

Dynamics of current induced magnetic superstructures in exchange-spring devices

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Thermoelectric manipulation of the magnetization of a magnetic layered stack in which a low-Curie temperature magnet is sandwiched between two strong magnets (exchange spring device) is considered. Controllable Joule heating produced by a current flowing in the plane of the magnetic stack (CIP configuration) induces a spatial magnetic and thermal structure along the current flow — a magnetothermal-electric domain (soliton). We show that such a structure can experience oscillatory in time dynamics if the magnetic stack is incorporated into an electric circuit in series with an inductor. The excitation of these magnetothermionic oscillations follow the scenario either of “soft” or “hard” instability: in the latter case oscillations arise if the initial perturbation is large enough. The frequency of the temporal oscillations is of the order of 10^5 – 10^7 s⁻¹ for current densities $j \sim 10^6$ – 10^7 A/cm².

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75.70.-i Magnetic properties of thin films, surfaces, and interfaces.

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Low-temperature magnetism and low-temperature transport in solids are two scientific fields in which the B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine (ILTPE) has gained worldwide recognition as a result of its many important contributions and groundbreaking discoveries.

The former director of ILTPE, Academician Viktor Valentinovich Eremenko — the founding father of low-temperature magnetic research at ILTPE — is now 80 years old. This paper, which is devoted to extending both of the mentioned branches of physics towards nanometre-sized conductors, is our modest contribution to the celebration of V.V. Eremenko's eightieth jubilee — a celebration of a truly great person and great scientist.

1. Introduction

The electric control of magnetization on the nanometer length scale became a subject of intensive study after the seminal suggestion by Slonzewsky [1,2] to use the spin torque transfer (STT) technique, where a current of spin-polarized electrons is injected into a magnetic material. Even though the magnetic tuning induced by the exchange interaction between the injected electrons and those that make the material magnetically ordered can only be achieved in a very small region, large current densities are needed to get a significant STT effect. Such current densities can be reached in electric point contacts where values of the order of 10^8 – 10^9 A/cm² have been achieved and the STT effect was detected [3,4]. By a further increase of

the current thermal Joule heating of the material becomes unavoidable. However, the fact that such heating can be precisely controlled by the bias voltage allows thermoelectrical manipulation of the magnetization direction based on the orientational phase transition in layered magnetic structures predicted in Refs. 5–7. It was shown there that in such structures with different Curie temperatures of the layers there can exist a finite temperature interval inside which the angle of the relative orientation of the layer magnetization Θ depends on temperature, $\Theta = \Theta(T)$, changing reversibly with a change of temperature in the whole interval of angles from the parallel to the antiparallel orientation. As a result, by controlling the Joule heating by the bias voltage V one may smoothly and reversibly control the relative magnetization angle in the whole angle interval $0 \leq \Theta [T(V)] \leq \pi$. There was also suggested a thermal-electronic oscillator device based on this magnetothermal effect in magnetic exchange spring structures.

In paper [8], for exchange-spring magnetic stacks [9] with an in-plane electric current flow (CIP) (see Fig. 1), it was predicted and investigated a possibility of generation of magnetic spatial superstructures (domains and solitons) the appearance of which and their parameters can be controlled by the bias voltage. In the present paper we study temporal dynamics of such spatial structures in the case that the exchange-spring magnetic stack is incorporated in an external circuit in series with an inductor [10]. Such oscillations can be understood as a realization of the bistability of an N-shaped current voltage (IV) curve of the device in the regime at which a large enough inductance of the circuit determines slow temporal variations of the current.

The structure of the paper is as follows. In Sec. 2 we present in short some peculiar features of the dependence

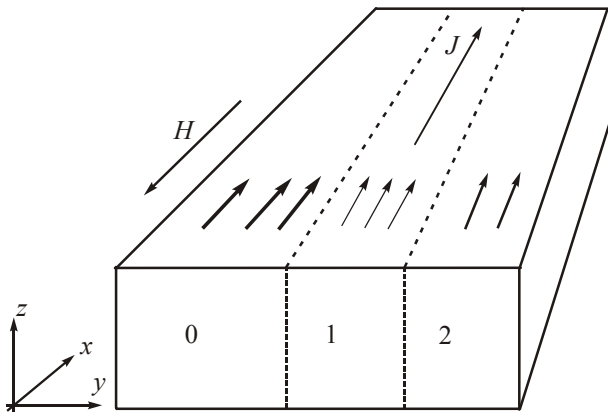


Fig. 1. Orientation of the magnetic moments in a stack of three ferromagnetic layers (0, 1, 2); the magnetic moments in layers 0, 1, and 2 (shown with short arrows) are coupled by the exchange interaction thus building an exchange spring tri-layer; H (shown with the arrow outside the stack) is an external magnetic field directed antiparallel to the magnetization in layer 0. The current J (shown with the long arrow) flows in the layer plane (that is along the x axis).

of the magnetization orientation angle in the stack on temperature T and some properties of the magnetic stack with a magnetic thermal-electric domain (MTED) inside it needed for further considerations. In Sec. 3 we show that MTED loses its stability if the stack is incorporated in an external circuit with a large enough inductance. We develop there an adiabatic theory which allows an analytical description of the MTED dynamics.

2. The magnetic-thermal-electric domain in the stack under Joule heating

The system under consideration has three ferromagnetic layers in which two strongly ferromagnetic layers 0 and 2 are exchange coupled through a weakly ferromagnetic spacer (layer 1) as is shown in Fig. 1. We assume that the Curie temperature $T_C^{(1)}$ of layer 1 is lower than the Curie temperatures $T_C^{(0,2)}$ of layers 0, 2; we also assume the magnetization direction of layer 0 to be fixed; this stack is under an external magnetic field H directed opposite to the magnetization of layer 0. We require this magneto-static field to be weak enough so that at low temperatures T the magnetization of layer 2 is kept parallel to the magnetization of layer 0 due to the exchange interaction between them via layer 1. At $H = 0$ and $T > T_C^{(1)}$ this tri-layer is similar to the spin-flip “free layer” widely used in memory device application [11]. The stack is incorporated into an external circuit along which the total current J flows through the cross-section of the layers:

$$J = \left[\frac{1}{R(\Theta)} + \frac{1}{R_0} \right] V \quad (1)$$

where $R(\theta)$ and R_0 are the magnetoresistance and the angle-independent resistance of the stack, Θ is the angle between the magnetization directions of layers 0 and 2, V is the voltage drop across the stack.

In paper [6], it was shown that parallel orientations of the magnetization in layers 0, 1 and 2 becomes unstable if the temperature exceeds some critical temperature $T_C^{(or)} < T_C^{(1)}$. The magnetization direction in layer 2 smoothly tilts with an increase of the stack temperature T in the temperature interval $T_C^{(or)} \leq T \leq T_C^{(1)}$. The dependence of the equilibrium tilt angle Θ between the magnetization directions of layers 0 and 2 on T and the magnetic field H is determined by the equation [6]

$$\Theta = D(H, T) \sin \Theta, \quad T < T_C^{(1)}$$

$$\Theta = \pm \pi, \quad T \geq T_C^{(1)} \quad (2)$$

where

$$D(H, T) = \frac{L_1 L_2 H M_2(T)}{4 \alpha_1 M_1^2(T)} \approx D_0(H) \frac{T_C^{(1)}}{T_C^{(1)} - T} \quad (3)$$

and

$$D_0(H) = \frac{\mu_B H}{k_B T_C^{(1)}} \left(\frac{L_1}{a} \right) \left(\frac{L_2}{a} \right). \quad (4)$$

Here $L_1, M_1(T)$ and $L_2, M_2(T)$ are the widths and the magnetic moments of layers 1 and 2, respectively; $\alpha_1 \sim I_1 / a M_1^2(0)$ is the exchange constant, I_1 is the exchange energy in layer 1, μ_B is the Bohr magneton, k_B is Boltzmann's constant, and a is the lattice spacing. $D(H, T)$ is a dimensionless parameter that determines the efficiency of the external magnetic in the misorientation effect under the consideration. It is a ratio between the energy of magnetic layer 2 in the external magnetic field and the energy of the indirect exchange between layers 0 and 2 (see Fig. 1).

The critical temperature of the orientation transition [12] $T_C^{(or)}$ is determined by the condition $D(T) = 1$ and is equal to

$$T_C^{(or)} = T_C^{(1)} \left(1 - \frac{\delta T}{T_C^{(1)}} \right), \quad \frac{\delta T}{T_C^{(1)}} = D_0(H). \quad (5)$$

Taking experimental values (see Ref. 6) $L_1 = 30$ nm, $L_2 = 12$ nm, $T_C^{(1)} = 373$ K, and magnetic field $H = 10\text{--}47$ Oe one finds $D_0 = 0.1\text{--}0.36$ and the temperature interval $\delta T = T_C^{(1)} - T_C^{(or)} = D_0 T_C^{(1)} \approx 37.3\text{--}134$ K.

If the stack is Joule heated by current J its temperature $T(V)$ is determined by the heat-balance condition

$$JV = Q(T), \quad J = V / R_{\text{eff}}(\Theta), \quad (6)$$

where

$$R_{\text{eff}}(\Theta) = \frac{R(\Theta)R_0}{R(\Theta) + R_0}, \quad (7)$$

and Eq. (2) which determines the temperature dependence of $\Theta(T)$. Here V is the voltage drop across the stack, $Q(T)$ is the heat flux flowing from the stack and $R_{\text{eff}}(\Theta)$ is the total stack magnetoresistance. Here and below we neglect the explicit dependence of the magnetoresistance on T considering the main mechanism of the stack resistance to be the elastic scattering of electrons by impurities. Below we drop the subscript ‘‘eff’’ at the magnetoresistance symbol R_{eff} . On the other hand, we consider the temperature changes caused by the Joule heating only in a narrow vicinity of $T_C^{(1)}$ which is sufficiently lower than both the critical temperatures $T_C^{(0,2)}$ and the Debye temperature.

Equations (6) and (2) define the current-voltage characteristics (IVC) of the stack

$$J_0(V) = \frac{V}{R[\Theta(V)]}, \quad (8)$$

where $\Theta(V) \equiv \Theta[T(V)]$. The differential conductance $G_{\text{diff}} \equiv dJ/dV$ is negative [6] if

$$\frac{d}{d\Theta} \frac{(1 - \bar{D} \sin \Theta / \Theta)}{R(\Theta)} < 0 \quad (9)$$

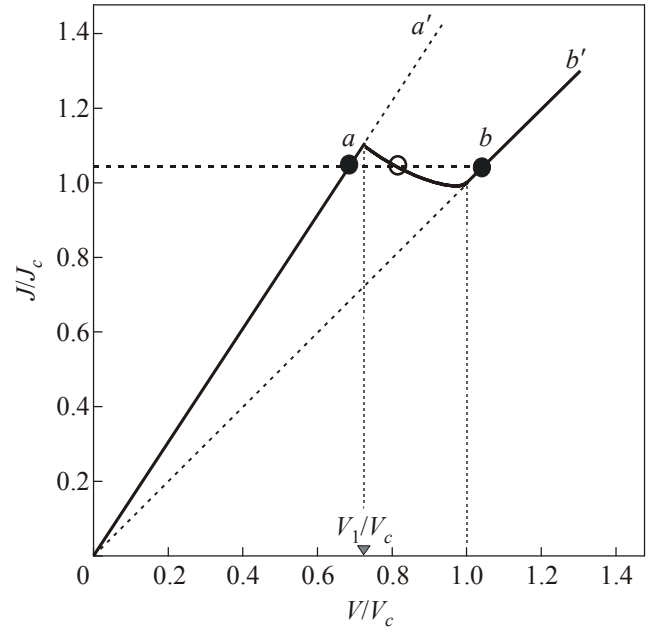


Fig. 2. Current-voltage characteristics of the magnetic stack of Fig. 1 in which the magnetization directions in the layers are homogeneously distributed along the x direction (that is along the stack). It was calculated for $R(\Theta) = R_+ - R_- \cos \Theta$, $R_-/R_+ = 0.2$, $D_0 = 0.2$; $J_c = V_c/R(\pi)$. The branches $0\text{--}a$ and $b\text{--}b'$ of the IVC correspond to parallel and antiparallel orientations of the stack magnetization, respectively (the parts $a\text{--}a'$ and $0\text{--}b$ are unstable); the branch $a\text{--}b$ corresponds to $0 \leq \Theta[T(V)] \leq \pi$.

where $\bar{D} = (T/Q)(dQ/dT)D_0 \approx D_0$. In this case the IVC of the stack is N-shaped as is shown in Fig. 2.

In paper [8] it was shown that a homogeneous in space distribution of the magnetization direction, temperature and electric field along the spring-type magnetic stack becomes unstable and a MTED spontaneously arises [14] if the electric current flows in the plane of the layers (CIP-configuration) and the length of the stack exceeds the critical length

$$L_c \approx \sqrt{\frac{\kappa T_C^{(1)}}{r \rho(0) j^2}} \quad (10)$$

where κ is the thermal conductivity, $\rho(\Theta)$ is the magnetoresistivity and $r = (\rho(0) - \rho(\pi)) / (\rho(0) + \rho(\pi))$. Parameters of the MTED which arises in the stack, depend on the stack length and the voltage drop across it that allows to create and control magnetic structures in magnetic devices. Using Eq. (10) and the Lorentz relation $\kappa/\sigma = \pi^2 k_B^2 T / 3e$ ($\sigma = \rho^{-1}$ and e is the electron charge) one finds that

$$L_c \sim \frac{\pi k_B T_C^{(1)}}{\sqrt{r} e \rho(0) j} \sim 10 \mu\text{m} \quad (11)$$

for a realistic experimental situation [26] $r \sim 0.1\text{--}0.3$, $T_C^{(1)} \sim 10^2$ K, $\rho(0) \sim 10^{-5} \Omega\text{-cm}$, $j \sim 10^6\text{--}10^7$ A/cm².

The magnetic-thermal-electric domain which determines the space distributions of the temperature $T_d(x)$ and

is the characteristic evolution time of the temperature [30]. Therefore, for the case $L \gg L_c$ this instability develops under conditions that the characteristic time of the current evolution $\tau_L \sim L/R$ is much longer than the temperature evolution time τ_T (their ratio is $\tau_L/\tau_T \sim (L/L_c) \exp L/L_c$). Then any small fluctuation develops in such a way that after a time $t \gg \tau_T$ the domain recovers its initial trapezoidal form and only its length L_{Π} varies slowly with time t . This fact makes it possible to obtain a reduced description of the nonlinear dynamics of a MTED as follows. We seek the solution of the first equation in Eq. (16) in the form

$$T(x,t) = T_d(x, j_d(t)) + T_1(x,t) \quad (18)$$

where $T_d(x, j_d)$ is the solution of the equation which is obtained from Eq. (12) by the substitution $j \rightarrow j_d(t)$; the parameter j_d governing the length of the domain $L_{\Pi}(j_d)$ is a slow function of the time t which has to be found; $T_1(x,t)$ is a correction which is small with respect to T_d . Inserting Eq. (18) into the first equation in Eq. (16) and linearizing it one gets the following equation for T_1 :

$$\hat{H}T_1 = -c_V \frac{\partial T_d}{\partial j_d} \frac{dj_d}{dt} + (j^2 - j_d^2)\rho(T_d) \quad (19)$$

where \hat{H} is a Hermitian operator with a periodic boundary condition:

$$\hat{H} = -\kappa \frac{d^2}{dx^2} + \left(-j_d^2 \rho'_T(T_d) + Q'_T(T_d) / \Omega_{st} \right) \quad (20)$$

(here $\rho(T) \equiv \rho[\Theta(T)]$ and $F'_T = dF/dT$). The requirement of the smallness of the ratio $|T_1/T_d| \ll 1$ allows to find the needed equations describing the adiabatic MTED evolution.

The solution of Eq. (19) is found in the form of expansion

$$T_1(x,t) = \sum_v A_v(t) \Psi_v(x) \quad (21)$$

where Ψ_v are the eigenfunctions of \hat{H} which satisfy the Sturm–Liouville equation

$$\left[-\kappa \frac{d^2}{dx^2} - j_d^2 \rho'_T(T_d) + Q'_T(T_d) / \Omega_{st} \right] \Psi_v = \lambda_v \Psi_v. \quad (22)$$

Multiplying the both sides of Eq. (19) by Ψ_v from the left and averaging over the period L one finds

$$\lambda_v A_v = \left\langle \left(-c_V \frac{\partial T_d}{\partial j_d} \frac{dj_d}{dt} + (j^2 - j_d^2)\rho(T_d) \right) \Psi_v \right\rangle \quad (23)$$

where $\partial T_d / \partial j_d$ in terms of eigenfunctions Ψ_v is presented in Appendix A Eq. (A.9).

As the eigenvalue λ_0 is exponentially small (see Eq. (A.5)), from Eq. (19) it follows that the requirement $|T_1/T_d| \ll 1$ is satisfied only when the right-hand side of the equation with $v = 0$ is equal to zero:

$$\left\langle \left(-c_V \frac{\partial T_d}{\partial j_d} \frac{dj_d}{dt} + (j^2 - j_d^2)\rho(T_d) \right) \Psi_0 \right\rangle = 0. \quad (24)$$

Using Eq. (A.3) one sees that the right-hand side of the equation (22) with $v = 1$ is equal to zero and hence the factor $\lambda_1 = 0$ in the left-hand side of it does not violate the above-mentioned requirement.

Taking into account the second equation in Eq. (16) and Eq. (24) together with Eqs. (A.8), (A.11) one finds a set of equations which describes the temporal dynamics of a MTED:

$$\begin{aligned} \frac{dj_d}{dt} &= \omega_0(j_d) \frac{j^2 - j_d^2}{2j_0}, \\ \bar{L} \frac{dj}{dt} &= V/L - \langle \rho(T_d(x, j_d)) j \rangle. \end{aligned} \quad (25)$$

Here $\bar{L} = S L/L$ and $\omega_0 = -\lambda_0/c_V$ (here λ_0 is defined by Eq. (A.11)) may be written as

$$\omega_0^{-1}(j) = -\tau_0 \frac{2r}{\rho_+} \frac{d\mathcal{E}_d}{dj} \quad (26)$$

where the constant τ_0 is

$$\tau_0 = \frac{c_V L^2 \langle (dT_d/dx)^2 \rangle}{8j^2 \int_{T_{\min}}^{T_{\max}} \rho(T) dT} \sim \tau_T, \quad (27)$$

being of the order of the characteristic evolution time of the temperature τ_T , and \mathcal{E}_d is defined in Eq. (A.6).

Throughout the range of the existence of a trapezoidal MTED one has $|j - j_0| \ll j_0$ and the set of equations Eq. (25) can be reduced to an equation for the electric field which is coupled to the current j_d via the current voltage characteristic $\mathcal{E} = \mathcal{E}_d(j_d)$ (see Eqs. (13), (A.6)). Differentiating the first equation in Eq. (25) multiplied by ω_0 with respect to t , and inserting in the resulting expression the above-mentioned electric field \mathcal{E} as a new variable together with the second equation in Eq. (25) one obtains the following equation in the form of a linear oscillator with a nonlinear nonconservative term:

$$\ddot{\mathcal{E}} + \left(\frac{\mathcal{E}}{\bar{L}j_0} + \frac{\rho_+}{\tau_0} \frac{dj_d}{d\mathcal{E}} \right) \dot{\mathcal{E}} + \frac{\rho_+}{\bar{L}\tau_0} (\mathcal{E} - V/L) = 0. \quad (28)$$

Here $j_d(\mathcal{E})$ is the IVC of the magnetic stack with a MTED.

The static points of Eq. (28) and the second equation in Eq. (25) correspond to the domain solution $\mathcal{E} = V/L$ and $j = j_d(V/L)$. As $dj_d/d\mathcal{E} < 0$, in the range of inductance $\bar{L} > \bar{L}_{c1}$ the factor in front of $\dot{\mathcal{E}}$ is negative and a domain is absolutely unstable. Here the critical inductance is

$$\bar{L}_{c1} \approx -\tau_0 \left(\frac{dj_d}{d\mathcal{E}} \right)_{\mathcal{E}=\mathcal{E}_{\inf}}^{-1} \sim \tau_0 \rho_+ \exp(L/L_c) \quad (29)$$

where \mathcal{E}_{\inf} is the electric field at which $-dj_d/d\mathcal{E}$ has a maximum that is at which $d^2j_d/d\mathcal{E}^2 = 0$. In this region

the voltage drop across the magnetic stack $V(t) = \mathcal{E}(t)L$ oscillates and hence the length of the domain $L_{\text{II}}(t)$ (see Eq. (14)) oscillates with an amplitude that increases in time. At the time when L_{II} reaches either values $0, L$ (depending on the initial fluctuation) the domain disappears and the sample becomes homogeneous with temperatures T_{min} or T_{max} , respectively. Further temporal evolution of the system is described by the set of equations Eq. (16) an analysis of which for the case $\tau_T/\tau_L \ll 1$ shows that the system exhibits stable large-amplitude spontaneous oscillations of the temperature T , current J , magnetization direction $\Theta(T)$, and the voltage drop across the stack $\tilde{V} = JR[\Theta(T)]$, the oscillations being the same as those predicted in Ref. 6 for a magnetic stack in the absence of MTED.

Investigation of stability of the domain solution of Eq. (28) with the Poincare method (see, e.g., Ref. 32) shows that at $\delta\mathcal{L} = \mathcal{L}_{c1} - \mathcal{L} \ll \mathcal{L}$ there is an unstable limiting cycle in the phase plane $(\mathcal{E}, \dot{\mathcal{E}})$ the radius of which $K \propto \sqrt{\delta\mathcal{L}/\mathcal{L}_{c1}}$ and hence it increases with a decrease of the inductance. From here and from the fact that at $\mathcal{L} = 0$ the domain is absolutely stable follows an existence of a second critical value of the inductance \mathcal{L}_{c2} which limits the interval of the hard excitation of oscillations. There-

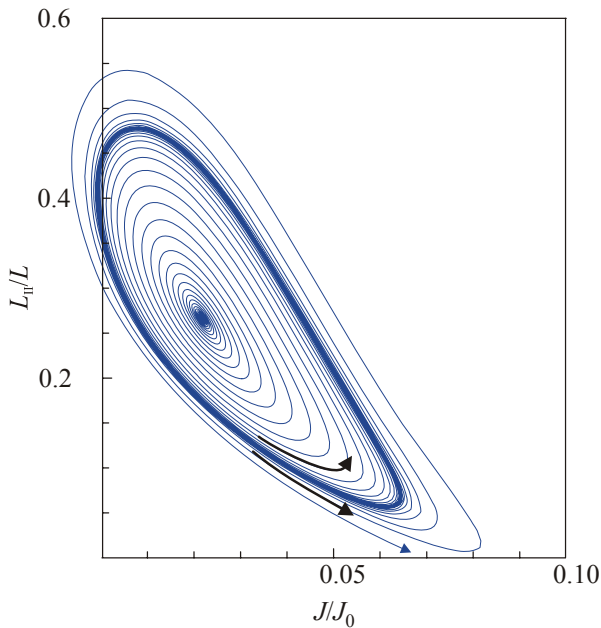


Fig. 4. Temporal evolution of the length of MTED $L_{\text{II}}(t)$ for the case $\mathcal{L}_{c1} > \mathcal{L} > \mathcal{L}_{c2}$. The unstable limiting cycle (shown with a thick line) separate the phase plane into two regions: any initial state inside the limiting cycle develops into the length of the steady stationary MTED (shown with a dot) while an initial state outside it results in oscillations of the MTED length with an increasing in time amplitude until the MTED disappears, that is L_{II} reaches either $L_{\text{II}} \approx 0$ or $L_{\text{II}} \approx L$. Calculations are made for $R_-/R_+ = 0.2$, $D_0 = 0.2$ and $\tau_T/\tau_L = 0.1$ where τ_T and τ_L are the characteristic times of the temperature and current developments, respectively; J_0 is the stabilization current.

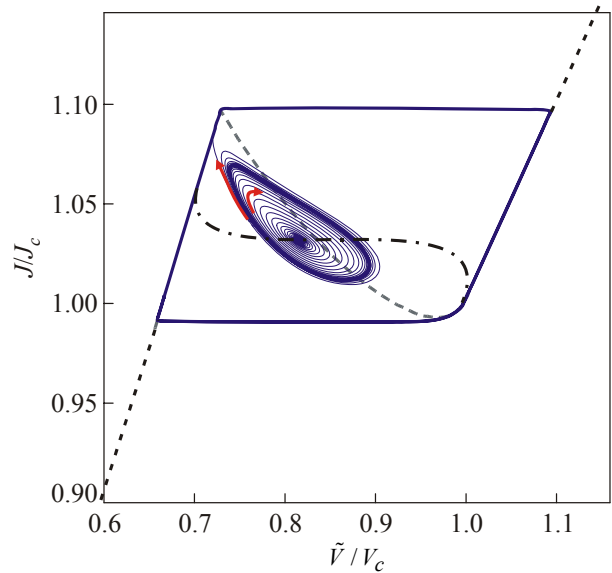


Fig. 5. Temporal evolution of the current $J(t)$ and voltage drop $\tilde{V}(t)$ in a MTED inside it for the case $\mathcal{L}_{c1} > \mathcal{L} > \mathcal{L}_{c2}$. The steady IVC of a homogeneous stack IVC with a MTED are shown with a dashed and dashed-dotted lines, respectively. The stack is in a bistable state: depending on the initial conditions the system goes either to a stable steady MTED (which is shown with a dot) or goes to a stable limiting cycle (the largest closed curve) corresponding to spontaneous oscillations of the current $J(t)$, voltage drop $\tilde{V}(t)$, temperature $T(t)$, and the magnetization direction $\Theta(t)$, the stack being in a homogeneous state (in which the MTED has disappeared). There is an unstable limiting cycle that separates the initial states which develop either to the MTED or the the oscillations as is shown by two black arrows. Calculations are made for the same parameters, as on the Fig. 4.

fore, the range of values of the inductance $\mathcal{L}_{c1} > \mathcal{L} > \mathcal{L}_{c2}$ is the region in which a stable MTED, an unstable limiting cycle, and a stable limiting cycle coexist. For this case, temporal developments of the MTED length L_{II} , current J and voltage drop \tilde{V} across the magnetic stack are shown in Fig. 4 and Fig. 5.

4. Conclusion

We have considered the temporal evolution of magneto-thermo-electric domains which spontaneously arise in an exchange-spring magnetic stack with the electrical current flowing along the layers (CIP configuration). For the case that such a stack is incorporated in an electric circuit in series with an inductor we have found critical values of the inductance at which the magneto-thermal domain loses its stability. We have developed a perturbation theory in an adiabatic parameter (which is the ratio between the temperature and current characteristic evolution times) theory which allows to investigate the evolution of this instability into an oscillatory in time regime. The excitation of such magneto-thermionic oscillations follow the scenario of either “soft” of a “hard” instability; in the former case any

perturbation results in a spontaneous transition to the oscillatory regime while in the latter case oscillations only appear if the initial perturbation is large enough. The frequency of the temporal oscillations of the magnetization direction in the stack is of the order of 10^5 – 10^7 Hz.

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Appendix A

According to Ref. 8 $T_d(x)$ can be written as

$$T_d(x) = \vartheta(x+x_+) + \vartheta(x_+-x) - T_{\max} \quad (\text{A.1})$$

where the function $\vartheta(x)$ is a domain-wall type solution of Eq. (12) at $j = j_0$ and $L \rightarrow \infty$ which has the following asymptotic

$$\begin{aligned} \lim_{x \rightarrow -\infty} \vartheta(x) &= T_{\min} \\ \lim_{x \rightarrow \infty} \vartheta(x) &= T_{\max}. \end{aligned} \quad (\text{A.2})$$

Here $\pm x_+$ are the points of deflections of the curve $dT_d(x)/dx$ so that $L_{\text{II}} = x_+ - (-x_+) = 2x_+$ is approximately the length of the “hot” section of a trapezoidal MTED having the maximal temperature T_{\max} and $L_{\text{I}} = L - L_{\text{II}}$ is the length of its “cold” section having the minimal temperature T_{\min} (see Fig. 3).

As one sees from Eq. (A.1) the function

$$U(x) = -j_d^2 \rho'_T(T_d) + Q'_T(T_d) / \Omega_{\text{st}}$$

in equation (22) consists of two symmetrical “potential” wells of the width $\sim L_c$, separated by a barrier of the width of order of L_{II} . In this case (see, e.g., Ref. 31) tunneling between the wells results in a splitting of each of the eigenvalues of a completely separated well into two neighboring ones, the splitting being proportional to $\exp(-L_{\text{II}}/L_c)$, while the corresponding eigenfunctions are symmetric and antisymmetric combinations of the corresponding eigenfunctions of the separated left and right wells.

From the translation symmetry of the time independent equation Eq. (12) it follows that the eigenfunctions of the Hermitian operator \hat{H} include the function $\Psi_1 = dT_d/dx$ corresponding to the eigenvalue $\lambda_1 = 0$ (it is easy to check inserting this function in the Eq. (22)). As it follows from Eq. (A.1) the eigenfunction $\Psi_1(x)$ is

$$\Psi_1(x) = \frac{dT_d}{dx} = \frac{d\vartheta(x+x_+)}{dx} + \frac{d\vartheta(x_+-x)}{dx} \quad (\text{A.3})$$

where two functions in the right-hand side are eigenfunctions of the left and right wells when the tunneling is ignored. This is an antisymmetric eigenfunction ($\Psi(-x) = -\Psi(x)$) corresponding to the eigenvalue $\lambda_1 = 0$ and hence the

nearest eigenvalue λ_0 is negative and the corresponding eigenfunction is symmetric:

$$\Psi_0(x) = \frac{d\vartheta(x+x_+)}{dx} - \frac{d\vartheta(x_+-x)}{dx} \quad (\text{A.4})$$

while

$$\lambda_0 \propto -\kappa \left(\frac{d\vartheta(x+x_+)}{dx} \frac{d^2\vartheta(x+x_+)}{dx^2} \right) \Big|_{x=0} \propto \exp(-L/L_c). \quad (\text{A.5})$$

To express the eigenvalue λ_0 in terms of the differential resistivity we use the following reasoning.

According to Eq. (13) the current voltage characteristic of the stack with a MTED in it is

$$\mathcal{E}_d(j) = j \langle \rho[\Theta(T_d(j))] \rangle. \quad (\text{A.6})$$

Differentiating the both sides of Eq. (35) and Eq. (12) with respect to j one finds the differential resistivity of the magnetic stack with a MTED as

$$\frac{d\mathcal{E}_d}{dj} = \langle \rho(T_d) \rangle + \left\langle \rho'_T(T_d) \frac{\partial T_d}{\partial j} \right\rangle. \quad (\text{A.7})$$

and an equation for $\partial T_d / \partial j$

$$\hat{H} \frac{\partial T_d}{\partial j} = 2j \langle \rho(T_d) \rangle \quad (\text{A.8})$$

where the operator \hat{H} is given by Eq. (20).

Expanding $\partial T_d / \partial j$ in the form of the eigenfunctions Ψ_v (see Eq. (22)) one finds

$$\frac{\partial T_d}{\partial j} = 2j \sum_v \frac{\langle \rho(T_d) \Psi_v \rangle}{\lambda_v} \Psi_v. \quad (\text{A.9})$$

Inserting it in Eq. (A.7) one finds

$$\begin{aligned} \frac{d\mathcal{E}_d}{dj} &= \langle \rho(T_d) \rangle + \\ &+ 2j_d^2 \sum_v \frac{\langle \rho'_T(T_d) \Psi_v \rangle \langle \rho(T_d) \Psi_v \rangle}{\lambda_v}. \end{aligned} \quad (\text{A.10})$$

Remembering that λ_0 is exponentially small and hence it gives the main contribution to the sum with respect to v one finds

$$\lambda_0 = \frac{16r(j_d^2 \rho_+) \int_{T_{\min}}^{T_{\max}} \rho[\Theta(T)] dT}{L^2 \langle (dT_d/dx)^2 \rangle d\mathcal{E}_d/dj}. \quad (\text{A.11})$$

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