Low Temperature Physics/Fizika Nizkikh Temperatur, 2015, v. 41, No. 5, pp. 520–525

Conformal phase transition: QCD like theories with a large number of fermion flavors and all that

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Received November 4, 2014, published online March 23, 2015

The notion of the conformal phase transiton (CPhT) is discussed. As its realization, the dynamics with an infrared stable fixed point in the conformal window in QCD like theories with a relatively large number of fermion flavors is reviewed. The emphasis is on the description of a clear signature for the conformal window, which in particular can be useful for lattice computer simulations of these gauge theories. A possibility of the relevance of the CPhT in graphene is mentioned.

PACS: 11.30.Rd Chiral symmetries;
11.30.Qc Spontaneous and radiative symmetry breaking;
12.38.Aw General properties of QCD (dynamics, confinement, etc.).

Keywords: conformal phase transition, infrared fixed point, scaling law for hadron masses, light glueballs.

1. Introduction

It is a pleasure to write this short review for the Proceedings devoted to the seventieth anniversary of Dima Loktev (when you know a man almost fifty years, it is difficult to call him formally).

I remember the day of our first meeting quite clearly. It was in March 1966, when both of us were proctoring the City Physics Olympiad in the famous Kiev University Red Building. Dima had just become a third year student in the Physics Department, moving there from the Radio-Physics one. I was a fourth year Physics student. We were young, full of energy and hopes...

This work reviews the conformal phase transition, whose concept was introduced in Ref. 1. The main reason of my choice is that this topic is rather general and relevant both for relativistic field theories and condensed matter. In particular, it might be relevant for graphene (see below).

The Landau, or σ-model-like, phase transition [2] is characterized by the following basic feature. Around the critical point $z = z_c$ (where z is a generic notation for parameters of a theory, as the coupling constant α, number of particle flavors N_f, etc.), an order parameter X is

$$X = \Lambda f(z), \tag{1}$$

where Λ is an ultraviolet cutoff and the function $f(z)$ has such a non-essential singularity at $z = z_c$ that $\lim f(z) = 0$ as z goes to z_c both in symmetric and non-symmetric phases. The standard form for $f(z)$ is $f(z) \sim (z - z_c)^\nu$, $\nu > 0$, around $z = z_c$ [for convenience, we assume that $z > z_c$ ($z < z_c$) in the nonsymmetric (symmetric) phase]*. The conformal phase transition [1] is a very different continuous phase transition. It is defined as a phase transition in which an order parameter X is given by Eq. (1) where $f(z)$ has such an essential singularity at $z = z_c$ that while

$$\lim_{z \to z_c} f(z) = 0 \tag{2}$$

as z goes to z_c from the side of the non-symmetric phase, $\lim f(z) \neq 0$ as $z \to z_c$ from the side of the symmetric phase (where $X \equiv 0$). Notice that since the relation (2) ensures that the order parameter $X \to 0$ as $z \to z_c$, the phase transition is continuous.

There are the following basic differences between the Landau phase transition (LPhT) and the CPhT one [1].

1. In the case of the LPhT, masses of light excitations are continuous functions of the parameters z around the critical point $z = z_c$ (though they are non-analytic at $z = z_c$). In the case of the CPhT, the situation is different: there is an abrupt change of the spectrum of light excitations, as the critical point $z = z_c$ is crossed. This implies that the effective actions describing low energy dynamics in the phases with $z < z_c$ and $z > z_c$ are different in a system with CPhT.

* Strictly speaking, Landau considered the mean-field phase transition. By the Landau phase transition, we understand a more general class, when fields may have anomalous dimensions [3].

2. Unlike the LPhT, the parameter z governing the CPhT is connected with a marginal operator [in the LPhT phase transition, such a parameter is connected with a relevant operator; it is usually a mass term].

3. The fact that the parameter z is connected with a marginal operator in the CPhT implies that in the continuum limit, when $z \to z_c + 0$, the conformal symmetry is broken by a marginal operator in nonsymmetric phase, i.e., there is a conformal anomaly.

4. Unlike the LPhT, in the case of CPhT, the structures of renormalizations (i.e., the renormalization group at high momenta) are different in symmetric phase and nonsymmetric one.

In relativistic field theory, the CPhT is realized in the two-dimensional Gross-Neveu (GN) model [4] at the critical coupling constant $g_c = 0$, reduced (or defect) QED [5,6], and quenched QED [7–10]. It was suggested that the chiral phase transition with respect to the number of fermion flavors N_f in QCD is a CPhT one [1,11]. In condensed matter physics, a CPhT like phase transition is realized in the Berezinskii–Kosterlitz–Thouless (BKT) model [12] and, possibly, graphene [13].

Recently, the interest to the dynamics with the CPht phase transition has essentially increased. It is in particular connected with a progress in numerical lattice studies of gauge theories with a varied number of fermion flavors (for reviews, see Refs. 14–17)*, the revival of the interest to the electroweak symmetry breaking based on the walking technicolor like dynamics [21,22] (for a recent review, see Ref. 23), and intensive studies of graphene, a single atomic layer of graphite (for a review, see Ref. 24).

2. Dynamics in the conformal window in QCD-like theories

2.1. General description

In this section, we will consider the problem of the existence of a nontrivial conformal dynamics in 3+1 dimensional non-supersymmetric vector like gauge theories, with a relatively large number of fermion flavors N_f. We will discuss their phase diagram in the $(\alpha^{(0)}, N_f)$ plane, where $\alpha^{(0)}$ is the bare coupling constant. We also discuss a clear signature for the conformal window in lattice computer simulations of these theories suggested quite time ago in Ref. 25.

The roots of this problem go back to a work of Banks and Zaks [26] who were first to discuss the consequences of the existence of an infrared-stable fixed point $\alpha = \alpha^*$ for $N_f > N_f^*$ in vector-like gauge theories [27]. The value N_f^* depends on the gauge group: in the case of SU(3) gauge group, $N_f^* = 8$ in the two-loop approximation. In Nineties, a new insight in this problem [1,11] was, on the one hand, connected with using the results of the analysis of the Schwinger-Dyson (SD) equations describing chiral symmetry breaking in quenched QED [7–10] and, on the other hand, with the discovery of the conformal window in $N = 1$ supersymmetric QCD [28].

In particular, Appelquist, Terning, and Wijewardhana [11] suggested that, in the case of the gauge group SU(N_c), the critical value $N_f^{\rm cr} \simeq 4N_c$ separates a phase with no confinement and chiral symmetry breaking ($N_f > N_f^{\rm cr}$) and a phase with confinement and with chiral symmetry breaking ($N_f < N_f^{\rm cr}$). The basic point for this suggestion was the observation that at $N_f > N_f^{\rm cr}$ the value of the infrared fixed point α^* is smaller than a critical value $\alpha_{\rm cr} \simeq \frac{\pi}{3} 2N_c / (N_c^2 - 1)$, presumably needed to generate the chiral condensate [7–10].

The authors of Ref. 11 considered only the case when the running coupling constant $\alpha(\mu)$ is less than the fixed point α^*. In this case the dynamics is asymptotically free (at short distances) both at $N_f < N_f^{\rm cr}$ and $N_f^{\rm cr} < N_f < N_f^{**} \equiv 11N_c / 2$. Yamawaki and the author [1] analyzed the dynamics in the whole $(\alpha^{(0)}, N_f)$ plane and suggested the $(\alpha^{(0)}, N_f)$-phase diagram of the SU(N_c) theory, where $\alpha^{(0)}$ is the bare coupling constant (see Fig 1 below)**. In particular, it was pointed out that one can get an interesting non-asymptotically free dynamics when the bare coupling constant $\alpha^{(0)}$ is larger than α^*, though not very large.

The dynamics with $\alpha^{(0)} > \alpha^*$ admits a continuum limit and is interesting in itself. Also, its better understanding can be important for establishing the conformal window in lattice computer simulations of the SU(N_c) theory with such large values of N_f. In order to illustrate this, let us consider the following example. For $N_c = 3$ and $N_f = 16$, the value of the infrared fixed point α^* calculated in the two-loop approximation is small: $\alpha^* \simeq 0.04$. To reach the asymptotically free phase, one needs to take the bare coupling $\alpha^{(0)}$ less than this value of α^*. However, because of large finite size effects, the lattice computer simulations of the SU(3) theory with such a small $\alpha^{(0)}$ would be unreliable. Therefore, in this case, it is necessary to consider the dynamics with $\alpha(\mu) > \alpha^*$.

In Ref. 25, this author suggested a clear signature of the existence of the infrared fixed point α^*, which in particular can be useful for lattice computer simulations. The signature is based on two characteristic features of the the spectrum of low energy excitations in the presence of a bare fermion mass in the conformal window: a) a strong (and simple) dependence of the masses of all the colorless bound states (including glueballs) on the bare fermion mass, and b) unlike QCD with a small N_f (N_f = 2 or 3), glueballs are lighter than bound states composed of fermions, if the value of the infrared fixed point is not too large.

* For pioneer papers in this area, see Refs. 18–20.

** This phase diagram is different from the original Banks–Zaks diagram [26].

2.2. Phase diagram

The phase diagram in the $(\alpha^{(0)}, N_f)$-plane in the SU(N_c) gauge theory is shown in Fig. 1. The left-hand portion of the curve in this figure coincides with the line of the infrared-stable fixed points $\alpha^*(N_f)$ [27]:

$$\alpha^{(0)} = \alpha^* = -\frac{b}{c}, \qquad (3)$$

where

$$b = \frac{1}{6\pi}(11N_c - 2N_f), \qquad (4)$$

$$c = \frac{1}{24\pi^2}\left(34N_c^2 - 10N_c N_f - 3\frac{N_c^2 - 1}{N_c}N_f\right). \qquad (5)$$

It separates two symmetric phases, S_1 and S_2, with $\alpha^{(0)} < \alpha^*$ and $\alpha^{(0)} > \alpha^*$, respectively. Its lower end is $N_f = N_f^{\rm cr}$ (with $N_f^{\rm cr} \simeq 4N_c$ if $\alpha_{\rm cr} \simeq \frac{\pi}{3} 2N_c/(N_c^2 - 1)$): at $N_f^* < N_f < N_f^{\rm cr}$ the infrared fixed point is washed out by generating a dynamical fermion mass (here N_f^* is the value of N_f at which the coefficient c in Eq. (5) becomes positive and the fixed point disappears).

The horizontal, $N_f = N_f^{\rm cr}$, line describes a phase transition between the symmetric phase S_1 and the phase with confinement and chiral symmetry breaking. As it was suggested in Ref. 11, based on a similarity of this phase transition with that in quenched QED_4 [8,9], there is the following scaling law for $m_{\rm dyn}^2$:

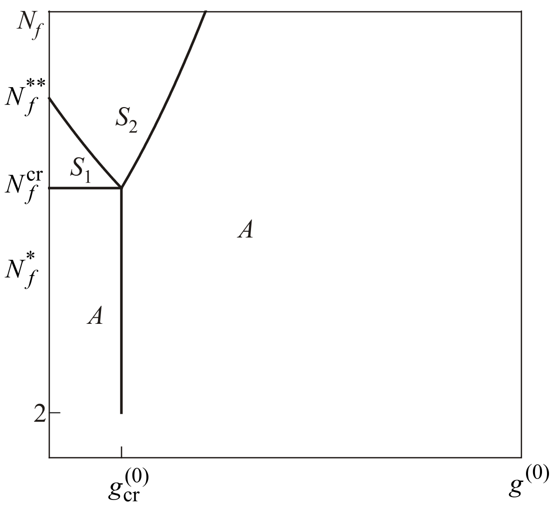

Fig. 1. The phase diagram in an SU(N_c) gauge model. The coupling constant $g^{(0)} = \sqrt{4\pi\alpha^{(0)}}$ and S and A denote symmetric and asymmetric phases, respectively.

$$m_{\rm dyn}^2 \sim \Lambda_{\rm cr}^2 \exp\left(-\frac{C}{\sqrt{\dfrac{\alpha^*(N_f)}{\alpha_{\rm cr}} - 1}}\right), \qquad (6)$$

where the constant C is of order one and $\Lambda_{\rm cr}$ is a scale at which the running coupling is of order $\alpha_{\rm cr}$. It is a CPhT phase transition with an essential singularity at $N_f = N_f^{\rm cr}$.

At last, the right-hand portion of the curve on the diagram occurs because at large enough values of the bare coupling, spontaneous chiral symmetry breaking takes place for any number N_f of fermion flavors. This portion describes a phase transition called a bulk phase transition in the literature, and it is presumably a first order phase transition*. The vertical line ends above $N_f = 0$ since in pure gluodynamics there is apparently no phase transition between weak-coupling and strong-coupling phases.

2.3. Signature for the conformal window

Up to now we have considered the case of a chiral invariant action. But how will the dynamics change if a bare fermion mass term is added in the action? This question is in particular relevant for lattice computer simulations: for studying a chiral phase transition on a finite lattice, it is necessary to introduce a bare fermion mass. As was pointed out in Ref. 25, adding even an arbitrary small bare fermion mass results in a dramatic changing the dynamics both in the S_1 and S_2 phases.

Recall that in the case of confinement SU(N_c) theories, with a small, $N_f < N_f^{\rm cr}$, number of fermion flavors, the role of a bare fermion mass $m^{(0)}$ is minor if $m^{(0)} \ll \Lambda_{QCD}$ (where Λ_{QCD} is a confinement scale). The only relevant consequence is that massless Nambu–Goldstone pseudoscalars get a small mass (the PCAC dynamics).

The reason for that is the fact that the scale Λ_{QCD}, connected with a conformal anomaly, describes the breakdown of the conformal symmetry connected both with perturbative and nonperturbative dynamics: the running coupling and the formation of bound state. Certainly, a small bare mass $m^{(0)} \ll \Lambda_{QCD}$ is irrelevant for the dynamics of those bound states.

Now let us turn to the phases S_1 and S_2, with $N_f > N_f^{\rm cr}$. There is still the conformal anomaly in these phases: because of the running of the effective coupling constant, the conformal symmetry is broken. It is restored only if $\alpha^{(0)}$ is equal to the infrared fixed point α^*. However, the essential difference with respect to confinement theories is that this conformal anomaly have nothing to do with the dynamics

* The fact that spontaneous chiral symmetry breaking takes place for any number of fermion flavors, if $\alpha^{(0)}$ is large enough, is valid at least for lattice theories with Kogut–Susskind fermions. Notice however that since the bulk phase transition is a lattice artifact, the form of this portion of the curve can depend on the type of fermions used in simulations.

forming bound states: Since at $N_f > N_f^{\rm cr}$ the effective coupling is relatively weak, it is impossible to form bound states from *massless* fermions and gluons (recall that the S_1 and S_2 phases are chiral invariant).

Therefore the absence of a mass for fermions and gluons is a key point for not creating bound states in those phases. The situation changes dramatically if a bare fermion mass is introduced: indeed, even weak gauge, Coulomb-like, interactions can easily produce bound states composed of massive constituents, as it happens, for example, in QED, where electron-positron (positronium) bound states are present. To be concrete, let us consider the case when all fermions have the same bare mass $m^{(0)}$. It leads to a mass function $m(q^2) \equiv B(q^2)/A(q^2)$ in the fermion propagator $G(q) = (\hat{q}A(q^2) - B(q^2))^{-1}$. The current fermion mass m is given by the relation

$$m(q^2)\,|_{q^2=m^2} = m. \tag{7}$$

For the clearest exposition, let us consider a particular theory with a finite cutoff Λ and the bare coupling constant $\alpha^{(0)} = \alpha(q)\,|_{q=\Lambda}$ being not far away from the fixed point α^*. Then, the mass function is changing in the "walking" regime [22] with $\alpha(q^2) \simeq \alpha^*$. It is

$$m(q^2) \simeq m^{(0)} \left(\frac{M}{q}\right)^{\gamma_m} \tag{8}$$

where γ_m is the anomalous dimension of the operator $\bar{\psi}\psi$: $\gamma_m = 3 - d_{\bar{\psi}\psi}$ with $d_{\bar{\psi}\psi}$ being the dynamical dimension of this operator. In the walking regime, $\gamma_m \simeq 1 - (1 - \alpha^*/\alpha_{\rm cr})^{1/2}$ (see Refs. 9, 22).

Equations (7) and (8) imply that

$$m \simeq \Lambda \left(\frac{m^{(0)}}{\Lambda}\right)^{\frac{1}{1+\gamma_m}}. \tag{9}$$

Recall that the anomalous dimension $\gamma_m \geq 0$, and $\gamma_m \lesssim 2$ in the "walking" regime.

There are two main consequences of the presence of the bare mass.

(a) Bound states, composed of fermions, occur in the spectrum of the theory. The mass of a n-body bound state is $M^{(n)} \simeq nm$. Therefore they satisfy the scaling

$$M^{(n)} \simeq nm \sim n\,(m^{(0)})^{\frac{1}{1+\gamma_m}}. \tag{10}$$

(b) At low momenta, $q < m$, fermions and their bound states decouple. There is a pure SU(N_c) Yang-Mills (YM) theory with confinement. Its spectrum contains glueballs.

To estimate glueball masses, notice that at momenta $q < m$, the running of the coupling is defined by the parameter $\bar{b}$ of the Yang-Mills theory,

$$\bar{b} = \frac{11}{6\pi} N_c. \tag{11}$$

Therefore the glueball masses M_{gl} are of order

$$\Lambda_{YM} \simeq m \exp\left(-\frac{1}{\bar{b}\alpha^*}\right). \tag{12}$$

For $N_c = 3$, we find from Eqs.(4), (5), and (11) that $\exp(-1/\bar{b}\alpha^*)$ is $6{\cdot}10^{-7}$, $2{\cdot}10^{-2}$, 10^{-1}, and $3{\cdot}10^{-1}$ for $N_f =$ = 16, 15, 14, and 13, respectively. Therefore at $N_f = 16$, 15 and 14, the glueball masses are essentially lighter than the masses of the bound states composed of fermions.

The situation is similar to that in confinement QCD with heavy (nonrelativistic) quarks, $m >> \Lambda_{QCD}$. However, there is now a new important point. In the conformal window, any value of $m^{(0)}$ (and therefore m) is "heavy": the fermion mass m sets a new scale in the theory, and the confinement scale Λ_{YM} (12) is less, and rather often much less, than this scale m. One could say that the latter plays a role of a dynamical ultraviolet cutoff for the pure YM theory.

This leads to a spectacular "experimental" signature of the conformal window in lattice computer simulations: the masses of all colorless bound states, including glueballs, decrease as $(m^{(0)})^{\frac{1}{1+\gamma_m}}$ with the bare fermion mass $m^{(0)}$ for all values of $m^{(0)}$ less than cutoff Λ. Moreover, one should expect that glueball masses are lighter than the masses of the bound states composed of quarks.

Few comments are in order.

1. The phases S_1 and S_2 have essentially the same long distance dynamics. They are distinguished only by their dynamics at short distances: while the dynamics of the phase S_1 is asymptotically free, that of the phase S_2 is not. Also, while around the infrared fixed point α^* the sign of the beta function is negative in S_1, it is positive in S_2.[1]. When all fermions are massive (with the current mass m), the continuum limit $\Lambda \to \infty$ of the S_2-theory is a non-asymptotically free confinement theory. Its spectrum includes colorless bound states composed of fermions and gluons. For $q < m$ the running coupling $\alpha(q)$ is the same as in pure SU(N_c) Yang-Mills theory, and for all $q \gg m$ $\alpha(q)$ is very close to α^* ("walking", actually, "standing" dynamics). For those values N_f for which α^* is small (as $N_f = 16$, 15 and 14 at $N_c = 3$), glueballs are much lighter than the bound states composed of fermions. Notice that unlike the case with $m = 0$, corresponding to the unparticle dynamics [29], there exists a conventional S-matrix in this theory.

2. In order to get the clearest exposition, we assumed such estimates as $N_f^{\rm cr} \simeq 4N_c$ for $N_f^{\rm cr}$ and $\gamma_m = 1 - \sqrt{1 - \alpha^*/\alpha_{\rm cr}}$ for the anomalous dimension γ_m. While the latter should be reasonable for $\alpha^* < \alpha_{\rm cr}$ (and especially for $\alpha^* << \alpha_{\rm cr}$) [9], the former is based on the assumption that $\alpha_{\rm cr} \simeq \frac{\pi}{3}(2N_c/(N_c^2 - 1))$ which, though

seems reasonable, might be crude for some values of N_c. It is clear however that the dynamical picture presented above is essentially independent of those assumptions.

2.4. Lattice computer simulations

During last two years, there has been an essential progress in the lattice computer simulations of gauge theories with a varied number of fermion flavors*. For reviews, see Refs. 16, 17.

The two first papers [30,31], which considered the SU(3) lattice gauge theory with the number of flavors $N_f = 8$ and $N_f = 12$, concluded that while the theory with $N_f = 12$ is in the conformal window, the theory with N_f is a theory with the chiral symmetry breaking. These works used different approaches: while the authors of Ref. 30 used the analysis based on the Schrödinger functional, the authors of Ref. 31 used the renormalization group analysis taking into account the form of the phase diagram in Fig. 1. Based on the fact that the sign of the beta function changes from negative to positive when the line between the S_1 and S_2 phases is crossed, the existence of the conformal window in QCD with $N_f = 12$ was revealed. The analysis of the hadron spectrum in this theory, based on the signature of the conformal window discussed in Sec. 2.3, supports this conclusion. On the other hand, the same analysis for the $N_f = 8$ case suggests that this theory is outside the conformal window.

The hadron spectrum of the SU(3) gauge theory with $N_f = 8$ has been recently studied in detail by the LatKMI Collaboration in Ref. 32. It was in particular shown that there is a flavor-singlet scalar meson as light as the pion. This result is in accord with the previous conclusion of the same Collaboration [33] that this theory yields an example of theories with the walking dynamics. Such a light scalar may be a "technidilaton", a pseudo-Nambu-Goldstone boson of the spontaneously broken approximate scale symmetry. As was shown in Ref. 34, the technidilaton is phenomenologically consistent with the current LHC data.

It is clear that lattice simulations of gauge theories with varied numbers of fermion flavors are crucial for further progress in our understanding of such dynamics. The important point is that CPhT is a long range interactions phenomenon, which is very sensitive to any screening and finite-size effects. The progress made in this area during last two years is certainly encouraging.

This work was supported by the Natural Sciences and Engineering Research Council of Canada.

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* For pioneer papers in this area, see Refs. 18, 19, 20.

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