

UDC 62-50

V. Romanenko, V. Gubarev, Yu. Miliavskyi

STAGES AND MAIN PROBLEMS OF THE CENTURY-LONG CONTROL THEORY AND SYSTEM IDENTIFICATION DEVELOPMENT. Part 5. PRINCIPLES AND PROBLEMS IN CONTROL AND IDENTIFICATION OF COMPLEX SYSTEMS OF VARIOUS NATURE BASED ON COGNITIVE MAPS IMPULSE PROCESSES MODELS

Viktor Romanenko

National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute»,
Institute for Applied Systems Analysis,

ipsa@kpi.ua; romanenko.viktorroman@gmail.com

Vyacheslav Gubarev

Space Research Institute of NAS of Ukraine and SSA of Ukraine, Kyiv,
<https://orcid.org/0000-0001-6284-1866>

v.f.gubarev@gmail.com

Yurii Miliavskyi

National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute»
Institute for Applied Systems Analysis,
<https://orcid.org/0000-0003-0882-3418>

yuriy.milyavsky@gmail.com

The article provides a review and generalization of the principles, methods and problems of designing discrete controllers for controlling complex systems of various nature, the dynamics of which are described using difference equations of cognitive maps (CM) impulse processes (Roberts equations). An external control vector is implemented by discrete controllers on the basis of varying nodes coordinates or weight coefficients of CM; the controllers are designed based on well-known methods of control theory. The article provides a solution to the following problems of controlling impulse processes in the complex systems CM: stabilization of unstable impulse processes in the CM of complex systems based on reference models of closed-loop control systems and on the basis of the modal control method; control of CM impulse process by varying weight coefficients; implementation of coordinating control of the ratios between the CM nodes coordinates in complex systems; suppression of external disturbances when controlling complex systems based on invariant ellipsoids method; control of impulse processes in the CM of complex systems with multirate sampling of nodes coordinates; identification of weight coefficients in CM impulse processes models with complete and incomplete measurement of nodes coordinates. The

solution to the above problems is based on the described new principles of controlling CM impulse processes in complex systems using automatic control theory methods.

Keywords: cognitive maps, impulse processes, reference models, invariant ellipsoids, identification, coordinating control.

Introduction

As a means for modeling complex systems of various nature with large dimension, cognitive maps (CM) are used, which are structural diagrams of cause-and-effect relationships between the components (coordinates, factors, concepts) of a complex system. From a mathematical point of view, a CM is a weighted directed graph, the nodes of which represent the coordinates of complex systems, and the edges describe the relationships between these coordinates [1, 2]. The construction of the CM is carried out by experts, which makes it possible to describe qualitatively the relationships between the components of a complex system and quantitatively display the influence of each CM coordinate on all others using the edges of directed graphs.

During the operation of a complex system with impulse-type behavior, under the influence of various disturbances, the coordinates of the CM change over time. In this case, each node l_i of the CM takes on a value $y_i(k)$ at discrete time moments $k = 0, 1, 2, \dots$. At the next sampling period, the value $y_i(k+1)$ is determined by the value $y_i(k)$ and information about whether other nodes l_j adjacent to l_i at a time instants k have increased or decreased their values. The change in the coordinate of the node l_i at a time moment k is called an impulse [2], which is specified by the difference $P_j(k) = y_j(k) - y_j(k-1)$, $k > 0$. An impulse $P_j(k)$ received at one of the nodes l_j will propagate along the CM chains to the remaining nodes, intensifying or attenuating. The process of propagation of disturbances along the nodes of the CM is determined by the difference equation

$$y_i(k+1) = y_i(k) + \sum_{j=1}^n a_{ij} P_j(k), \quad i = 0, 1, 2, \dots, n, \quad (1)$$

where a_{ij} is the weight coefficient of the directed graph edge that connects the j -th with i -th one. If there is no edge from node l_j to node l_i , then the corresponding coefficient is $a_{ij} = 0$.

The rule for changing the coordinates of the nodes of the CM (1) is usually formulated in the form of a first-order difference equation in increments of variables (Roberts equations) [2]

$$\Delta y_i(k+1) = \sum_{j=1}^n a_{ij} \Delta y_j(k), \quad (2)$$

which describes the impulse process in the CM, and $\Delta y_i(k) = y_i(k) - y_i(k-1)$, $i = 0, 1, 2, \dots, n$.

In vector form, expression (2) is written as follows:

$$\Delta \bar{y}(k+1) = A \Delta \bar{y}(k), \quad (3)$$

where A is the weight adjacency matrix, and $\Delta \bar{y}(k)$ is the vector of increments of the coordinates of the nodes $\bar{y}$ of the CM.

From the point of view of control theory, model (3) describes the dynamics of a multidimensional system in discrete time in the free motion of the coordinates of the CM nodes.

1. Principles of controlling impulse processes in cognitive maps of complex systems

In [3], axiomatic definitions of the basic principles of control for complex systems are introduced, if their mathematical model is presented in the form (2), (3).

The first principle is the formation of an external control vector based on variable vertices coordinates of the CM nodes of a complex system. To do this, it is necessary to formulate an equation for the forced motion of the system during an impulse process

$$\Delta y_i(k+1) = \sum_{j=1}^n a_{ij} \Delta y_j(k) + b_i \Delta u_i(k),$$

where $\Delta u_i(k) = u_i(k) - u_i(k-1)$ is the increment of the control action. In vector form this equation can be written as follows:

$$\Delta \bar{y}(k+1) = A \Delta \bar{y}(k) + B \Delta \bar{u}(k), \quad (4)$$

where $\Delta \bar{u}$ is the vector of increments of control actions $\Delta \bar{u}(k) = \bar{u}(k) - \bar{u}(k-1)$.

Depending on the physical nature of the complex system under study, we identify those CM nodes coordinates (values) which can be changed at discrete moments of time by the decision maker (dm) by changing the available resources. In general, for various CM of complex systems of various nature, these can be financial, energy, intellectual, information, economic, technological, administrative, defense, social, scientific, political, educational, environmental and other resources that can be changed at each set sampling period as external controls influencing specific nodes of the CM. In this case, external controls must have the same physical nature as the nodes on which they influence. The matrix B in (4) usually contains units and zeros, and the elements of this matrix corresponding to the controls in the vector $\bar{u}$ are equal to units.

Thus, when forming a vector $\Delta \bar{u}$, it is necessary to select the coordinates of the nodes of the CM, which can be influenced by the decision maker by changing the available resources. For example, for a complex system of socio-economic type, the following ones can act as control actions:

- financial costs (capital investments);
- changes in the standard time for performing certain work;
- variations in prices for certain goods;
- development of new types of products;
- increasing the level of scientific research;
- improving the staff qualification through retraining;
- administrative commands of the management of a particular organization.

The second principle is the implementation of a closed-loop control system, including a multidimensional discrete controller designed on the basis of methods of automatic control theory, which forms a vector of controls acting directly on the nodes of the CM as the output controlled coordinates of a complex system. In [4], the main properties of models (3) were studied. It is shown that if the CM is stable from the point of view of the theory of cognitive modeling, then the corresponding model (4) will be asymptotically stable from the point of view of control theory (if we imagine it as a model in state space). A study of controllability was also carried out and it was shown that if the system is controllable, then it is possible to form a state control vector in a closed system, as a result of which the corresponding CM goes into a stable static state.

The third principle is the use of the possibility of varying the weight coefficients of the matrix A in (3) to implement control actions $\Delta u_i(k)$ in a closed-loop control system. This principle must be applied in cases where it is impossible or undesirable to vary the coordinates of the CM nodes (resources) during formation $\Delta u_i(k)$ in accordance with the first and second principles. Varying the weight coefficient can take place when it is possible to change the degree of sensitivity of the influence of one CM value on another one. The decision maker can implement this principle by changing the transmission coefficients when forming administrative, financial, political, educational, information interactions between the coordinates of a complex system, presented in the form of nodes of the CM. When controlling the impulse CM process by varying the weight coefficients, we change the degree of influence on the coordinate y_i of the remaining coordinates y_j . In this case, the magnitude of the control action $\Delta u_i(k)$ is formed at the expense of changing the influence of the remaining coordinates $\Delta u_i(k)$ on the node Δy_i .

When implementing all three of the above control principles, it is necessary to measure (fix) accurately the coordinates of all nodes of the CM. However, many CM nodes in various complex systems are difficult to formalize and cannot be measured in real time. These include:

- competitiveness of products;
- level of technology development;
- shadow connections between the economy and business;
- effectiveness of information propaganda;
- level of democratization of the state;
- level of corruption;
- integral level of population health in a given region, etc.

For this case, the work [3] proposed a **fourth principle** for controlling impulse processes of the CM, which involves the decomposition of the original CM into two interconnected parts. The first part of the CM is compiled of the measurable coordinates of the original CM nodes, and the second part includes the unmeasured coordinates of the nodes. Then, for the first part of the CM, the first model is compiled, which describes the dynamics of the impulse process of the measured coordinates of the CM:

$$\Delta y_i(k+1) = \sum_{j=1}^P a_{ij} \Delta y_j(k) + \sum_{\mu=p+1}^n a_{i\mu} \Delta y_\mu(k), \quad (5)$$

where y_i , $i=1, 2, \dots, P$ are the measured coordinates of the CM in real time; y_μ , $\mu = p+1, \dots, n$ are unmeasured coordinates of the nodes of the original CM.

The second model is compiled to describe the dynamics of the impulse process of unmeasured nodes of the CM:

$$\Delta y_\mu(k+1) = \sum_{j=p+1}^n a_{\mu j} \Delta y_j(k) + \sum_{j=1}^P a_{\mu j} \Delta y_j(k). \quad (6)$$

In this case, unmeasured coordinates are considered as disturbances in model (5). Expressions (5), (6), respectively, can be written in vector-matrix form:

$$\Delta \bar{y}_1(k+1) = A_{11} \Delta \bar{y}_1(k) + A_{12} \Delta \bar{y}_2(k), \quad (7)$$

$$\Delta \bar{y}_2(k+1) = A_{21} \Delta \bar{y}_1(k) + A_{22} \Delta \bar{y}_2(k), \quad (8)$$

where the adjacency matrices A_{11} , A_{12} , A_{21} , A_{22} have dimensions $(p \times p)$, $(p \times (n-p))$, $((n-p) \times p)$, $(n-p) \times (n-p)$ respectively.

In the future, the impulse process model (7) will be used for control.

The fifth control principle involves multirate sampling of the coordinates of the nodes of the CM for the case when the measurement of all coordinates of the CM cannot be performed with one sampling period, the choice of which is carried out depending on the rate of change of the most rapidly changing coordinate of the CM. During the operation of a complex system, these coordinates can be measured (fixed) at discrete moments in time with different sampling periods. To determine these periods, it is necessary to have information about the possible rates of change of each coordinate of the nodes of a complex system. Then the choice of period T_0 , according to [5], is performed on the basis

$$T_{0i \min} \leq \frac{\varepsilon}{\left| \frac{dy_i(t)}{dt} \right|_{\max}},$$

where ε is the specified absolute error of the coordinate y_i , which arises due to the inaccuracy of measurement y_i and sampling of the continuous function $y_i(t)$. The quantity ε has the physical dimension of coordinates y_i .

Let us consider a model of an impulse process of a complex system, which has the property of functioning on two time scales. In this case, one part of the coordinates is measured with a sampling period T_0 , and the other one with a period $h = mT_0$, where m is an integer greater than one. Then the original model (1) can be written as

$$\Delta y_i \left[\left[\frac{k}{m} \right] h + (l+1)T_0 \right] = \sum_{j=1}^n a_{ij} \Delta y_j \left[\left[\frac{k}{m} \right] h + lT_0 \right], \quad (9)$$

where $l = 0, 1, \dots, m-1$; $i = 1, 2, \dots, n$; $\left[\frac{k}{m} \right]$ is integer part of k divided by m .

Let us assume that in the CM of a complex system with dimension n there are p coordinates of nodes y_i that are measured with a sampling period T_0 , and $(n-p)$ coordinates can be measured with an increased period $h = mT_0$. Then the model of the impulse process of CM should be described with multirate sampling of coordinates as follows:

$$\Delta y_i \left[\left[\frac{k}{m} \right] h + (l+1)T_0 \right] = \sum_{j=1}^P a_{ij} \Delta y_j \left[\left[\frac{k}{m} \right] h + lT_0 \right] + \sum_{\gamma=1}^{n-P} \beta_{i\gamma} \Delta \tilde{y}_{p+\gamma} \left[\left[\frac{k}{m} \right] h \right]; \quad (10)$$

$$\Delta y_r \left[\left(\left[\frac{k}{m} \right] + 1 \right) h \right] = \sum_{j=1}^P a_{rj} \Delta \tilde{y}_j \left[\left[\frac{k}{m} \right] h + lT_0 \right] + \sum_{\varphi=1}^{n-P} \beta_{r\varphi} \Delta y_{p+\varphi} \left[\left[\frac{k}{m} \right] h \right], \quad (11)$$

where $i = 1, 2, \dots, p$; $r = (p+1), \dots, n$; $l = 0, 1, \dots, (m-1)$; $\Delta \tilde{y}_{p+\gamma} \left[\left[\frac{k}{m} \right] h \right]$ equal to the increment of the slowly changing coordinate $y_{p+\gamma}$ at $l = 0$ and equal to zero at $l = 1, \dots, m-1$.

Equations (10), (11) can be written in a generalized vector-matrix form:

$$\Delta \bar{y}_1 \left[\left[\frac{k}{m} \right] h + (l+1)T_0 \right] = W_{11} \Delta \bar{y}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right] + W_{12} \Delta \tilde{y}_2 \left[\left[\frac{k}{m} \right] h \right]; \quad (12)$$

$$l = 0, 1, \dots, (m-1);$$

$$\Delta \bar{y}_2 \left[\left(\left\lfloor \frac{k}{m} \right\rfloor + 1 \right) h \right] = W_{21} \Delta \tilde{y}_1 \left[\left\lfloor \frac{k}{m} \right\rfloor h + l T_0 \right] + W_{22} \Delta \bar{y}_2 \left[\left\lfloor \frac{k}{m} \right\rfloor h \right]; \quad (13)$$

$$l = 0, 1, \dots, (m-1),$$

where the matrices have dimensions $W_{11}(p \times p)$; $W_{12}(p \times (n-p))$; $W_{21}((n-p) \times p)$; $W_{22}((n-p) \times (n-p))$. Coordinates $\bar{y}_2$ that are measured with a sampling period h will be constant over time $\left\lfloor \frac{k}{m} \right\rfloor h \leq t < \left(\left\lfloor \frac{k}{m} \right\rfloor + 1 \right) h$.

In work [3], the following statements were formulated and proven.

Statement 1. When calculating $\Delta \bar{y}_2 \left[\left(\left\lfloor \frac{k}{m} \right\rfloor + 1 \right) h \right]$ in model (13), the component

$\Delta \tilde{y}_1 \left[\left\lfloor \frac{k}{m} \right\rfloor h + l T_0 \right]$ is taken into account according to the formula

$$\Delta \tilde{y}_1 \left[\left\lfloor \frac{k}{m} \right\rfloor h + l T_0 \right] = \bar{y}_1 \left[\left\lfloor \frac{k}{m} \right\rfloor h + (m-1) T_0 \right] - \bar{y}_1 \left[\left\lfloor \frac{k}{m} \right\rfloor h - T_0 \right].$$

Statement 2. If the first inverse differences in (9) are defined as

$$\Delta y_i \left[\left\lfloor \frac{k}{m} \right\rfloor h + (l+1) T_0 \right] = y_i \left[\left\lfloor \frac{k}{m} \right\rfloor h + (l+1) T_0 \right] - y_i \left[\left\lfloor \frac{k}{m} \right\rfloor h + l T_0 \right],$$

$$\Delta y_i \left[\left\lfloor \frac{k}{m} \right\rfloor h + l T_0 \right] = y_i \left[\left\lfloor \frac{k}{m} \right\rfloor h + l T_0 \right] - y_i \left[\left\lfloor \frac{k}{m} \right\rfloor h + (l-1) T_0 \right],$$

then the transition of these differences to their representation with a large sampling period h is carried out on the basis of:

$$\Delta y_i \left[\left(\left\lfloor \frac{k}{m} \right\rfloor + 1 \right) h \right] = \sum_{\mu=1}^m \Delta y_i \left[\left\lfloor \frac{k}{m} \right\rfloor h + \mu T_0 \right];$$

$$\Delta y_i \left[\left\lfloor \frac{k}{m} \right\rfloor h \right] = \sum_{\mu=1}^m \Delta y_i \left[\left(\left\lfloor \frac{k}{m} \right\rfloor - 1 \right) h + \mu T_0 \right].$$

The formulated principles are intended for the subsequent stabilization of unstable impulse processes in the CM, control of the ratios between the coordinates of the CM nodes and for identifying the coefficients of the CM adjacency matrix.

2. Problems of stabilization of unstable impulse processes in cognitive maps of complex systems

2.1. Stabilization of unstable impulse processes in CM based on reference models. It is assumed that the original dynamic model of the impulse process of CM (3) is unstable. In this case, the eigenvalues of the adjacency matrix A will be $|\lambda_i| > 1$. To stabilize an unstable impulse process, the problem of designing a discrete controller to create a vector of external controls, which, according to a certain law, influences the nodes y_i of the CM in a closed-loop control system, was solved in [6]. For this purpose, the CM model (4) is presented in increments of variables in the «input–output» form as

$$(I - Aq^{-1}) \Delta \bar{y}(k) = Bq^{-1} \Delta \bar{u}(k), \quad (14)$$

where q^{-1} is the reverse shift operator by one sampling period. Assume $\dim \Delta \bar{u} = \dim \Delta \bar{y}$, control actions are applied to all nodes of the CM. The control law of a discrete controller is formulated as

$$\Delta \bar{u}(k) = D_0(I + F_1 q^{-1})^{-1}(\Delta \bar{G}(k) - \Delta \bar{y}(k)), \quad (15)$$

where $\Delta \bar{G}(k)$ is vector of increments of set-point values. In practice in stabilization systems $\Delta \bar{G}(k) = 0$. Based on (14), (15), the equation of the multidimensional closed-loop control system is presented over the « $\Delta \bar{G} \rightarrow \Delta \bar{y}$ » channel in the form

$$\Delta \bar{y}(k) = (I + F_1 q^{-1})\{(I - A q^{-1})(I + F_1 q^{-1}) + q^{-1} B D_0\}^{-1} B q^{-1} (I + F_1 q^{-1})^{-1} \Delta \bar{G}(k). \quad (16)$$

To ensure the stability of the closed-loop system (16), a reference model of the characteristic polynomial is used

$$I + A_{M_1} q^{-1} + A_{M_2} q^{-2} = (I - A q^{-1})(I + F_1 q^{-1}) + q^{-1} B D_0. \quad (17)$$

In this case, the roots of the reference model $\det(I + A_{M_1} q^{-1} + A_{M_2} q^{-2}) = 0$ of the reference model are selected with a modulus less than one. From equation (17) the matrices of the control law (15) are determined

$$F_1 = -A^{-1} A_{M_2}; \quad D_0 = B^{-1}(A + A^{-1} A_{M_2} + A_{M_1}). \quad (18)$$

Thus, at each sampling period with measured increments of coordinates $\Delta \bar{y}(k)$ of the nodes of the CM, based on the control law (15) and controller parameters (18), the vector of increments of control actions is determined for $\Delta \bar{G}(k) = 0$ as

$$\Delta \bar{u}(k) = -B^{-1}(A + A^{-1} A_{M_2} + A_{M_1}) \cdot (I - A^{-1} A_{M_2} q^{-1})^{-1} \Delta \bar{y}(k).$$

The disadvantage of this method is equality $\dim \Delta \bar{u} = \dim \Delta \bar{y}$, so that to implement control it is necessary to vary the resources of all nodes of the CM. This is possible only in rare cases. For example, this algorithm was used in [6] to stabilize an unstable process in a financial bank (in the event of its bankruptcy).

2.2. Stabilization of an unstable impulse process in a CM based on modal control.

When forming control actions to stabilize unstable processes in complex systems based on models of impulse processes, the CM can be varied only by some of the coordinates of the CM nodes. In work [7] to control the impulse process (4)

$$\Delta \bar{y}(k+1) = A \Delta \bar{y}(k) + B \Delta \bar{u}(k)$$

controller applied

$$\Delta \bar{u}(k) = -K_p \Delta \bar{y}(k),$$

which is designed based on the modal control method [8] for the case when $\dim \Delta \bar{u}(k) < \dim \Delta \bar{y}(k)$. To do this, the desired spectrum of the closed-loop system $\Delta \bar{y}(k+1) = [A - B K_p] \Delta \bar{y}(k)$ is set

$$\bar{\lambda} = \begin{bmatrix} \lambda_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_n \end{bmatrix},$$

where all $|\lambda_j| < 1$. In this case, all eigenvalues λ_j of the matrix $(A - B K_p)$ are chosen to be different, and among λ_j there are no eigenvalues of the matrix A of weight

coefficients of the CM. It is assumed that the control vector $\bar{u}$ has dimension m , where $m < n$. To design the controller, eigenvectors $\bar{R}_j$, $j = 1, \dots, n$ of the state matrix $A - BK_p$ of the closed-loop system are introduced, for which the equation $(A - BK_p)\bar{R}_j = \lambda_j \bar{R}_j$ holds. This equality can be written in the form

$$(A - \lambda_j I)\bar{R}_j = BK_p \bar{R}_j = B\bar{P}_j, \quad j = 1, \dots, n, \quad (19)$$

where the vectors $\bar{P}_j = K_p \bar{R}_j$ have dimension m .

An arbitrary matrix $P(m \times n)$ is specified so that it has full rank and does not have zero columns: $P = (\bar{P}_1, \bar{P}_2, \dots, \bar{P}_n)$. Then from expression (19) we can determine

$$\bar{R}_j = (A - \lambda_j I)^{-1} B \bar{P}_j, \quad j = 1, \dots, n \quad (20)$$

and form a matrix $R = (\bar{R}_1, \dots, \bar{R}_n)$ of dimension $(n \times n)$, which will be non-degenerate. Since $\bar{P}_j = K_p \bar{R}_j$, the feedback matrix of the closed-loop control system is calculated as follows:

$$K_p = PR^{-1}. \quad (21)$$

By construction K_p provides the desired set of roots $\lambda_1, \lambda_2, \dots, \lambda_n$ of the characteristic equation of a closed-loop control system $\det[Iq - A + BK_p] = 0$.

Based on the modal control method, work [9] carried out stabilization of an unstable mode in the impulse process of the CM of a student's social-educational process, and work [10] implemented an adaptive system for stabilizing an unstable cryptocurrency rate based on a model of the impulse process of a cognitive map.

3. The problem of controlling the impulse process of CM by varying the weight coefficients

In [11], a new approach to controlling the CM process is proposed. It is implemented by varying the weight coefficients of the CM edges when forming control actions $\bar{u}(k)$. Varying the weight coefficients is possible when it is possible to change the influence of one of the CM nodes on another one. The paper considers the problem of stabilizing the coordinates of the nodes y_i of the CM at given levels G_i at each sampling period. In this case, control actions are formed by changing the increments of the weighting coefficients $\Delta a_{i_{\mu_l}}(k)$ on the edges of the CM, which connect the node $y_{\mu_l}(k)$ with $y_i(k)$. For this purpose, the equation of the forced motion of the impulse process of the CM is written in the full values of the coordinates $\bar{y}(k)$ of the CM nodes in the following form:

$$\bar{y}(k+1) = (I + A - Aq^{-1})\bar{y}(k) + L(k)\Delta\bar{a}(k) + \bar{\xi}(k), \quad (22)$$

where the vector of increments of weighting coefficients $\Delta\bar{a}(k)$ of dimension $(m \times 1)$, $m \leq n$, contains only non-zero elements $\Delta a_{i_{\mu_l}}(k) \neq 0$, where μ is the number of the node of the CM, which affects the i -th node y_i through the edge of the CM with a coefficient $a_{i_{\mu}}$, and the index $l = 1, 2, \dots, m$ denotes the sequence number of this coefficient in the vector $\Delta\bar{a}(k)$. If some weight coefficient $a_{i_{\mu}}$ does not vary, then its increment $\Delta a_{i_{\mu}}(k) = 0$ is not included in the vector $\Delta\bar{a}(k)$.

The matrix $L(k)$ contains only the measured coordinates of the nodes $y_{\mu_i}(k)$ of the CM, which influence the nodes $y_i(k+1)$, $i=1, 2, \dots, n$, through the edges of the CM with variable coefficients $\Delta a_{i_{\mu_l}}(k)$. The matrix $L(k)$ ($n \times m$) contains at most one non-zero element y_{μ_i} on each i row, which affects the node y_i through the increment of the weight coefficient $\Delta a_{i_{\mu_l}}(k)$. The number of the column in the matrix $L(k)$ in which this element is located is equal to the number μ of the element $\Delta a_{i_{\mu_l}}(k)$ in the vector $\Delta \bar{a}(k)$ of equation (22).

The design of the optimal control vector $\Delta \bar{a}(k)$ is implemented based on the minimization of the quadratic optimality criterion

$$J(k+1) = E\{[\bar{y}(k+1) - \bar{G}]^T [\bar{y}(k+1) - \bar{G}] + \Delta \bar{a}^T(k) R \Delta \bar{a}(k)\}, \quad (23)$$

where $\bar{G}$ is the set-point vector; R is diagonal positive definite matrix. As a result of minimization $J(k+1) \rightarrow \min_{\Delta \bar{a}(k)}$, the controller control law is found

$$\Delta \bar{a}(k) = -[L^T(k)L(k) + R]^{-1} L^T(k) \{(I + A - Aq^{-1}) \bar{y}(k) + \bar{\xi}(k) - \bar{G}\}. \quad (24)$$

Based on equations (22), (23), the equation of the closed-loop control system for the impulse CM process has the form

$$\begin{aligned} \bar{y}(k+1) &= \{I - L(k)[L^T(k)L(k) + R]^{-1} L^T(k)\} \times \\ &\times (I + A - Aq^{-1}) \bar{y}(k) + L(k)[L^T(k)L(k) + R]^{-1} L^T(k) \bar{G} + \\ &+ \{I - L(k)[L^T(k)L(k) + R]^{-1} L^T(k)\} \bar{\xi}(k), \end{aligned} \quad (25)$$

where $\bar{\xi}(k)$ is the external disturbance.

The stability of the closed-loop system (25) is determined by the location of the eigenvalues λ_i of the matrix $\{I - L(k)[L^T(k)L(k) + R]^{-1} L^T(k)\}$. The following statement was formulated and proven in [11].

Statement 3. Let the equation of the forced motion of the CM impulse process have the form (22), where $\Delta \bar{a}(k)$ is the m -dimensional vector of increments of weight coefficients, and the matrix $L(k)$ ($n \times m$) is formed at each sampling period based on the coordinates $y_{\mu_l}(k)$, where $l=1, 2, \dots, m$ is the number of the corresponding increments $\Delta a_{i_{\mu_l}}(k)$ in the vector $\Delta \bar{a}(k)$, then the product of the dimensional ($m \times m$) matrices $L^T(k)L(k)$ in the control law (24) will be a diagonal matrix with elements $y_{\mu_l}^2(k)$.

Corollary 1. Since the weight matrix R ($m \times m$) in (24) is diagonal and positive definite, the inverse matrix $[L^T(k)L(k) + R]^{-1}$ in (25) will be diagonal with elements $\frac{1}{y_{\mu_l}^2(k) + R_{ll}}$, $l=1, 2, \dots, m$, which are always positive.

Corollary 2. The matrix $M(k) = \{I - L(k)[L^T(k)L(k) + R]^{-1} L^T(k)\}$ in (25) dimension ($n \times n$) will be diagonal with elements

$$1 - \frac{y_{\mu_l}^2(k)}{y_{\mu_l}^2(k) + R_{ll}} = \frac{R_{ll}}{y_{\mu_l}^2(k) + R_{ll}} < 1$$

in rows that correspond to non-zero values $y_{\mu_i}(k)$ and equal to one in other cases. Thus, the eigenvalues of the matrix $M(k)$ will always be $0 < \lambda_i \leq 1$, $i = 1, 2, \dots, n$, and the closed loop system (25) will be stable.

In work [12], the problem of combined control of impulse processes in a CM was solved based on varying the weight coefficients and resources of the coordinates of the CM nodes.

4. The problem of coordinating control of the ratios between the coordinates of complex system CM in a stochastic environment

The problem of controlling coordinate ratios arises in multidimensional automatic control systems [13]. A huge number of complex systems represented by CM, must be described in a stochastic environment due to the following circumstances:

a) many coordinates of socio-economic, political, administrative systems cannot be measured and they can only be taken into account as stochastic disturbances;

b) some coordinates of the CM are measured with a large error.

In [14], the problem of controlling the ratios between the coordinates of the nodes of CM in a stochastic environment (coordinating control) is considered. The impulse process of CM (4) is presented in full coordinates of the nodes by analogy with [15]

$$[I - (I + A)q^{-1} + Aq^{-2}] \bar{y}(k) = Bq^{-1} \Delta \bar{u}(k) + \bar{\xi}(k), \quad (26)$$

where q^{-1} is the reverse shift operator; A ($n \times n$) is the adjacency matrix of the CM with eigenvalues $\lambda_i < 1$, the control matrix B is usually assumed $B = I$ (it is possible to vary the resources of all nodes of the CM); $\bar{\xi}(k)$ is the vector of stochastic disturbances or errors in measuring the coordinates of nodes in the form of white noise, uncorrelated with the coordinates $\bar{y}(k)$ and controls $\Delta \bar{u}(k)$. For convenience, it is considered in further calculations

$$\tilde{\bar{y}}(k+1) = (I + A)\bar{y}(k) - A\bar{y}(k-1), \quad (27)$$

which is known part of $\bar{y}(k+1)$ at the time k .

First, a standard criterion for the generalized variance of the discrepancy between the coordinates $\bar{y}(k+1)$ and the set-point vector $\bar{G}$ and of control actions $\Delta \bar{u}(k)$ is introduced:

$$J_G(k+1) = E\{[\bar{y}(k+1) - \bar{G}(k)]^T [\bar{y}(k+1) - \bar{G}(k)] + \Delta \bar{u}^T(k) R \Delta \bar{u}(k)\}, \quad (28)$$

where E is the operator of conditional mathematical expectation based on all information available at a given time k . The matrix R is chosen to be symmetrical, positive definite and such that $(B^T B + R)$ it is not degenerate. To minimize (28) $\bar{y}(k+1)$ is split into known and unknown parts

$$\bar{y}(k+1) = \tilde{\bar{y}}(k+1) + B \Delta \bar{u}(k) + \bar{\xi}(k+1).$$

Then criterion (28) will have the form

$$J_G(k+1) = [\tilde{\bar{y}}(k+1) - \bar{G}(k) + B \Delta \bar{u}(k)]^T [\tilde{\bar{y}}(k+1) - \bar{G}(k) + B \Delta \bar{u}(k)] + \Delta \bar{u}^T(k) R \Delta \bar{u}(k) + 2E\{[\tilde{\bar{y}}(k+1) - \bar{G}(k) + B \Delta \bar{u}(k)]^T \bar{\xi}(k+1)\} + E\{\bar{\xi}^T(k+1) \bar{\xi}(k+1)\}. \quad (29)$$

Minimization of this criterion by vector $\Delta \bar{u}(k)$ leads to the equation of the optimal controller

$$\frac{\partial J_G(k+1)}{\partial \Delta \bar{u}(k)} = 2B^T (B \Delta \bar{u}(k) + \tilde{\bar{y}}(k+1) - \bar{G}(k)) + 2R \Delta \bar{u}(k) = 0.$$

It was shown in [14] that the solution to this equation is a minimum point. Then the control law is calculated by the formula

$$\Delta \bar{u}_G(k) = -(B^T B + R)^{-1} \{B^T [\tilde{y}(k+1) - \bar{G}(k)]\}.$$

The problem of coordinating control is stated based on given M ratios

$$S\bar{y}(k) = \bar{b}, \quad (30)$$

where $\bar{b}$ is a given vector with dimension M , S is a given matrix with dimension $(M \times n)$ for $M < n$. Here $\text{rank}(SB) = M$. The requirement of coordinating control is that ratios (30) should be satisfied as accurately as possible at each sampling period.

To implement coordinating control, the following optimality criterion is proposed

$$J_b(k+1) = E\{[S\bar{y}(k+1) - \bar{b}]^T [S\bar{y}(k+1) - \bar{b}]\} \rightarrow \min. \quad (31)$$

Let us write this criterion similarly to the previous criterion (29)

$$\begin{aligned} J_b(k+1) = & [S\tilde{y}(k+1) + SB\Delta\bar{u}(k) - \bar{b}]^T [S\tilde{y}(k+1) + SB\Delta\bar{u}(k) - \bar{b}] + \\ & + 2E\{[S\tilde{y}(k+1) + SB\Delta\bar{u}(k) - \bar{b}]^T S\bar{\xi}(k+1)\} + E\{\bar{\xi}(k+1)^T S^T S\bar{\xi}(k+1)\}. \end{aligned}$$

The last term does not depend on $\Delta\bar{u}(k)$, so it can be ignored. The penultimate term is equal to zero due to $E\{\bar{\xi}(k+1)\} = 0$. The first term is equal to the square of the Euclidean norm of the vector $S\tilde{y}(k+1) + SB\Delta\bar{u}(k) - \bar{b}$. It is known that the squared norm of a vector is non-negative and equals to zero if and only if the vector is zero. Therefore, if the equation $SB\Delta\bar{u}(k) = -[S\tilde{y}(k+1) - \bar{b}]$ has a solution, then this solution is the global minimum point of this criterion. By condition, a vector $\Delta\bar{u}(k)$ has dimension n , and a vector $[S\tilde{y}(k+1) - \bar{b}]$ has dimension $M < n$, and $\text{rank}(SB) = M$. Therefore, according to the Kronecker-Capelli theorem, this equation has many solutions, at which the minimum of criterion (31) is achieved, namely

$$SB\Delta\bar{u}(k) = -[S\tilde{y}(k+1) - \bar{b}]. \quad (32)$$

In general, the control for $\Delta\bar{u}_g(k)$ does not satisfy equality (32). Thus, we have a multicriteria optimization problem $\Delta\bar{u}(k)$ based on criteria (28) and (31), and criterion (31) by condition has higher priority, but has an ambiguous solution.

In [14], a method of conditional minimization of the discrepancy of ratios and the generalized variance of CM coordinates is considered. In this case, equation (32) is considered as a constraint that must be satisfied when minimizing criterion (28), that is, the problem of unconditional multicriteria optimization is reduced to the problem of conditional single-criteria optimization, which is solved by the Lagrange multiplier method [16], and then the Frobenius theorem is applied to address block matrix [17]. As a result, the following theorem was formulated and proven.

Theorem 1. *Let the CM impulse process be specified by equation (26) and it is necessary at each time moment to create such a control that will minimize the optimality criteria (31) and (28), and criterion (31) has higher priority, then the optimal control $\Delta\bar{u}(k)$ will be determined as follows:*

$$\begin{aligned} \Delta\bar{u}(k) = & -(B^T B + R)^{-1} \{[I - B^T S^T L^{-1} SB(B^T B + R)^{-1}] \times \\ & \times [B^T (\tilde{y}(k+1) - \bar{G}(k))] + B^T S^T L^{-1} [S\tilde{y}(k+1) - \bar{b}]\} \end{aligned} \quad (33)$$

if the matrices $(B^T B + R)$ and $L = SB(B^T B + R)^{-1} B^T S^T$ are nonsingular, and $\tilde{y}(k+1) = (I + A)\bar{y}(k) - A\bar{y}(k-1)$.

In [18], the problem of adaptive coordinating control of the ratios of the coordinates of the nodes of interacting CM in the mode of impulse processes was solved.

5. The problem of suppressing external disturbances of impulse processes in cognitive maps of complex systems based on invariant ellipsoids

In [19], a study was carried out on the use of the invariant ellipsoid method for suppressing limited disturbances with partial control of dynamic processes in complex systems of various nature, represented by mathematical models of impulse processes of CM. According to the fourth control principle, the original CM is decomposed into two interconnected parts. The first part of the CM is compiled of the measured coordinates of the nodes of the original CM (equation (7)), and the second part (8) describes the dynamics of the unmeasured coordinates of the nodes. Let us write models (7), (8) in a more convenient form [19]

$$\Delta\bar{x}(k+1) = A\Delta\bar{x}(k) + D\Delta\bar{y}(k), \quad (34)$$

$$\Delta\bar{y}(k+1) = C\Delta\bar{y}(k) + \Psi\Delta\bar{x}(k), \quad (35)$$

where $\bar{x}$ is the vector of measured coordinates of the CM; $\bar{y}$ is the vector of unmeasured coordinates. In this case, the matrices D and Ψ display the interconnections between the first (34) and second (35) parts of the original CM. In this case, changes in unmeasured coordinates $\Delta\bar{y}(k)$ are considered in the first system of equations (34) of the CM for impulse processes of the CM with measurable parameters as external limited unmeasured disturbances with unknown probabilistic characteristics.

In [20, 21], a method for suppressing limited external disturbances based on invariant ellipsoids for models of control objects in state space is considered. If we consider $\Delta\bar{y}(k)$ as a disturbance in model (34), then its norm limitation l_∞ will have the form

$$\|\Delta\bar{y}(k)\|_\infty = \sup_{k \geq 0} [\Delta\bar{y}^T(k)\Delta\bar{y}(k)]^{1/2} \leq 1. \quad (36)$$

In this case, to describe the characteristics of the influence of disturbances of type (36) on the trajectory of motion of a dynamic discrete system (34), invariant ellipsoids along coordinates $\Delta\bar{x}(k)$ are used in the following form:

$$\varepsilon_{\Delta\bar{x}} = \{\Delta\bar{x}(k) \in \mathbf{R}^n: \Delta\bar{x}^T(k)P^{-1}\Delta\bar{x}(k) \leq 1\}, \quad P > 0, \quad (37)$$

if the condition $\Delta\bar{x}(0) \in \varepsilon_{\Delta\bar{x}}$ implies the fulfillment of the condition $\Delta\bar{x}(k) \in \varepsilon_{\Delta\bar{x}}$ for all discrete moments of time $k=1, 2, 3, \dots$. The matrix P is called an ellipsoid matrix $\varepsilon_{\Delta\bar{x}}$. In [20] it is shown that the ellipsoid (37) $\varepsilon_{\Delta\bar{x}}$ will be invariant for the dynamic system (34) with l_∞ -bounded perturbations (36) if and only if the matrix P satisfies the linear matrix inequality

$$\frac{1}{\alpha} APA^T - P + \frac{DD^T}{(1-\alpha)} \leq 0, \quad P \geq P_0, \quad \alpha \in (0, 1). \quad (38)$$

In [19], a design of static state feedback was carried out, which minimizes the size of invariant ellipsoids for a controlled dynamic system, the model of which, based on (34), is supplemented with a control vector $\Delta\bar{u}(k)$

$$\Delta\bar{x}(k+1) = A\Delta\bar{x}(k) + B\Delta\bar{u}(k) + D\Delta\bar{y}(k). \quad (39)$$

The control vector $\Delta\bar{u}(k)$ is formed by the state controller

$$\Delta\bar{u}(k) = -K_p\Delta\bar{x}(k), \quad (40)$$

which implements static state feedback.

In the model of a controlled impulse process CM (39), as in the state equation, the components of the vector $\Delta\bar{x}(k)$ are completely measured.

The optimality criterion for implementing the controller (40) is minimizing the size of the invariant ellipsoid (37) based on

$$\text{tr } P(\alpha) \rightarrow \min, \alpha^* \leq \alpha < 1 \quad (41)$$

with maximum suppression of disturbances $\Delta\bar{y}(k)$ that are limited by the maximum range (36).

Naturally, minimizing the size of an invariant ellipsoid is equivalent to the problem of minimizing the trace of the matrix (41) under a constraint of the type of linear matrix inequality (38).

Based on model (39) and control law (40), the equation of the closed-loop control system for the impulse CM process will take the form

$$\Delta\bar{x}(k+1) = (A - BK_p)\Delta\bar{x}(k) + D\Delta\bar{y}(k). \quad (42)$$

It is assumed that the pair (A, B) in model (39) is controllable. Then the linear matrix inequality (38) for the closed system (42) takes the form

$$\frac{1}{\alpha}(A - BK_p)P(A - BK_p)^T - P + \frac{DD^T}{(1-\alpha)} \leq 0, \quad (43)$$

and after multiplying the terms

$$\frac{1}{\alpha}(APA^T - BK_pPA^T - APBK_p^T B^T + BK_pPK_p^T B^T) - P + \frac{DD^T}{(1-\alpha)} \leq 0. \quad (44)$$

This inequality is nonlinear with respect to P and K_p . In [20] replacement $L = K_pP$ was proposed, and the matrix $R = R^T$ was introduced, for which an additional constraint is satisfied

$$\begin{bmatrix} R & L \\ L^T & P \end{bmatrix} \geq 0. \quad (45)$$

Since, according to the Schur formula for $P > 0$, inequality (45) is equivalent to $R \geq LP^{-1}L^T = K_pPK_p^T$, then to satisfy inequality (44) it is sufficient that

$$\frac{1}{\alpha}(APA^T - BLA^T - AL^TB^T + BRB^T) - P + \frac{DD^T}{(1-\alpha)} \leq 0. \quad (46)$$

Minimization of criterion (41) under restrictions (45), (46) is performed for variables P , L , R using the semidefinite programming method, for example, by using SeDuMi Toolbox based on Matlab. Let $\hat{\alpha}$, $\hat{P}$, $\hat{L}$, $\hat{R}$ provide the minimum of (41) under restrictions (45), (46). Then the matrix $\hat{K}_p$ of the optimal controller (40) is found as

$$\hat{K}_p = \hat{L}\hat{P}^{-1}. \quad (47)$$

Thus, changes in unmeasured CM coordinates $\Delta\bar{y}(k)$ are taken into account as limited disturbances in model (34) for impulse CM processes with measured coordinates. These disturbances are suppressed by the controller (47).

In [19], a study of the impulse process control system in the cognitive map of an IT company was carried out, in which unmeasured coordinates are presented as disturbances for the controlled part of the CM. Compared to an uncontrolled process, when applying designed control actions (40), (47), the development time of IT projects was significantly reduced, the quality of the project was improved, and sales volumes increased.

6. The problem of controlling impulse processes in the CM of complex systems with multirate sampling of node coordinates

The work [22] describes a method for multirate sampling of the coordinates of the nodes of CM for the case when measuring all coordinates with one sampling period T_0 is impossible. Based on the fifth principle [3], it is assumed that the first part of the node coordinates $\bar{y}_1$ in model (17) is measured at discrete time with a sampling period T_0 , and the rest $\bar{y}_2$ with a period $h = mT_0$, where m is an integer greater than one. To control the impulse process in model (12), (13), control action vectors $\bar{u}_1$, $\bar{u}_2$ are introduced, which directly affect the nodes $\bar{y}_1$ and $\bar{y}_2$. In this case, the controlled model of the impulse process (12), (13) will have the form:

$$\begin{aligned} \Delta \bar{y}_1 \left[\left[\frac{k}{m} \right] h + (l+1)T_0 \right] &= W_{11} \Delta \bar{y}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right] + \\ &+ W_{12} \Delta \tilde{\bar{y}}_2 \left[\left[\frac{k}{m} \right] h \right] + B_{11} \Delta \bar{u}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right]; \end{aligned} \quad (48)$$

$$\Delta \bar{y}_2 \left[\left(\left[\frac{k}{m} \right] + 1 \right) h \right] = W_{21} \Delta \tilde{\bar{y}}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right] + W_{22} \Delta \bar{y}_2 \left[\left[\frac{k}{m} \right] h \right] + B_{22} \Delta \bar{u}_2 \left[\left[\frac{k}{m} \right] h \right];$$

where $l = 0, 1, \dots, (m-1)$, $\left[\frac{k}{m} \right]$ is the integer part of division k by m , B_{11} , B_{22} are diagonal matrices that are selected by the automation system developer. The dynamics of vectors $\bar{y}_1(k)$, $\bar{y}_2(k)$ in model (48) are presented in full values of the variables as follows:

$$\begin{aligned} \bar{y}_1 \left[\left[\frac{k}{m} \right] h + (l+1)T_0 \right] &= (I_{11} + W_{11} - W_{11}q_1^{-1}) \cdot \bar{y}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right] + \\ &+ B_{11} \Delta \bar{u}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right] + W_{12} \Delta \tilde{\bar{y}}_2 \left[\left[\frac{k}{m} \right] h \right]; \end{aligned} \quad (49)$$

$$\begin{aligned} \bar{y}_2 \left[\left(\left[\frac{k}{m} \right] + 1 \right) h \right] &= (I_{22} + W_{22} - W_{22}q_2^{-1}) \cdot \bar{y}_2 \left[\left[\frac{k}{m} \right] h \right] + \\ &+ B_{22} \Delta \bar{u}_2 \left[\left[\frac{k}{m} \right] h \right] + W_{21} \Delta \tilde{\bar{y}}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right], \end{aligned} \quad (50)$$

where q_1 , q_2 are the reverse shift operators for the sampling periods T_0 and $h = mT_0$, respectively. In equations (49), (50), the terms $W_{12} \Delta \tilde{\bar{y}}_2 \left[\left[\frac{k}{m} \right] h \right]$ and $W_{21} \Delta \tilde{\bar{y}}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right]$ are perturbations respectively. To create the first control vector $\Delta \bar{u}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right]$, which is implemented with a sampling period T_0 , the following optimality criterion is formulated in [22]:

$$J_1 \left[\left[\frac{k}{m} \right] h + (l+1)T_0 \right] = E \left\{ \left[\bar{y}_1 \left[\left[\frac{k}{m} \right] h + (l+1)T_0 \right] - \bar{G}_1 \right]^T \cdot \right. \\ \left. \cdot \left[\bar{y}_1 \left[\left[\frac{k}{m} \right] h + (l+1)T_0 \right] - \bar{G}_1 \right] + \Delta \bar{u}_1^T \left[\left[\frac{k}{m} \right] h + lT_0 \right] R_1 \Delta \bar{u}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right] \right\}, \quad (51)$$

where $\bar{G}_1$ is the set-point vector for controlling the coordinates of the nodes $\bar{y}_1$ of the CM. Based on the minimization of this criterion with respect to vector $\Delta \bar{u}_1$, taking into account (49), the equation of the first controller was written.

$$\frac{\partial J_1 \left[\left[\frac{k}{m} \right] h + (l+1)T_0 \right]}{\partial \Delta \bar{u}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right]} = 2B_{11}^T \left\{ (I_{11} + W_{11} - W_{11}q_1^{-1}) \cdot \bar{y}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right] + \right. \\ \left. + B_{11} \Delta \bar{u}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right] + W_{12} \Delta \bar{y}_2 \left[\left[\frac{k}{m} \right] h \right] - \bar{G}_1 \right\} + 2R_1 \Delta \bar{u}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right] = 0,$$

where the control law of the first controller is created

$$\Delta \bar{u}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right] = - (B_{11}^T B_{11} + R_1)^{-1} B_{11}^T \left\{ (I_{11} + W_{11} - W_{11}q_1^{-1}) \cdot \right. \\ \left. \cdot \bar{y}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right] + W_{12} \Delta \bar{y}_2 \left[\left[\frac{k}{m} \right] h \right] - \bar{G}_1 \right\}, \quad l = 0, 1, \dots, (m-1), \quad (52)$$

which, after expanding the difference $\Delta \bar{u}_1$, has the form

$$\bar{u}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right] = \bar{u}_1 \left[\left[\frac{k}{m} \right] h + (l-1)T_0 \right] - (B_{11}^T B_{11} + R_1)^{-1} B_{11}^T \times \\ \times \left\{ (I_{11} + W_{11} - W_{11}q_1^{-1}) \cdot \bar{y}_1 \left[\left[\frac{k}{m} \right] h + lT_0 \right] + W_{12} \Delta \bar{y}_2 \left[\left[\frac{k}{m} \right] h \right] - \bar{G}_1 \right\}.$$

To create the second control vector $\Delta \bar{u}_2 \left[\left[\frac{k}{m} \right] h \right]$, which is implemented with a period $h = mT_0$, the second optimality criterion is formulated

$$J_2 \left[\left(\left[\frac{k}{m} \right] + 1 \right) h \right] = E \left\{ \left[\bar{y}_2 \left[\left(\left[\frac{k}{m} \right] + 1 \right) h \right] - \bar{G}_2 \right]^T \cdot \right. \\ \left. \cdot \left[\bar{y}_2 \left[\left(\left[\frac{k}{m} \right] + 1 \right) h \right] - \bar{G}_2 \right] + \Delta \bar{u}_2^T \left[\left[\frac{k}{m} \right] h \right] R_2 \Delta \bar{u}_2 \left[\left[\frac{k}{m} \right] h \right] \right\}, \quad (53)$$

where $\bar{G}_2$ is the set-point vector for controlling the coordinates of the nodes of the CM $\bar{y}_2$. Based on the minimization of criterion (53) with respect to vector $\Delta \bar{u}_2$, taking into account (50), the equation of the second controller was created

$$\begin{aligned} \frac{\partial J_2 \left[\left(\left\lceil \frac{k}{m} \right\rceil + 1 \right) h \right]}{\partial \Delta \bar{u}_2 \left[\left\lceil \frac{k}{m} \right\rceil h \right]} &= 2B_{22}^T \left\{ (I_{22} + W_{22} - W_{22}q_2^{-1}) \cdot \bar{y}_2 \left[\left\lceil \frac{k}{m} \right\rceil h \right] + \right. \\ &+ B_{22}\Delta \bar{u}_2 \left[\left\lceil \frac{k}{m} \right\rceil h \right] + W_{21}\Delta \tilde{y}_1 \left[\left\lceil \frac{k}{m} \right\rceil h + lT_0 \right] - \bar{G}_2 \left. \right\} + 2R_2\Delta \bar{u}_2 \left[\left\lceil \frac{k}{m} \right\rceil h \right] = 0, \end{aligned}$$

where the control law of the second controller is found

$$\begin{aligned} \bar{u}_2 \left[\left\lceil \frac{k}{m} \right\rceil h \right] &= \bar{u}_2 \left[\left(\left\lceil \frac{k}{m} \right\rceil - 1 \right) h \right] - (B_{22}^T B_{22} + R_2)^{-1} B_{22}^T \cdot \\ &\cdot \left\{ (I_{22} + W_{22} - W_{22}q_2^{-1}) \cdot \bar{y}_2 \left[\left\lceil \frac{k}{m} \right\rceil h \right] + W_{21}\Delta \tilde{y}_1 \left[\left\lceil \frac{k}{m} \right\rceil h + lT_0 \right] - \bar{G}_2 \right\}. \end{aligned}$$

In the impulse process model (50), in which the vector $\bar{y}_2$ is considered at discrete times with a period h , frequently measured coordinates $\Delta \tilde{y}_1 \left[\left\lceil \frac{k}{m} \right\rceil h + lT_0 \right]$ based on statement 1 are taken into account as follows

$$\Delta \tilde{y}_1 \left[\left\lceil \frac{k}{m} \right\rceil h + lT_0 \right] = \bar{y}_1 \left[\left\lceil \frac{k}{m} \right\rceil h + (m-1)T_0 \right] - \bar{y}_1 \left[\left\lceil \frac{k}{m} \right\rceil h - T_0 \right].$$

In [23], a new principle for controlling impulse processes in the CM of complex systems is considered based on varying the coordinates of the nodes and the weight coefficients of the CM when forming control actions in closed-loop control systems. In this case, the coordinates of the nodes of the CM and the weight coefficients of the edges change in discrete time with multirate sampling.

In [24], a cognitive map of cause-and-effect relationships was developed in the process of the spread of population morbidity due to Covid-19 in a specific region, models of impulse processes of CM subsystems with multirate sampling were developed, and combined control subsystems with multirate sampling were designed based on the formation of control actions by varying node coordinates and CM weight coefficients. Experimental studies of control systems for impulse processes in the CM of morbidity for Covid-19 have been carried out.

7. Problems of identifying models of impulse processes in cognitive maps of complex systems

7.1. Identification in CM in the impulse processes mode with complete information. The dynamics of a complex system changes widely during its operation because of crisis phenomena, emergence of non-standard and conflict situations, human factor, changes in social and political relationships, etc. If difference equations of impulse processes CM (2) are used as a mathematical model of a complex system, then the weighting coefficients a_{ij} must be periodically assessed based on identification methods.

The identification problem is performed during the impulse CM process, when all coordinates of the CM nodes are in a transition mode or under the influence of external testing influences. For this purpose, the equation of the impulse process (2) is written in the form

$$\Delta y_i(k+1) = \sum_{j=1}^n a_{ij} [\Delta y_j(k) + \Delta q_j(k)], \quad (54)$$

where $y_i(k)$ is the coordinate value of the i -th node of the CM at time $t = kT_0$, $\Delta y_i(k) = y_i(k) - y_i(k-1)$ is the magnitude of the impulse at the i -th node, $i = 1, 2, \dots, n$; $q_j(k)$ is generated known testing influence on the j -th node during identification. The impulse process (54) for the entire CM is written in vector-matrix form

$$\Delta \bar{y}(k+1) = A[\Delta \bar{y}(k) + \Delta \bar{Q}(k)], \quad (55)$$

where A is the weight adjacency matrix of the CM, $\Delta \bar{y}(k)$ is the vector of increments of coordinates of the CM nodes, $\Delta \bar{Q}(k)$ is the vector of increments of testing influences according to a given program.

In [6, 25], to estimate variable matrix A of coefficients a_{ij} , the recurrent least squares method (RLSM) was used, which gives unbiased estimates provided that the disturbances in (54) are discrete white noise. This condition is not always met during the impulse CM process. When the structure of the CM changes, which is expressed in the appearance of new weighting coefficients a_{ij} that were not previously established by experts, the RLSM does not evaluate these coefficients at all, since they will be absent in the RLSM algorithm.

The works [26, 27] consider various options for solving the problem of parametric identification of the coefficients of the CM adjacency matrix, when all CM nodes coordinates are measured.

Parametric identification of adjacency matrix in the absence of disturbances.

The equation of the impulse process (55) for moments in time $k = 1, 2, \dots, N-1$ can be represented as a sequence of systems of equations

$$\begin{aligned} \Delta \bar{y}(1) &= A[\Delta \bar{y}(0) + \Delta \bar{Q}(0)], \\ \Delta \bar{y}(2) &= A[\Delta \bar{y}(1) + \Delta \bar{Q}(1)], \\ &\dots\dots\dots \\ \Delta \bar{y}(N) &= A[\Delta \bar{y}(N-1) + \Delta \bar{Q}(N-1)]. \end{aligned} \quad (56)$$

In the deterministic case, based on (56), the problem of determining the matrix A from measurement data is solved when there is no noise, i.e. sequences of vectors $\{\Delta \bar{y}(k)\}$ are specified exactly. In this case, the impacts $\{\Delta \bar{Q}(k)\}$ are formed according to the program.

Then system (56) can be written in a unified form

$$\Delta \bar{y}_i = [\Delta Y + \Delta \bar{Q}] \bar{a}_i, \quad i = 1, 2, \dots, n, \quad (57)$$

where $\Delta \bar{y}_i$ is the vector representing the dynamics of the i -th node of the CM from the 1st to the n -th sampling period

$$\Delta \bar{y}_i^T = [\Delta y_i(1) \ \Delta y_i(2) \ \dots \ \Delta y_i(n)], \quad (58)$$

and $\bar{a}_i^T = (a_{i_1} \ a_{i_2} \ \dots \ a_{i_n})$ is a vector containing the elements of the i -th row of the matrix A . The matrix ΔY is composed of discrete measurements of the coordinates of the nodes of the CM from the zero to the $(n-1)$ -th sampling periods as follows:

$$\Delta Y = \begin{bmatrix} \Delta y_1(0) & \Delta y_2(0) & \dots & \Delta y_n(0) \\ \Delta y_1(1) & \Delta y_2(1) & \dots & \Delta y_n(1) \\ \vdots & \vdots & & \vdots \\ \Delta y_1(n-1) & \Delta y_2(n-1) & \dots & \Delta y_n(n-1) \end{bmatrix}. \quad (59)$$

The matrix of known testing influences is formed according to

$$\Delta Q = \begin{bmatrix} \Delta q_1(0) & \Delta q_2(0) & \dots & \Delta q_n(0) \\ \Delta q_1(1) & \Delta q_2(1) & \dots & \Delta q_n(1) \\ \vdots & \vdots & & \vdots \\ \Delta q_1(n-1) & \Delta q_2(n-1) & \dots & \Delta q_n(n-1) \end{bmatrix}. \quad (60)$$

Thus, according to (57), the weighting coefficients of the adjacency matrix A are determined based on

$$\bar{a}_i = (\Delta Y + \Delta Q)^{-1} \Delta \bar{y}_i, \quad i = 1, 2, \dots, n. \quad (61)$$

Solvability of (61) is ensured when $\det(\Delta Y + \Delta Q) \neq 0$. When implementing (61), it is assumed that the matrix A elements remain constant over the time interval $k = 1, 2, \dots, n$.

When forming sequences of testing influences q_i , $i = 1, 2, \dots, n$ it is necessary to take into account that not all nodes of the CM can be varied according to the program established by the decision maker. In this case, some Δq_i in equation (57) will be equal to zero and will not be taken into account when estimating coefficients $\bar{a}_i$ based on (61). So during identification the corresponding coordinates y_i will change only when influenced through weighting coefficients a_{ij} when varying the coordinates y_j to which testing influences q_j will be applied.

Identification when there is noise in the measured data. In this case, instead of the exact values of the matrix ΔY and vector $\Delta \bar{y}_i$, there is information about the approximate values of the matrix $\Delta \tilde{Y}$ and vector $\Delta \tilde{\bar{y}}_i$. Therefore, when solving (61), an approximate estimate of the coefficients $\hat{\bar{a}}_i$ is obtained. When carrying out identification, it is very important to know how approximate information differs from the exact values. It should be noted that the estimate $\hat{\bar{a}}_i$ significantly depends on the condition number of the summary matrix $(\Delta Y + \Delta Q)$. If this matrix is ill-conditioned or close to degenerate, then even small disturbances or inaccuracies in measuring the coordinates of the nodes of the CM can lead to the identification problem becoming incorrectly posed. Poor conditioning depends on two factors: the initial state $\Delta \bar{y}(0)$, namely the signal level of each of the modes of the system, and the parameters of the system generating the data. In this case, the condition number, even with equal excitation of all modes of the CM, grows rapidly with increasing dimension of the CM. Therefore, for large n , it is necessary to evaluate the deviation of the resulting approximate solution from the exact one.

Solving the problem using the combinatorial method. Measuring the coordinates of the CM nodes in the presence of noise is specified in the form

$$\Delta \tilde{y}_i = \Delta \bar{y}_i + \bar{\xi}_i, \quad \|\bar{\xi}_i\| \leq \varepsilon, \quad (62)$$

where $\Delta \bar{y}_i$ is the exact increment of the coordinate vector (58), $\bar{\xi}_i$ is the vector of errors in measuring the i -th coordinate at discrete moments in time over the observation interval $k=1, 2, \dots, n$, and ε is a fairly small value.

Consider a system of linear quadratic equations

$$\Delta \tilde{y}_i = \Delta \tilde{Y}_1 + \hat{a}_i, \quad (63)$$

where $\Delta \tilde{Y}_1 = \Delta \tilde{Y} + \Delta Q$. In system (63), the matrix $\Delta \tilde{Y}$ and vector $\Delta \tilde{y}_i$ are specified with an error, and

$$\Delta \tilde{y}_i - \bar{\xi}_i = (\Delta Y_1 + \Xi) \bar{a}_i \quad (64)$$

corresponds to an exact equation in which Ξ is an additive matrix with elements that are the measurement errors of the coordinates of the nodes of the CM, taken with a minus sign. Then, to obtain a component-wise estimate of the membership intervals of exact values, the following theorem is applied [28].

Theorem 2. *Let an approximate non-degenerate system (63) and an exact system (64) be given such that*

$$\|\Xi\|_\infty \leq \|\delta \Delta \tilde{Y}_1\|_\infty, \quad \|\bar{\xi}_i\|_\infty \leq \delta \|\Delta \tilde{y}_i\|_\infty. \quad (65)$$

If the condition number $\rho_\infty(\Delta \tilde{Y}_1)$ satisfies the condition

$$\delta \rho_\infty(\Delta \tilde{Y}_1) = r < 1, \quad (66)$$

then $\Delta Y_1 + \Xi$ it is also non-degenerate and

$$\|\hat{a}_i - \bar{a}_i\|_\infty \leq \frac{2\delta}{1-r} \|\Delta \tilde{Y}_1^{-1} \cdot \|\Delta \tilde{Y}_1\|_\infty \cdot \|\hat{a}_i\|_\infty. \quad (67)$$

The value $\rho_\infty(\Delta \tilde{Y}_1) = \|\Delta \tilde{Y}_1\|_\infty \|\Delta \tilde{Y}_1^{-1}\|_\infty$ in (66) is the condition number of the matrix $\Delta \tilde{Y}_1$ according to the « ∞ » norm, and $\|\Delta \tilde{Y}_1^{-1} \cdot \|\Delta \tilde{Y}_1\|_\infty$ in (66) it is also called the Shkeel condition number. In this theorem, specific realizations Ξ and $\bar{\xi}_i$ are unknown, and, therefore, it is not possible to determine δ and r from (65), (66). However, if we focus on the most unfavorable realization, the probability of which is small, then, based on (65), we can obtain a guaranteed membership interval of the exact value for any realizations that satisfy (62). With this realization we have

$$\Xi = \varepsilon \Xi_1, \quad \bar{\xi}_i = \varepsilon \bar{e}, \quad (68)$$

where Ξ_1 is a matrix in which all elements are equal to one, and $\bar{e}$ is a vector with all unit elements. Then

$$\|\Xi\|_\infty = n\varepsilon, \quad \|\bar{\xi}_i\|_\infty = \varepsilon. \quad (69)$$

The obtained result allows us to apply a combinatorial method for solving the identification problem using approximate data, which is implemented when condition (66) of the above theorem is satisfied. Therefore, its feasibility is first checked. To do this, find the smallest value δ at which the loose inequalities are satisfied

$$\varepsilon \leq \delta \frac{\|\Delta \tilde{Y}\|_\infty}{n}, \quad \varepsilon \leq \delta \|\Delta \tilde{y}_i\|_\infty. \quad (70)$$

After this, the feasibility of strict inequality (66) is checked. In this case, it should be expected that if the matrix $\Delta\tilde{Y}$ is poorly conditioned, the value r at the smallest δ will not be less than one. This can actually mean a large spread of solutions obtained based on quadratic equations formed from (56), that is, the solution method based on a guaranteed result cannot be implemented. In this case, when (66) is satisfied for the set of quadratic systems formed from (56), the combinatorial method will be effective.

Its description is given in [29]. The advantage of the method is the successful combination of the properties of statistics with guaranteed interval estimation. This makes it possible, based on multiple estimates and with certain properties of statistics, to significantly reduce the guaranteed membership intervals of the exact values of the vector components $\bar{a}_i$.

The solution algorithm using the combinatorial method is as follows:

- from the overdetermined system (56) we create a set of square equations, discarding unnecessary equations in an arbitrary manner. There will be a total C_N^n of such combinations;
- we leave only those systems for which condition (66) is satisfied, that is, non-degenerate and well conditioned systems. Let such systems remain S ;
- we solve the remaining quadratic systems and find approximate estimates $\hat{a}_{ij}$ of the parameters $\bar{a}_{ij}$.

Each s -th square system $s = 1, 2, \dots, S$ according to (67) is characterized by a guaranteed component-wise error

$$\varepsilon_{is} = \frac{2\delta}{1-r} \left\| \left| \Delta\tilde{Y}_1^{-1} \right| \cdot \left| \Delta\tilde{Y}_1 \right| \right\|_{\infty} \cdot \left\| \hat{a}_i \right\|_{\infty}.$$

Inequality (67) in this case allows us to write an element-wise estimate for $\bar{a}_{ij}$ in the form $\left| \hat{a}_{ij}^s - \bar{a}_{ij}^s \right| \leq \varepsilon_{is}$, or

$$\hat{a}_{ij}^s - \varepsilon_{is} \leq \bar{a}_{ij} \leq \hat{a}_{ij}^s + \varepsilon_{is}, \quad (71)$$

For different s and fixed i, j (71) gives a system of guaranteed estimates, from which it follows that the exact value must belong to the truncated smallest interval

$$\max_s (\hat{a}_{ij}^s - \varepsilon_{is}) \leq \bar{a}_{ij} \leq \min_s (\hat{a}_{ij}^s + \varepsilon_{is}). \quad (72)$$

From (72) it follows that with increasing S and spread of realizations $\hat{a}_{ij}^s$, the accuracy of estimation should improve. It is advisable to take the middle of the interval $[\max_s (\hat{a}_{ij}^s - \varepsilon_{is}); \min_s (\hat{a}_{ij}^s + \varepsilon_{is})]$ as an estimate $\bar{a}_{ij}$ integrated over S , and half of its width will characterize the accuracy of the estimate.

Solving the problem using least squares method. The combinatorial method is not applicable when condition (66) is not satisfied, since estimate (67) will not be fair. In this case, it is proposed to use ordinary or weighted OLS. The sequence of equations (56) allows us to construct an overdetermined system of equations (63) with the same desired vector $\hat{a}_i$, in which $\Delta\tilde{y}_i = [\Delta\tilde{y}_i(1) \ \Delta\tilde{y}_i(2) \ \dots \ \Delta\tilde{y}_i(N)]^T$,

$$\Delta\tilde{Y} = \begin{bmatrix} \Delta\tilde{y}_1(0) & \Delta\tilde{y}_2(0) & \dots & \Delta\tilde{y}_n(0) \\ \Delta\tilde{y}_1(1) & \Delta\tilde{y}_2(1) & \dots & \Delta\tilde{y}_n(1) \\ \vdots & \vdots & & \vdots \\ \Delta\tilde{y}_1(N-1) & \Delta\tilde{y}_2(N-1) & \dots & \Delta\tilde{y}_n(N-1) \end{bmatrix},$$

$$\Delta Q = \begin{bmatrix} \Delta q_1(0) & \Delta q_2(0) & \dots & \Delta q_n(0) \\ \Delta q_1(1) & \Delta q_2(1) & \dots & \Delta q_n(1) \\ \vdots & \vdots & & \vdots \\ \Delta q_1(N-1) & \Delta q_2(N-1) & \dots & \Delta q_n(N-1) \end{bmatrix}.$$

Here it is assumed that the input impulses $\Delta q_i(k)$ are known exactly. However, all the results that will be obtained below remain valid even if instead ΔQ we have an approximate matrix $\Delta\tilde{Q}$.

If we use the least squares method to solve an overdetermined system of linear algebraic equations with an approximate right-hand side $\Delta\tilde{y}_i$ and matrices $\Delta\tilde{Y}$ and ΔQ , then the vector estimate $\hat{a}_i$ is determined as

$$\hat{a}_i = [(\Delta\tilde{Y} + \Delta Q)^T (\Delta\tilde{Y} + \Delta Q)]^{-1} (\Delta\tilde{Y} + \Delta Q)^T \Delta\tilde{y}_i. \quad (73)$$

When using a weighted least squares method, in which the weighting coefficients are specified by the elements of the matrix D (diagonal, dimension K), formula (73) takes the form

$$\hat{a}_i = [(\Delta\tilde{Y} + \Delta Q)^T D (\Delta\tilde{Y} + \Delta Q)]^{-1} (\Delta\tilde{Y} + \Delta Q)^T D \Delta\tilde{y}_i. \quad (74)$$

Both these methods are most effective when the matrix ΔQ is formed from permanently exciting input impulses [30]. Regarding the problem under consideration, they are defined as follows. Let us have a sequence of input impulses $\Delta\bar{Q}$, presented in the form

$$\Delta\bar{Q}^T = [\Delta q_1(0) \dots \Delta q_n(0) \Delta q_1(1) \dots \Delta q_n(1) \dots \Delta q_1(N-1) \dots \Delta q_n(N-1)].$$

A sequence $\Delta\bar{Q}$ is considered permanently exciting if

$$\text{rank}(\Delta\bar{Q}\Delta\bar{Q}^T) = nN. \quad (75)$$

When the signal $\Delta\bar{Q}$ is stationary zero-mean white noise, the sequence $\Delta\bar{Q}(j)$ has the statistical property

$$E \begin{bmatrix} \Delta\bar{Q}(j) \\ \Delta\bar{Q}(j+1) \\ \dots \\ \Delta\bar{Q}(j+k-1) \end{bmatrix} [\Delta\bar{Q}^T(j) \Delta\bar{Q}^T(j+1) \dots \Delta\bar{Q}^T(j+k-1)] = \sigma_n^2 \begin{bmatrix} I_n & 0 & \dots & 0 \\ 0 & I_n & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & I_n \end{bmatrix},$$

where $\Delta\bar{Q}(j) = [\Delta q_1(j) \dots \Delta q_n(j)]^T$, E is the mathematical expectation of a stationary random sequence, and σ_n^2 is the variance.

Regularized solution. Under certain conditions, the identification problem may be significantly incorrectly formulated. This happens when n is large and the information matrix $\Delta\tilde{Y} + \Delta Q$ is ill-conditioned. This leads to the solution becoming sensitive to errors in the input data. In such cases, it is advisable to use additional information about the desired solution and, on its basis, introduce a regularization procedure into the solution algorithm, which makes it possible to find stable solutions with respect to error variations. Additional information about the values of the adjacency matrix, that is, about the connections between the nodes of the CM, can be very effective. The absence of certain connections means that in the matrix A coefficients a_{ij} are equal to zero. This leads to a decrease in the dimension of the vector $\bar{a}_i$ and the dimension of the information matrices in (73). If this is not enough to obtain a stable solution and in the absence of other additional information, one can use Tikhonov's regularization method [31] as applied to solving ill-conditioned systems of linear algebraic equations with inaccurately specified right-hand side and the main information matrix.

A stabilizer is introduced:

$$\Omega(\bar{a}_i) = \|\bar{a}_i\|_2^2, \quad (76)$$

in which $\|\cdot\|$ is the Euclidean norm. In accordance with [31], the smoothing functional is considered

$$M^a(\bar{a}_i, \Delta\tilde{Y}_i, \Delta\tilde{Y} + \Delta Q) = \|(\Delta\tilde{Y} + \Delta Q)\bar{a}_i - \Delta\tilde{Y}_i\|_2^2 + \alpha\|\bar{a}_i\|_2^2. \quad (77)$$

A decreasing sequence $\{\alpha_j\}$ is suggested, for example, geometrically decreasing. After this, a sequence of problems is solved for these α_j . The regularized solution will be at the smallest value α_j , which is found from the residual principle. According to this principle, the minimum value α_j is determined from the inequality

$$\|(\Delta\tilde{Y} + \Delta Q)\bar{a}_i - \Delta\tilde{Y}_i\|_2 \leq \varepsilon(\|\Xi_1\|_2 + \|\bar{e}\|_2) \quad (78)$$

if for α_{j-1} expression (78) is not satisfied. The element $\hat{\bar{a}}_i$ that provides the minimum (77) will be the regularized value of the parametric identification problem, that is, a solution robust to errors in the data.

7.2. Identification in CM in the impulse processes mode with incomplete measurement of node coordinates [32]. It is assumed that some of the measured nodes of the CM can be directly influenced by the decision maker by applying external impulses (test signals or controls) to them. Then the equation of forced motion (4) is written in the form

$$\Delta\bar{y}(k+1) = A\Delta\bar{y}(k) + B\bar{u}(k). \quad (79)$$

Let the vector $\bar{u}$ have dimension r , then the matrix $B(n \times r)$ is composed of zeros and ones: $B = \begin{bmatrix} I_r \\ 0 \end{bmatrix}$, where I_r is the unit matrix of dimension r .

It is assumed that some of the coordinates y_i of the nodes are measurable. Then the measured subvector $\bar{z}$ (dimension p) can be described in the form of a measurement equation

$$\Delta\bar{z}(k) = C\Delta\bar{y}(k) + \bar{\xi}(k), \quad (80)$$

where the matrix $C(p \times n)$ is obviously composed of zeros and ones $C = [I_p \ 0]$, where I_p is the unit matrix of size p .

Thus, equations (79), (80) is a system of equations for a discrete dynamic system in state space.

Identification of CM using the 4SID method. To identify systems in the subspace state space, the so-called 4SID methods [30] are most often used — subspace state space system identification methods. Let us adapt these methods to the problem at hand.

For identification the inputs and outputs of the system are observed over a time interval $N_1 + N_2$ of sampling periods. The values N_1 , N_2 are chosen so that $N_2 \geq N_1(r + p)$. Then, Hankel matrices are composed of input and output values on the considered interval:

$$U_{1,N_1} = \begin{bmatrix} \Delta \bar{u}(0) & \Delta \bar{u}(1) & \dots & \Delta \bar{u}(N_2 - 1) \\ \Delta \bar{u}(1) & \Delta \bar{u}(2) & \dots & \Delta \bar{u}(N_2) \\ \vdots & \vdots & \ddots & \vdots \\ \Delta \bar{u}(N_1 - 1) & \Delta \bar{u}(N_1) & \dots & \Delta \bar{u}(N_2 + N_1 - 2) \end{bmatrix}, \quad (81)$$

$$Y_{1,N_1} = \begin{bmatrix} \Delta \bar{z}(0) & \Delta \bar{z}(1) & \dots & \Delta \bar{z}(N_2 - 1) \\ \Delta \bar{z}(1) & \Delta \bar{z}(2) & \dots & \Delta \bar{z}(N_2) \\ \vdots & \vdots & \ddots & \vdots \\ \Delta \bar{z}(N_1 - 1) & \Delta \bar{z}(N_1) & \dots & \Delta \bar{z}(N_2 + N_1 - 2) \end{bmatrix}. \quad (82)$$

Test signals $\Delta \bar{u}$ are selected so that the matrix U_{1,N_1} is full-rank and well-conditioned. It is also advisable to choose N_2 so that the composite matrix of inputs and outputs $\begin{bmatrix} U_{1,N_1} \\ Y_{1,N_1} \end{bmatrix}$ does not differ much from the square one.

Quite important is the question of the nature of the input (test) influences. It is known that informative signals of infinite order are signals such as white noise or δ -functions. Therefore, white noise is used as input testing signals.

An RQ-decomposition of a composite matrix $\begin{bmatrix} U_{1,N_1} \\ Y_{1,N_1} \end{bmatrix}$ is carried out with a block representation of the decomposition in the form

$$\begin{bmatrix} U_{1,N_1} \\ Y_{1,N_1} \end{bmatrix} = \begin{bmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}.$$

For the block R_{22} , its SVD (Singular Value Decomposition) is calculated, that is

$$R_{22} = U \Sigma V^T, \quad (83)$$

where Σ is a square diagonal matrix consisting of singular numbers R_{22} on the main diagonal.

The decomposition matrix (83) is represented in the following block form

$$R_{22} = [U_1 \mid U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}$$

where the matrix U_1 has dimension $N_1 p \times n$, and matrix U_2 — $N_1 p \times (N_1 p - n)$, matrix Σ_1 — $(n \times n)$, Σ_2 — $(N_1 p - n) \times (N_1 p - n)$.

According to the realizations theory, some realization (A', B', C') is found for system (79), (80), without immediately trying to find the original system (A, B, C) . In this case, the matrix A' is found from an overdetermined system of matrix equations

$$U_1^{(1)} A' = U_1^{(2)}, \quad (84)$$

where matrix $U_1^{(1)}$ is a submatrix of the matrix U_1 in which the last p rows are crossed out, and matrix $U_1^{(2)}$ is a submatrix of the matrix U_n in which the first p rows are crossed out. When obtaining (84), the property of shift invariance is used. In this case, the matrix U_1 is considered as an observability matrix for some realization, which follows from the realizations theory. Then the matrix C' for the same realization will be a matrix consisting of the first p rows of the matrix U_1 .

To find B' , an auxiliary matrix $\Xi = U_2^T R_{21} R_{11}^{-1}$ is introduced (test signals must be selected so that R_{11} is invertible) and the following equation is solved

$$\begin{bmatrix} U_2^T(1:p) & U_2^T(p+1:2p) & \dots & U_2^T(p(N_2-1)+1:pN_2) \\ U_2^T(p+1:2p) & U_2^T(2p+1:3p) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ U_2^T(p(N_1-1)+1:pN_1) & 0 & \dots & 0 \end{bmatrix} \times$$

$$\times \begin{bmatrix} I_p & 0 \\ 0 & U_1^{(1)} \end{bmatrix} \times \begin{bmatrix} 0 \\ B' \end{bmatrix} = \begin{bmatrix} \Xi(1:r) \\ \Xi(r+1:2r) \\ \vdots \\ \Xi(r(N_1-1)+1:rN_1) \end{bmatrix},$$

where for all matrices $D(a:b)$ denote the submatrix of the matrix D from the a -th to the b -th columns. Thus, the system has been identified. However, for the practical use of the results, it is necessary to go to the original realization, that is, the one in which the matrices B, C have the known form $B = \begin{bmatrix} I_r \\ 0 \end{bmatrix}$, $C = [I_p \ 0]$.

To do this, equations from the realizations theory are used, which relate different realizations to each other using a non-singular transformation matrix T :

$$A = T^{-1} A' T, \quad (85)$$

$$B' = TB, \quad C = C' T. \quad (86)$$

To find the matrix T connecting the original realization (A, B, C) and the resulting realization (A', B', C') , it is necessary to solve the system of equations (86), in which the matrix T (dimension $(n \times n)$) is unknown. The system will be underdetermined if the number of unknowns is greater than the number of equations, that is if $n^2 > n + p$, which is almost always true. Thus, it is enough to find any of the set of its solutions to obtain T , and then find A from (85) in the original realization.

To assess the quality of identification of coefficients of the adjacency matrix CM, comparison of the original and resulting matrices A is not always sensible, since the 4SID method ensures equivalence of the original and identified systems by output, that is, to any identical input influence both models provide the same response; but the method does not guarantee the identity of the realizations of the state matrices. Therefore, it is advisable to compare the original and identified systems on the basis of some invariants that do not depend on the specific realization.

From (85) it is obvious that the most natural invariants are the eigenvalues, which are identical for the pair A, A' .

The works [33, 34] consider the problem of structural-parametric identification of a complex multidimensional discrete system in the class of state-space models. It is assumed that only the input and output coordinates of the system over a certain time interval and the measurement error range are known. The selected subspace method is used as a basis, which assumes that the dimension of the system (state vector) is known. However, this is not always true in practice, especially when developing cognitive maps of complex systems. In addition, due to the dependence on the noise level, it is impossible to identify correctly a high-dimensional system. Therefore, in [33] it was proposed to consider dimension as a regularizing parameter. Three methods have been developed for selecting the approximate dimension of the model depending on the length of the observation interval and the possibility of an active experiment. The proposed methods are developed using the example of problems of identifying a cognitive map of a commercial bank in an impulse process.

Conclusion

The problem of controlling the dynamics of complex systems of various nature based on methods of automatic control theory developed for technical objects is a relevant problem. There are works where cognitive maps are used as models of complex systems, and control is implemented using fuzzy controllers.

This article provides a review and generalization of the principles and main problems of a new direction in the control of complex systems based on modified mathematical models of impulse processes in cognitive maps (Roberts difference equations), which were developed to describe the dynamics of the free motion of complex systems in discrete time.

The novelty of this direction lies in the modernization of models of CM impulse processes by introducing a control vector based on the external control actions, which are generated by a discrete controller and implemented by varying the coordinate resources of the nodes or changing the weighting coefficients of the CM in closed-loop control systems. The article discusses the main problems of designing discrete controllers for control of complex systems: stabilization of unstable processes, coordinating control of the ratios of the CM nodes coordinates, suppression of external disturbances with unknown probabilistic characteristics affecting the coordinates of a complex system, control of complex systems with multirate sampling of various output controlled variables, identification CM weighting coefficients. In this case, these problems are solved for two options, namely with fully measured coordinates of the CM nodes and

with incomplete measurement. To design discrete controllers, well-known methods of control theory are used: modal control, linear-quadratic control, linear matrix inequalities technique, etc.

The main difficulty in implementing control systems for complex systems based on models of CM impulse processes is the organization of the execution of control actions associated with active intervention of the decision maker into different processes of a complex system, by means of varying the resources of the CM nodes coordinates or by changing the degree of effect of some CM nodes on others at discrete time moments.

В.Д. Романенко, В.Ф. Губарев, Ю.Л. Мілявський

**ЕТАПИ ТА ОСНОВНІ ЗАДАЧІ
СТОЛІТНЬОГО РОЗВИТКУ ТЕОРІЇ СИСТЕМ
КЕРУВАННЯ ТА ІДЕНТИФІКАЦІЇ.
Частина 5. ПРИНЦИПИ І ЗАДАЧІ КЕРУВАННЯ
ТА ІДЕНТИФІКАЦІЇ В СКЛАДНИХ СИСТЕМАХ
РІЗНОЇ ПРИРОДИ НА ОСНОВІ МОДЕЛЕЙ
ІМПУЛЬСНИХ ПРОЦЕСІВ КОГНІТИВНИХ КАРТ**

Романенко Віктор Демидович

Національний технічний університет України «Київський політехнічний інститут імені Ігоря Сікорського», Навчально-науковий інститут прикладного системного аналізу, м. Київ,

ipisa@kpi.ua; romanenko.viktorroman@gmail.com

Губарев Вячеслав Федорович

Інститут космічних досліджень НАН України та ДКА України, м. Київ,

v.f.gubarev@gmail.com

Мілявський Юрій Леонідович

Національний технічний університет України «Київський політехнічний інститут імені Ігоря Сікорського», Навчально-науковий інститут прикладного системного аналізу, м. Київ,

yuriy.milyavsky@gmail.com

Виконано огляд і узагальнення принципів, методів та задач проектування дискретних регуляторів для керування складними системами різної природи, динаміка яких описана за допомогою різницевих рівнянь імпульсних процесів когнітивних карт (рівнянь Робертса). В ці рівняння введено вектор керування, який реалізується на основі варіювання координат вершин або вагових коефіцієнтів когнітивних карт (КК), сформованого дискретними регуляторами, які проєктуються на основі відомих методів теорії керування. В статті наведено розв'язок наступних задач керування імпульсними процесами в КК складних систем: стабілізація нестійких імпульсних процесів в КК складних систем на основі еталонних моделей замкнених систем керування та на основі методу модального керування; керування імпульсним процесом КК на основі варіювання ваговими коефіцієнтами; реалізація координувального управління співвідношеннями координат вершин КК складних систем; приглушення зовнішніх збурень при керуванні складними системами на основі методу інваріантних еліпсоїдів; керування імпульсними процесами в КК складних систем з різнотемповою дискретизацією координат вершин КК; ідентифікація вагових коефіцієнтів матриці суміжності

в моделі імпульсних процесів КК при повному і неповному вимірюванні координат вершин КК. Розв'язування наведених задач виконується на основі описаних нових принципів керування імпульсними процесами в КК складних систем на основі методів теорії автоматичного керування.

Ключові слова: когнітивні карти, імпульсні процеси, еталонні моделі, інваріантні еліпсоїди, ідентифікація, координуюче керування, різнометрова дискретизація.

REFERENCES

1. Axelrod R. The structure of decision cognitive maps of political elites. Princeton University Press, 1976. 404 p.
2. Roberts F.S. Discrete mathematical models with applications to social biological and environmental problems. Prentice-Hall, Inc. Englewood Cliffs, New Jersey, 1976.
3. Zgurowsky M.Z., Romanenko V.D., Milyavskiy Yu.L. Principles and methods of impulse processes control in cognitive maps of complex systems. Part 1. *Journal of Automation and Information Sciences*. 2016. Vol. 48, N 3. P. 36–45. DOI: <https://doi.org/10.1615/JAutomatInfScien.v48.i3.40>
4. Романенко В.Д., Мильявський Ю.Л. Обеспечение устойчивости импульсных процессов в когнитивных картах на основе моделей в пространстве состояний. *Системні дослідження та інформаційні технології*. 2014. № 1. С. 26–42. DOI: <https://ela.kpi.ua/handle/123456789/10267>
5. Шевченко А.М. Методы расчета и адаптации частоты выдачи решений в управляющей ЦВМ. *Автоматика и телемеханика*. 1975. № 7. С. 143–153.
6. Romanenko V.D., Milyavskiy Y.L., Reutov A.A. Adaptive control method for unstable impulse processes in cognitive maps based on reference model. *Journal of Automation and Information Sciences*. 2015. Vol. 47, N 3. P. 11–23. DOI: <https://doi.org/10.1615/JAutomatInfScien.v47.i3.20>
7. Романенко В.Д., Мильявський Ю.Л. Стабилизация импульсных процессов в когнитивных картах сложных систем на основе модальных регуляторов состояния. *Кибернетика и вычислительная техника*. 2015. Вып. 179. С. 43–55.
8. Методы классической и современной теории автоматического управления. Учебник в 3-х томах. Том 2: Синтез регуляторов и теория оптимизации систем автоматического управления / Под ред. Н.Д. Егупова. М. : Изд-во МГТУ имени Н.Э. Баумана, 2000. 736 с.
9. Романенко В.Д., Мильявський Ю.Л. Когнітивне моделювання динаміки прийняття рішень для стабілізації нестійких режимів у соціально-навчальному процесі студента. *Наукові вісті НТУУ «КПІ»*. 2016. № 5. С. 48–53.
10. Романенко В.Д., Мильявський Ю.Л., Канцедал Г.О. Адаптивная система стабилизации неустойчивого курса криптовалюты на основе модели импульсного процесса когнитивной карты. *Международный научно-технический журнал «Проблемы управления и информатики»*. 2021. № 2. С. 11–23. DOI: <https://doi.org/10.34229/1028-0979-2021-2-2>
11. Romanenko V., Milyavsky Y.L. Control method in cognitive maps based on weights increments. *Кибернетика и вычислительная техника*. 2016. Вып. 184. С. 44–55.
12. Romanenko V.D., Milyavsky Yu.L. Control automation method in cognitive maps based on the synthesis of increments of weighting coefficients and nodes coordinates. *System Research and Information Technologies*. 2019. N 3. P. 89–99. DOI: <https://doi.org/10.20535/SRIT.2308-8893.2019.3.08>
13. Бойчук Л.М. Синтез координирующих систем автоматического управления. М. : Энергоатомиздат, 1991. 160 с.
14. Романенко В.Д., Мильявський Ю.Л. Координуюче керування імпульсним процесом когнітивної карти у стохастичному середовищі. *Міжнародний науково-технічний журнал Проблеми керування та інформатики*. 2022. № 4. С. 49–58. DOI: <https://doi.org/10.34229/2786-6505-2022-4-4>
15. Romanenko V.D., Milyavsky Yu.L. Coordinates ratio control for a cognitive model of a complex system under an unstable impulse process. *System Research and Information Technologies*. 2015. N 1. P. 121–129.
16. Сухарев А.Г., Тимохов А.В., Федоров В.В. Курс методов оптимизации. М. : Наука, 1986. 248 с.
17. Магнус Я.Р., Нейдекер Х. Матричное дифференциальное исчисление с приложением к статистике и эконометрике. М. : Физматлит, 2002. 495 с.
18. Romanenko V.D., Milyavsky Y.L. Adaptive coordinating control of interacting cognitive maps nodes' relations in impulse mode. *System Research and Information Technologies*. 2015. N 3. P. 109–120.

19. Romanenko V.D., Milyavsky Y.L. Control automation of impulse processes in cognitive maps with constrained disturbance suppression based on invariant ellipsoids method. *System research and information technologies*. 2017. N 2. P. 29-39. DOI: <https://doi.org/10.20535/SRIT.2308-8893.2017.2.03>
20. Nazin S.A., Polyak B.T., Topunov M.V. Rejection of bounded exogenous disturbances by the method of invariant ellipsoids. *Autom Remote Control*. 2007. Vol. 68. P. 467–486. DOI: <https://doi.org/10.1134/S0005117907030083>
21. Поляк Б.Т., Хлебников М.В., Щербаков П.С. Управление линейными системами при внешних возмущениях (техника линейных матричных неравенств). М. : Наука, 2013. 546 с.
22. Zgurowsky M.Z., Romanenko V.D., Milyavskiy Y.L. Principles and methods of impulse processes control in cognitive maps of complex systems. Part II. *Journal of Automation and Information Sciences*. 2016. Vol. 48, N 7. P. 4–16. DOI: <https://doi.org/10.1615/JAutomatInfScien.v48.i7.20>
23. Romanenko V., Milyavsky Yu. Combined control of impulse processes in complex systems cognitive maps with multirate sampling. *2017 IEEE 9th International Conference on Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications (IDAACS)*. Bucharest. 2017. P. 8–13. DOI: <https://doi.org/10.1109/IDAACS.2017.8094500>
24. Romanenko V., Miliavskiy Y. Combined control of multirate impulse processes in cognitive map of COVID-19 morbidity. *System Research & Information Technologies*. 2022. N 3. P. 46–56. DOI: <https://doi.org/10.20535/SRIT.2308-8893.2022.3.03>
25. Zgurowsky M., Romanenko V., Milyavsky Y. Adaptive control of impulse processes in complex systems cognitive maps with multirate coordinates sampling. *Advances in Dynamical Systems and Control, Studies in Systems. Decision and Control*. 69. Springer International Publishing, Switzerland, 2016. P. 363–374. DOI: http://dx.doi.org/10.1007/978-3-319-40673-2_19
26. Gubarev V.F., Romanenko V.D., Milyavskiy Y.L. Identification in cognitive maps in the impulse processes mode with full information. *Journal of Automation and Information Sciences*. 2018. Vol. 50, N 8. P. 1–15. DOI: <https://doi.org/10.1615/JAutomatInfScien.v50.i8.10>
27. Gubarev V., Romanenko V., Miliavskiy Y. Identification and control automation of cognitive maps in impulse process mode. *Control Systems: Theory and Applications*. River Publishers. 2018. P. 43–64
28. Голуб Дж., Ван Лоун Ч. Матричные исчисления. М. : Мир, 1999. 548 с.
29. Gubarev V.F., Melnichuk S.V. Guaranteed state estimation algorithms for linear systems in the presence of bounded noise. *Journal of Automation and Information Sciences*. 2015. Vol. 47, N 3. P. 1–10. DOI: <https://doi.org/10.1615/JAutomatInfScien.v47.i3.10>
30. Verhagen M., Dewilde P. Subspace model identification. Part 1. The output-error state-space mode identification class of algorithms. *International Journal of Control*. 1992. Vol. 56, N 3. P. 1187–1210. DOI: <https://doi.org/10.1080/00207179208934363>
31. Тихонов А.Н., Арсенин В.Я. Методы решения некорректных задач. М. : Наука, 1979. 285 с.
32. Miliavskiy Y.L. Identification on cognitive maps in impulse process mode with incomplete measurement of nodes coordinates. *Кибернетика и вычислительная техника*. 2019. № 1 (195). С. 49–63. DOI: <https://doi.org/10.15407/kvt195.01.049>
33. Gubarev V.F., Romanenko V.D., Miliavskiy Yu.L. Methods for finding a regularized solution when identifying linear multivariable multiconnected discrete systems. *Cybernetics and System Analysis*. 2019. Vol. 55. P. 881–893. DOI: <https://doi.org/10.1007/s10559-019-00198-5>
34. Romanenko V., Miliavskiy Y., Kondratenko Y., Kuntsevich V., Gubarev V. (eds.) Advanced identification of impulse processes in cognitive maps. *Advanced Control Systems: Theory and Applications*. River Publishers, 2021. P. 231–256.

Submitted 07.05.2024