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dc Josephson current for *d*-wave superconductors with charge density waves

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dc Josephson tunnel current I_c between a $d_{x^2-y^2}$-wave superconductor with charge density waves (CDWs) — e.g., a high-T_c oxide and a conventional isotropic superconductor — was considered theoretically. A directionality of tunneling was taken into account. It was found that the dependence of I_c on the angle γ between the superconducting lobe direction and the normal to the junction plane is significantly altered by CDWs. For certain doping levels it may be even nonmonotonic, which can be readily found experimentally. For the sake of comparison, the corresponding results obtained for the tunnel junction between a CDW *s*-superconductor and a conventional isotropic superconductor are also presented.

PACS: 74.20.Fg BCS theory and its development;
74.20.Rp Pairing symmetries (other than s-wave);
74.50.+r Tunneling phenomena; Josephson effects;
71.45.Lr Charge-density-wave systems,

Keywords: charge-density-wave superconductors, *d*-pairing, cuprates, dc Josephson effect, tunnel directionality.

The article is devoted to the memory of great scientist Igor Yanson, who made outstanding contributions to Josephson effect physics.

Introduction

The Josephson tunnel current reveals coherent properties of superconductors, depending on the phase of a superconducting order parameter (to be more precise, on the phase difference between superconducting order parameters in both electrodes of the tunnel junction) [1–3]. Its significance and specificity for the superconductivity physics can be compared with the role played by the general phenomenon of tunneling in quantum mechanics, first implicitly demonstrated by Mandelshtam and Leontovich [4]. dc and ac Josephson currents demonstrate plenty of dramatic features [1–10] and have numerous applications in technology [11–18]. Similar phenomena were observed in superfluid ^{4}He [19–21] and ^{3}He [19–22], as well as in quantum gases [21–25].

Manifestations of the coherent Cooper pair tunneling become even more varied for superconductors with anisotropic order parameters discovered during recent decades, in particular, for cuprates, where the $d_{x^2-y^2}$ pairing is usually considered at least as a predominant one [15, 26–33], with the ghosts of the conventional *s*-wave contibutions still being revealed in electron tunneling [34–41]. Therefore, there is no wonder that a number of dissidents still exist defending the isotropic *s*-wave (or extended *s*-wave) nature of superconductivity in high-T_c oxides against the mainstream opinion [42–55], although the *d*-wave ideology and machinery has already penentrated even into technical devices [3,11,13,56–58].

On the other hand, cuprates are not simple in a different respect as well: they reveal the so-called pseudogaps [59–63]. Here, various phenomena manifesting themselves in resistive, magnetic, optical, photoemission (ARPES), and tunnel (STM and break-junction) measurements are considered as a consequence of the " pseudogap"-induced depletion of the electron density of states, in analogy to what is observed in quasi-one-dimensional compounds [64,65].

Notwithstanding large theoretical and experimental efforts, the pseudogap nature still remains unknown [61,66–83]. Namely, some scholars associate them with precursor order parameter fluctuations, which might be either superconducting or competing (charge density waves — CDWs, spin density waves — SDWs, etc.) ones. Another viewpoint consists in relating pseudogaps to those competing orderings, but treating them, on the equal footing with superconductivity, as well-developed states that can be made allowance for in the mean field approximation. We think that observations support the latter viewpoint. Moreover, although undoped cuprates are antiferromagnetic insulators [84], CDW seem to be a more suitable candidate responsible for the pseudogap phenomena that competes with Cooper pairing in doped high-T_c oxide samples [76] (contrary to what seems most probable for iron-based pnictides and chalcogenides [85]).

Bearing in mind all the aforesaid, we present here the following scenario of the tunnel dc Josephson current between cuprates and ordinary s-wave superconductors. The Fermi surface (FS) of the former is considered two-dimensional with $d_{x^2-y^2}$-wave four-lobe superconductivity and CDW-related doping-dependent pseudogaps emerging inside the lobes in their antinodal directions (see Fig. 1). Thus, the CDW order parameter Σ competes with its superconducting counterpart Δ over the whole territory of their coexistence, which gives rise to an interesting phenomena of temperature (T-) reentrant Σ [76,82,83,86,87]. In this paper, the first one in a series of papers applying this scenario to the Josephson effect, we restrict ourselves to the case $T = 0$, the main objective of studies being angular dependences, which might be observed in the framework of the adopted model. Of course, any admixture of Cooper pairing with a symmetry different from $d_{x^2-y^2}$-one [30,35,41,88,89] may alter the results. Moreover, the order parameter symmetry might be doping-dependent [90]. To obtain some insight into such more cumbersome situations, we treat here the pure isotropic s-wave case as well. Other possibilities for predominantly d-wave superconductivity coexisting with CDWs lie somewhere between those two extremes.

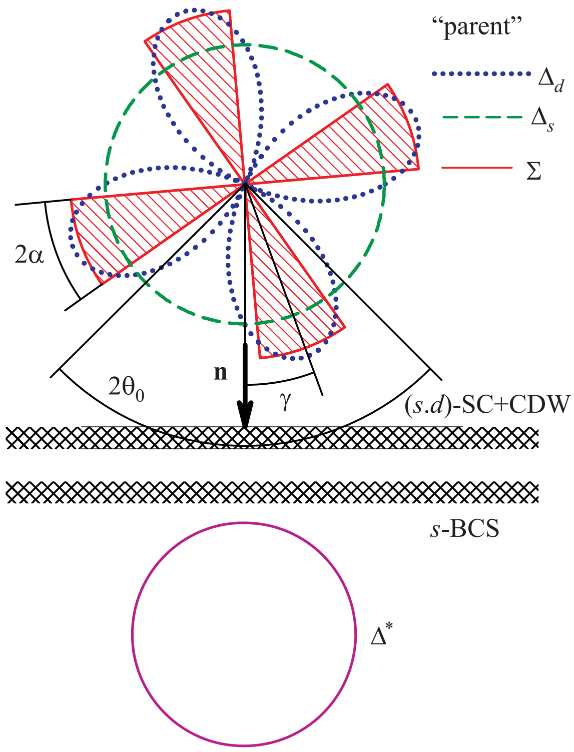

Fig. 1. Geometry of the junction between a conventional s-wave superconductor and a high-T_c $d_{x^2-y^2}$- or s-superconductor partially gapped by charge density waves (CDWs, induced by dielectric, i.e. electron-hole, pairing). The angle α denotes the half-width of each of four angular sectors at the Fermi surface, where the CDW gap appears. The profiles of the CDW dilelectric (Σ), and s-wave (Δ_s) and d-wave (Δ_d) superconducting order parameters of the high- T_c superconductor are shown for the "parent" CDW metal and BCS d-superconductor. Δ^* is the gap of the conventional superconductor; γ is the angle between the coincident bissetrices of the closest superconducting lobe (if any) and CDW-gapped sector, on the one hand, and the normal **n** to the junction plane, on the other hand. θ_0 is the directional sector of effective tunneling in two models, with step-like and graduate tunneling directionality (see explanations in the text).

Formulation

The starting point is the well known expression for the dc critical Josephson tunnel current between two superconductors [1,8] applied to the studied case

$$I_c(T) = 4eT\sum_{\mathbf{pq}}\left|\tilde{T}_{\mathbf{pq}}\right|^2 \sum_{\omega_n} \mathrm{F}^{+}_{HTSC}(\mathbf{p};\omega_n)\mathrm{F}_{OS}(\mathbf{q};-\omega_n). \quad (1)$$

Here $\tilde{T}_{\mathbf{pq}}$ are matrix elements of the tunnel Hamiltonian corresponding to various FS sections, $\mathbf{p}$ and $\mathbf{q}$ are the transferred momenta, $e > 0$ is the elementary electrical charge, $F_{HTSC}(\mathbf{p};\omega_n)$ and $F_{OS}(\mathbf{q};-\omega_n)$ are Gor'kov Green's functions for d-wave high-T_c (CDW gapped!) and ordinary s-wave superconductor, respectively, the internal summation is carried out over the discrete fermionic "frequencies" $\omega_n = (2n+1)\pi T$, $n = 0, \pm 1, \pm 2, \ldots$. The external summation should take into account both the anisotropy of the high-T_c-superconductor electron spectrum $\xi(\mathbf{p})$ in the manner suggested long time ago for all kinds of anisotropic superconductors [91], the directionality of tunneling [92–96], and the concomitant dielectric (CDW) gapping of the nested FS sections [97].

In what follows, we shall assume that the ordinary superconductor has the isotropic order parameter $\Delta^*(T)$, whereas the order parameter of high-T_c CDW superconductor has the properly shifted (see Fig. 1) pure d-wave form $\Delta_{HT}(T)\cos\left[2(\theta-\gamma)\right]$, the angle θ being reckoned from the normal $\mathbf{n}$ to the junction plane and γ is a tilt angle between $\mathbf{n}$ and the centre of the nearest lobe for the cuprate $d_{x^2-y^2}$ order parameter. In agreement with the previous studies [92–96,98], the tunnel matrix elements $\tilde{T}_{\mathbf{pq}}$ in Eq. (1) must make allowance for the tunnel directionality (the angle-dependent probability of penetration through the barrier) [99–101]. We isolate the corresponding directionality coefficient $w(\theta)$. The weight factor $w(\theta)$ effectively disables the FS outside a certain given sector around n, thus governing the magnitude and the sign of the Josephson tunnel current. In particular, we tried two models for $w(\theta)$, a continuous,

$$w_{\rm co}(\theta)=\exp\left[-\left(\frac{\tan\theta}{\tan\theta_0}\right)^2\right], \qquad (2)$$

where θ_0 is an angle describing the effective width of the directionality sector, and a steep one, $w_{\rm st}(\theta)$, with $w_{\rm st}(\theta)=1$ inside the sector $|\theta|\le\theta_0$ and 0 otherwise. The dielectric order parameter $\Sigma(T)$ corresponds to the checkerboard system of mutually perpendicular CDWs (observed in various high-T_c oxides [102–104]). In the adopted model, it is nonzero inside four sectors, each of the width 2α, with their centres coinciding with those of the superconducting lobes [76,82,83,86,87]. Another, unidirectional symmetry-breaking CDW geometry often existing in cuprates [105–107] can be treated in a similar way. We emphasize that, for tunneling between two anisotropic superconductors, two different coefficients $w(\theta)$ associated with **p**- and **q**-distributions in the corresponding electrodes come into effect [92].

According to the previous treatment of partially-gapped *s*-wave CDW superconductors [97,108–116] and its generalization to their *d*-wave counterparts [76,82,83,86,87,117] and in line with the basic theoretical framework for unconventional superconductors [118,119], the anomalous Gor'kov Green's functions for high-T_c oxides are different for angular sectors with CDWs and superconductivity (*d* sections of the FS) and the purely superconducting rest of the FS (nd sections)

$$F_{HTSC,nd}(\mathbf{p};\omega_n)=\frac{\Delta(T)\cos\left[2(\theta-\gamma)\right]}{\omega_n^2+\Delta^2(T)\cos^2\left[2(\theta-\gamma)\right]+\xi_{nd}^2(\mathbf{p})}, \qquad (3)$$

$$F_{HTSC,d}(\mathbf{p};\omega_n)=\frac{\Delta(T)\cos\left[2(\theta-\gamma)\right]}{\omega_n^2+\Delta^2(T)\cos^2\left[2(\theta-\gamma)\right]+\Sigma^2(T)+\xi_d^2(\mathbf{p})}. \qquad (4)$$

Here, we explicitly took into account a possible rotation of the $d_{x^2-y^2}$ angular factor of the superconducting order parameter with respect to **n** due to the misalignment between the grain boundary (the single crystal facet) in operation and the junction plane. The concomitant rotation of the CDW sectors is made allowance for implicitly. The quasiparticle specrtra $\xi_d(\mathbf{p})$ and $\xi_{nd}(\mathbf{p})$ correspond to "hot" and "cold" spots of the cuprate FS, respectively (see, e.g., Refs. [59,120–122]).

Substituting Eqs. (2), (3), and (4) into Eq. (1) and carrying out standard transformations [1,8], we obtain

$$I_c(T)=\frac{\Delta(0)\Delta^*(0)}{2eR_N}i_c(T), \qquad (5)$$

$$i_c(T)=\frac{1}{2\pi}\int_{\theta_d} w(\theta)\cos\left[2(\theta-\gamma)\right]P\left[\Delta^*(T),\sqrt{\Sigma^2+\Delta^2(T)\cos^2\left[2(\theta-\gamma)\right]}\right]d\theta+$$
$$+\frac{1}{2\pi}\int_{\theta_{nd}} w(\theta)\cos\left[2(\theta-\gamma)\right]P\left[\Delta^*(T),\left|\Delta(T)\cos 2(\theta-\gamma)\right|\right]d\theta. \qquad (6)$$

Here, R_N is the normal-state resistance of the tunnel junction, determined by $|T_{\mathbf{pq}}|^2$ without the isolated multiplier $w(\theta)$, the integration is carried out over the CDW-gapped and CDW-free FS sections (the FS-arcs θ_d and θ_{nd}, respectively, in the two-dimentional problem geometry), $\Delta^*(T)$ is the order parameter of the ordinary isotropic superconductor, whereas the function $P(\Delta_1,\Delta_2)$ is given by the expression [91,97]

$$P(\Delta_1,\Delta_2)=\int_{\min\{\Delta_1,\Delta_2\}}^{\max\{\Delta_1,\Delta_2\}}\frac{dx\tanh(x/2T)}{\sqrt{\left(x^2-\Delta_1^2\right)\left(\Delta_2^2-x^2\right)}}. \qquad (7)$$

At $w(\theta)\equiv 1$ (the absence of tunnel directionality), $\Sigma\equiv 0$ (the absence of CDW-gapping), and putting $\cos 2(\theta-\gamma)\equiv 1$ (actually, it is a substitution of an isotropic *s*-superconductor for the *d*-wave one), Equation (6) expectedly reproduces the famous Ambegaokar–Baratoff result for tunneling between *s*-wave superconductors [1,2,8,123].

Note that, in Eq. (6), the directionality is made allowance for only by entering the angular function $w(\theta)$ reflecting the barrier peculiar transparency. On the other hand, the tunneling process should also take into account the factors $|\mathbf{v}_{g,nd}\cdot\mathbf{n}|$ and $|\mathbf{v}_{g,d}\cdot\mathbf{n}|$, responsible for extra directionality [95,100,101], where $\mathbf{v}_{g,nd}=\nabla\xi_{nd}$ and $\mathbf{v}_{g,d}=\nabla\xi_d$ are the quasiparticle group velocities for proper FS sections. Those factors can be considered as proportional to a number of electron attempts to penetrate the barrier [124]. They were introduced decades ago in the general framework of the tunneling in heterostructures [125–127]. Nevertheless, we omitted here the group-velocity-dependent multiplier, since it requires to specify the FS shape, i.e., a superconducting oxide *per se*, thus going beyond our semiphenomenological scheme, as well as beyond similar semiphenomenological approaches of other groups [118,124,128–130]. We shall take it into account in subsequent publications, being fully aware of the phenomenological nature of both $|\mathbf{v}_g \cdot \mathbf{n}|$ and $w(\theta)$ functions.

It should be recognized that, as is well known [131], in the absence of some kind of directionality, the Josephson tunneling between *d*-wave and *s*-wave superconductors is averaged out due to the cosine multiplier in Eq. (6). On the other hand, it was found experimentally that the dc Josephson current between $Bi_2Sr_2CaCu_2O_{8+\delta}$ and Pb [36], $Bi_2Sr_2CaCu_2O_{8+\delta}$ and Nb [132], $YBa_2Cu_3O_{7-\delta}$ and PbIn [133], $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$ and Pb [34] can differ from zero. Hence, either a subdominant *s*-wave component of the superconducting order parameter does exist in cuprate materials, as was discussed above, or the introduction of directionality is inevitable to reconcile any theory dealing with tunneling of quasiparticles from (to) high-T_c oxides and the experiment.

Hereafter, we restrict ourselves to the case $T = 0$, when formula (7) is reduced to elliptic functions [1,134]. Hence, all effects concerning *T*-dependent interplay between Δ and Σ including possible reentrance of $\Sigma(T)$ [76,82,83,86,87,117] will be left for future studies. The main purpose of calculations presented in this paper is to investigate the CDW influence on the angular dependences of the dc Josephson current involving *d*-wave superconductors.

Results and discussion

In Fig. 2, the dependences of $i_c(T = 0)$ on the tilt angle γ are shown for various values of the parameter θ_0 describing the degree of directionality. Since $T = 0$, there is no need to self-consistently solve the equation set for $\Sigma(T)$ and $\Delta(T)$ for partially CDW-gapped *s*-wave [115] or *d*-wave [82] superconductors. Instead, for definiteness, we chose the experimental values $\Sigma(0) = 36.3$ meV and $\Delta(T) = 28.3$ meV appropriate to slightly overdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ samples [135] as input parameters. The half-width of the CDW sectors was rather arbitrarily chosen as $\alpha = 15^\circ$, because it is heavily dependent on the doping extent and cannot be unambiguously extracted even from the most precise angle-resolved photoemission spectra (ARPES) [136,137]. Thus, hereafter we consider α as a phenomenological parameter on the same footing as θ_0. The zero-*T* energy gap $\Delta^*(0)$ for the *s*-wave superconductor was taken as 1.4 meV as was found for Nb [132].

Of course, in the case of *d*-wave pairing, at the complete (90°) or the zero-value spread of the effective angular sector θ_0, the Josephson current disappears, irrespective of which weight function $w(\theta)$ — continuous or steep — is taken into consideration, due to the exactly mutually compensating contributions from superconducting order parameter lobes with different signs [3,128,131]. Intermediate θ_0's correspond to non-zero Josephson tunnel current of either sign (conventional 0-and π-junctions [57]) except at the tilt angle $\gamma = 45^\circ$. In this connection, one should recognize that the energy minimum *for non-conventional anisotropic superconductors* can occur, in principle, at any value of the order parameter phase [138]. The angular behavior of the Josephson current does not depend on the existence of CDWs in cuprates for the actual $d_{x^2-y^2}$-case, in which the bisectrices of CDW sectors and superconducting lobes coincide. For the hypothetical

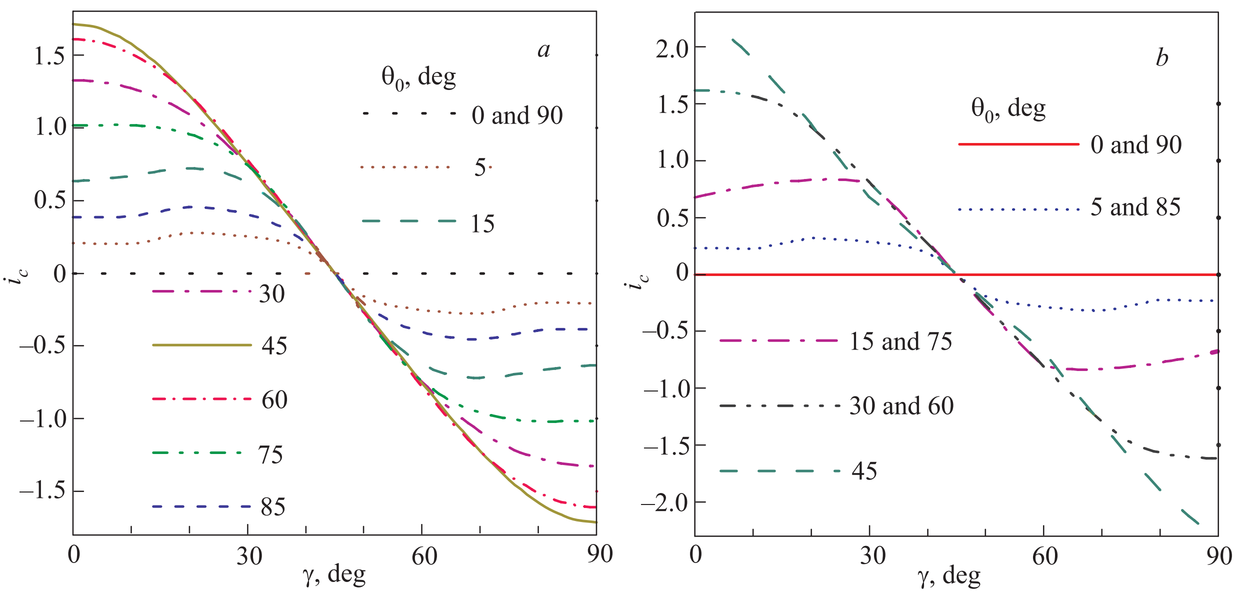

Fig. 2. Dependences on γ of the dimensionless dc Josephson current i_c for the smooth (*a*) and step-like (*b*) models of tunnel directionality at $\alpha = 15°$ and for various θ_0. The zero-*T* values of the gaps are $\Sigma(0) = 36.3$ meV and $\Delta(0) = 28.3$ meV, whatever the symmetry of the oxide superconducting order parameter; $\Delta^*(0) = 1.4$ meV (the specific gap values correspond to the experimental data for $Bi_2Sr_2CaCu_2O_{8+\delta}$ and Nb, respectively). Further explanations see in the text.

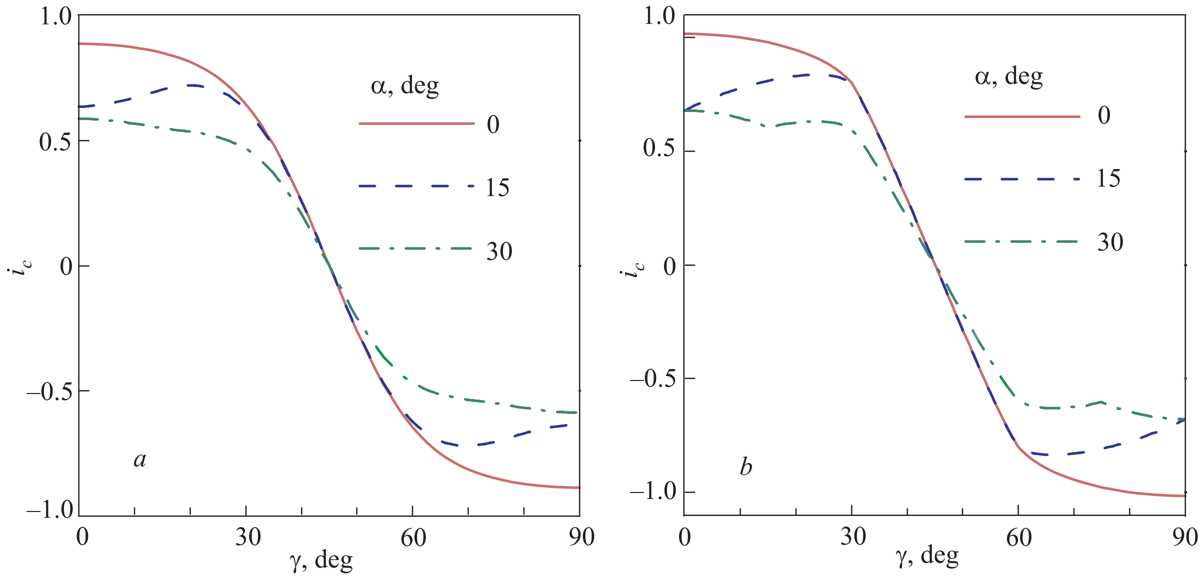

Fig. 3. The same as in Fig. 2, but for $\theta_0 = 15^\circ$ and various α's.

d_{xy} order parameter symmetry, the situation would have been drastically different. Nevertheless, Σ does influence the γ-dependences of i_c. Specifically, when the values θ_0 and α are close, the dependence $i_c(\gamma)$ becomes nonmonotonic, as is seen in Fig. 2. This circumstance may ensure the finding of CDWs (pseudogaps) by a set of relatively simple transport measurements.

One should also pay attention that, in the case of the step-like $w(\theta)$, the plots $i_c(\gamma)$ for paired θ_0-values, which complement each other to 90°, coincide (Fig. 2,*b*). Surely, for the more realistic weight function $w(\theta)$ describing the directionality of tunneling, this equality disappears (Fig. 2,*a*).

The role of CDW (or, equivalently, crystal) orientation with respect to the junction plane (the angle γ) is most clearly seen for varying α, which is shown in Fig. 3. A peculiar "resonance" between θ_0 and α leads to a nonmonotonic behavior of $i_c(\gamma)$, being especially pronounced for a steep type of directionality.

As for the dielectric gapping-degree dependence $i_c(\alpha)$, it is a rapidly dropping one (see Fig. 4 calculated for $\gamma = 0^\circ$), because a widening CDW-induced gap reduces the electron density of states available to the superconducting pairing until α becomes equal to θ_0. A further increase of the pseudogapped FS arc has no influence on i_c, since it falls outside the effective tunneling sector. We note that the α-dependence of i_c for cuprates can be, in principle, nonlinearly mapped onto the doping dependence of the pseudogap [136,137]. It is remarkable that, qualitatively, the results are very similar to those for the assumed *s*-wave order parameter (curves marked by *s*).

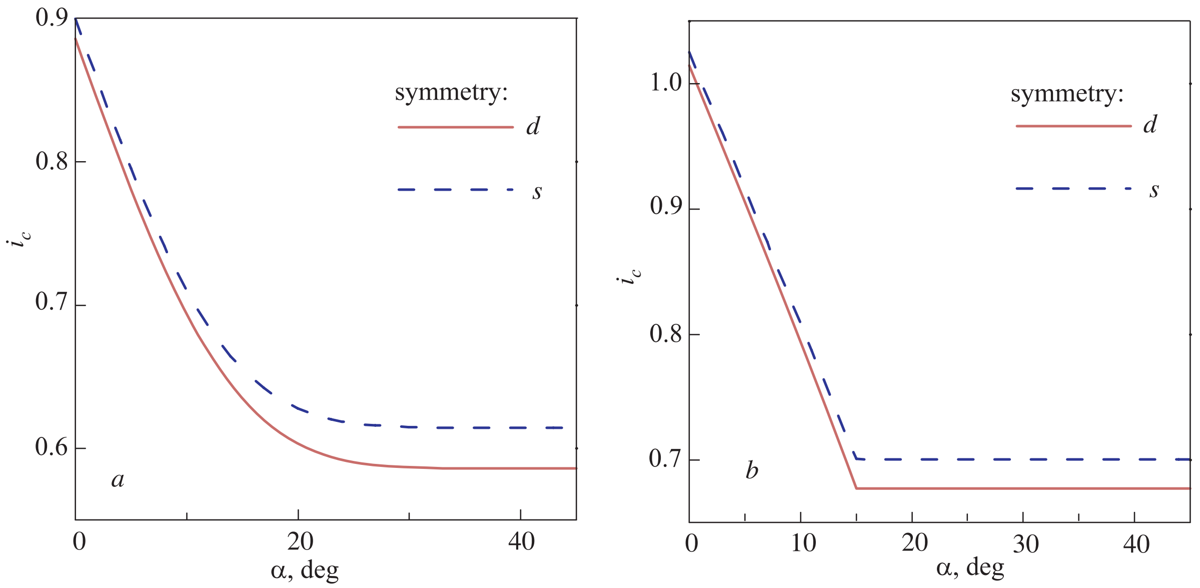

Fig. 4. The dependences $i_c(\alpha)$ for $\gamma = 0$ and $\theta_0 = 15^\circ$ for *d*- and *s*-symmetries of high-T_c oxide order parameter for the smooth (*a*) and step-like (*b*) models of tunnel directionality.

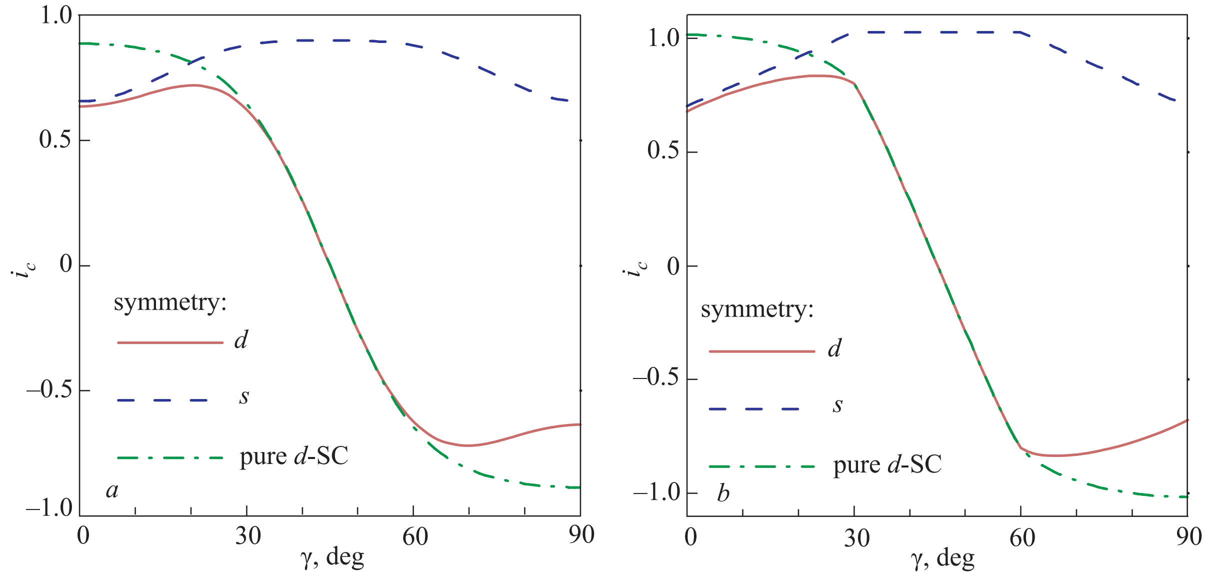

Fig. 5. The same as in Fig. 2, but for $\theta_0 = 15^\circ$ and various symmetries of the superconducting order parameter coexisting with the CDW order parameter (*s* and *d* curves) and for a $d_{x^2-y^2}$-wave superconductor without CDWs (pure *d*-SC curve).

These results confirm that the Josephson current, probing coherent superconducting properties [1–3,6,10,139–141], is always suppressed by the competing electron-hole pairing [97,110,111,142–145]. As for the quasiparticle current, the results are more ambiguos. In particular, the states on the FS around the nodes of the *d*-wave superconducting order parameter are engaged into CDW gapping [76,82,83,86,87,117,146], so that the ARPES or tunnel spectroscopy feels the overall energy gaps being larger than their superconducting constituent.

If one considers possible scenarios for the actual electron spectrum of cuprates, it becomes clear that the emerging CDWs should distort the dependence $i_c(\gamma)$ for cuprates with both *s*- and *d*-wave superconducting order parameters. Indeed, it is the case for the latter, which is seen from a comparison of dependences denoted by *d* and pure *d*-SC in Fig. 5. The pure *d*-SC curves correspond to $\Sigma = 0$ (no CDWs). It is easily seen that for equal (or almost equal) θ_0 and α, CDWs make the $i_c(\gamma)$ curves nonmonotonic and quantatively different from their CDW-free $d_{x^2-y^2}$ counterparts. Namely, i_c values are conspicuously smaller for $\Sigma \neq 0$. The required resonance between θ_0 and α can be ensured by the proper doping, i.e., a series of samples and respective tunnel junctions should be prepared with attested tilt angles γ, and the Josephson current should be measured. Of course, such measurements would be very cumbersome, although they are quite realistic to be performed.

In the case, when an *s*-wave contribution to the actual order parameter in a cuprate sample is dominant up to the complete disappearance of the *d*-wave component, the $i_c(\gamma)$ dependences for junctions involving CDW superconductors should be almost flat with a moderate maximum at $\gamma = 45^\circ$, as is depicted in Fig. 5 (curves marked by *s*). Therefore, the influence of CDW gaps (pseudogaps) on the angular behavior of Josephson current is much stronger for a constituent *d*-wave superconducting electrode than in the set-up, where both electrodes are isotropic.

To summarize, measurements of the Josephson current between an ordinary superconductor and a *d*-wave one (e.g., a high-T_c oxide) would be useful to detect a possible CDW influence on the electron spectrum of the latter. Similar studies of iron-based superconductors with doping-dependent spin density waves (SDWs) would also be of benefit (see, e.g., a recent Review [147]), since CDW and SDW superconductors have similar, although not identical, properties [112–114].

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