Fig. 4 illustrates the results of the first test. Fig. 4, *a* shows the dependence of the value δ from $\gamma l^2/4$ for any of the three parabolic cracks, the distance between which is 10^6 of their length. Here $p_1 = 1, \quad p_2 = \frac{\pi}{\gamma}$. As in [13], curves 1–3 correspond to the cases of half-spaces with fixed and free boundaries, as well as of infinite space ($A = 0$). The results were in complete agreement. Fig. 4, *b* shows a dependence that coincides with a similar result from [16] for the case of a straight crack at $A = 0$.

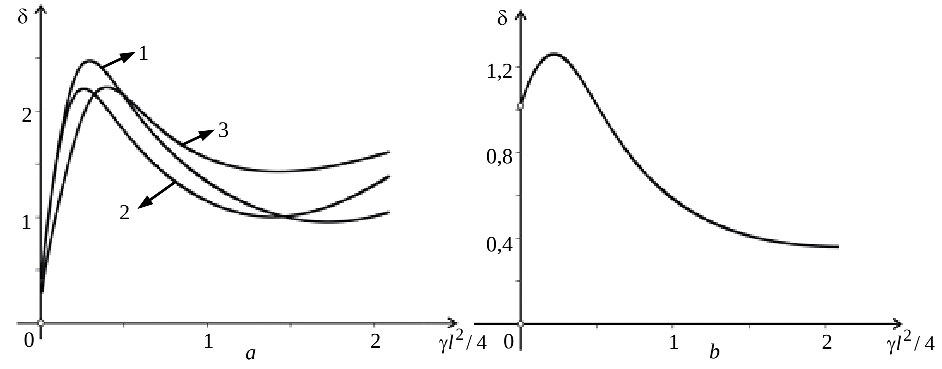

Fig. 4

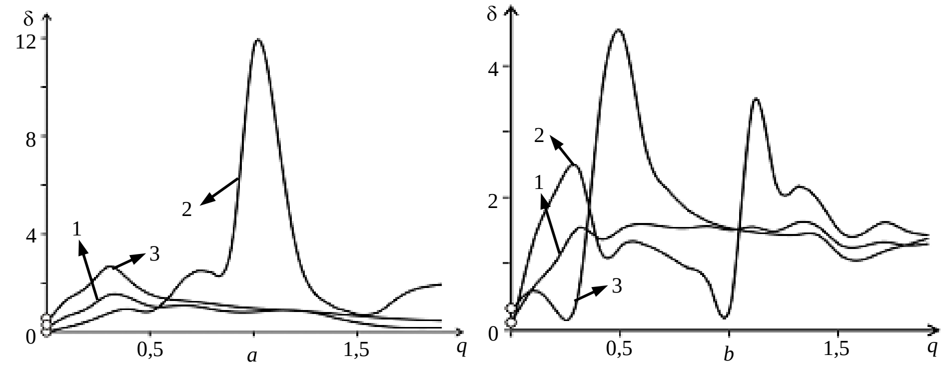

Fig. 5

In this paper, all dependences are given for the central object. Fig. 5, *a* shows the dependence of the intensity factor on the parameter *q* according to formula (26) [10, 13]. Curves 1–3 correspond to the cases $A=0$, $A=-1$, $A=1$, the coefficient of curvature of the parabola $p_2=0,5$. Nine objects were used in the calculations, with parameters (Fig. 2) $h=0{,}05$, $d=1$. Curves 1–3 (Fig. 5, *b*) correspond to the cases $A=0$, $A=-1$, $A=1$, $p_2=0,5$; 5 objects, $h=2$, $d=1$.

Curves 1–3 (Fig. 6, *a*) correspond to the cases $A=0$, $A=-1$, $A=1$, $p_2=0,5$, 5 objects, $h=6$, $d=1$. For curves 1–3 (Fig. 6, *b*) there are $A=0$, $A=-1$, $A=1$, $p_2=0,5$; 5 objects, $h=10$, $d=1$.

Curves 1–3 (Fig. 7, *a*) correspond to the cases of curvature $p_2=0,5;\ -0,5;\ 0,$ and 0 respectively, 5 objects, $h=10$, $d=1$, $A=0$. For curves 1–3 (Fig. 7, *b*) there are $p_2=0,5;\ -0,5;\ 0$ respectively; 5 objects, $h=10$, $d=1$, $A=-1$.

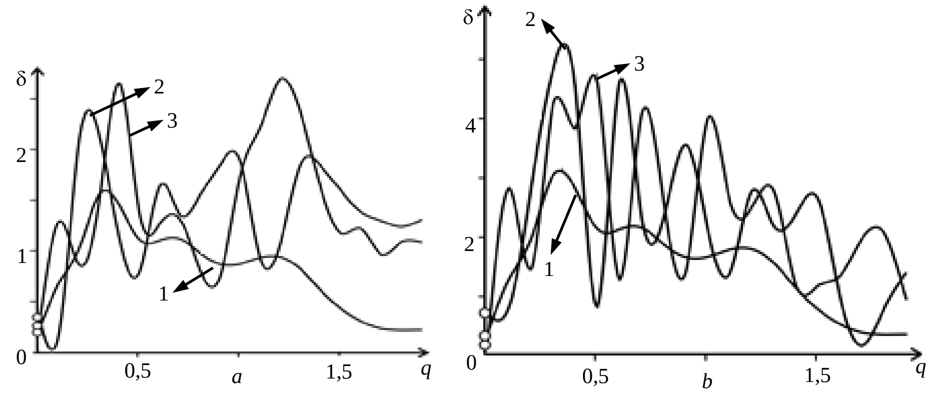

Fig. 6

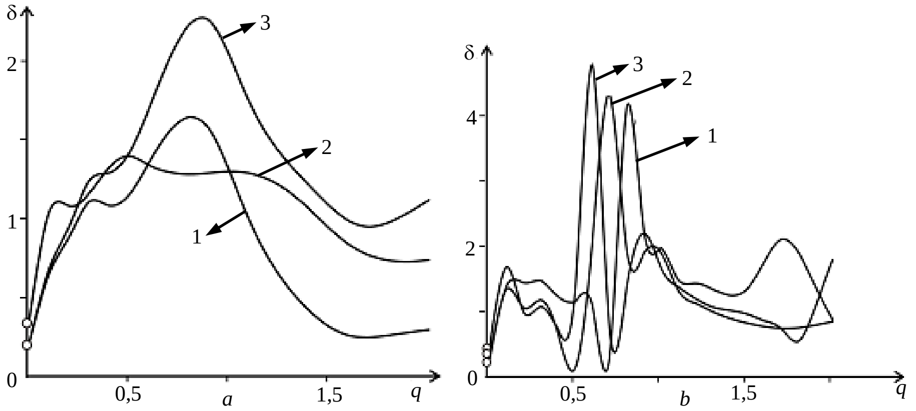

Fig. 7

Curves 1–3 (Fig. 8) correspond to $p_2 = 0{,}5;\ -0{,}5;\ 0,$ 5 objects, $h = 10$, $d = 1$, $A = 1$.

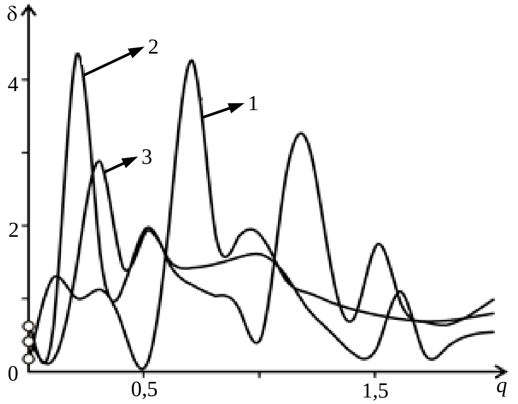

Fig. 8

Conclusion

In the problem of SH-wave diffraction by a system of cracks in an isotropic half-space, parallel algorithms can significantly reduce the computation time and analyze the characteristics of the wave field in more detail.

It was found that the parallel algorithm has an effective number of nodes from 150 to 200 (similarly to [9]), which allows obtaining the accuracy of multiple wave field studies up to 10^{-12}.

The combination of the SIE method, which reduces the dimension of the problem by one, and significant savings in computational time due to parallelization of computational procedures leads to a significant increase in the efficiency of the proposed algorithm.

Note that the updated results are provided here using the results of work [17].

Б.Є. Панченко, Л.М. Буката,
Д.Г. Багачук, Л.Я. Мартинович, О.М. Зуй

КЛАСТЕРНЕ МОДЕЛЮВАННЯ ВЗАЄМОДІЇ СТАЦІОНАРНИХ SH-ХВИЛЬ З СИСТЕМОЮ КРИВОЛІНІЙНИХ ТРІЩИН У НАПІВПРОСТОРІ

Панченко Борис Євгенійович

Одеський національний університет ім. Мечникова, Одеса,

pr-bob@ukr.net

Буката Людмила Миколаївна

Державний університет інтелектуальних технологій і зв’язку, Одеса,

ygrikluda@gmail.com

Багачук Денис Генадійович

Державний університет інтелектуальних технологій і зв’язку, Одеса,

bagachukdg@gmail.com

Мартинович Лариса Ярославівна

Одеський національний університет ім. Мечникова, Одеса,

larysa.yaroslavna@onu.edu.ua

Зуй Оксана Миколаївна

Одеський національний університет ім. Мечникова, Одеса,

oks.zuj@gmail.com

Запропоновано метод розв’язування задач математичної фізики для напівнескінченних середовищ, що містять системи криволінійних розрізів. На основі сингулярних інтегральних рівнянь (СІР) розроблено єдиний підхід до розв’язання задачі. Чисельна реалізація виконана за допомогою розпаралелювання. Наведено графік тривалості початкової кластерної години для розрахунку масиву шуканих функцій у розрізі параболічної форми як функції кількості процесів для одного варіанту реалізації. Показано, що весь алгоритм добре масштабується та має ефективну кількість процесів. Очікуваний результат було отримано, оскільки структура процесу розрахунку в моделях на основі СІР здебільшого не змінена та побудована на чітко визначених процедурах. Виявилося, що результативними є 150–200 процесів. Точність 10^{-12} було досягнуто при кількості точок колокації контуру кожної ділянки $N = 300$, оскільки в алгоритмі чисельного розв’язку задачі використовується інтерполяція поліномами Чебишева відповідно до того, що невідома функція має ключову особливість на кінцях розділу, що зумовлює більш високу швидкість збіжності алгоритму. Вивчення питання подальшого підвищення точності результату не проводилося. Досліджено відповідні динамічні крайові задачі для перенапруженої та безсилової півплощини. Досліджено вплив кривизни дефектів, їх взаємодії та близькості межі на величину та характер зміни коефіцієнтів інтенсивності динамічних напружень. Для перевірки надійності алгоритму проведено два тести: видалення трьох неоднорідностей на відстані 10^6 їх довжини одна від одної та насичення системи збільшенням кількості геометрично рівних відбивачів до 13–15. При видаленні характеристики кожного відбивача прагнуть до характеристик окремого, а при насиченні — до результатів відповідної періодичної задачі. Згода між результатами показала хорошу надійність алгоритму. Запропонований метод може бути використаний для оцінки впливу різних механічних або геометричних факторів на міцність тіл з дефектами.

Ключові слова: крайова задача, зсувні хвилі, сингулярні інтегральні рівняння, паралельні обчислення, система тріщин-зрізів.

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