UDC 532.529

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MODELING OF THE PROCESS OF ASH REMOVAL FROM GAS IN A VENTURI SCRUBBER WITH REGARD FOR THE TURBULENT FLUCTUATIONS OF PARTICLE VELOCITY

We generalize our model of gas cleaning from suspended solid particles in a Venturi scrubber with regard for the influence of turbulent fluctuations of gas flow. We develop a method for calculating the effective velocity of slip between two fractions of particles, including their averaged and fluctuation motion.

Keywords: drops, solid particles, interaction, turbulence, kinetic energy of turbulent fluctuations, Venturi scrubber, gas cleaning.

Various technological processes in many branches of industry are connected with the formation of exhaust gases, which contain solid particles and are thrown out to the atmosphere. Power engineering based on coal makes its substantial contribution to the contamination of atmospheric air. Numerous power units are equipped with apparatus for the wet cleaning of gases from ash particles, among which Venturi scrubbers should be considered as the most efficient and promising [1]. Nevertheless, in many cases, such apparatus do not provide the necessary purity of gases thrown out to the atmosphere, and, hence, the search for ways of enhancing the efficiency of gas cleaning from solid particles in Venturi scrubbers represents an important ecological problem. The most real way of the solution of this problem is connected with mathematical modeling. We have constructed such a model [2, 3], but one quite significant factor is here not taken into account.

It is customary to think that, under usual conditions, fluctuation velocities (caused by turbulence) are much lower than averaged ones. However, as shown in [4], this is correct for *absolute* velocities of suspended particles, but not for their *relative* velocities. Even in the case of a channel of constant cross-section, the fluctuation velocities of slip between two fractions of particles can have the same order of magnitude as the averaged slip velocities [4]. This feature is still more clearly pronounced for channels of variable cross-section (e.g., Venturi tubes): at some domains of the flow, the curves of averaged velocities of two fractions can intersect, and fluctuation slip velocities can here be much more than averaged ones. Therefore, the aim of this paper is to generalize model [2, 3] with regard for turbulent motion of particles.

First, we give a short characteristic of model [2, 3]. It is based on the continuous approach to the description of particle interaction (coalescence and breakup). Here, collisions of a given fraction i with smaller and greater particles are described in different ways: in the first case, particle i preserves its individuality (the substance continues to belong to the same fraction) and loses it in the second. Two polydisperse ensembles are considered: drops with small solid inclusions and solid particles (SP) with (possible) liquid shells. The state of each fraction is described by five quantities: particle mass (m, M), specific mass flow rate (g, G), velocity (u, U; small letters refer to drops, and capital to SP), temperature, and the mass content of «foreign» phase. According to the continuous approach, four types of interaction are considered: two fractions of drops, small drop – large SP, small SP – large drop, two SP.

As an example, we present equations for drop mass m_i and SP specific flow rate G_j. The first consists of four terms taking into account phase transition, interaction of drops i with smaller ones, coalescence with smaller SP, and liquid sticking onto SP that do not coagulate with our drops:

$$\left(\frac{dm_i}{dx}\right)_1 = -\frac{\pi\delta_i^2 V_i}{u_i};$$
$$\left(\frac{dm_i}{dx}\right)_2 = \frac{\pi}{4u_i}\sum_{j=1}^{i-1}\Phi_{ji}K_{ji};$$
$$\left(\frac{dm_i}{dx}\right)_3 = \frac{\pi}{4u_i}\sum_{j=1}^{N_s}\Psi_{ji}L_{ji};$$
$$\left(\frac{dm_i}{dx}\right)_4 = -\frac{\pi}{4u_i}\sum_{j=1}^{N_s}\beta_{ji}(1-\Psi_{ji})L_{ji}, \quad (1)$$
$$K_{ji} = \frac{E_{ji}(\delta_i+\delta_j)^2|u_j-u_i|g_j}{u_j};$$
$$L_{ji} = \frac{E_{ji}(\delta_i+\Delta_j)^2|U_j-u_i|G_j}{U_j},$$

where δ, Δ are the sizes of drops and SP, V is the intensity of phase transition, E is the collision efficiency, K, L are the interaction constants, Φ, Ψ are the parameters of coalescence (and breakup), and β is the coefficient of liquid sticking.

Equation for G_j has the form

$$\frac{dG_j}{dx} = -\frac{\pi G_j}{4U_j}\sum_{\delta_i>\Delta_j}N_{ji}\left[\Psi_{ji}-\beta_{ji}(1-\Psi_{ji})\right]/m_i + \frac{\pi G_j}{4M_jU_j}\sum_{\delta_i<\Delta_j}Q_{ij}X_{ij}; \quad (2)$$
$$N_{ji} = \frac{E_{ji}(\delta_i+\Delta_j)^2|U_j-u_i|g_i}{u_i}$$

(Q_{ij} can be obtained from N_{ji} by substitution of E_{ij} instead of E_{ji}, X is the parameter of coalescence and breakup for interaction small drop – large SP).

We now calculate the velocity of slip between two fractions with regard for turbulence. According to [5], the root-wean-square fluctuation velocity of a particle is equal to

$$\langle v_p^2\rangle = \langle v_g^2\rangle\frac{\gamma}{\psi+\gamma}, \quad (3)$$

where γ^{-1} is the particle relaxation time, and ψ^{-1} is the integral time scale of turbulence. For calculating the average velocity of fluctuation slip between two fractions of particles, it is reasonable to use the approach [6], where only monodisperse particles are considered. The function of joint distribution of the particles of two fractions (i and p) by fluctuation velocities is

$$f_{ip}(\mathbf{V}_i, \mathbf{V}_p) \approx \approx \int f_{gip}(\mathbf{V}_g, \mathbf{V}_i, \mathbf{V}_p)d\mathbf{V}_g, \quad (4)$$

where vector quantities are denoted by bold letters, and f_{gip} is the function of joint distribution of gas moles and two fractions (integration is performed over the entire space of fluctuation velocities):

$$f_{gip}(\mathbf{V}_g, \mathbf{V}_i, \mathbf{V}_p) \approx \approx f_i^*(\mathbf{V}_i|\mathbf{V}_g)f_p^*(\mathbf{V}_p|\mathbf{V}_g)f_{gi}^*(\mathbf{V}_g). \quad (5)$$

Here, two first multipliers characterize the conditional probability that the particle velocity takes a certain value if the gas velocity is equal to $\mathbf{V}_g$, and the third represents the distribution function of gas velocities. As follows from the definition of conditional probabilities, we may write

$$f_{gn}(\mathbf{V}_g, \mathbf{V}_n) \approx \approx f_n^*(\mathbf{V}_n|\mathbf{V}_g)f_{gn}^*(\mathbf{V}_g), \; n = i, p, \quad (6)$$

where f_{gn} is the function of joint distribution of gas moles and particles n by fluctuation velocities. Hence, using (5) and (6), we obtain

$$f_{gip} = f_{gi}(\mathbf{V}_g, \mathbf{V}_i)f_{gp}(\mathbf{V}_g, \mathbf{V}_p)/f_{gi}^*(\mathbf{V}_g). \quad (7)$$

Further, following [6], we approximate the velocity distributions of particles and gas by Maxwellian functions:

$$f_n(\mathbf{V}_n) = \frac{\nu_n}{(4\pi k_n/3)^{3/2}}\exp\left(-\frac{\mathbf{V}_n^2}{4k_n/3}\right);$$
$$f_{gn}^*(\mathbf{V}_g) = \frac{1}{(4\pi k_g/3)^{3/2}}\exp\left(-\frac{\mathbf{V}_g^2}{4k_g/3}\right), \quad (8)$$

where ν_n is the number of particles per unit volume. The simplest variant of joint distribution of gas and particles, corresponding to (8), is

$$f_{gn}\left(\mathbf{V}_g, \mathbf{V}_n\right) = \\ = \nu_n A_{gn} \exp\left(-A_g \mathbf{V}_g^2 - A_n \mathbf{V}_n^2 + A_{gn}^{\circ} \mathbf{V}_g \cdot \mathbf{V}_n\right), \quad (9)$$

where

$$A_{gn} = \frac{1}{\left(4\pi k_g/3\right)^{3/2}\left(4\pi k_n/3\right)^{3/2}} \cdot \frac{1}{\left(1-\xi_{gn}^2\right)^{3/2}};$$

$$A_s = \frac{1}{4k_s/3} \cdot \frac{1}{1-\xi_{gn}^2}; \quad s = g, n;$$

$$A_{gn}^{\circ} = \frac{1}{\left(2k_g/3\right)^{1/2}\left(2k_n/3\right)^{1/2}} \cdot \frac{\xi_{gn}}{1-\xi_{gn}^2};$$

$$\xi_{gn}^2 = \frac{k_n}{k_g}.$$

We now substitute functions (8) and (9) in (7) and obtain

$$f_{gip}\left(\mathbf{V}_g, \mathbf{V}_i, \mathbf{V}_p\right) = \\ = \frac{\nu_i \nu_p}{(4\pi/3)^{9/2}\left(k_g k_i k_p\right)^{3/2}\left(1-\xi_{gi}^2\right)^{3/2}\left(1-\xi_{gp}^2\right)^{3/2}} \times \\ \times \exp\left(-A_i \mathbf{V}_i^2 - A_p \mathbf{V}_p^2\right) \exp\left(-a\mathbf{V}_g^2 + \mathbf{V}_g \cdot \mathbf{b}\right);$$

$$a = \frac{3\left(1-\xi_{gi}^2 \xi_{gp}^2\right)}{4k_g\left(1-\xi_{gi}^2\right)\left(1-\xi_{gp}^2\right)}; \quad (10)$$

$$\mathbf{b} = \frac{3}{2\sqrt{k_g}}\left[\frac{\mathbf{V}_i \xi_{gi}}{\sqrt{k_i}\left(1-\xi_{gi}^2\right)} + \frac{\mathbf{V}_p \xi_{gp}}{\sqrt{k_p}\left(1-\xi_{gp}^2\right)}\right].$$

The integration of function (10) over the space of fluctuation velocities gives

$$f_{ip}\left(\mathbf{V}_i, \mathbf{V}_p\right) = \\ = E \exp\left[-a_i \mathbf{V}_i^2 - a_p \mathbf{V}_p^2 + \frac{3\xi_{gi}\xi_{gp}\mathbf{V}_i \cdot \mathbf{V}_p}{2\sqrt{k_i k_p}\left(1-\xi_{gi}^2 \xi_{gp}^2\right)}\right];$$

$$E = \frac{\nu_i \nu_p}{(4\pi/3)^3\left(k_i k_p\right)^{3/2}\left(1-\xi_{gi}^2 \xi_{gp}^2\right)^{3/2}}; \quad (11)$$

$$a_n = \frac{3}{4k_n\left(1-\xi_{gi}^2 \xi_{gp}^2\right)}.$$

Obviously, the average velocity of fluctuation slip between particles i and p is equal to

$$\left\langle g_{ip}\right\rangle = \frac{1}{\nu_i \nu_p}\iint\left|\mathbf{V}_i - \mathbf{V}_p\right| f_{ip}\left(\mathbf{V}_i, \mathbf{V}_p\right) d\mathbf{V}_i\, d\mathbf{V}_p. \quad (12)$$

For calculating the integrals in (12), it is convenient to pass to new variables **G** and **g**:

$$a_0 \mathbf{G} = a_i \mathbf{V}_i + a_p \mathbf{V}_p; \\ a_0 = a_i + a_p; \; \mathbf{g} = \mathbf{V}_p - \mathbf{V}_i. \quad (13)$$

Then we obtain instead of (11):

$$f_{ip}(\mathbf{G}, \mathbf{g}) = E \exp\left[-\left(a_0 - h\right)\mathbf{G}^2 - \right. \\ \left. - \frac{a_i a_p}{a_0}\left(1 + \frac{h}{a_0}\right)\mathbf{g}^2 + h\frac{a_i - a_p}{a_0}\mathbf{G} \cdot \mathbf{g}\right]; \quad (14)$$

$$h = 2\xi_{gi}\xi_{gp}\sqrt{a_i a_p}.$$

It is easy to verify that the corresponding Jacobian is $\partial(\mathbf{G}, \mathbf{g})/\partial(\mathbf{V}_i, \mathbf{V}_p) = 1$. Further, we pass to spherical coordinates G, ψ°, θ°, g, ψ, θ and calculate the distribution function for pairs of particles by the modulus of vector **g**:

$$f(g) = \frac{E}{\nu_i \nu_p}\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} G^2 \sin\psi^{\circ} dG\, d\psi^{\circ}\, d\theta^{\circ} \times \\ \times \int_0^{2\pi}\int_0^{\pi}\exp\left[-\left(a_0 - h\right)G^2 - \frac{a_i a_p}{a_0^2}\left(a_0 + h\right)g^2 + \right. \quad (15) \\ \left. + h\frac{a_i - a_p}{a_0} Gg\cos\psi\right]\sin\psi\, d\psi\, d\theta$$

(here, the vector **G** is first fixed so that $\psi = (\mathbf{G}, \mathbf{g})$, and then integration by its direction and modulus is performed). Having calculated the integrals in (15), we arrive at the following result:

$$f(g) = \frac{27}{16\sqrt{\pi}(a_0 - h)^{3/2}(k_i k_p)^{3/2}(1 - \xi_{gi}^2 \xi_{gp}^2)^{3/2}} \times \exp\left[-\frac{a_i a_p (1 - \xi_{gi}^2 \xi_{gp}^2)}{a_0 - h} g^2\right]. \quad (16)$$

Finally, calculating $\langle g_{pi} \rangle = \int_0^\infty g^3 f(g) dg$ and carrying out the corresponding transformations, we obtain

$$\langle g_{pi} \rangle = \left[\frac{32}{3\pi}\left(\frac{k_i + k_p}{2} - \xi_{gi}\xi_{gp}\sqrt{k_i k_p}\right)\right]^{1/2} \quad (17)$$

The last part of our study is connected with combining average and fluctuation slip velocities. This problem was first solved in [7] for the case of isotropic pseudoturbulence. Here, for simplification, the actual distribution of the moduli of fluctuation velocities (e.g., (16)) was replaced by delta function

$$f(g) = \delta(g - \langle g_{pi} \rangle), \quad (18)$$

which led to the following result for effective slip velocity:

$$\langle w_{pi} \rangle = \begin{cases} u + \langle g_{pi} \rangle^2 / (3u), & u \geq \langle g_{pi} \rangle; \\ \langle g_{pi} \rangle + u^2 / (3\langle g_{pi} \rangle), & u < \langle g_{pi} \rangle, \end{cases} \quad (19)$$

where u is the averaged slip velocity between particles i and p.

Let us now take, instead of (18), a more actual distribution of pairs of particles by fluctuation slip velocities (16). For brevity, we denote the coefficients of this distribution by D (pre-exponential multiplier) and B. Then, using, as earlier, spherical coordinates g, ψ, θ, we write

$$\langle w_{pi} \rangle = \frac{1}{4\pi}\int_0^{2\pi} d\psi \int_0^\infty Dg^2 \exp(-Bg^2) dg \times \int_{-\pi/2}^{\pi/2} (u^2 + g^2 - 2ug\sin\theta)^{1/2} \cos\theta \, d\theta. \quad (20)$$

The way of calculating the internal integral in (20) (we denote it by I) depends on the relation between u and g, and, hence, we must divide the second integral in (20) into two: $\int_0^u$ and $\int_u^\infty$. Simple calculations give

$$I = \begin{cases} 2g^2/(3u) + 2u, & g \leq u; \\ 2u^2/(3g) + 2g, & g > u \end{cases} \quad (21)$$

(cf. (19)). Substituting (21) in (20) and carrying out remaining integration, we finally obtain

$$\langle w_{pi} \rangle = 0{,}5D\left[\frac{1 + 2u^2 B}{4u B^{5/2}}\sqrt{\pi}\,\mathrm{erf}(uB^{1/2}) + \frac{4u^2 B + 3}{6B^2}\exp(-u^2 B)\right], \quad (22)$$

where $\mathrm{erf}(y) = \frac{2}{\sqrt{\pi}}\int_0^y \exp(-t^2) dt$ is the error function.

Calculations show that the new approach enables one to refine the average slip velocity and, hence, the intensity of catching of solid particles by drops (see (2)). In Table, we present the values of $\langle w_{pi} \rangle$ calculated according to (19) and (22) for the interaction of water drops (δ_i =0.25 mm) with ash particles (Δ_j = 5 μm) at a gas velocity of 60 m/s (here, $\langle g_{pi} \rangle$ = 9.22 m/s).

It is seen that, in the case where averaged and fluctuation slip velocities are comparable, the refined method (22) give results different from the approximate approach [7] by 15–18%. At the same

Table **– Effective slip velocities (m/s)**

u, m/s	3	5	7	10	13	16	20
$\langle w_{pi} \rangle$ by (19)	9.55	10.13	10.99	12.83	15.18	17.77	21.47
$\langle w_{pi} \rangle$ by (22)	10.25	11.90	13.45	15.3	16.77	18.56	21.72

time, for very low of very high u, this difference becomes not so important.

Note that the described results form a tool for the search for the optimal conditions of gas cleaning from SP in Venturi scrubbers.

CONCLUSIONS

In three-phase mixtures flowing in channels of variable cross-section (e.g., Venturi tubes), the fluctuation velocities of slip between two fractions of particles can be not only comparable with the averaged slip velocities, but also exceed them substantially. Therefore, we have generalized our model of three-phase polydisperse flow in apparatus for the wet cleaning of combustion products from ash particles with regard for turbulent fluctuations. First, approximating the distribution of fluctuation velocities of gas and particles by Maxwellian functions, we determine the average velocity of fluctuation slip between two fractions. Second, based on the same approximation, we find the effective slip velocity of particles, taking into account their average and fluctuation motion. We determine the domain where the proposed method of calculating the effective slip velocity between two fractions enables one to obtain more exact data as compared with the known, approximate approach.

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Надійшла до редколегії 29.05.2014