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THE METHODOLOGY FOR MODELING LOGISTICS SYSTEMS: IMPLEMENTATION PRINCIPLES AND EXAMPLES

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The Methodology for Modeling Logistics Systems: Implementation Principles and Examples

The article deals with the development of a methodology for constructing a model of an enterprise's logistics system engaged in production and marketing activities. The introduction notes the importance of general theoretical research methods. A universal method for constructing a logistics system is developed, starting with the construction and analysis of the simplest reservoir systems (J. Forrester's approach). The construction of the logistics system is carried out in stages, as a result of the consistent refinement of the reservoir model. At the first stages, the model is modified with consideration for the most important functions of any logistics system. At each stage of the construction, calculations of the main characteristics of the logistics system are made. Based on the analysis of the calculations results, the conclusions on the ways for further refinement of the model are drawn. The model is used to solve optimization problems in order to determine the optimal value of the parameters of the constructed logistics systems. The optimization problems solved in the article by their formulation are similar to Pontryagin's maximum principle used in optimal control theory. To solve them, the method for computation of optimization problems of this class is applied. The developed methodology is universal and applicable for any theoretical study of the logistics system.

Keywords: *logistics system, production capacity, economic and mathematical model.*

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Шерстенников Ю. В. Методологія моделювання логістичних систем: принципи та приклади

Роботу присвячено розробці методології побудови моделі логістичної системи підприємства, що веде виробничо-збутову діяльність. У введенні зазначено важливість загальних теоретичних методів дослідження. Розроблений універсальний метод побудови логістичної системи, починаючи з побудови й аналізу найпростішої систем резервуарів (підхід Дж. Форрестера). Побудова логістичної системи ведеться поетапно, як результат послідовного уточнення моделі резервуарів. На перших етапах виконується модифікація моделі для урахування найбільш важливих функцій будь-якої логістичної системи. На кожному етапі побудови виконуються розрахунки основних характеристик логістичної системи, на підставі яких робляться висновки про способи подальшого уточнення моделі. Модель використовується для розв'язку оптимізаційних завдань із метою визначення оптимального значення параметрів побудованих логістичних систем. Завдання оптимізації, які вирішені в статті, по своїй постановці аналогічні принципу максимуму Л. С. Понтрягіна в теорії оптимального керування. Для їхнього розв'язку був застосований метод чисельного розв'язку оптимізаційних завдань цього класу. Розроблена методологія є універсальною й застосовна для будь-якого теоретичного дослідження логістичної системи.

Ключові слова: *логістична система, виробнича потужність, економіко-математична модель.*

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Шерстенников Ю. В. Методология моделирования логистических систем: принципы и примеры

Работа посвящена разработке методологии построения модели логистической системы предприятия, ведущего производственно-сбытовую деятельность. Во введении отмечена важность общих теоретических методов исследования. Разработан универсальный метод построения логистической системы, начиная с построения и анализа простейшей систем резервуаров (подход Дж. Форрестера). Построение логистической системы ведется поэтапно, как результат последовательного уточнения модели резервуаров. На первых этапах выполняется модификация модели для учета наиболее важных функций любой логистической системы. На каждом этапе построения выполняются расчеты основных характеристик логистической системы, на основании анализа которых делаются выводы о способах дальнейшего уточнения модели. Модель используется для решения оптимизационных задач с целью определения оптимального значения параметров построенных логистических систем. Задачи оптимизации, решенные в статье, по своей постановке аналогичны принципу максимума Л. С. Понтрягина в теории оптимального управления. Для их решения был применен метод численного решения оптимизационных задач такого класса. Разработанная методология является универсальной и применима для любого теоретического исследования логистической системы.

Ключевые слова: *логистическая система, производственная мощность, экономико-математическая модель.*

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Problem statement. The study deals with the development of a methodology for constructing a logistics system model. It begins with examining a simple system of reservoirs, and further, as a result of a consistent refinement of the model, a transition to more realistic logistics systems is carried out. The idea of reservoirs in considering logistics systems was proposed by J. Forrester in the 50s of the last century [1]. However, Forrester himself, omitting the stage of analyzing the simplest systems, immediately began to apply this idea to complex types of enterprises engaged in production and marketing activities.

The practice of using this approach in scientific research as well as for educational purposes encounters a certain amount of mistrust on the part of both scientific and student community. A frequently asked question is the following, "How do the results obtained in calculating the proposed model correlate to experimental data for real enterprises". But what is an experiment on an economic system? First, this experiment is usually extremely expensive. In addition, no experiment on a real-life economic system can be repeated twice, and therefore it does not make sense to conduct it once. In scientific research, experiment is very important and, ultimately, experiment plays a decisive role in verifying any theory. But in the course of the proving most of scientific provisions, researchers try not to resort to experiment since usually a particular case can be verified in an experiment, and the theory always strives for the most general conclusions. For example, the well-known rule of addition of two forces, the so-called "parallelogram law", which is sometimes presented as an experimental fact in the school curriculum, was actually obtained as a result of a series of consecutive logical reasoning, the so-called "thought experiments".

The starting point of this reasoning was knowledge of the frictional force acting on the body on an inclined plane. The magnitude of the frictional force itself, which can easily be determined experimentally, was determined by T. Stevin as a result of a "thought experiment" as well [2]. When measuring the frictional force, it is important to determine not only its absolute value but also the functional dependence on the inclination angle of the plane. To perform the second task experimentally is quite difficult since the functional dependencies that will give almost identical results are virtually unlimited. To be fair, it should be noted that in science there are cases when rather complex laws were established experimentally, e.g., the formula of the force acting on a current carrying conductor in a magnetic field (Ampere's force law). The formula for this force was developed by Ampere when performing four of his famous experiments [3]. Although, it should be noted that he carried out these experiments only to confirm his ingenious guesses about the nature of electromagnetic interactions.

Analysis of recent publications on the problem studied and identification of the parts unsolved. Presently, much attention is paid to the problems of modeling and management of logistics systems [4 - 12]. It is noted that today many issues in the management and design of logistics systems remain theoretically and practically unsolved [4]. Servicing the resources of operational activities of industrial enterprises and strategic measures for their development is transforming from quantitative to qualitative, which causes the emergence of new models of the management system [5]. In work [6], research methods based on the use of mathematical tools are proposed. In particular, this approach is applied to modeling development of the management system of an industrial enterprise [7]. Work [8] presents a model for optimizing an enterprise's activities through the use of more universal resources by expanding their functions in order to increase the total amount of finished goods.

In [9–11], it is noted that logistics activities take place in a very dynamic environment, and the state of the logistics system should be constantly checked, analyzed and evaluated. Study [12] deals with an important and at the same time complex problem – the process of designing logistics systems, which is one of the key components of an enterprise's performance.

However, the considered works pay too much attention to particular details that are characteristic only for a particular logistics system under study, while the logic of the functioning of a logistics system and internal connections remain in the shadow of specific functions and tasks that are solved by this particular logistics system.

In the work, the focus is not on individual functions of the logistics system but on the construction of the entire logistics system as an interconnected and interdependent system of components (links).

The aim of the research is the development of a methodology for constructing a logistics system model.

1. Constructing a balanced logistics system of an enterprise.

1.1. Constructing and analyzing a simple model of the logistics system.

We will begin building a simple logistics system model by applying J. Forrester's idea of reservoirs (or levels) and the flows into and out of any reservoir (which are characterized by rates). To designate the reservoir levels, we will use capital English letters, the flow rates – lowercase letters. Levels can be the amount of products/goods or raw materials. Rates are the speed of increase (decrease) in the corresponding quantity of products/goods or raw materials.

Let us consider an enterprise's logistics system (production-distribution system – using the terminology of J. Forrester) presented in Fig. 1.

The system includes nine components – four reservoirs and five transport links Tpj (j = 1–5). We will build a discrete time model. The time variable i determines the whole-number values within the interval [0; T] ($I = \overline{0,\ T}$). As usual, we will choose T = 365 (a year).

Based on Figure 1 we will define the key variables describing the logistics system:

q_i – flow of raw materials to the raw material warehouse (the number of units of raw materials that enter the raw material warehouse in i^{th} period);

X_i – level of raw material inventory at the raw material warehouse (the number of units of raw materials at the raw material warehouse) in the i^{th} period;

x_i – flow of raw materials out of the raw material warehouse in the i^{th} period;

Y_i – level of work in progress inventory (it is determined by the enterprise's work equipment, and in turn determines its production capacity);

y_i – flow of the products manufactured (the number of production units manufactured in the i^{th} period;

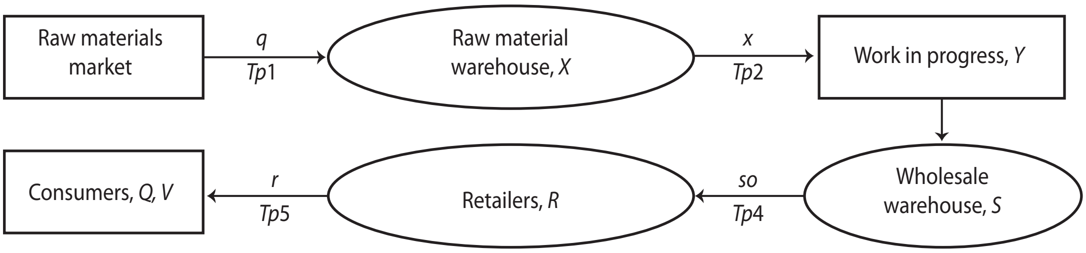

Fig. 1. Scheme of the enterprise's logistics system (LS)

S_i – level of inventory at the wholesale warehouse (factory warehouse – using the terminology of J. Forrester) in the i^{th} period;

so_i – flow of goods from the finished goods warehouse to retailers in the i^{th} period;

R_i – level of inventory at retail in the i^{th} period;

r_i – sales flow in the i^{th} period;

V_i – level of inventory in the hands of the consumer (not consumed yet) in the i^{th} period;

Q_i – number of potential consumers in the i^{th} period.

We will define a unit of raw material as the amount of the raw material required for manufacturing a production unit.

Now a mathematical model of reservoirs that simulates the operation of the logistics system can be written as a system of equations:

$$X_{i+1} = X_i + q_i - x_i, \tag{1}$$

$$x_{i+1} = \frac{X_i}{tX}, \tag{2}$$

$$Y_{i+1} = Y_i + x_i - y_i, \tag{3}$$

$$y_{i+1} = \frac{Y_i}{tY}, \tag{4}$$

$$S_{i+1} = S_i + y_i - so_i, \tag{5}$$

$$so_{i+1} = \frac{S_i}{tS}, \tag{6}$$

$$R_{i+1} = R_i + so_i - r_i, \tag{7}$$

$$r_{i+1} = \frac{R_i}{tR}, \tag{8}$$

$$V_{i+1} = V_i + r_i - k_i * V_i. \tag{9}$$

In equation (9), the parameter $k1$ stands for the rate of the product consumption.

To determine the net operating profit of the enterprise, we will apply the following formula:

$$M_i = (1-kp)*[(1-kad)*p*r_i - p*c*yp_i - \\ -k2*S_i - z*Rm - Pq\cdot q_i], \tag{10}$$

where c is the share of the prime cost in the price for products; p is the price for a production unit; $k2$, z are the costs for the storage of a production unit during one period at retail and wholesale warehouse, respectively; kp is the income tax rate; kad is the value-added tax rate; Pq is the price for a unit of raw material.

Task 1. Assuming that the raw material purchase is continued q_i =3.2 for all periods ($i = \overline{0.365}$) during the year, and based on the following values of the parameters in the system of equations (1) – (9):

Constants:

$$k1 = 0.33,\ tX = 10,\ tY = 8,\ tS = 20,\ tR = 18,$$

Initial values:

$$x_0 = 0,\ y_0 = 0,\ so_0 = 1,\ r_0 = 1,\ X_0 = 0,\ Y_0 = 0. \tag{11}$$

$$S_0 = 0,\ R_0 = 0,\ V_0 = 0,$$

1) to determine the behavior of the key economic indicators of the logistics system;
2) to explain the behavior of the key economic indicators.

Solution.

1) We perform mathematical calculations for system (1) – (9) in the Mathcad15 environment. The behavior of the key economic indicators of the logistics system is calculated using the values of the parameters given in (11). We present the behavior of the key economic indicators of the logistics system in Figures 2 – 5.

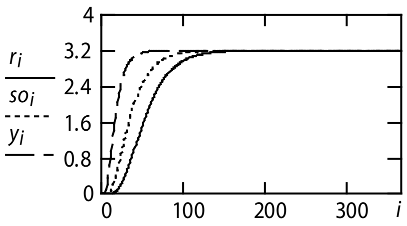

Fig. 2. Behavior of basic rates of the LS

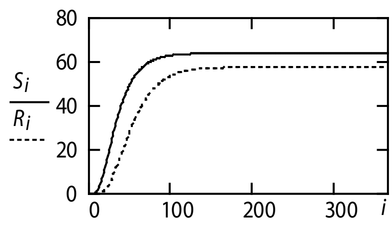

Fig. 3. Behavior of basic levels of the LS

2) Explanation of the behavior of the key economic indicators. Figure 2 shows that the increase in the production rate y_i exceeds that in the delivery rate so_i, whose growth is faster than that of the sales rate r_i. This correlation of rates is understandable. All rates close to the 140th period reach the rate of raw material purchase q_i = 3.2 (as it must be in case of a balanced logistics system). Figure 2 demonstrates that the raw material warehouse X_i is the first to reach its maximum, then follows

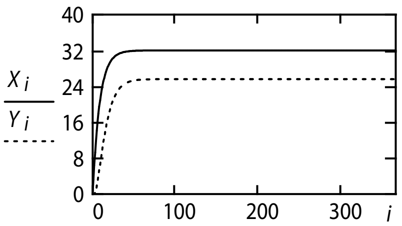

Fig. 4. Comparison of the behavior of the inventory level X_i with the behavior of the level of work in progress inventory Y_i

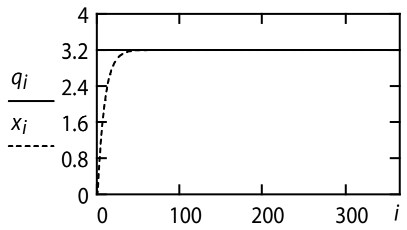

Fig. 5. Comparison of the behavior of the rate of raw material purchase q_i with the rate of delivery of raw material X_i to the production component of the LS

the wholesale warehouse S_i, and retail inventory R_i reaches its peak value by the 120th period. After reaching their maximum values, the levels stop changing. It is the result that is desirable for any enterprise – stable operation after the end of the initial transitional period.

Thus, despite the extremely simplified model of the reservoir system, this logistics model allows to correctly display some of the main features of any logistics system.

As seen from Figure 4, the increase in the stock of raw materials occurs earlier than that in the level of work in progress inventory. In the general case, this is how it must be in a real enterprise. Figure 5 explains why the sequence of changes in these levels takes place. Raw material purchase starts from the very beginning of the project, while supplies of raw materials to production gradually increase from zero to the rate of raw material purchase.

This lag can be explained by the fact that in case with the raw material warehouse it takes some time to account the raw materials that arrive and to form the batches of raw materials that are sent to production. Since, as a rule, raw materials comprise several ingredients, the fulfillment of orders of the production component will be simpler (and faster) if the raw material warehouse is fully loaded.

This can be seen from the comparison of Figure 4 and Figure 5. It is at the moment when the raw material inventory X_i reaches its maximum that the rate of delivery of raw materials to production becomes equal to the rate of raw material purchase.

Similar considerations also explain the behavior of the production rate (Fig. 6). If the project begins, as we assume, with zero capacity utilization $Y_0 = 0$, it takes a certain amount of time for the production to reach its planned capacity.

Figure 4 and Figure 6 show that the level of work in progress inventory Y_i and the production rate y_i reach their peak simultaneously. The level of work in progress inventory characterizes the capacity utilization. It is clear that at full utilization of all links of production the production capacity will be maximum.

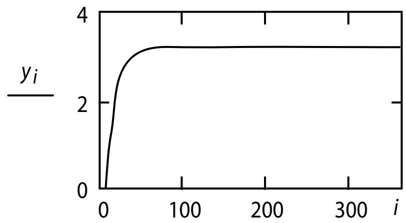

Fig. 6. Behavior of the production capacity

Figure 7 demonstrates the behavior of the inventory level in the hands of the consumer. The behavior of the value, as can be seen from equation (9) is completely determined by the behavior of the sales flow rate (Fig. 8).

Figure 9 illustrates the dependence of the net operating profit on time.

Figure 9 shows that at the beginning of the operation the profit is negative. This feature is characteristic for the initial period of any project.

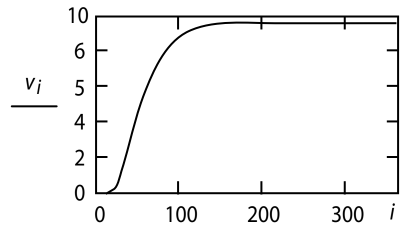

Fig. 7. Behavior of the inventory level in the hands of the consumer

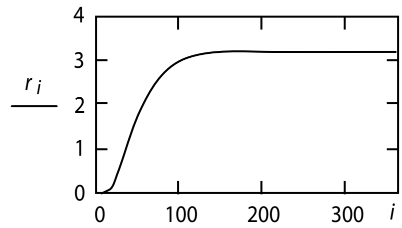

Fig. 8. Behavior of the sales rate of goods

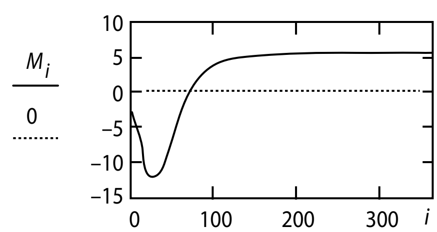

Fig. 9. Behavior of the net operating profit of the enterprise

The solution of the problem is complete.

It is worth noting that our consideration, based on an extremely simplified logistics system model, is only preliminary. Further, the model will be refined stage by stage. But in Problem 1, only general properties of logistics systems are considered.

These properties will also take place in the following more realistic models.

1.2. The first refinement of the reservoir model: accounting for the number of potential consumers.

The first drawback of the simple reservoir model is that it does not reflect the demand for the product, i.e., it does not include the number of potential consumers of the product Q.

This drawback can be eliminated if the formal equation (8) of the reservoir model is replaced by a more meaningful from an economic point of view equation that reflects the real economic nature of the sale of goods. The refinement is proposed in [13]:

$$r_{i+1} = n \cdot R_i \cdot (Q_i - V_i), \tag{12}$$

where r_i is the sales rate of goods (units per time period) in the i^{th} period; n is the parameter determined by the average sales level for the previous quarter (or year); R_i is the inventory level at retail in the i^{th} period; Q is the number of potential consumers in the i^{th} period; V_i is the inventory level in the hands of the consumer (not consumed yet).

Problem 2. Considering the consequences of the first refinement of the reservoir model. Additional parameters for making calculations by formula (12) are determined as follows:

$$n = 1.305\ 10^{-4},\ Q_i = Q = 500. \tag{13}$$

Now the behavior of the values shown in Figure 2 and Figure 3 will look like that in Figure10 and Figure 11, which demonstrate only the first 150 periods, since then the system starts working in a steady-state mode.

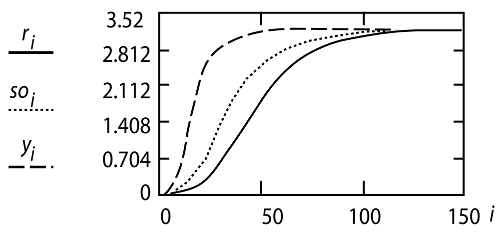

Fig. 10. Behavior of the basic rates of the LS

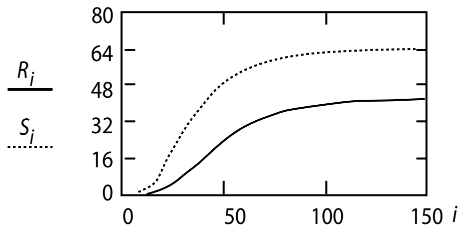

Fig. 11. Behavior of the basic levels of the LS

From the comparison presented in Figure 2 and Figure 10, we see that the behavior of R_i has undergone the greatest change. Using formula (10), we will calculate the behavior of the net operating profit for this case (Fig. 12).

The solution of the problem is complete.

Formula (12) makes it possible to take into account the demand for products since it contains the potential number of their consumers. It also allows to determine the effect of advertising on the enterprise's production efficiency.

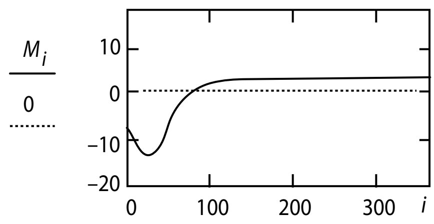

Fig. 12. Behavior of the net operating profit

1.3. The second refinement of the reservoir model: accounting for the requirement of maximum possible inventory level at retail.

The enterprise's chief executives decide to attract a retail network with a maximum capacity of Rm (units) to sell products.

At the same time, they put a requirement for managers: to organize the work of the logistics system so that to ensure maximum possible utilization of the retail network in order not to lose potential sales revenue. Thus, requirement can be met if the transport link $Tp4$ (Fig. 1) will provide the rate of delivery in accordance with the formula:

$$so_{i+1} = \begin{cases} y_i, & if\ i < ty \\ r_i \cdot \left(1 + \dfrac{Rm - R_i}{Rm}\right), & otherwise \end{cases} \tag{14}$$

Formula (14) demonstrates that the priority task in the initial periods of operation ($0 < i < ty$) is to fill the retail network with products. Formula (14) means that under the established operating mode ($i > ty$) the quantity of goods that must be transported in the $(i + 1)^{\text{th}}$ period is determined by the quantity of goods sold in the i^{th} period r_i as well as the level of utilization of the retail network R_i in the i^{th} period.

Problem 3. Assuming that the daily purchase equals $q0 = 3.5$ and taking $Rm = 50$, calculate the behavior of all the basic characteristics of the logistics system for this maximum capacity of the retail network and explain the differences from the previous cases.

If we apply for the calculations instead of formal equation (6) equation (14) and chose the daily purchase equal to $q_i = 3.5$, we will obtain the results presented in Figure 13 and Figure 14.

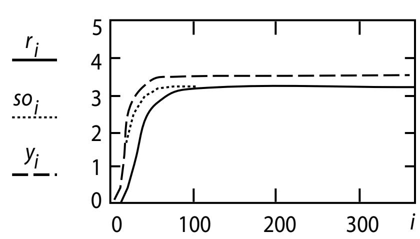

Fig. 13. Behavior of the basic rates of the LS

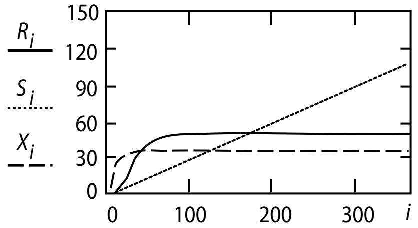

Fig. 14. Behavior of the basic levels of the LS

Separately, we calculate the behavior of the inventory level at retail (Fig. 15).

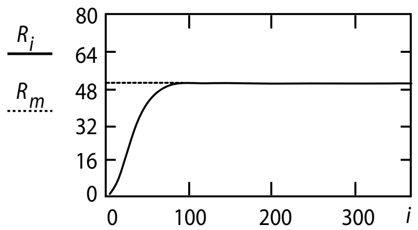

Fig. 15. Behavior of the inventory level at retail R_i at reaching its maximum value *Rm*

The situation shown in Figure 15 is desirable for any enterprise. The total net operating profit calculated by formula (10) for the whole project period T = 365 is

$$\sum M_i = 37{,}5. \qquad (15)$$

The solution of the problem is complete.

Such a low value of the net operating profit received from the project is explained by the fact that in each period of the project duration (0; 365) the production rate y_i exceeds the rate of sales of goods r_i (Fig. 13), which leads to a monotonous increase in the inventory level in the wholesale warehouse, as can be seen from Figure 14. Of course, this product will also be sold sometime, but it may have to be sold with some (possibly significant) price reduction.

1.4. The third refinement of the model. Accounting for the limited storage capacity.

Thus, we obtain a logistics system model, which is defined by the system of equations (1) – (5), (9), (12) and (14). The work of the logistics system in accordance with the indicated equations will lead to the behavior of its basic levels that is shown in Figure 14. In this case, by the time the project is completed (i = T =365), the inventory level at the warehouse will equal S_T = 109,5 units. However, firstly, it may not be profitable for the enterprise to have significant storage facilities, and secondly, to have large stock of finished products in the wholesale warehouse by the end of the project does not make sense either. If, e.g., the maximum capacity of the wholesale warehouse is Sm = 90, then, in order to account for this limitation, the enterprise management must take some measures. There are two obvious possibilities: 1) reducing the daily purchase of raw materials $q0$; 2) increasing the sales rate r_i. We will consider each of these possibilities. Let us assume that the decision to decrease the daily purchase from 3.5, e.g., to 3.2 is taken. In this case, the behavior of the levels calculated in accordance with the model will be as shown in Figure 16. Thus, the total net operating profit calculated by formula (10), for the whole project period T = 365 will equal

$$\sum M_i = 975. \qquad (16)$$

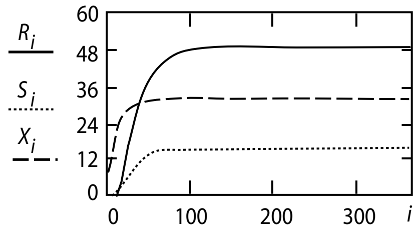

Fig. 16. Behavior of the basic levels of the LS with qo = 3.2

Another possibility to avoid overfilling warehouses is to increase the sales rate. Since, in accordance with formula (12), the sales rate r_i directly depends on the inventory level at retail R_i, a growth in the sales rate can be achieved by increasing the inventory level at retail. For this purpose, new outlets can be created or contracts concluded with operating shopping centers for selling goods manufactured by the enterprise. We assume that the increase in the maximum level of inventory at retail occurs simultaneously in the period tR:

$$Rv_{i+1} = \begin{cases} Rm, & if\ i < tR \\ qR \cdot Rm, & otherwise \end{cases}, \qquad (17)$$

where tR is the period of expanding the retail network; qR is the coefficient of retail network increase.

The task has tree new parameters: $q0$, tR, qR. It is obvious that these parameters should be determined in solving the optimization problem:

$$F(q0, tR, qR) = \sum M_i \to \max. \qquad (18)$$

Optimization problem (18) should be solved accounting for the limitations that are presented as the system of equations (1) – (5), (9), (12) and (17) that defines the operation of the logistics system. The solution of optimization problem (18) will be as follows:

$$\begin{pmatrix} q0 \\ tR \\ qR \end{pmatrix} = \begin{pmatrix} 3.4 \\ 194.8 \\ 1.15 \end{pmatrix}, \quad F = 1037. \qquad (19)$$

When calculating the economic result (19), the costs of creating additional outlets were not taken into account. Optimal solution (19) leads to the following behavior of the basic economic indicators of the logistics system (Fig. 17 and Fig. 18).

As seen from Figure 18, until the 194th period, the inventory level at the wholesale warehouse increases since the sales rate r_i within the time interval (0; 194), i.e., before the expansion of the retail network R_i, is lower in comparison with the production rate y_i (Fig. 17). But after increasing the retail network R_i (i.e. for i > 194), the correlation of the sales and production rate reverses $r_i > y_i$, as can be seen from Figure 17. This circum-

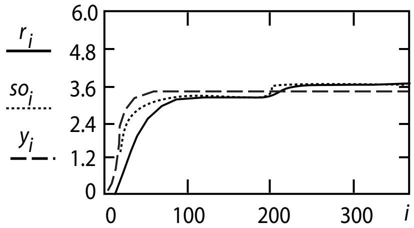

Fig. 17. Behavior of the basic rates of the LS for optimal solution (19)

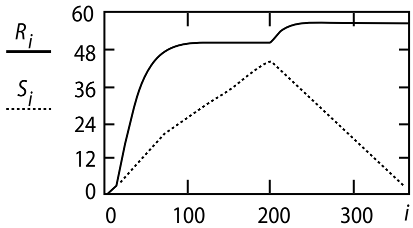

Fig. 18. Behavior of the basic levels of the LS for optimal solution (19)

stance leads to a decrease in the finished products inventory at the wholesale warehouse. Let us consider separately the behavior of the inventory level at retail (Fig. 19).

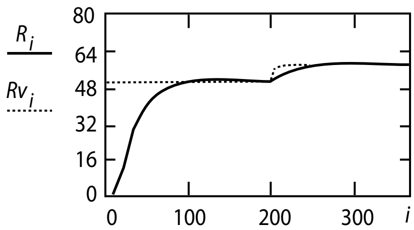

Fig. 19. Behavior of the inventory level at retail for optimal solution (19)

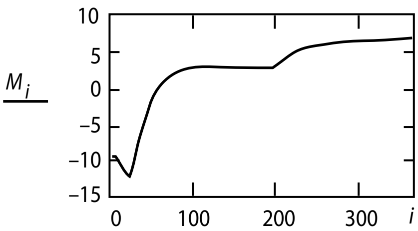

Fig. 20. Behavior of the daily profit for optimal solution (19)

Figure 19 demonstrates that within each of the time intervals (0; 194) and (194; 365) the inventory level at retail R_i reaches the maximum level Rv_i of each interval. Figure 20 demonstrates the behavior of the daily profit for optimal solution (19). As seen from the figure, in the initial periods, the profit is negative, as it is supposed to be. Moreover, it can be seen that increasing the level of inventory at retail R_i in the 194th period results in an increase in the daily profit. Now the enterprise's chief executives must analyze whether the economic effect of creating new outlets for selling the goods will be greater than the cost of creating them.

1.5. The fourth refinement of the model. Limiting the time of raw material purchase

As Figure 14, Figure 15 and others demonstrate, by the time the project is completed, either previously purchased raw materials or finished products remain at the warehouse. It is clear that it makes no sense for the enterprise to continue purchasing raw materials up to the end of the project. The raw material purchase should be ceased at a certain point Tq ($Tq < T = 365$).

Thus, the system of equations (1) – (5), (9), (12) and (17) describing the logistics system model should be supplemented by another equation:

$$q_{i+1} = q0 * \begin{cases} 1, & if\ i < Tq \\ 0, & otherwise \end{cases}, \tag{20}$$

Now, the problem, along with the three parameters $q0$, tR, qR, has one more new parameter Tq. All the four parameters must be determined when solving the optimization problem:

$$F(q0, tR, qR, Tq) = \sum M_i \to \max. \tag{21}$$

Optimization problem (21) should be solved accounting for limitations that are defined by a system of equations (1) – (5), (9), (12), (17) and (20).

Optimization problems (18) and (21) in their formulation are similar to Pontryagin's maximum principle used in optimal control theory [14]. To solve them, the author developed a method for computation of optimization problems of this class.

The solution of optimization problem (21) will be as follows:

$$\begin{pmatrix} q0 \\ tR \\ qR \\ Tq \end{pmatrix} = \begin{pmatrix} 3.3 \\ 195.5 \\ 1.1 \\ 317 \end{pmatrix}, \quad F = 1363. \tag{22}$$

The economic result in this case exceeds the economic result in (19) by 31.4 %. Thus, due to ceasing the purchase of raw materials at $Tq = 317$, i.e., 48 days before the end of the project, a significant economic effect is obtained. But discontinuing the raw material purchase does not require any additional costs. This result proves the importance of creating mathematical models of logistics systems.

Behavior of the basic rates of the LS for optimal solution (22)

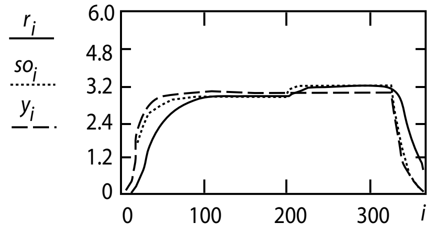

Fig. 21. Behavior of the basic rates of the LS for optimal solution (22)

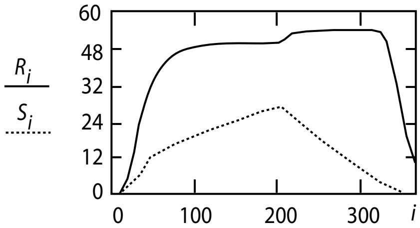

Fig. 22. Behavior of the basic levels of the LS for optimal solution (22)

Figure 23 demonstrates the behavior of the daily profit for optimal solution (22). The sharp increase in profits after the 317th period is due to the cessation of raw material purchase.

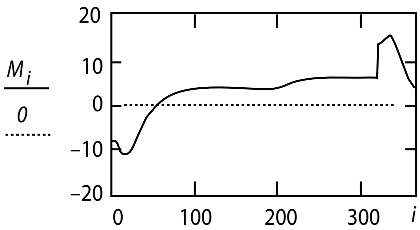

Fig. 23. Behavior of the daily profit for optimal solution (22)

Discussion. Above, one of the possible ways to refine reservoir model (1) - (9) was developed for the logistics system that is schematically shown in Figure 1. However, Figure 1 demonstrates only the simplest logistics system. Real logistics system can have a much more complex structure. Moreover, the logistics system has a wide range of priorities that may greatly differ, depending on the goals of the enterprise's management.

This means that there are many possible ways to construct an initial reservoir model as well as many ways to refine the initial model. Actually, in practice, there is no need to literally build the initial model and go through all phases of refining it. Usually, the model is built immediately and is aimed to fully reflect the aspects of the enterprise's operation that are of interest for the researcher. However, the methodology for the phased analysis and refinement of the model developed above will be useful for constructing and refining any logistics system model.

Since one of the parameters in equation (12) is the number of potential consumers of products, the model makes it possible to take into account the effect of advertising on the enterprise's production efficiency.

Conclusions. The article presents a methodology that allows constructing a logistics system model as a result of consistent consideration of the most important functions of the logistics system. The methodology is universal and applicable for any theoretical study of logistics systems.

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