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Exciton–polariton laser

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We present a review of the investigations realized in the last decades of the phenomenon of the Bose–Einstein condensation (BEC) in the system of two-dimensional cavity polaritons in semiconductor nanostructures. The conditions at which the excitons interacting with cavity photons form new type of quasiparticles named as polaritons are described. Since polaritons can form in a microcavity a weakly interacting Bose gas, similarly to the exciton gas in semiconductors, the microcavity exciton–polariton BEC emerged in the last decades as a new direction of the exciton BEC in solids, promising for practical applications. The high interest in BEC of exciton–polaritons in semiconductor microcavities is related to the ultra-low threshold lasing which has been demonstrated, in particular, for an electrically injected polariton laser based on bulk GaN microcavity diode working at room temperature.

PACS: **71.35.–y** Excitons and related phenomena;
71.36.+c Polaritons (including photon–phonon and photon–magnon interactions).

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1. Introduction

Our review article is dedicated to the memory of Professor Kirill Borisovich Tolpygo commemorating the centenary of his birthday. S.A. Moskalenko (S.A.M) was fortunate to be doctoral student at the Kiev Institute of Physics (IOP) of the Academy of Science of USSR in 1956–1959 years under the supervision of Professor K.B. Toplygo. It was a period of time marked by the fundamental achievements in physics such as the creation of the microscopic theories of the superfluidity and superconductivity, the discovery of excitons in semiconductors, and by the elaboration of the lasers. All these events created an exceptional scientific atmosphere stimulating the search of new interconnected physical phenomena. In addition to these favorable conditions Kirill Borisovich Taplygo happened to be a scientist with vast and profound knowledge in physics, democratic and benevolent supervisor providing an unlimited amount of consultations to his doctoral students. To complete the characterization of that time in IOP it is necessary to mention the existence of an excellent library and the picturesque situation of the Institute near the Goloseevskii forest. The candidatus scientiarum thesis fulfilled by S.A.M. under the guidance of K.B. Toplpygo included some suggestions concerning the collective properties of high density excitons in semiconductors also. The suggestions arose under the influence of the microscopic theory of superfluidity elaborated by N.N. Bogoliubov. They concerned to the Bose–Einstein condensation of the excitons in semiconductors. This topic made the contents of our report at the 3rd International Conference on Nanotechnologies and Biomedical Engineering (ICNBME-2015) held in Chisinau, Moldova on September 23–26, 2015. Our review article has been written on the base of this report.

2. Bose–Einstein condensation. Fundamental concepts

Bose introduced the concept of an ideal photon gas and deduced in this model the Planck's distribution formula for the black-body radiation [1]. Einstein generalized these results applying them to the model of an ideal monoatomic gas. He came to the conclusion that in the conditions of the thermodynamic equilibrium the number of atoms in the given volume and at a given temperature cannot exceed a well defined density and the surplus of atoms in the system will accumulate in a macroscopic number in the lowest state with zero kinetic energy [2]. Due to the contributions of Bose and Einstein the concepts of the ideal Bose gas, Bose–Einstein condensation (BEC), and Bose–Einstein statistics were established in physics. Kapitsa discovered He II superfluidity phenomenon below the λ-point [3] and London [4] suggested the idea that the superfluidity and the superconductivity discovered earlier both are due to the phenomenon of Bose–Einstein condensation.

In the microscopic theory of superfluidity Bogoliubov [5] introduced the notion of the amplitude of the condensate considering the Bose operators a_0^+ and a_0 with wave vector $\mathbf{k} = 0$ as macroscopic values with the given phases, the mean values of which $\langle a_0^+ \rangle, \langle a_0 \rangle$, named later quasi-averaged, remain different from zero if the gauge symmetry $U(1)$ of the system is broken. Bogoliubov introduced the unitary canonical transformation for the operators a_0^+, a_0 named as displacement transformation which is equivalent to the Glauber [6] unitary transformation with the operator

$$D\left(\sqrt{N_0}\right) = \mathrm{e}^{\sqrt{N_0}(a_0^+ - a_0)}, \quad D a_0 D^+ = \sqrt{N_0} + a_0 \qquad (1)$$

which was introduced in quantum optics much latter. It determines the coherent states. Because N_0 is proportional to the volume V, it means that Bogoliubov introduced the concept of coherent macroscopic states. The field operators a_0^+, a_0 are equivalent to the quantum macroscopic wave functions Ψ^*, Ψ or to the order parameter in the Ginzburg–Landau [7] theory of superconductivity. In the presence of the gauge symmetry when the Hamiltonian commutes with the operators $\hat{N}$ describing the full number of particles, the ground state of the system is invariant under the action of the unitary transformation $\hat{U}(\varphi) = \mathrm{e}^{i\hat{N}\varphi}$ changing the phase of the ground state wave function. There are many degenerate ground states of the system, the wave functions of which differ only by their phases. The potential $V(\varphi)$ represented in Fig. 1(a) has the minima positioned along the ring. Breaking of the gauge symmetry of the system leads to the fixation of the phase and to the selection of one point in the ring where the position of the ground state is localized. The excitation of this ground state with changing only its phase does not need energy in the limit of zero wave vector **k**. Such elementary excitations are referred as massless Goldstone [8] modes or as Nambu–Goldstone gapless modes discussed by Nambu in Ref. 9. This phenomenon is general and takes place in any order of the perturbation theory with some exceptions. The spontaneous breaking of the system continuous symmetry leads to the existence of one Nambu–Goldstone mode in the spectrum of its elementary excitations. It is true not only in the case of gauge symmetry but also in the cases of a continuous rotational symmetry. Just such type of elementary excitations was obtained by

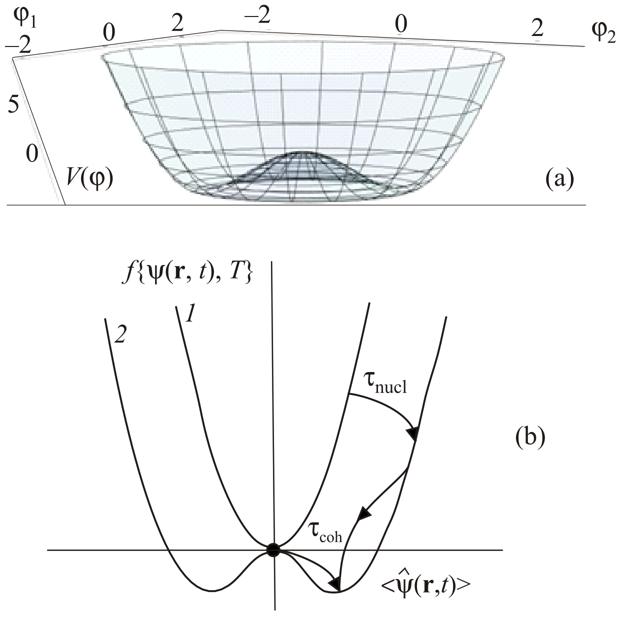

Fig. 1. The potential $V(\varphi)$ with the minima at $|\varphi| = a$ and a local maximum at $\varphi = 0$ (a). The free energy density functional $f(\Psi(\mathbf{r}, t), T)$ in dependence on the order parameter $\langle \Psi(\mathbf{r}, t) \rangle$ (b) [This drawing is reproduced from the paper by H.T.C. Stoof published in Bose–Einstein condensation, A. Griffin, D.W. Snoke, and S. Stringari (eds.), Cambridge U. Press, Cambridge (1995)].

Bogoliubov in the model of a weakly nonideal Bose gas with the Hamiltonian

$$\mathcal{H} = \sum_{\mathbf{k}} \left(T_{\mathbf{k}} - \mu\right) a_{\mathbf{k}}^+ a_{\mathbf{k}} + \frac{g}{2V} \sum_{\mathbf{p},\mathbf{q},\mathbf{k}} a_{\mathbf{p}}^+ a_{\mathbf{q}}^+ a_{\mathbf{q}+\mathbf{k}} a_{\mathbf{p}-\mathbf{k}} \qquad (2)$$

where $T_{\mathbf{k}} = (\hbar^2 \mathbf{k}^2)/2m$ is the kinetic energy and μ is the chemical potential. The state of Bose–Einstein condensation was introduced by the displacement canonical transformation

$$a_k = \sqrt{N_0}\,\delta_{k,0} + \alpha_k \qquad (3)$$

which leads to the equation of motion for the condensate annihilation operator

$$\frac{i\hbar da_0}{dt} = -\mu a_0 + g\left|a_0\right|^2 a_0 \qquad (4)$$

and to the Bogoliubov spectrum of the elementary excitations

$$E\left(k\right) = \pm\sqrt{T_k^2 + 2T_k L_0}; \quad L_0 = \frac{N_0}{V} g = n_0 g\,. \qquad (5)$$

In the case of inhomogeneous in real-space and in time Bose–Einstein condensate the Eq. (4) transforms into the Gross–Pitaevskii equation of motion of the condensate wave function $\psi(r, t)$

$$i\hbar \frac{d\psi(\mathbf{r}, t)}{dt} = -\frac{\hbar^2}{2m} \Delta\psi\left(\mathbf{r}\right) + \frac{g}{V} \left|\psi(\mathbf{r}, t)\right|^2 \psi(\mathbf{r}, t)\,. \qquad (6)$$

It describes also the vortices formation in a boson systems discussed by Gross [10] and Pitaevskii [11]. The nucleation and the building of the Bose condensate was investigated by Stoof [12] who argued that the problem related to the nucleation of BEC does not lie in the study of the kinetic stages of the phase transition. The Boltzmann equation describing the kinetic stages is unable to treat the buildup of the coherence. It cannot lead to a macroscopic occupation of the single-particle ground state, since the production rate of the condensate fraction in the thermodynamic limit $N \to \infty$, $V \to \infty$ is nonzero only if a condensate already exists. If the gas is quenched sufficiently far into the critical region of the phase transition, the first stage of its evolution is then kinetic stage. It leads to the thermalization of the gas, but does not lead to phase coherence. The first kinetic stage leads to transformation of the initially highly nonequilibrium distribution into a quasi-equilibrium one. The dominant time scale of the kinetic process is determined by the elastic relaxation time $\tau_{\rm el}$. During this stage in the range of critical temperature the number of elementary excitations in the system becomes so high that their interactions cannot be neglected. Taking them into account and using mean-field approximation Stoof has obtained another dispersion law different from the Bogoliubov dispersion relation:

$$\hbar\omega(\mathbf{k}) = \sqrt{(T_{\mathbf{k}})^2 - (\tilde{g}n_0)^2}\,, \qquad (7)$$

where $\tilde{g}$ plays the role of an effective interaction constant.

The modes for which $T_{\mathbf{k}}$ is less than $\tilde{g}n_0$ are unstable and their population decreases. It leads to the buildup of the condensate during the coherent stage with the coherence time $\tau_{\rm coh}$ of the same order as $\tau_{\rm el}$. It is demonstated in the Fig. 1(b).

This coherence stage gives rise to the appearance of the condensate embryo in a form of a seed which is not in the thermodynamic quasi-equilibrium with the noncondensed part of the Bose gas. This goal has to be achieved in the last stage of the phase transition, which is also a kinetic stage. Its duration $\tau_{\rm rel}$ is much longer than the first kinetic stage and is limited in the case of excitons or polaritons by their lifetime $\tau_{\rm life}$. In the condition $\tau_{\rm rel} > \tau_{\rm life}$, the complete quasi-thermalization cannot be achieved.

3. BEC of excitons in semiconductors

The fundamental concepts of the Bose gases formed from the matter particles (helium atoms, Cooper electron pairs and atomic gases) and photons were investigated in the conditions of thermodynamic equilibrium. The coherent macroscopic states arising in the conditions of BEC due to the thermalization processes were supplemented by the laser radiation arising due to the photon stimulated emission in the media with inverse particle distribution instead of thermal equilibrium. The created physical backgrounds of the microscopic theory of superfluidity elaborated by Bogoliubov, as well as of the theory of superconductivity elaborated by Bardeen, Cooper and Schrieffer [13] and refined by Bogoliubov [5] from the one hand and the creation of the lasers and coherent sources of radiation in the middle of the 20th century from the other hand stimulated enormously the search of similar phenomena with the participation of quasiparticles with finite lifetimes in quasiequilibrium states but far from the thermodynamic equilibrium. The first attempt in this direction was the investigation of the BEC phenomenon and the superfluidity of the excitons created by the bound electron-hole pairs in semiconductors initiated in the Refs. 14–16. The following step was the study of the mixed exciton–photon systems composed from the dipole-active excitons interacting with the photons captured together in the resonators, when the full number of excitons and photons is conserved. In such mixed systems the half-matter-half-light [17–20] quasiparticles named as polaritons can be formed. A variant described in [19,20] is the most effective at present time as will be shown below. Side by side with the exciton-exciton interaction there is a wide range of optical nonlinearity sources. In this case instead of polaritons we deal with the cavity photons interacting between themselves through the optical nonlinearities and forming the quantum fluids of light [20].

The present report is related to the mixed exciton–photon systems enclosed into the resonators when the total number of enclosed and photons can be conserved due to high quality mirrors. Their description can be made basing on a Hamiltonian written in the exciton and photon representation with a common chemical potential for the both kinds of particles in a similar way as in Refs. 17,18, or passing directly to the polariton representation in special conditions when the polariton creation is not destroyed by the frequent exciton–exciton scattering processes [19,20]. The quantum states in the mixed exciton–photon systems largely depend on the ratio between the exciton–exciton scattering rate and the rate of the exciton–photon conversion process. The first process is nonlinear and its rate is proportional to the exciton density, whereas the last one is a linear optical process depending on the value of exciton–photon interaction constant and without any dependence on the particle densities. The polaritons cannot be formed at high exciton density when the rate of exciton–exciton scattering is greater than the rate of exciton–photon conversion as it was considered in the Refs. 17, 18. The BEC of dipole active excitons takes place firstly in the exciton subsystem generating posteriori a coherent macroscopic polariton mode [17] named in Ref. 18 as giant polariton. To avoid this obstacle and to reach the polariton formation on the level of individual single-particle formations it was necessary to increase the exciton–photon conversion rate even in the case of GaAs-type quantum well (QW) embedded into the resonator, which is characterized by a weak exciton–photon interaction. It was achieved by an ingenious proposal [19–27] to introduce into the antinodes of the cavity not one but many QWs with a considerable number $N_{QW} \approx \approx 20–30$ in the condition of the Dicke [28] superradiant state, when the rate of the photon emission by N_{QW} QWs increases by the factor N_{QW}. It means that the Rabi frequency and the rate of the exciton–photon conversion in such conditions will increase in $\sqrt{N_{QW}}$ and N_{QW} respectively and the polariton formation can be possible even at a high exciton density. The cavity polariton formation is characterized by a very small effective mass. It is on four orders of magnitude smaller than the free electron mass and on nine orders of magnitude less than that of Rb atoms. The small effective mass opens the possibility to reach the conditions necessary for BEC even at the room temperatures. The small polariton mass leads also to high polariton–polariton elastic scattering rate and to small thermalization time of the polaritons on the lower polariton branch of about same tenth of picoseconds. In spite of a small lifetime of polaritons in microcavity (tens picoseconds), due to the smaller thermalization time the BEC of polaritons in the point with $\mathbf{k}_{||} = 0$ on the lower polariton branch succeeded to be realized. In such a way the BEC of cavity polaritons emerges in the last decades of our century as a more perspective variant of the exciton BEC in solids. The results obtained in last decades by many collectives of researchers investigating in details the physical processes related with BEC of cavity exciton–polaritons were represented in the review articles [20–28] and in some initial papers [19,30–33]. The investigations of this problem in the precedent period of time, developing in the second half of the 20th century, have been reflected in the monograph [34]. The review article [35] is devoted to the special case of excitons BEC in Cu_2O crystal. Surprisingly, the BEC of excitons was observed in the conditions of the fractional quantum Hall effects [33]. The coherent macroscopic state of dipole-active excitons gives rise to the coherent macroscopic beam of photons and their reciprocal transformation gives rise to the macroscopic coherent mode as it is argued in the Refs. 17,18. In the same way the coherent macroscopic wave created by cavity polaritons condensed on the lower polariton branch in the point with the in-plane wavevector $\mathbf{k}_{||} = 0$ may emit photons with the same in-plane wave vector $\mathbf{k}_{||} = 0$, but with the quantized value π/L_c of the component k_z oriented along the axis of the cavity and perpendicular to the surface of the QW embedded in it [19,20]. The emitted wave formed by the escaped photons from the composition of the condensed polaritons has the same coherent macroscopic property as the condensed polariton wave and looks outside the resonator as a laser radiation. It is named as the exciton–polariton laser radiation. The system of Bose–Einstein condensed cavity polaritons arising due to the establishment of the quasi-equilibrium in the system and emitting outside the cavity the laser radiation works as a device, which can be named as polariton laser. Usually the electron–hole pairs in the QW are excited by the light of the pumping laser. But the electron-hole pairs can be injected electrically. In this case we deal with the electrically pumped polariton laser. The creation and the physics of the room temperature electrically injected exciton-polariton laser is the main topic of our report. The information concerning this topic is gathered from the Refs. 36–41.

The polariton decay by leakage through the DBR mirrors gives rise to the photons carrying the same energy and momentum as the polaritons in the QWs. The energy can be measured by spectroscopic method, while the momentum can be measured by the angular direction of the photon propagation. Below condensation threshold there is a broad distribution of exciton-polaritons in both energy and momentum scales of the lower polariton dispersion branch. As the pumping power is increased, there is a sudden narrowing in both distributions and a large population of polariton occupies the zero-momentum mode of the system consistent with the formation of a BEC. Below threshold the population at $\mathbf{k} = 0$ increases linearly with the pump power, whereas after condensation it increases nonlinearly. After the transition region the population returns to linear increasing and the condensate population dominates the polariton-population. The macroscopic occupation of the ground state, the nonlinear threshold behavior and the narrowing of the linewidth are all expected in the case of BEC transition. All these peculiarities are represented in Fig. 2.

All of them are needed but they are not sufficient features to declare unambiguously about the BEC establishing. The supplementary arguments in the favour of BEC are the measurements of the degree of coherence and of the correlation functions of the condensate fraction using the interferometer and the photon counting setup. Some supplementary information concerning the polaritons and the two-dimensional Bose gas is needed and it is added below.

4. Polaritons

In the Timofeev [28] review article it was fairly mentioned once again that the interaction of the electromagnetic field oscillations with the transverse optical phonons in polar dielectric crystals leading to the formation of the mixed half-matter-half-electromagnetic field states was investigated for the first time by Kirill Borisovich Tolpygo [42] and by Huang Kun [43,44]. Later such mixed states were named as phonon–polaritons similarly with the exciton–polariton states described by Hopfield [45]. Hopfield introduced the concept of polariton as a new quasiparticle in the solid state physics. It arises in the conditions when the exciton–photon conversion takes place in the form of multiple, reversible mutual transformations whereas another irreversible quantum processes such as the usual emission or the absorption of the light by the exciton state in crystal, as well as the exciton–phonon or exciton–exciton scatterings do not. The new exciton–photon states are representatives of the half-matter-half-light elementary

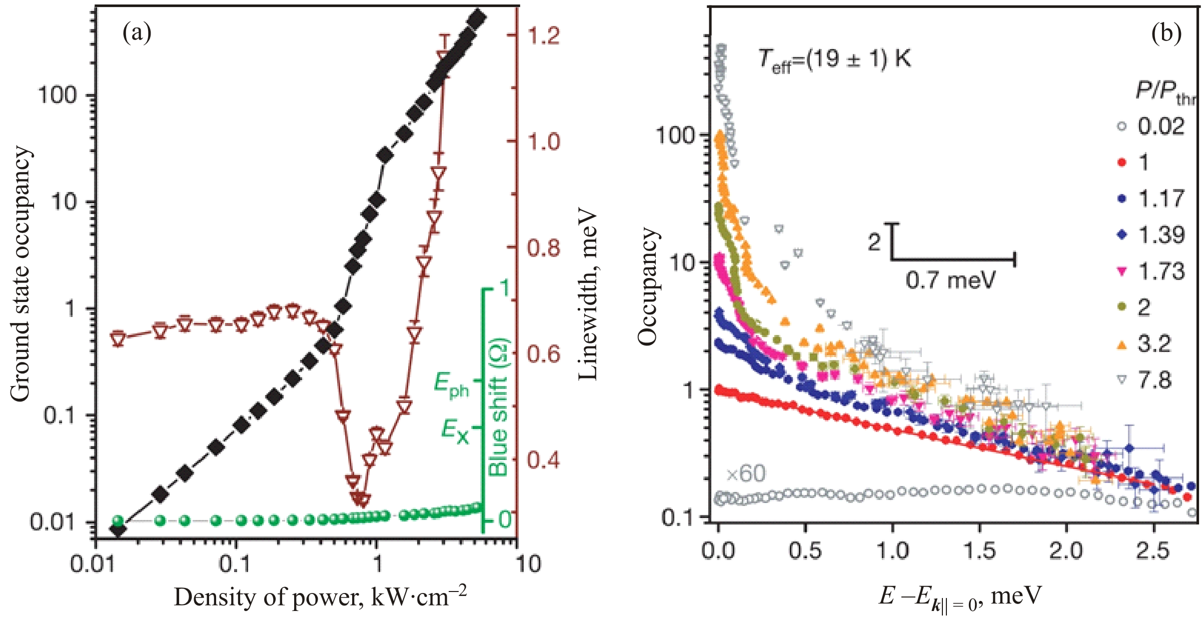

Fig. 2. (Color online) Polariton occupancy measured at 5 K. (a) Occupancy of the $\mathbf{k}_{\|} = 0$ ground state (solid black diamonds), its energy blue shift (solid green circles) and linewidth (open red triangles) versus the excitation power. The blue shift is plotted in units of the Rabi splitting Ω = 26 meV. (b) Polariton occupancy in ground- and excited-state levels is plotted in a semi-logarithmic scale for various excitation powers. For each excitation power, the zero of the energy scale corresponds to the energy of the $\mathbf{k}_{\|} = 0$ ground state. The figure adapted from the paper by J. Kasprzak *et al*., *Nature* **443**, 409 (2006).

excitations. Their mathematical formulation is written in the form of the coherent superposition of their wave functions or of the corresponding field operators and will be presented below. The Hamiltonian describing the quantum states of the system includes the free-particle terms and the exciton–photon interaction. Last has the type **A∇**, where **A** is the vector–potential of the electromagnetic field. Interaction terms of higher order of **A** and antiresonance type are excluded from the Hamiltonian for simplicity. Free-particle terms include the 2D excitons in the quantum well embedded into the microresonator and the cavity photons captured in the spacer of the microresonator. Resulting Hamiltonian has a quadratic form:

$$\mathcal{H} = \sum_{\mathbf{k}_{\|},j} [\hbar\omega_{\mathrm{ex},j}(\mathbf{k}_{\|})a^{\dagger}_{\mathbf{k}_{\|},j}a_{\mathbf{k}_{\|},j} + \hbar\omega_c(\pi/L_c,\mathbf{k}_{\|})c^{\dagger}_{\pi/L_c,\mathbf{k}_{\|},j}c_{\pi/L_c,\mathbf{k}_{\|},j} + \hbar\varphi_j(\pi/L_c,\mathbf{k}_{\|})(a^{\dagger}_{\mathbf{k},j}c_{\pi/L_c,\mathbf{k}_{\|},j} + c^{\dagger}_{\pi/L_c,\mathbf{k}_{\|},j}a_{\mathbf{k}_{\|},j})], \quad (8)$$

where $\omega_{\mathrm{ex},j}(\mathbf{k}_{\|})$ is the frequency of the QW exciton with polarization j and in-plane (in regard to QW plane) wave vector $\mathbf{k}_{\|}$, $\omega_c(\pi/L_c,\mathbf{k}_{\|})$ is the frequency of the cavity photons, and $\varphi_j(\pi/L_c,\mathbf{k}_{\|})$ is the constant of the exciton–photon interaction. Photon wave vector $\mathbf{k} = \mathbf{k}_{\|} + \mathbf{a}_3 k_z$ consists of two parts: in-plane $\mathbf{k}_{\|}$ and size-quantized normal component $\mathbf{a}_3 k_z$, where $\mathbf{a}_3$ is the unit vector oriented along the axis of the microresonator and perpendicular to the surface of the QW embedded in it, and $k_z = \pm\pi/L_c$, (L_c is the microresonator length).

The dispersion relations for the both frequencies are: $\omega_{\mathrm{ex},j}(\mathbf{k}_{\|}) = \omega_{\mathrm{ex},j}(0) + (\hbar^2\mathbf{k}_{\|}^2)/2m_{\mathrm{ex}}$,

$$\omega_c(\pi/L_c,\mathbf{k}_{\|}) = \frac{c}{n_c}\sqrt{\left(\frac{\pi}{L_c}\right)^2 + \mathbf{k}_{\|}^2}\,. \quad (9)$$

Here n_c is the refractive index of the spacer inside the microresonator. If length of the in-plane wave vector $\mathbf{k}_{\|}$ is small in comparison with π/L_c, the cavity photon dispersion relation looks as:

$$\omega_c(\pi/L_c,\mathbf{k}_{\|}) = \omega_c + \frac{\hbar\mathbf{k}_{\|}^2}{2m_c}, \quad \omega_c = \frac{c\pi}{n_c L_c},$$

$$m_c = \frac{\hbar\pi n_c}{cL_c}, \quad |\mathbf{k}_{\|}| < \frac{\pi}{L_c}. \quad (10)$$

Using values $L_c = 10^{-5}$ cm and $n_c = 3$ we obtain $m_c = 3\cdot 10^{-5} m_0$, where m_0 is the free electron mass. The polaritons from the lower branch inherit the excessive small effective mass of the cavity photons, so their effective mass becomes small also. One can remember that the small effective mass of the lower polariton branch plays a crucial role and facilitates the establishing of the polariton BEC.

The polariton annihilation operator $\hat{P}_{\mathbf{k}_{\|},j}$ can be expressed as a coherent superposition of the exciton and the cavity photon annihilation operators:

$$\hat{P}_{\mathbf{k}_{\|},j} = u_j(\mathbf{k}_{\|}) a_{\mathbf{k}_{\|},j} + v_j(\mathbf{k}_{\|}) c_{\frac{\pi}{L_c},\mathbf{k}_{\|},j}, \; u_j^2(\mathbf{k}_{\|}) + v_j^2(\mathbf{k}_{\|}) = 1 . \tag{11}$$

Instead of the two bare exciton and cavity photon branches, two other mixed-type branches appear with the dispersion laws

$$\omega_{P_{\mathbf{k}_{\|},j}} = \frac{\omega_{\text{ex},j}(\mathbf{k}_{\|}) + \omega_c(\pi/L_c, \mathbf{k}_{\|})}{2} \pm$$

$$\pm \frac{1}{2}\sqrt{(\omega_{\text{ex},j}(\mathbf{k}_{\|}) - \omega_c(\pi/L_c, \mathbf{k}_{\|}))^2 + 4\,|\,\varphi_j(\pi/L_c, \mathbf{k}_{\|})\,|^2} \; . \tag{12}$$

They are named as upper and lower polariton branches with a given *j* polarization.

5. Two-dimensional Bose-gas

In contrast to the three-dimensional (3D) ideal Bose-gas and its fundamental concepts mentioned above, in the case of cavity polaritons new aspects related with their finite lifetime and with the two-dimensionality of the system arise. The two-fold dimensionality has an important role, because as was established in Ref. 46 the BEC of the two-dimensional (2D) homogeneous Bose gas extended on an infinite surface area $S \to \infty$ cannot occur at finite temperature. It is possible only at the temperature $T = 0$, which cannot be achieved experimentally. An uniform 2D system of bosons in the thermodynamic limit at finite temperatures has no phase transition to the highly ordered states, since the long wavelength thermal fluctuations destroy any long-range order. In such a way the experimental realization of the BEC phenomena with the participation of the 2D cavity polaritons at finite temperatures faced with an obstacle of principle. Nevertheless, the ingenuity of the experimental physics succeeded to avoid this obstacle and to reveal the basic properties of the nature hidden by such restrictions. The solution was to study the 2D systems with finite area using the pumping light with finite area spot for excitation. It should be noted, however, that the confinement of the Bose gas on a finite area gives rise to the uncertainty of the in-plane wave vector $\mathbf{k}_{\|}$ of the confined polaritons. The condensate wave vector also exhibits an uncertainty. It means that the condensate stretches to a finite wave vector that is disregarded, for example, by the standard Bogoliubov theory of superfluidity [5], but can be taken into account introducing a trap for the 2D polaritons. The spatial extent of the condensed polaritons leads also to the size-dependent critical density of the condensation. When the Bose-gas is confined by a spatially varying potential, the constant density of states well known for the uniform 2D system is strongly modified. As a result the BEC phase at finite temperatures is recovered [20–32].

Practically any experimental system has a finite size and a finite number of particles on the single-particle states. In a 2D box with size L the critical conditions for BEC can be fulfilled at $T \neq 0$. If the system is small, the finite size BEC phase transitions are expected to occur before the Berezinskii–Kosterlitz–Touless (BKT) phase transition. This possibility was foreseen and introduced independently by Berezinskii [47] and Kosterlitz and Thouless [48] in the cited papers. As was mentioned above, the highly focused spot for optical excitations can provide a localization of cavity polaritons. Due to their very small effective mass, the observation of their 3D quantization is possible for trap diameters of the order of a few μms. For the trap of 1–2 μm the quantized energy of the polariton equals few meV [20–25]. The lateral confinement of 2D polaritons creates opportunities for realization of the future quantum light emitters as sources of indistinguishable single photons. Here is an opportunity following the Refs. 20–28 to remember that the polariton system has a unique feature compared to the dilute atomic BEC and dense superfluids. It is related to direct experimental accessibility to measure the quantum statistical properties of the condensate. The main decay channel of the cavity polaritons consists in the photon leakage from the cavity. During this process two conservation laws are fulfilled. The energy and the in-plane wave vectors of each cavity polariton and leaked photon are fully coincident. It is a new experimental tool to study the non-equilibrium open system. As was mentioned in the Ref. 22 the photon portion of the polariton when leaving the microcavity keeps all relevant information concerning not only the polariton population, but also its phase that reflects the degree of coherence of the formed Bose–Einstein condensate. The cavity polaritons are an unique system for exploring both the cavity quantum electrodynamics and the many-body physics, opening the possibility to realize experimentally different phase transitions [20–28]. The phase properties along the spot can be measured studying interferometrically the interfence fringes arising from the photons leaked from different points of the spot. The emitted photons conveyed to the researchers the information about the polaritons occupancy at the given wave vectors $\mathbf{k}_{\|}$, about their phase, their coherence, noise, polarization and polariton dispersion law. The second order time and space correlation functions can be determined using the Hanbury–Brown and Twiss setup. The convincing proofs and evidences of the experimental realization of the BEC phenomenon do exist. The made measurements testify that the macroscopic population and the establishing of the coherent phase of the condensate over whole illuminated spot take place simultaneously as an unique interdependent process exactly as it was argued by Stoof [12]. It can be repeated again following [22] that the ease with which the internal properties of the condensate can be assessed is due to the photon portion of the polaritons that leak out the cavity. The initial demonstrations of the condensation in the lower polariton branch were made by some research groups such as the groups of Deng [30] and Kasprzak [31] in Grenoble. They used CdTe-based microcavities. The BEC under the nonresonant excitation of the system relies on many observations such as: (i) the nonlinear threshold in the accumulation of the polaritons in the point $\mathbf{k}_{||}=0$ with vanishing chemical potential $\mu \to 0$; (ii) the increase of the temporal coherence with the simultaneous decrease of the linewidth; (iii) the second order temporal correlation function measured by the Hanbury–Brown and Twiss photon-counting setup; (iv) the long-range spatial order measured interferometrically. The latter means the interference between two shifted one to another images of condensate that allows to probe the long-range spatial coherence over the entire spot. The ability to obtain in a very direct way by interferometric measurements the information about the polariton population is a major advantage of polaritons and has served as a basis for many different observations. Below threshold the coherence of the polariton could typically extends over microns, approximately equal to the de Broglie wavelength of polaritons at the measured temperature 19 K. Above the condensation threshold, correlations were observed over most of the excitation spot more than 20 μm.

6. The driven dissipative nature of the polariton condensate

The polaritons in microcavity have a lifetime comparable to the thermalization time that gives them an inherently nonequilibrium nature as was mentioned in the Refs. 20–32. It raises fundamental questions especially concerning the possibility to guarantee the effectuation of the relaxation processes and of establishing thermodynamic quasi-equilibrium. Unexpectedly, due to an excessive small polariton effective mass and as a result of a very small elastic polariton–polariton scattering time, many doubts in this direction were removed and the cavity polaritons exhibited many features that could be expected for the equilibrium Bose–Einstein condensates. Spontaneous coherence embracing of a macroscopic number of particles is a fascinating phenomenon. It is represented by laser radiation, by BEC of ultra cold atoms, by the formation of the Cooper pairs in the superconductors as well as by the formation of the superfluid fraction in superfluid ^{4}He. Now to this list one can add the BEC of microcavity polaritons and of the interacting cavity photons [20,49]. A large number of particles initially possessing random phases become coherent due to their interaction and the thermalization processes when such parameters as density and temperature crossed their threshold values. Exciton polaritons undergoing BEC incorporate new physics due to their intrinsically non-equilibrium nature. In many experiments [20–28] the exciton-polariton condensates were produced in the vicinity of 10 K in GaAs and CdTe semiconductors, whereas in GaN, ZnO and organic semiconductors the polariton condensates were realized at room temperatures [20,25]. They stimulated the development of new physical aspect not only in the fundamental perspectives but also in quantum technological devices [20,25]. The Bose–Einstein condensed cavity polaritons with short lifetime, their evolution in time and in the real space taking into account the gain and loss terms were described in Ref. 20 by the generalized Gross–Pitaevskii equation (GPE):

$$i\hbar\frac{d\psi(rt)}{dt}=\left\{-\frac{\hbar^2}{2m_{LP}}\Delta+V_d(\mathbf{r},t)+\frac{i\hbar}{2}\left(R\left(n_R(\mathbf{r},t)\right)-\gamma\right)+\hbar g\left|\psi(\mathbf{r},t)\right|^2+2\hbar g n_R(\mathbf{r},t)\right\}\psi(\mathbf{r},t). \quad (13)$$

Here $\psi(\mathbf{r},t)$ is the macroscopic wave function of the condensate on the lower polariton branch with effective mass m_{LP}, $V_d(\mathbf{r},t)$ is a local potential. The gain is effectuated by the reservoir with the density of the particles $n_R(\mathbf{r},t)$ satisfying the kinetic equation and the lost γ is due to the leakage of photon portion of the polaritons through the mirrors of the microcavity. In the review article by Deveaud [22] the experiments were described, when the pulse duration of the off-resonant excitation was about 100 fsec. The delayed buildup of the population of polaritons in the lower branch was observed with a very rapid buildup of the long-range phase coherence in the condensate soon after the stimulated scattering. The onset of long- range coherence appears even for points at $10\,\mu\text{m}$ distance. As was mentioned by Deveaud [22], this result is amazing because in some earlier expressed opinions just the phase relaxation of the condensate was supposed to be in delay comparable with the relaxation time of the number of the condensed particles N_0. This result confirms also the theoretical statements by Stoof [12] that the relaxation of the number N_0 and of the condensed particle phase are unique processes taking place simultaneously without delay between them. When the initial random phase was introduced in the lower polariton branch, then the set of the vortices in the excitation spot was observed. The annihilation of the counter-rotating vortices and the restoration of the phase coherence take place during only a few picoseconds.

The stability of the elementary excitation spectrum in non-equilibrium conditions was firstly discussed in Ref. 50 using a Keldysh diagram technique. Another route based on the nonequilibrium GPE was used in Ref. 20 leading to similar results. The main feature is the diffusive behavior of the Bogoliubov dispersion at low wave vectors **k** and the restoration of the standard Bogoliubov picture at greater values of **k**. The stability of the condensate is guaranteed by the fact that the imaginary part of the dispersion law is negative for all values of **k** [20]. As was reminded in Ref. 22, the criterion formulated by Penrose and Onsager [51] for the condensate single-particle density matrix $\rho(x,x') = \psi^{+}(x)\psi(x')$ does exist. Its limit at $|x-x'| \to \infty$ must be different from zero. It was proved experimentally, that despite the dissipative nature of the polaritons their distribution follows the criterion proposed by Penrose and Onsager, because above the thresholds the macroscopic occupancy of a single-particle level was clearly evidenced [22]. In the Ref. 22 it is reminded another outstanding feature of the Bogoliubov theory of superfluidity, which consists in the superposition of wave functions of the corresponding operators of the propagating particle and of its counter-propagating antiparticle or hole. In the superfluid state there are two branches of the Bogoliubov energy spectrum. One is positive and is named as an energy branch whereas another is negative and is named as an quasi-energy branch or even as a ghost branch. Both branches in the range of small wave vectors have the linear dispersion relations. The experiments performed in the Ref. 22 were based on the idea to probe the dispersion law of the superfluid excitations using the four-wave mixing (FWM) method. The superfluid fraction of polaritons was induced in the point $\mathbf{k}_{\parallel} = 0$ by the pumping beams. The properties of the system were probed through the diffraction of the probe beam sent at an angle to the normal to QW. The ghost branch appeared only above the threshold density for the superfluidity of the polariton gas. To describe the behavior of the condensate in the case of the FWM it was used the generalized GPE with the wave function $\psi(\mathbf{r},t)$ of the form $\psi(\mathbf{r},t) = \varphi_0(\mathbf{r},t) + U(\mathbf{r},t)e^{i\mathbf{kr}} + V^{*}(\mathbf{r},t)e^{-i\mathbf{kr}}$ where $\varphi_0(\mathbf{r},t)$ describes the coherent polariton superfluid created by the pump beams in the point $\mathbf{k} = 0$, whereas $U(\mathbf{r},t)$ and $V^{*}(\mathbf{r},t)$ are the wave functions describing the counter-propagating excitations of the system. At enough large densities of the polaritons, the ghost branch shows the same intensity as the normal branch [22].

7. Superfluidity and vortices

In this section we will follow the review articles with Refs. 20, 22, 25. The criterion of superfluidity was originally proposed by Landau [52] to explain the phenomenon of superfluidity of the liquid Helium below the temperature of the λ-point discovered by Kapitsa [3]. Landau supposed that the energy spectrum of the elementary excitation in the superfluid helium has a roton-type behavior with a critical velocity determined as

$$V_c = \min_{\mathbf{k}} \frac{\omega(\mathbf{k})}{|\mathbf{k}|}. \quad (14)$$

It concerns to the liquid in a rest state. When the liquid is moving with respected to the laboratory reference frame, the energy spectrum due to the Galilean transformation looks as

$$\omega_{Lab}(\mathbf{k}) = \omega(\mathbf{k}) + \mathbf{Vk}. \quad (15)$$

It can obtain negative values if $V > V_c$ and $(\mathbf{k}\cdot\mathbf{V}) < 0$. In the flowing liquid with supersonic velocity V, the possibility to emit unlimitedly elementary excitations and to break the superfluidity appears. If the moving condensate has the parabolic dispersion law, its particles encountering the obstacle will find the resonant states to scatter and to transmit their energy. In his theory of superfluidity Bogoliubov obtains the energy spectrum with a needed linear dependence in the range of small wave vectors with the sound velocity

$$V_c = \sqrt{\frac{gn_0}{m_{LP}}}, \quad (16)$$

where $gn_0 > 0$ determines the blue detuning of the exciton energy level due to the exciton–exciton interaction. In the case of $gn_0 \approx 1$ meV and $m_{LP} \approx 10^{-4} m_0$ the Bogoliubov sound velocity has the value $V_c = 10^{8}$ cm/s. The liquid with flow velocity V smaller than the Bogoliubov sound velocity V_c has no resonant states to scatter on the weak impurity or obstacle. The superfluidity is due to the suppression of the scattering processes. But this statement is true only if the impurities or the obstacles have a weak influence on the fluid and do not change essentially its density in their surroundings. In the opposite case the vortices in the liquid near the surfaces of the impurities and obstacles appear. The question arises whether the Bogoliubov type dispersion law does exist in the conditions of the driven-dissipative system. It was investigated on the base of generalized Gross–Pitaevskii equation in the Ref. 53 taking into account the gain effectuated by an incoherent reservoir the dynamical variables of which modify strongly the dispersion law of the condensate. As a result the instability in the spectrum appeared in the range of small wave vectors [53]. Another analysis on the base of the Green's function method proposed in Ref. 54 arrived to the conclusion about the existence of the normal and superfluid components. The investigations of the quantum vortices in the superfluid liquid helium were initiated by Feynman [55–57].

Now the vortices formation in the 2D superfluid polariton gas will be discussed. As was mentioned above in the infinite homogeneous 2D system the off-diagonal long-range order associated with the BEC phenomenon does not exist. Its existence breaks down being vulnerable to the thermal fluctuation at nonzero temperatures. However such systems can exhibit quasi-long-range-order preserving superfluidity. Berezinskii [47], Kosterlitz and Thouless [48] investigated the formation of the vortices in such system due to the quantum fluctuation. The creation of one vortex at low temperature is unlikely due to the necessity of an extensive amount of energy. In spite of it, the creation of a vortex-anti-vortex (V-AV) pair is possible at low temperature and does not need an extensive amount of energy, because the phase disturbances created by the vortex the anti-vortex are canceled out. The pairs V-AV may exist in free or in bound states. The transition of the V-AV pairs from the unbound to bound state is named as the Berezinskii–Kosterlitz-Thouless (BKT) phase transition. The possible crossover from the BKT and BEC transitions can be observed changing the density of the Bose gas. We can expect the existence of the free vortices above the BKT critical temperature, the presence of the bound V-AV pairs in the BKT phase and the absence of the vortices in the BEC phase. At the same time the change of the density of the electron–hole (*e*–*h*) pairs leads to another type BEC–BCS crossover. It takes place due to the changes of electron structure of the quasiparticles rather than the vortex or soliton formations. At low densities of the *e*–*h* pairs we deal with the exciton formation leading to cavity-polariton existence. At high density of *e*–*h* pairs due to the screening of the *e*–*h* Coulomb interaction the excitons as well as the exciton polaritons do not exist. Instead of them we deal with the 2D degenerate electron and hole Fermi-gases and with their Fermi energies. The electrons and holes from the surroundings of their Fermi energies can take place in the coherent pairing, forming the Cooper-type *e*–*h* pairs as was established by Keldysh and Kopaev [58] similar with the electron Cooper pairs in the theory of superconductivity. At high density of *e*–*h* pairs we can expect the formation of the 2D photon gas instead of 2D polariton gas with a strong indirect interaction between the photons mediated by their direct interactions with *e*–*h* pairs. Such conclusions were made in the Ref. 59. The photon BEC was investigated experimentally in the Ref. 49. The direct interaction of the photons with the *e*–*h* pairs can create not only the indirect interaction between the photons, but it can create also the supplementary effective attraction between the electrons and holes [25].

8. GaN-based exciton polariton laser

This section contains the information concerning the Galium Nitride room-temperature electrical-injection polariton laser elaborated and described in the original paper [36] and in the review articles [25,27,39]. The interest in this laser is determined by the fact that it represents the first realization of the ultra-low-energy-coherent light source. In the previous decades a fast development of the growth and of the technology of the GaN based devices took place [60–64]. The industrial production of lasers emitting in the blue and ultraviolet ranges of spectrum was realized. These achievements facilitated the creation of the GaN polariton laser. The previous GaN lasers were based on the band-to-band quantum transitions and can be named as band-edge lasers. They take into account the quantum transitions between the conduction and valence band in the condition of the inversion population, when the electrons from the valence band are preliminary excited into the conduction band so that the bottom of the conduction band is completely filled by electrons whereas the top of the valence band is completely filled by the holes. As is frequently mentioned the band-edge laser needs an electron inversion population with regard to the ground state of the crystal. The ground state of the crystal means the absence of the electron-hole pairs at all. Such-type inversion population remains the same in the case of high density *e*–*h* pairs, when they being bound in the form of Wannier–Mott excitons form a weakly non-ideal exciton gas. In this case a set of kinetic and dynamic processes in the presence of the pump and loss occur. The quasi-equilibrium thermalization and the condensate formation take place without any supplementary inversion and the lasers constructed in these conditions are named as inversionless lasers, in spite of the fact that the initial inversion with regards the ground state of the crystal persists. The creation of a nonequilibrium exciton gas, with the density by some orders of magnitude smaller than the density of *e*–*h* pairs in the case of the band-edge laser, needs smaller excitation energy by the same order of magnitude. As was underlined in the Ref. 36 the current density threshold needed for the creation of the electrically-injected polariton laser is almost hundred times smaller than for the GaN-based edge-emitting and surface-emitting lasers. The edge-emitting laser, as was mentioned by [39], is due to the stimulated emission of the photons and this process imposed stringent requirements on the amount of energy needed for its realizations. In contrast to this in the case of exciton–polariton BEC the condensate emits a coherent-light directly due to the leakage of the photons from the cavity and without any supplementary stimulation processes. As a result the polariton laser offers a more effective way to generate coherent light at very low threshold [39]. Some data concerning the GaN diode are needed. The GaN diodes consist from a 430 nm thick GaN *p*–*n* junction. The energy of the cavity mode equals to E_c = 3.57 eV with a linewidth γ_c = = 1.7 meV corresponding to cavity mode lifetime 0.387 ps. The quality factor of the cavity is Q = 1911. The energy of the exciton transition at 10 K equals to $E_{\rm ex}$ = 3.65 eV with a linewidth 4.8 meV. The blue shift of the polariton emission line caused by the polariton–polariton interaction equals to $\delta E = (1.9 \pm 0.28)$ meV when the exciton concentration was $n_{\rm ex}^{3D} = (1.53 \pm 0.13)\cdot 10^{17}$ cm^{-3}. The Mott transition density is $(1–2)\cdot 10^{19}$ cm^{-3}. In GaN the exciton has a binding energy of the order of 30 meV and is characterized by extremely high oscillator strength [24]. Wannier Mott excitons with small binding energy of about 10 meV are unsuitable for many practical applications [24]. Strong oscillator strength permits to achieve the strong-coupling regime necessary to obtain the reversible exciton–photon conversion and the formation of the polaritons even at elevated concentrations of excitons. A dominant radiation in the case of polariton laser is observed below the exciton resonance $E_x = = 3.65$ eV. The value of the normalized chemical potential equals to

$$\alpha = -\mu/k_B T = \ln\left(1+\frac{1}{N_0}\right) = 0.59 .$$

The small value of α confirms the establishing of the quantum degeneracy in the system [36]. One can once again mention that the process of the polariton laser creation is very different from the stimulated emission of radiation in the conventional photon laser with inversion distribution. The novelty and advantages of the polariton devices elaborated in Ref. 36 are related to the successful fabrication of the low resistance diode and to strong feedback which can be achieved with high quality DBR mirrors. In those conditions both polariton and photon lasers can be constructed. The polariton laser, as was underlined in the Ref. 36, originates from a nonequilibrium BEC obtained by dynamical balance between injection and loss. A condensate with a size of about 1.5 μm was achieved. At low current injection level the emission is depolarized. Above threshold the polarization of about 30% develops spontaneously. The long-range spatial coherence along the condensate extent was measured by the Mach–Zehnder interferometer [36]. As was mentioned in [39] electrically pumped polariton lasers are a new generation of coherent light sources and represent a model system for the fundamental understanding of strong-coupled quantum electrodynamics. They promote the development of nanophotonics.

9. Conclusions

As was underlined in the previous section, the Bose–Einstein condensate is characterized not only by a macroscopic population of the ground state but, that is very important, by the existence or by the formation of a coherent macroscopic state described by a macroscopic quantum wave function Ψ named also as the order parameter, and this wave function has the properties

$$\Psi^\dagger\Psi = N_0, \quad \Psi = \sqrt{N_0}e^{i\alpha}, \quad N_0 \sim V. \tag{17}$$

The fixed or the well defined value of the phase α is a very important property of the Bose–Einstein condensate. If the Bose–Einstein condensate is slowly inhomogeneous in space and in time, in this case we have the functions $N_0(x,t)$ and $\alpha(x,t)$, which are changing following the Gross–Pitaevskii equation. But $\alpha(x,t)$ is well defined and is not changing randomly as in the case of noncondensed particles. Their distribution functions $n_{\mathbf{p}}(x,t)$ with wave vector $\mathbf{p}$ different from the condensate wave vector $\mathbf{k}(\mathbf{p}\neq\mathbf{k})$ are determined by the Boltzmann kinetic equations. In the homogeneous system and in the case of the BEC in the single-particle state with wave vector $\mathbf{k}\neq 0$ the coherent macroscopic state is characterized by the wave function $\Psi(x,t) = e^{-i\mu t}e^{-i\mathbf{kx}}\sqrt{N_0}$, where μ is the chemical potential. They lead to the nondiagonal elements of the single-particle density matrix

$$\rho^{(1)}(x,x') = \frac{1}{V}\Psi^\dagger(x,t)\Psi(x',t) = \frac{N_0}{V_0}e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')} = n_0 e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')}. \tag{18}$$

Their absolute value in the thermodynamic limit $N_0 \to \infty$ and $V \to \infty$ and in the limit $|\mathbf{x}-\mathbf{x}'| \to \infty$ equals to $n_0 \neq 0$. It means that the system has the off-diagonal long-range order and satisfies to the condition formulated by Penrose and Onsager [47]. This criterion is a central concept of the theory of BEC phenomenon [25]. In the review articles [24,25] and in the original experimental investigations described in the Refs. 30, 31 a special attention was paid to the interferometric measurements of the phase coherence of the cavity polariton condensates. On this way, in Ref. 31 it was shown that the coherence time of N_0 condensed polaritons is greater than the coherence time of a single polariton. In such a way it was confirmed the existence of the macroscopic coherent condensation fraction. The temporal coherence of the polariton laser radiation has a collective nature due to the polariton-polariton interaction in the condition of the pump and of the loss and, that is very important, it exceeds the lifetime of a single polariton. Side by side with the measurements of the population distributions, the interferometric data testify the establishing of the polariton condensate as a collective coherent state. Its formation in the condition of pump and loss is a marvelous event saying us that every decayed polariton was compensated in the frame of condensate by another pumped polariton exactly with the same phase. Such substitutions are possible due to the bosonic induced scattering processes, the rate of which is proportional to the number of particles in the final state. In the investigations represented in the Refs. 30, 31, 24, 25 not only the temporal coherence of the polariton condensates but also their spatial coherence were studied. Toward this end the photons leaked from the microcavity from different sites of the extended in space condensate were directed to the interferometer to determine the degree of coherence along the full surface of the condensate. It was shown experimentally [31] by the interferometric measurements that the spatial coherence of the condensate and emitted radiation is much greater than the spatial coherence of a single photon. Below threshold the spatial coherence of a single particle is determined by its de Broglie wavelength λ_{dB} [24]. The dispersion law of the emitted photons escaped due to the leakage from the cavity coincides with the curve of the lower polariton branch and does not follow the cavity photon dispersion [24]. Excitons in GaAs-type crystals have the resulting spin structure with the projections $j_z^{\rm ex}$ =±1 for the bright dipole-active states and with projections $j_z^{\rm ex} = \pm 2$ for the forbidden dark states. The bright states determine the polarization of the emitted radiation used in the devices of the spin-optronics. The interaction of the polaritons gives rise to the shift of the polariton energy levels, which is enhanced due to the small volume of the microcavity and to the stronger exciton–photon interaction in it than in an unlimited 3D space. The self-repulsion of the polaritons leads to their blue shift and to extremely high optical nonlinearity. A few numbers of photons in microcavity with small volume can form a polariton gas with considerable density. The repulsion between the polaritons can be used to organize the pushing force and photon blockade [24]. In such a way the polariton condensates are an excellent platform to study the physics of quantum gases, quantum phase transitions and quantum vortices [24]. In another review article in Ref. 25 the difference between the coherence in the case of the conventional lasers and in the case of polariton condensates is analyzed. In the first case we deal with the coherence of the emitted photons in conditions when the media remains incoherent. In the case of polariton BEC the coherence is accumulated in the condensed fraction of the polariton gas, not in the cavity photons [25]. The coherent photon radiation outside the microcavity appears due to the direct transformation of the condensed polaritons into the leaked photons without any other stimulation processes. The distinction between two variants concerns the particle species undergoing the coherence. Laser becomes coherent by stimulated emission of radiation, whereas the polariton condensates are coherent owing to the bosonic stimulated scattering processes. This distinction is clear if the polariton lifetimes are much greater than the thermalization time. In many cases it is not so and the intermediate regimes do exist [25]. Such regime when the strong-coupling exciton–photon interaction in the cavity does exist and the coherent macroscopic polariton condensate with a well defined fixed phase and with a macroscopic population of the ground state did appear but without a full thermalization process was proposed to be named as polariton laser. To this proposal it is necessary to add one remark. At least a partial thermalization is obligatory needed for the condensate formation. Without many stimulated bosonic scattering processes the common phase for all condensed particles could not appear. In fact the Stoof [12] scenario of the BEC phenomenon is more suitable to describe these processes. Stoof [12] mentioned that the phase relaxation and the condensed particle number N_0 establishing take place simultaneously in three stages, namely in two kinetic stages and in one coherence stage. After the first kinetic stage with the short characteristic time $\tau_{\rm el}$ and the coherence stage with the same duration, during which the condensate embryo appears, there is a second kinetic stage with a longer duration, possibly longer than the polariton lifetime. During this final stage the condensate embryo has to grow and to behave in equilibrium with the noncondensed fraction of the Bose gas. This last kinetic stage is not completely realized in the case of the cavity polariton condensates. Nevertheless the reliable establishing of the condensate phase coherence in the frame of the polariton BEC [30,31] was observed.

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