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Spin-orbit interaction in the supersymmetric antiferromagnetic *t*-*J* chain with a magnetic impurity

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The effect of spin-orbit interaction in the strongly correlated exactly solvable electron model with magnetic impurity is studied. The considered magnetic impurity reveals the property of a “mobile” one. It is shown that the asymptotics of correlation functions, calculated in the framework of the conformal field theory and finite size corrections of the Bethe ansatz exact solution, are strongly affected by both, the spin-orbit coupling, and by the magnetic impurity.

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71.10.Pm Fermions in reduced dimensions (anyons, composite fermions, Luttinger liquid, etc.);
75.10.Pq Spin chain models;
75.70.Tj Spin-orbit effects.

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1. Introduction

Recently the interest in systems with the spin-orbit interaction has been grown. For example, the spin-orbit interaction manifests itself in the effect of an electric field on a moving particle with spin. Such interest is connected with spintronics, where the spin of electrons in electronic devices has to be manipulated and detected. In many devices of spintronics an interaction between electrons also has to be taken into account, hence, it is very important to investigate the effects of the spin-orbit coupling together with the antiferromagnetic spin-spin interaction between spin of electrons. In low-dimensional semiconductor structures the internal electric field may exist [1] so that the spin-orbit coupling takes place even without external electric fields. Recently systems, where the spin-orbit interaction plays the crucial role in low-dimensional electron systems, like edge or surface states of topological insulators [2], or semiconductor nanowires [3] have attracted much attention of researchers.

The strong electron–electron coupling can essentially change the properties of electron systems with spin-orbit interaction, especially in low-dimensional electron systems, where fluctuations are strongly enhanced due to the peculiarities in the density of states, and, hence, exact theoretical results are very important. On the other hand, magnetic impurities affect the behavior of electron systems, the famous Kondo effect is the prime example [4].

The aim of the present work is twofold. First, we want to study the effect of the spin-orbit coupling and electron–electron interaction in a low-dimensional correlated electron system. Second, we decide to investigate how the magnetic impurity can affect the exponents for correlation functions in such a strongly correlated electron chain. As the model for studying we choose the model, which, on the one hand, has both of these features, i.e., it has spin-orbit and electron–electron interaction, and the magnetic impurity can change the properties of such a system considerably. On the other hand, that model permits to obtain an exact solution. Exact solutions are rare, and, some of the features of exactly solvable Hamiltonians are not very realistic, however it is very important to obtain results for exactly solvable models, because they give the only opportunity to check the validity of the results of approximate methods applied to more realistic models. Also, sometimes the results of exactly solvable models can be directly applied to real systems, systems of ultracold atoms being the prime example of such an application [5] (for the recent review, use, e.g., Ref. 6). It is important to add that the study of artificially designed spin-orbit interaction in ultracold atomic gases, mimicking effects from condensed matter physics, is now coming of age, making our research timely.

In this paper, using the exact Bethe ansatz method and the conformal field theory technique we have studied the properties of a strongly correlated electron chain, the supersymmetric *t-J* model with spin-orbit interaction. We show that the spin-orbit coupling manifests itself mostly in finite-size effects there. However, those finite-size effects determine the values of critical exponents for the considered system. In particular, we show that the spin-orbit coupling strongly renormalizes exponents for density-density, field-field, spin-spin, and spin-singlet and spin-triplet pair correlation functions of the considered chain. Also, we show that the features of the exactly solvable magnetic impurity provide influence of the impurity on exponents of the correlation functions, which does not depend on the position of the impurity. This feature is caused by the non-reflective character of the integrable impurity. Our investigation shows that the impurity can drastically change the values of critical exponents for correlation functions in the correlated electron chain with the internal spin-orbit interaction.

2. The model

The hopping of the one-dimensional (1d) tight-binding electron system in the presence of the internal spin-orbit interaction can be written in the form [7] $\sum_{j,\sigma,\sigma'} t' S_{\sigma,\sigma'}(\psi^{\dagger}_{j+1,\sigma}\psi_{j,\sigma'} + \text{h.c.})$, where $S_{\sigma,\sigma'}$ is the SU(2)-symmetric matrix. This hopping term can be transformed to the diagonal term by a unitary transformation [7], which rotates the spin space. The matrix $S_{\sigma,\sigma'}$ is a unitary one, thus its eigenvalues can be presented as exponentials $\exp(\pm i\lambda)$. Parameters t' and $S_{\sigma,\sigma'}$ can be site-dependent, however, the diagonalization of the hopping term is possible in that case, too, cf. Ref. 7. This transformation can be also applied for any form of the electron–electron interaction, if the latter respects SU(2) symmetry. In what follows we will essentially use that property.

Let us start with the consideration of the 1d lattice version of the electron gas with the internal spin-orbit interaction, the Hamiltonian of which can be written as [7]

$$\mathcal{H}_0 = \sum_{j,\sigma}[t'(\psi^{\dagger}_{j+1,\sigma}\psi_{j,\sigma} + \text{h.c.}) + \alpha\sigma(i\psi^{\dagger}_{j+1,\sigma}\psi_{j,\sigma} + \text{h.c.}) - \mu n_{j,\sigma}] = \sum_{j,\sigma}[\sqrt{(t')^2+\alpha^2/4}(\psi^{\dagger}_{j+1,\sigma}\psi_{j,\sigma}e^{i2\pi\sigma^z\varphi} + \text{h.c.}) - \mu n_{j,\sigma}], \quad (1)$$

where $\psi^{\dagger}_{j,\sigma}$ creates the electron with the spin projection $\sigma = \pm 1/2$ at the lattice site j ($j = 1,\ldots,L$, L is the number of sites), t' is the hopping integral, α is the parameter of the spin-orbit coupling (see, e.g., Refs. 2, 3), and $n_{j,\sigma} = \psi^{\dagger}_{j,\sigma}\psi_{j,\sigma}$. Then the "usual" spin-independent hopping has the magnitude $t\cos(2\pi\varphi)$, and the internal spin-orbit coupling has the magnitude $t\sin(2\pi\varphi)$, $t = \sqrt{(t')^2+\alpha^2/4}$. Worth noting that for $\varphi = 1/4$ there is no "usual" hopping, and only spin-dependent transfer of electrons persists, as for edge states of 2d topological insulators [2] (notice, however, that for topological insulators the spin-dependent transfer is chiral, i.e., electrons with spin up can move only to the right, and those with spin down to the left, or vice versa). Using the (spin-dependent) gauge transformation we can remove the explicit dependence on the phase factor from the 1d Hamiltonian to twisted boundary conditions. Such a transformation can be used also for Hamiltonians, which interaction part also respects U(1) symmetry (it, naturally, persists for SU(2)-symmetric interactions). For instance, it takes place for the supersymmetric *t-J* chain with the antiferromagnetic interaction between neighboring electrons. Supersymmetry implies the fixed ratio between the parameter of the electron hopping t and the antiferromagnetic exchange integral J [8]. The Hamiltonian for the supersymmetric antiferromagnetic *t-J* chain with internal spin-orbit coupling has the form $\mathcal{H}_{\text{host}} = t\sum_j \mathcal{H}_{j,j+1}$, where

$$\mathcal{H}_{j,j+1} = -\sum_{\sigma}\mathcal{P}(\psi^{\dagger}_{j,\sigma}\psi_{j+1,\sigma} + \psi^{\dagger}_{j+1,\sigma}\psi_{j,\sigma})\mathcal{P} + \psi^{\dagger}_{j,\downarrow}\psi_{j,\uparrow}\psi^{\dagger}_{j+1,\uparrow}\psi_{j+1,\downarrow} + \psi^{\dagger}_{j,\uparrow}\psi_{j,\downarrow}\psi^{\dagger}_{j+1,\downarrow}\psi_{j+1,\uparrow} - n_{j,\uparrow}n_{j+1,\downarrow} - n_{j,\downarrow}n_{j+1,\uparrow}, \quad (2)$$

where the hopping $t = \sqrt{(t')^2+\alpha^2/4}$, and the standard for *t-J* chains projector operator $\mathcal{P} = (1-n_{j,-\sigma})(1-n_{j+1,-\sigma})$ excludes double occupation at each site of the chain. The electron–electron interaction term has the standard Heisenberg antiferromagnetic form $\mathbf{S}_j \cdot \mathbf{S}_{j+1}$ written using the operators of creation and destruction of electrons. The total exactly solvable by Bethe ansatz (BA) Hamiltonian of the system with a magnetic impurity [8] can be written as $\mathcal{H} = \mathcal{H}_{\text{host}} + \mathcal{H}_{\text{imp}}$. The impurity's part of the Hamiltonian (for an impurity situated at the site labeled as *imp*, for example, between the sites labelled by the numbers m and $m+1$, see Fig. 1) is

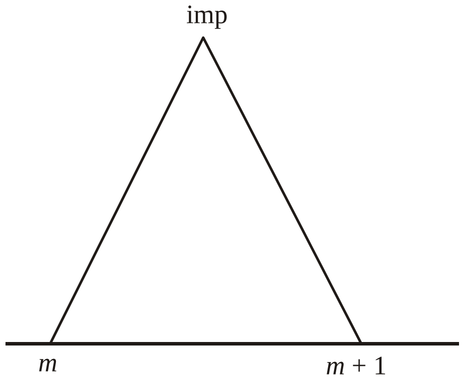

Fig. 1. The illustration of the coupling of the impurity to the host chain in the considered model. The impurity is situated between sites m and $m+1$ of the host chain.

$$\mathcal{H}_{\rm imp} = \frac{(M,\sigma|M+\sigma)}{\theta^2+(S+1/2)^2}(\mathcal{H}_{m,{\rm imp}}+\mathcal{H}_{{\rm imp},m+1} -$$

$$-2S(S-1)\mathcal{H}_{m,m+1}+\{\mathcal{H}_{m,{\rm imp}},\mathcal{H}_{{\rm imp},m+1}\}-$$

$$-2i\theta[\mathcal{H}_{m,{\rm imp}},\mathcal{H}_{{\rm imp},m+1}]), \qquad (3)$$

where $\{.,.\}$ ($[.,.]$) denote anticommutator (commutator) and $(M,\sigma|M+\sigma)$ denotes the Clebsch–Gordan coefficient $(\frac{1}{2}\sigma,S'M|\frac{1}{2}S'SM+\sigma)$ with $S'=S-1/2$. S defines the spin of the impurity. The real parameter θ regulates the impurity-host coupling. For $\theta\to\infty$ the impurity is totally decoupled from the chain. On the other hand, for $S=1/2$ and $\theta=0$ we have a simple addition of one site to the chain, i.e., $L\to L+1$. Terms in the Hamiltonian $\mathcal{H}_{\rm imp}$, proportional to the commutator and anticommutator, are irrelevant from the renormalization group viewpoint (though they are important for the exact solvability), and they can be neglected in the long-wave limit. If the impurity is situated at the edge of the open chain (i.e., for $m=L$) we see that the Hamiltonian becomes simpler: The impurity is coupled to the last site of the chain with the coupling strength, which is determined by two parameters, S and θ, which distinguish the impurity site from other sites of the chain. It can be checked that the gauge transformation, which removes the spin-orbit phase shift from the Hamiltonian for open boundary conditions and transfers it to twisted boundary conditions for the closed chain, can be applied also when the impurity interaction is included.

3. Bethe ansatz description

The stationary Schrödinger equation for the considered model can be solved exactly by BA for the magnetic field H applied along the distinguished by the spin-orbit coupling direction. For any other direction of the field H excitations become gapped with the gap value $\propto\sqrt{H^2+\alpha^2/4}$. For twisted (due to the internal spin-orbit coupling) boundary conditions BA equations for two sets of quantum numbers (rapidities) v_j ($j=1,\dots,N$, where N is the number of electrons) and λ_α ($\alpha=1,\dots,M$, where M is the number of electrons with spin down) can be written as

$$\frac{v_j-\theta+iS}{v_j-\theta-iS}\left[\frac{v_j+i/2}{v_j-i/2}\right]e^{-2\pi\varphi}=\prod_{\alpha=1}^{M}\frac{v_j-\lambda_\alpha+i/2}{v_j-\lambda_\alpha-i/2},$$

$$\frac{\lambda_\alpha-\theta+i(2S-1)/2}{\lambda_\alpha-\theta-i(2S-1)/2}\prod_{j=1}^{N}\frac{\lambda_\alpha-v_j+i/2}{\lambda_\alpha-v_j-i/2}=$$

$$=-e^{4\pi\varphi}\prod_{\beta=1}^{M}\frac{\lambda_\alpha-\lambda_\beta+i}{\lambda_\alpha-\lambda_\beta-i}. \qquad (4)$$

The energy of the eigenstate is equal to

$$E=-\mu\frac{N}{L}-H\frac{N-2M}{L}+2t\sum_{j=1}^{N}\frac{1-4v_j^2}{1+4v_j^2}, \qquad (5)$$

i.e., rapidities parametrize eigenvalues (as well as eigenfunctions) of the Hamiltonian for given N and M.

The problem can be also solved exactly for open boundaries. In that case the spin-dependent phase factor can be totally removed from the 1d Hamiltonian with the help of the gauge transformation [8]. In that case eigenvalues and eigenvectors of the Hamiltonian $\mathcal{H}$ depend on the spin-orbit coupling only in the trivial way $t=\sqrt{(t')^2+\alpha^2/4}$, like for the homogeneous Hubbard chain [9].

We can introduce the low-energy spin scale $T_K\sim t\exp(-\pi|\theta|)$, which plays the role of the Kondo temperature for spin degrees of freedom of the considered model [8]. The parameters of the internal spin-orbit coupling appear in BA equations twofold: As the renormalization of the hopping constant t due to α, and, hence, of the Kondo temperature T_K, and via the phase factor in BA equations. The former can be trivially taken into account in magnetic and temperature dependences of the magnetic moment of the impurity. Contrary, the latter can manifest itself only in finite size corrections [8]. The most important manifestation of the latter can be seen in the asymptotics of correlation functions and in persistent currents. For persistent currents the phase factor φ reveals itself in the initial phase of charge and spin (Aharonov–Bohm–Casher effect [10]) persistent currents [11]. The former can be calculated with the help of the finite-size corrections to the BA ground-state eigenvalues using the conformal field theory. The ground state of the considered models is organized as usual for all Fermi systems: All states with negative energies (Fermi seas) are filled, while other states are non-occupied. For the supersymmetric *t-J* model with antiferromagnetic interactions between spin of neighboring electrons there are two Fermi seas [8]: for unbound electron excitations, which carry charge e and spin 1/2, and for spin-singlet pairs, which carry only charge $2e$ and no spin.

4. Finite-size corrections

Let us start first with the homogeneous case, which can be obtained for $\theta\to\infty$. We can introduce two sets of counting functions ($i=1,2$)

$$z_{i,L}(x)=\frac{1}{2\pi}\left(p_i^0(x)-\frac{1}{L}[2\pi\varphi_i+\sum_{j=1}^{2}\sum_{l=1}^{N_j}\varphi_{ij}^0(x,u_{j,l})]\right), \qquad (6)$$

where $\varphi_1=\varphi$, $\varphi_2=0$, and $\sigma_{i,L}=\partial z_{i,L}(x)/\partial x$ with $\varphi_{ij}^0(x,y)=-\varphi_{ji}^0(y,x)$, which satisfy BA equations, cf. Eqs. (4) $z_{i,L}(\lambda_j)=J_{i,j}/L$, where $J_{1,j}=N_2/2$ (mod 1) and $J_{2,j}=(N_1+N_2+1)/2$ (mod 1), $u_{j,l}$ are rapidities.

Here we use $N_1 = M$ and $N_2 = N - 2M$, the number of unbound electrons ($2M$ is the number of paired ones). The functions are

$$p_1^0(x) = 2\tan^{-1} 2x\,, \quad p_2^0(x) = 2\tan^{-1} x\,,$$

$$\varphi_{12}^0(x,y) = 2\tan^{-1}[2(x-y)]\,, \quad \varphi_{11}^0 = 0\,,$$

$$\varphi_{22}^0(x,y) = 2\tan^{-1}(x-y)\,. \tag{7}$$

The momentum and energy of the state with N electrons, M of which having their spin down are

$$P = \frac{2\pi}{L}\sum_{i=1}^{2}\sum_{j=1}^{N_i} J_{ij}\,, \quad E = E_0 + \sum_{i=1}^{2}\sum_{j=1}^{N_i}\varepsilon_i^0(u_{i,j}), \tag{8}$$

where

$$\varepsilon_1^0(x) = -\mu - \frac{H}{2} - 2t[1 - \pi a_1(x)]\,,$$

$$\varepsilon_2^0(x) = -2\mu - 2t[2 - \pi a_2(x)]\,, \tag{9}$$

which are energies of an unbound electron excitation and a pair, respectively, if we neglect interactions, and $a_m(x) = 2m/\pi[4x^2 + m^2]$. Let us choose two sets of numbers $J_1^+ - J_1^- = L - N + M$ and $J_2^+ - J_2^- = L - N$, $J_1^+ + J_1^- = 2D_1 - 2\varphi_1$, and $J_2^+ + J_2^- = 2D_2 - 2\varphi_2$ that determine numbers of particles in each Dirac sea for low-lying excitations and numbers of particles which are transferred from the left Fermi point of excitations of each kind to the right Fermi point. With this choice $J_{i,j}$ are all numbers satisfying the conditions $J_{i,j} < J_i^-$, $J_{i,j} > J_i^+$, $i = 1, 2$. By using the Euler–Maclaurin formula, we can derive the following equations:

$$\sigma_{i,L}(x) = \frac{1}{2\pi}\left[\frac{dp_i^0(x)}{dx} - \sum_{j=1}^{2}\left(\int dy K_{ij}(x,y)\sigma_{j,L}(y) - \right.\right.$$

$$\left.\left.-\frac{1}{24L^2}\left(\frac{1}{\sigma_{j,L}(\Lambda_j^+)}\frac{\partial K_{ij}(x,\Lambda_j^+)}{\partial x} - \frac{1}{\sigma_{j,L}(\Lambda_j^-)}\frac{\partial K_{ij}(x,\Lambda_j^-)}{\partial x}\right)\right)\right], \tag{10}$$

where $K_{ij}(x,y) = \partial\varphi_{ij}^0(x,y)/\partial x$. Here $\Lambda_i^\pm$ satisfy the equations $z_{i,L}(\Lambda_i^\pm) = J_i^\pm/L$. Notice that for the case of a supersymmetric antiferromagnetic *t-J* chain integrations are performed not from Λ_j^- to Λ_j^+, as, e.g., for the Hubbard chain, but from $-\infty$ to Λ_j^- and from Λ_j^+ to ∞. The equations for $\sigma_{i,L}(x)$ can be written in the form

$$\sigma_{i,L}(x) = \sigma_i(x\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-) +$$

$$+\frac{1}{24L^2}\sum_{j=1}^{2}\left[\frac{\rho_j(x\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-)}{\sigma_{j,L}(\Lambda_j^+)} - \frac{\rho_j(-x\,|\,-\Lambda_{1,2}^-,-\Lambda_{1,2}^+)}{\sigma_{j,L}(\Lambda_j^-)}\right], \tag{11}$$

where $\sigma(x\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-)$ and $\rho_i(x\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-)$ are the solutions of the following linear integral equations:

$$\sigma_i(x\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-) = \frac{1}{2\pi}\left(\frac{dp_i^0(x)}{dx} + \right.$$

$$\left. + \sum_{j=1}^{2}\int dy K_{ij}(x,y)\sigma_j(y\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-)\right),$$

$$\rho_i(x\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-) = \frac{1}{2\pi}\sum_{j=1}^{2}\left(\frac{\partial K_{ij}(x,\Lambda_j^+)}{\partial x} + \right.$$

$$\left. + \int dy K_{ij}(x,y)\rho_j(y\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-)\right). \tag{12}$$

We can convert the integrals from $-\infty$ to Λ_j^- and from Λ_j^+ to ∞ to the ones from Λ_j^- to Λ_j^+ by using a Fourier transformation. That conversion implies the formal changes $K_{11} \to -K_{22}$, $K_{22} \to 0$, $p_2^0 \to 0$, $dp_1^0(x)/dx \to K_{12}(x)$, $\varepsilon_1^0(x) \to H - tK_{21}(x,0)$, and $\varepsilon_2^0(x) \to 2t + \mu - (H/2)$, and $\varphi_1 \to -\varphi_1$. After such a transformation we get

$$\int_{\Lambda_1^-}^{\Lambda_1^+} dx\sigma_{1,L}(x) = 1 - \frac{N-M}{L}, \quad \int_{\Lambda_2^-}^{\Lambda_2^+} dx\sigma_{2,L}(x) = 1 - \frac{N}{L}. \tag{13}$$

Then the energy of the state can be written as

$$E = E_0 + L\varepsilon_\infty(\Lambda_i^+,\Lambda_i^-) +$$

$$+\frac{1}{24L}\sum_{i=1}^{2}\left(\frac{e_i(\Lambda_i^+,\Lambda_i^-)}{\sigma_{i,L}(\Lambda_i^+)} + \frac{e_i(-\Lambda_i^-,-\Lambda_i^+)}{\sigma_{i,L}(\Lambda_i^-)}\right), \tag{14}$$

where

$$\varepsilon_\infty(\Lambda_i^+,\Lambda_i^-) = \sum_{j=1}^{2}\int_{\Lambda_j^-}^{\Lambda_j^+} dx\varepsilon_j^0(x)\sigma_j(x\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-)\,, \tag{15}$$

and

$$e_i(\Lambda_i^+,\Lambda_i^-) = \frac{d\varepsilon_i^0(\Lambda_i^+)}{d\Lambda_i^+} -$$

$$-\sum_{j=1}^{2}\int_{\Lambda_j^-}^{\Lambda_j^+} dx\varepsilon_j^0(x)\rho_j(x\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-)\,. \tag{16}$$

Naturally, these equations can be re-written in terms of dressed energies

$$\varepsilon_\infty(\Lambda_i^+,\Lambda_i^-)=\frac{1}{2\pi}\sum_{i=1}^{2}\int_{\Lambda_i^-}^{\Lambda_i^+}dx\varepsilon_i(x\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-)\frac{dp_i^0(x)}{dx}, \quad (17)$$

where the dressed energies $\varepsilon_i(x\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-)$ satisfy the set of equations

$$\varepsilon_i(x\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-)=\varepsilon_i^0(x)-$$

$$-\sum_{j=1}^{2}\int_{\Lambda_j^-}^{\Lambda_j^+}dyK_{ij}^t(x,y)\varepsilon_j(y\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-), \quad (18)$$

where the index t denotes transposition, which implies

$$e_i(\Lambda_i^+,\Lambda_i^-)=\frac{\partial\varepsilon_i^0(x\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-)}{\partial x}\Big|_{x=\Lambda_i^+}. \quad (19)$$

In the infinite chain $\varepsilon_\infty(\Lambda_i^+,\Lambda_i^-)$ is minimal with respect to $\Lambda_i^\pm$ at given μ and H. This condition leads to

$$\varepsilon_i(\Lambda_i^\pm\,|\,\Lambda_{1,2}^+,\Lambda_{1,2}^-)=0, \quad (20)$$

which is the determination of the ground-state Fermi points for dressed energies. Expanding $\varepsilon_\infty(\Lambda_i^+,\Lambda_i^-)$ to the second order in $(\Lambda_i^\pm\mp\Lambda_i)$, we find

$$\varepsilon_\infty(\Lambda_i^+,\Lambda_i^-)=\varepsilon_\infty(\Lambda_i,-\Lambda_i)+$$

$$+\sum_{j=1}^{2}\frac{\frac{\partial}{\partial x}\varepsilon_j(x\,|\,\Lambda_{1,2},-\Lambda_{1,2})\,|_{x=\Lambda_j}}{\sigma_j(\Lambda_j\,|\,\Lambda_{1,2},-\Lambda_{1,2})}\times$$

$$\times\frac{1}{2}([\sigma_j(\Lambda_j\,|\,\Lambda_{1,2},-\Lambda_{1,2})(\Lambda_j^+-\Lambda_j)]^2+$$

$$+[\sigma_j(\Lambda_j\,|\,\Lambda_{1,2},-\Lambda_{1,2})(\Lambda_j^-+\Lambda_j)]^2). \quad (21)$$

This equation is written with the accuracy of L^{-2}. It turns out that

$$\frac{1}{\sigma_j(\lambda_j\,|\,\Lambda_{1,2},-\Lambda_{1,2})}\frac{\partial}{\partial x}\varepsilon_j(x\,|\,\Lambda_{1,2},-\Lambda_{1,2})\,|_{x=\Lambda_j}=2\pi v_j^F, \quad (22)$$

where v_j^F are Fermi velocities of low-lying excitations. It is easy to check that for $\Lambda_i^\pm=\pm\Lambda_i$ the equations for $\sigma_i(x\,|\,\Lambda_{1,2},-\Lambda_{1,2})$ and $\varepsilon_i(x\,|\,\Lambda_{1,2},-\Lambda_{1,2})$ coincide with the standard definitions of densities and dressed energies for the ground state in the thermodynamic limit. Let us denote $\nu_i=N_i/L$, $\delta_i=(D_i-\varphi_i)/L$ and calculate

$$\frac{\partial\nu_i}{\partial\Lambda_j^+}=-\frac{\partial\nu_i}{\partial\Lambda_j^-}=\sigma_j(\Lambda_j\,|\,\lambda_j,-\Lambda_j)Z_{ji},$$

$$\frac{\partial\delta_i}{\partial\Lambda_j^+}=\frac{\partial\delta_i}{\partial\Lambda_j^-}=\frac{1}{2}\sigma_j(\Lambda_j\,|\,\lambda_j,-\Lambda_j)(Z_{ji}^t)^{-1}, \quad (23)$$

where we introduced dressed charge matrix Z_{ij}. Dressed charge matrix can be expressed as $Z_{ij}=\xi_{ij}(\Lambda_i)$, where $\xi_{ij}(x)$ satisfy the set of integral equations

$$\xi_{ij}(x)=\delta_{ij}+\sum_{l=1}^{2}\int_{-\Lambda_l}^{\Lambda_l}dyK_{il}^t(x,y)\xi_{lj}(y). \quad (24)$$

Again, the coefficients of the dressed charge matrix satisfy the relation $\xi_{ij}(x)=\partial\varepsilon_i(x)/\partial\mu_j$, where μ_i are effective chemical potentials for low-lying excitations, $\mu_1=H$ and $\mu_2=\mu-(H/2)$. A dressed charge matrix measures how strong the interaction is in a system. For the noninteracting electron chain without magnetic impurities the dressed charge matrix is the unity matrix. At the half-filling one has only one Fermi sea for unbound electron excitations. The results can be obtained for the formal limit $v_2^F=0$ and $Z_{11}=Z_{21}=Z_{12}=0$, and $Z_{22}=\xi_s(\Lambda_1)$. Here $\xi_s(\lambda)$ is the solution of the equation

$$\xi_S(\lambda)=\frac{1}{2}+\int_{|\lambda'|\ge\Lambda_1}d\lambda'\xi_s(\lambda')\int_{-\infty}^{\infty}\frac{dx}{2\pi}\frac{\exp(-i[\lambda-\lambda']x)}{1+\exp(|x|)}. \quad (25)$$

In this limit the supersymmetric *t-J* correlated electron chain is equivalent to the antiferromagnetic spin-1/2 Heisenberg chain (electrons cannot move from site to site, and the only possible movement is the one for spin flips). For the spin chain the spin-orbit coupling is similar to the Dzyaloshinskii–Moriya interaction [12].

By using dressed charges and velocities of low-lying excitations it is easy to write

$$E=E_0+L\varepsilon_\infty(\Lambda_i,-\Lambda_i)-\frac{\pi}{6L}\sum_{i=1}^{2}v_i^F+\frac{2\pi}{L}\sum_{i=1}^{2}v_i^F\Delta_i, \quad (26)$$

where

$$\Delta_1=\left(\sum_{j=1}^{2}Z_{1j}(D_j-\varphi_j-\delta_jL)\right)^2+$$

$$+\frac{1}{4(\det Z)^2}[Z_{22}(N_1-\nu_1(\mu,H)L)-Z_{21}(N_2-\nu_2(\mu,H)L)]^2,$$

$$\Delta_2=\left(\sum_{j=1}^{2}Z_{2j}(D_j-\varphi_j-\delta_jL)\right)^2+$$

$$+\frac{1}{4(\det Z)^2}[Z_{12}(N_1-\nu_1(\mu,H)L)-$$

$$-Z_{11}(N_2-\nu_2(\mu,H)L)]^2. \quad (27)$$

We can introduce particle–hole excitations by removing $J_{i,j}$ from a Dirac sea for low-lying excitations and introducing $J_{i,j}$ outside the sea. In order not to change M_i and D_i, in other words, the total number of quasiparticles and the number of quasiparticles moved from the left Fermi point to the right one in each Dirac sea for low-lying excitations, the number of particles and holes for particle–hole excitations should be equal both in the vicinity of the left and right Fermi points. We characterize holes and particles in the vicinity of $J_i^{\pm}$ as $J_{i,p}^{\pm} = J_i^{\pm} \pm n_{i,p}^{\pm}$, $J_{i,h}^{\pm} = J_i^{\pm} \mp n_{i,h}^{\pm}$ $(i = 1, 2)$, where the numbers $n_{i,p,h}^{\pm} > 0$ are half integers. We then introduce total numbers as $n_i^{\pm} = \sum (n_{i,p}^{\pm} + n_{i,h}^{\pm})$, where $n_i^{\pm}$ are integers since $J_{i,p}^{\pm} = J_{i,h}^{\pm}$.

This description is modified when one considers finite size corrections for a chain with an integrable impurity. Calculations, similar to the above, yield $\Delta_i \to \Delta_{i,\text{imp}}$, where

$$\Delta_{1,\text{imp}} = \left(\sum_{j=1}^{2} Z_{1j}^t (D_j - \varphi_j - d_{j,\text{imp}} - \delta_j L)\right)^2 +$$

$$+ \frac{1}{4(\det Z)^2} [Z_{22}(N_1 - n_{1,\text{imp}} - \nu_1(\mu, H)L) -$$

$$- Z_{21}(N_2 - n_{2,\text{imp}} - \nu_2(\mu, H)L)]^2 ,$$

$$\Delta_{2,\text{imp}} = \left(\sum_{j=1}^{2} Z_{1j}^t (D_j - \varphi_j - d_{j,\text{imp}} - \delta_j L)\right)^2 +$$

$$+ \frac{1}{4(\det Z)^2} [Z_{12}(N_1 - n_{1,\text{imp}} - \nu_1(\mu, H)L) -$$

$$- Z_{11}(N_2 - n_{2,\text{imp}} - \nu_2(\mu, H)L)]^2, \tag{28}$$

where $n_{1,2,\text{imp}}$ are related to the valence n_{imp} and the magnetization m_{imp}^z of an impurity. For the supersymmetric antiferromagnetic *t-J* chain we have $m_{\text{imp}}^z = 1/2 n_{1,\text{imp}}$ and $n_{\text{imp}} = n_{1,\text{imp}} + 2n_{2,\text{imp}}$, i.e., $n_{1,\text{imp}} = 2m_{\text{imp}}^z$ and $n_{2,\text{imp}} = (n_{\text{imp}}/2) - m_{\text{imp}}^z$. As for $d_{\text{j,imp}}$, they define shifts of the total momentum of a correlated electron chain caused by an integrable impurity as

$$d_{i,\text{imp}} = -\frac{1}{2}\left(\int_{\Lambda_i}^{\infty} dx \sigma_i^{(1)}(x) - \int_{-\infty}^{-\Lambda_i} dx \sigma_i^{(1)}(x)\right) +$$

$$+ \frac{1}{4\pi}[x_i(\infty) + x_i(-\infty)] , \tag{29}$$

where

$$x_1(x) = 2\tan^{-1}[(x-\theta)/S] +$$

$$+ 2\int_{-\Lambda_2}^{\Lambda_2} dy \tan^{-1}[2(x-y)]\sigma_{2,h}^{(1)}(y) ,$$

$$x_2(x) = 2\int_{-\Lambda_1}^{\Lambda_1} dy \tan^{-1}[2(x-y)]\sigma_{1,h}^{(1)}(y) +$$

$$+ 2\tan^{-1}[2(x-\theta)/(2S+1)] +$$

$$+ 2\int_{-\Lambda_2}^{\Lambda_2} dy \frac{1}{1+(x-y)^2}\sigma_{2,h}^{(1)}(y) \tag{30}$$

for periodic boundary conditions, where $\sigma_i^{(1)}$ satisfies the equation for density of an impurity of order of L^{-1}. In the ground-state integral equations for densities of impurities are

$$\sigma_1(p) + \sigma_{1,h}(p) + \int_{-\Lambda_2}^{\lambda_2} d\lambda a_1(p-\lambda)\sigma_{2,h}(\lambda) = a_{2S}(p-\theta),$$

$$\sigma_{2,h}(\lambda) + \sigma_2(\lambda) + \int_{-\Lambda_2}^{\Lambda_2} d\lambda' a_2(\lambda-\lambda')\sigma_{2,h}(\lambda') +$$

$$+ \int_{-\Lambda_1}^{\Lambda_1} dp a_1(p-\lambda)\sigma_{1,h}(p) = a_{2S+1}(\lambda-\theta). \tag{31}$$

We point out that the impurity introduces nonzero ground-state momentum via $d_{j,\text{imp}}$. It implies that the studied impurity has the properties of the "mobile" impurities [13].

The valence of an impurity (for $H = 0$) is

$$n_{\text{imp}} = \int dp \sigma_{1,h}(p) + 2\int d\lambda \sigma_{2,h}(\lambda), \tag{32}$$

which is equal to $n_{\text{imp}} = (2S+1)\Lambda_2 / 2\pi(\Lambda_2^2 - \theta^2)$ for large Λ_2 (we assumed that $\Lambda_2 >> |\theta|$), i.e., for low electron density, and $n_{\text{imp}} = 1 - O(\Lambda_2)$ for small Λ_2, i.e., for the electron density close to half-filling. As a function of the band filling the valence of an impurity smoothly varies between 0 (for $N \to 0$) and 1 (for $N \to L$)). The valence is a decaying function of θ for fixed band filing: Larger θ pertain to weaker coupling of an impurity to the host. The valence is maximum for $\theta = 0$, which is the resonance situation (the impurity level is situated at the Fermi point for the Dirac sea of pairs). The impurity valence also decreases as a function of S close to half-filling, and increases for higher values of the impurity spin for small total number of electrons in the system. The magnetization of an impurity for $H = 0$ is $m_{\text{imp}}^z = S - 1/2$.

For small $H \neq 0$ the valence of an impurity for $S-(1/2) >> |Q-\theta|$ is $n_{\rm imp} \approx 2\sqrt{|\Lambda_2-\theta|}/\pi(2S-1)$, and for the opposite case $S-(1/2) << |\Lambda_2-\theta|$ it is

$$n_{\rm imp} = 1/2 + (1/\pi)[\ln 2\sqrt{|\Lambda_2-\theta|} - (2S-1)/2\sqrt{|\Lambda_2-\theta|}],$$

where

$$\Lambda_2^2 = \frac{2}{3\zeta(3)}[2t\ln 2 - \mu + (H^2/4\pi t)].$$

When switching on the magnetic field the valence of an impurity becomes smaller than unity even at half-filling, as the manifestation of correlations between electrons in the host.

The ground-state magnetization comes from two origins, the magnetization arising from the valence admixture, and the one due to spin degrees of freedom of an impurity. The magnetic field is usually much smaller than the band width, the former contribution is small (and linear in *H*), and can be neglected. Then the Fredholm equation, which describes only the "Kondo"-like spin excitations is for $\theta >> 1$

$$\sigma_{1,h}(p) + \sigma_1(p) - \int dp' G_1(p-p')\sigma_{1,h}(p') = G_{2S-1}(p-\theta). \tag{33}$$

Then the Kondo temperature can be introduces as $\pi(\theta-\Lambda_1) = \ln(H/T_K)$, and we obtain the solution for the magnetization of an impurity

$$m_{\rm imp}^z = \mu_i\left(1 \pm \frac{1}{2|\ln(H/T_K)|} - \frac{\ln|\ln(H/T_K)|}{4\ln^2(H/T_K)} + \cdots\right), \tag{34}$$

where we use for $H >> T_K$ the lower sign and $\mu_i = S$, and for $H << T_K$ we use the upper sign and $\mu_i = S-(1/2)$ for $S > 1/2$, and $\mu_i = H/T_K$ for $S = 1/2$. The impurity spin is underscreened at low fields to the value $S-1/2$ for $S > 1/2$, while for $S = 1/2$ it is totally screened with the finite magnetic susceptibility (inverse proportional to the "Kondo" temperature). For high enough values of the magnetic field the impurity spin behaves as an asymptotically free spin *S*. The "Kondo" temperature depends on the band filling via Λ_1. If charge fluctuations are totally suppressed, for $N = L$, at the half-filling, the "Kondo" temperature is $T_K = v_1^F \exp(-\pi|\theta|)$, where v_1^F is the Fermi velocity of spin-carrying excitations. The corrections due to the mixed valence of an impurity shift the value of the "Kondo" temperature, e.g., as $T_K \to T_K(1+2\zeta(3)\Lambda_2^3)$ for $\Lambda_2 << 1$, as the additional manifestation of correlations between electrons in the host. At $H \geq H_s$, in the spin-saturation phase, the magnetization of an impurity is equal to $M_{\rm imp}^z = S + (n_{\rm imp}-1)/2$, where $n_{\rm imp}$ is the valence of an impurity.

Naturally, the values $m_{j,\rm imp}$ and $d_{j,\rm imp}$ are defined modulo 1. They determine shifts of the values $\Delta M_i = M_i - L\nu_i(\mu,H)$ and $\Delta D_i = D_i - \varphi_i - L\delta_i$ due to a single impurity. It is important to emphasize that a dressed charge matrix of a correlated electron chain with a single impurity also does not depend on the parameters of the impurity.

Notice that for open boundary conditions finite-size corrections can be obtained as above with the formal substitution $L \to 2L$ with $D_i = \varphi_i = 0$, and with the contribution from particle–hole excitations from only one of Fermi edges, say n_i^+. In that case charge and spin persistent currents are obviously zero, because the Aharonov–Bohm–Casher phase factors (as well as the spin-orbit one) can be removed from the Hamiltonian using a gauge transformation [8,11].

5. Correlation functions

Let us now turn to the calculation of asymptotics of correlation functions. Within the conformal field theory they can be then written as ($x = ja$ where *a* is the lattice constant)

$$\langle \mathcal{O}(x,t)\mathcal{O}(0,0)\rangle \sim \frac{e^{-2iD_2\mathcal{P}_\uparrow^F x}\, e^{-2i(D_2+D_1-\varphi)\mathcal{P}_\downarrow^F x}}{\prod_{j=1,2}(x-iv_j^F t)^{2\Delta_j^+}(x+iv_j^F t)^{2\Delta_j^-}}, \tag{35}$$

where the Fermi momenta $\mathcal{P}_{\uparrow(\downarrow)}^F = (\pi/2L)[N \pm (N-2M)]$. For small nonzero temperatures T one has to replace the values $(x \mp iv_i^F t)$ by $v_i^F \sinh[\pi T(x \mp iv_i^F t)/v_i^F]/\pi T$ in Eq. (35). In the conformal field theory asymptotic of correlation functions are determined up to the (constant) multipliers (form factors), which do not depend on t and *x*. It is clear from Eq. (35) that nonzero φ produces additional oscillating factor

$$\exp(-2i\varphi x[\mathcal{P}_\uparrow^F - \mathcal{P}_\downarrow^F]) = \exp(-i2\pi\varphi x[N-2M]/L),$$

which phase is proportional to the magnetic moment of the system $(N-2M)/2L$.

The lowest exponents for correlation functions can be obtained for $n_{1,2}^\pm = 0$. On the other hand, the numbers D_i due to BA equations are restricted to $D_1 = \Delta N_2/2$ mod 1 and $D_2 = (\Delta N_1 + \Delta N_2)/2$ mod 1, where we introduced $\Delta N_{1,2} = N_{1,2} - \nu_{1,2}(\mu,H)$ and use $\delta_i = 0$.

For the homogeneous supersymmetric *t-J* chain without internal spin-orbit coupling we can use the following values (cf. Ref. 14) of quantum numbers $\Delta N_{1,2}$, $D_{1,2}$, and $n_{1,2}^\pm$. For the $\psi^\dagger$-ψ correlation function we use $\Delta N_2 = 1$ with half-integer D_1 and $\Delta N_1 = 1(0)$ and integer (half-integer) D_2 for spin-up (down) states. For density-density (or spin-spin $z-z$) correlations we use $\Delta N_{1,2} = 0$ and integer $D_{1,2}$. For spin-singlet (-triplet) pairs the choice is $\Delta N_1 = 1(2)$, $\Delta N_2 = 2$ with integer D_1 and half-integer (integer) D_2. The role of the phase φ caused by the internal spin-orbit interaction is in the renormalization of D_1

values, so that for the maximum value of the spin-orbit coupling $\varphi = 1/4$ the values of D_1 are changed from integers to half-integers and vice versa. On the other hand, the magnetic impurity changes in general all values ΔN_i and D_i due to nonzero n_i and d_i.

In the general case $H \neq 0$ and non-half-filled band all components of the dressed charge matrix are nonzero, as well as $n_{i,\mathrm{imp}}$ and $d_{i,\mathrm{imp}}$. Let us consider two limiting cases, at which some analytic results can be obtained.

First, consider the situation with $H = 0$, at which we have $Z_{11} = 1\sqrt{2}$, $Z_{12} = 0$ and $Z_{22} = 2Z_{21} = \xi_c(\Lambda_2)$, where ξ_c is the solution of the following integral equation:

$$\xi_c(\lambda) = 1 + \int_{-\Lambda_2}^{\Lambda_2} d\lambda' \xi_c(\lambda') \int_{-\infty}^{\infty} \frac{dx}{2\pi} \frac{\exp(-ix(\lambda - \lambda'))}{1 + \exp(|x|)}. \quad (36)$$

It is easy to see that $\xi_c(\Lambda_2)$ decays from the value $\sqrt{2}$ at $N = 0$ to 1 at $N = L$. It is convenient to introduce $\alpha_c = 2\xi_c^2(\Lambda_2)$, which decays from 4 to 2 with the growth of the band filling from zero to half-filling. On the other hand, we have $n_{1,\mathrm{imp}} = 0$ and $n_{2,\mathrm{imp}} = n_{\mathrm{imp}}/2$ (there are no unbound electron excitations) with $4\pi d_{1,\mathrm{imp}} = x_1(\infty) + x_1(-\infty)$ for $H = 0$. For the equal-time correlations in the ground state the decay of correlation functions is proportional to $x^{2[\Delta_c^+ + \Delta_c^- + \Delta_s^+ + \Delta_s^-]}$, and at nonzero temperatures the decay is proportional to $\prod_{j=1,2} (\pi T/v_j^F)^{2\Delta_j^+ + \Delta_j^-} \exp(-x/R_j)$, where we introduced the correlation radia $R_j = v_j^F/[2\pi T(\Delta_j^+ + \Delta_j^-)]$.

We have for the homogeneous chain with the internal spin-orbit coupling $\psi^\dagger$-ψ correlation functions proportional to

$$\cos(\pi N x/2L) x^{-[(1/\alpha_c)+(1/4)+[(\alpha_c/4)+1](1-2\varphi)^2/4]}.$$

It implies that the power-law singularity for $\langle n_k \rangle$ is proportional to

$$|P - \mathcal{P}^F|^{[(1/\alpha_c)-3/4+[(\alpha_c/4)+1](1-2\varphi)^2/4]}.$$

For density-density correlation functions the main contribution is proportional to $(N/L)^2$, and the next-to main corrections are proportional to

$$\cos(2\pi N x/L) x^{-[(\alpha_c/4)+1](1-\varphi)^2}$$

and $$\cos(\pi N x/L) x^{-[(\alpha_c/4)+1](1-\varphi)^2}$$

or $$\cos(\pi N x/L) x^{-[(\alpha_c/4)(2-\varphi)^2+\varphi^2]},$$

depending on the band filling and the strength of the internal spin-orbit coupling. Notice that there can exist the term, proportional to $x^{-2-\varphi^2-(\alpha_c/4)\varphi^2}$ caused by possible particle–hole excitations. For the spin-spin correlation functions the asymptotics are similar without the constant term. For the spin-singlet pair correlation function the main asymptotic is proportional to

$$\cos(\pi N x/L) x^{-[(4/\alpha_c)+(\alpha_c/4)(1-\varphi)^2+\varphi^2]}.$$

Finally for the spin-triplet pair correlation function the asymptotic is proportional to $x^{-[1+(4/\alpha_c)+[(\alpha_c/4)+1]\varphi^2]}$. We can see that the spin-orbit interaction, as a rule, can enhance field-field, spin-spin and density-density correlation functions. On the other hand, pair correlation functions decay stronger due to the spin-orbit coupling. It implies that the spin-orbit interaction favors the charge density wave or spin density wave quasi-ordering (the real ordering is excluded in one-dimensional systems at nonzero temperatures).

At small nonzero temperatures the same exponents as in the ground state determine the temperature dependences and the correlation radii for the asymptotics of correlation functions. For example, for the spin-triplet pair correlation function the low-temperature asymptotics is given by

$$e^{-[x(R_{t1}+R_{t2})/R_{t1}R_{t2}]} (\pi T/v_1^F)^{1+\varphi^2} (\pi T/v_2^F)^{(4/\alpha_c)+(\alpha_c/4)\varphi^2},$$

where

$$R_{t1} = [v_1^F/2\pi T(1+\varphi^2)]$$

and $$R_{t2} = (v_2^F/2\pi T[(4/\alpha_c) + (\alpha_c/4)\varphi^2]).$$

The presence of the magnetic impurity can strongly renormalize the behavior of correlation function exponents according to Eqs. (28). Here we point out that the renormalization of the exponents due to the integrable impurity does not depend on the position of the impurity. Such a property is caused by the reflectionless nature of the integrable impurity. The manifestation of the latter is seen in the nonzero momentum, caused by the impurity (i.e., the influence of the impurity is spread along all the chain without dissipation).

The other case, in which analytical answers can be obtained, is the half-filling, $\Lambda_2 = 0$. Here we have only one Fermi sea for unbound electron excitations. This case for the homogeneous situation is equivalent to the studied earlier repulsive Hubbard chain with the internal spin-orbit interaction at half-filling [9].

In a number of recent publications for correlated spinless fermions [15] and for correlated electron models [16] the conformal field theory result was corrected by taking into account the curvature of the dispersion law of low-energy excitations. Formally their corrections [15,16] were related to the fictitious impurity, which description is similar to the impurity, introduced in our model. The difference between our impurity and the fictitious one is in the definition of θ and S. For our case θ is determined by the impurity-host coupling. For the fictitious impurity instead of θ the rapidity of the high-energy excitation is introduced. (In such a case the fictitious impurity can have a nonzero momentum, i.e., it is similar to the "mobile" one.) Then, it is easy to generalize our results for the nonzero curvature of the low-energy excitation. It means, that we have to add $n^{f}_{i,\mathrm{imp}}(\Lambda_h)$ and $d^{f}_{i,\mathrm{imp}}(\Lambda_h)$ together with $n_{i,\mathrm{imp}}(\theta)$ and $d_{i,\mathrm{imp}}(\theta)$, where Λ_h defines the rapidity of the high-energy excitation. They are related [15,16] to the value of the dressed charge matrix $Z_{i,j}$. For example, $n^{f}_{1,\mathrm{imp}}(\Lambda_h) = 1 - \xi_{11}(\Lambda^h)$, $n^{f}_{2,\mathrm{imp}}(\Lambda_h) = -\xi_{21}(\Lambda_h)$. It is important to emphasize that (i) $n^{f}_{i,\mathrm{imp}}$ and $d^{f}_{i,\mathrm{imp}}$ are also determined modulo 1; (ii) critical exponents for correlation functions are determined by the *minimal* choice of quantum numbers ΔN_i and D_i.

6. Conclusions

In summary, using the exact Bethe ansatz solution and conformal field theory we have calculated the asymptotic behavior of correlation functions for the supersymmetric antiferromagnetic *t-J* chain with the internal spin-orbit interaction and with a magnetic impurity. We have shown that the spin-orbit coupling strongly renormalizes critical exponents for correlation functions even for the homogeneous case for periodic boundary conditions. On the other hand, for open chains the contribution of the spin-orbit interaction to correlation functions is trivial. In particular, the spin-orbit coupling, as a rule, causes the increase of critical exponents for pair-pair correlation functions, while for spin-spin and density-density correlations the critical exponents decrease with the growth of the spin-orbit interactions. The magnetic impurity also can play the decisive role in the behavior of correlation functions, because it renormalizes critical exponents in the way, similar to the contribution from curvatures of dispersion laws of low-energy excitations of the correlated electron system in the Luttinger liquid approach. This feature is the manifestation of the reflectionless nature of the integrable impurity. The nonzero momentum, caused by the impurity implies that the influence of the impurity is spread along all the chain without dissipation. Such a property of the impurity, together with the highest entanglement of the antiferromagnetic ground state of the considered model makes the latter as the promising candidate for the application of similar systems in quantum computation and spintronics.

Our findings are generic for one-dimensional correlated electron systems with the internal spin-orbit coupling and magnetic impurities, and can be used in studies of, e.g., semiconducting wires, where the spin-orbit interaction is relatively large. In particular, the influence of such spin-orbit effects can be very important for spintronics, e.g., in spin-filtering current states. On the other hand, we have shown that in general critical exponents depend on the valence and the magnetic moment of the magnetic impurity.

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1. G. Dresselhaus, *Phys. Rev. B* **100**, 580 (1955); Y.A. Bychkov and E.I. Rashba, *J. Phys. C* **17**, 6039 (1984).
2. M.Z. Hasan and C.L. Kane, *Rev. Mod. Phys*. **82**, 3045 (2010); X.-L. Qi and S.-C. Zhang, *Rev. Mod. Phys*. **83**, 1057 (2011).
3. R.M. Lutchyn, J.D. Sau, and S. Das Sarma, *Phys. Rev. Lett*. **105**, 077001 (2010); Y. Oreg, G. Refael, and F. von Oppen, *Phys. Rev. Lett*. **105**, 177002 (2010).
4. N. Andrei, K. Furuya, and J.H. Lowenstein, *Rev. Mod. Phys*. **55**, 331 (1983); A.M. Tsvelick and P.B. Wiegmann, *Adv. Phys*. **32**, 453 (1983).
5. Y.A. Liao, A.S.C. Rittner, T. Paprotta, W.H. Li, G.B. Partridge, R.G. Hulet, S.K. Baur, and E.J. Mueller, *Nature* **467**, 567 (2010); M.W. Zwierlein, A. Schirotzek, C.H. Schunk, and W. Ketterle, *Science* **311**, 492 (2006); C.H. Schunk, Y. Shin, A. Schirotzek, M.W. Zwierlein, and W. Ketterle, *Science* **316**, 867 (2007).
6. P. Schlottmann and A.A. Zvyagin, *Mod. Phys. Lett. B* **26**, 1230009 (2012).
7. Y. Meir, Y. Gefen, and O. Entin-Wohlman, *Phys. Rev. Lett*. **63**, 798 (1989).
8. See, e.g., A.A. Zvyagin, *Finite-Size Effects in Correlated Electron Systems: Exact Results*, Imperial College Press, London (2005), and references therein.
9. A.A. Zvyagin, *Phys. Rev. B* **86**, 085126 (2012).
10. Y. Aharonov and D. Bohm, *Phys. Rev*. **115**, 485 (1959); Y. Aharonov and A. Casher, *Phys. Rev. Lett*. **53**, 319 (1984).
11. See, e.g., A.A. Zvyagin and I.V. Krive, *Fiz. Nizk. Temp*. **21**, 687 (1995) [*Low Temp. Phys*. **21**, 533 (1995)].
12. I.E. Dzyaloshinskii, *Sov. Phys. JETP* **6**, 1259 (1957); T. Moriya, *Phys. Rev*. **120**, 91 (1960); A.A. Zvyagin, *Fiz. Nizk. Temp*. **15**, 977 (1989) [*Sov. J. Low Temp. Phys*. **15**, 540 (1989)]; A.A. Zvyagin, *J. Phys.: Condens. Matter* **3**, 3865 (1991).
13. T. Ogawa, A. Furusaki, and N. Nagaosa, *Phys. Rev. Lett*. **68**, 3638 (1992); H. Castella and X. Zotos, *Phys. Rev. B* **47**, 16186 (1993); A.H. Castro-Neto and M.P.A. Fisher, *Phys. Rev. B* **53**, 9713 (1996); S. Sorella and A. Parola, *Phys. Rev. Lett*. **76**, 4604 (1996); S. Sorella and A. Parola, *Phys. Rev. B* **57**, 6444 (1998); Y. Tsukamoto, T. Fujii, and N. Kawakami, *Phys. Rev. B* **58**, 3633 (1998); L. Balents, *Phys. Rev. B* **61**, 4429 (2000).
14. N. Kawakami and S.-K. Yang, *J. Phys.: Condens. Matter* **3**, 5983 (1991).
15. M. Pustilnik, M. Khodas, A. Kamenev, and L.I. Glazman, *Phys. Rev. Lett*. **96**, 196405 (2006); R.G. Pereira, J. Sirker, J.S. Caux, R. Hagemans, J.M. Maillet, S.R. White, and I. Affleck, *Phys. Rev. Lett*. **96**, 257202 (2006); M.B. Zvonarev, V.V. Cheianov, and T. Giamarchi, *Phys. Rev. Lett*. **99**, 110404 (2007); M. Khodas, M. Pustylnik, A. Kamenev, and L.I. Glazman, *Phys. Rev. Lett*. **99**, 110405 (2007); A. Imambekov and L.I. Glazman, *Phys. Rev. Lett*. **102**, 126405 (2009); A. Imambekov and L.I. Glazman, *Science* **323**, 228 (2009).
16. T.L. Schmidt, A. Imambekov, and L.I. Glazman, *Phys. Rev. Lett*. **104**, 116403 (2010); F.H.L. Essler, *Phys. Rev. B* **81**, 205120 (2010).