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Influence of unconventional current-phase relation of Josephson junction on the escape rate in macroscopic quantum tunneling regime

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In this study, we carried out the analysis of the quantum fluctuations on Josephson junction (JJ) dynamics with unconventional current-phase relation (CPR). We analyzed two case CPR: anharmonic term case and the case of Majorana term. The expression was obtained for the escape rate in macroscopic tunneling regime. It is shown that the changing of the escape rate JJ with unconventional CPR is determined by the renormalized critical current of JJ.

Keywords: quantum fluctuations, Josephson junction, Majorana fermion.

Introduction

Macroscopic quantum tunneling is an intriguing phenomenon in quantum mechanics appearing at the macroscopic level, which was observed in various field of physics [1–6] (see also [7]). The phenomenological theory of this phenomena was developed in the corresponding study [1]. The microscopic derivation of the effective action for a tunnel JJ was conducted in [2,3]. Macroscopic quantum tunneling in JJ has attracted interests in condensed physics from the point of application to a Josephson phase qubit [8,9]. It is well known that macroscopic quantum tunneling in JJ was realized in switching experiments of at low temperatures [4,10].

In all studies of the macroscopic quantum tunneling in JJ (see [1–10]), it was considered that CPR has a harmonic character:

$$I = I_{c0} \sin \varphi. \tag{1}$$

Where I_{c0} refers the critical current. Relationship (1) is fulfilled with a high accuracy for JJs on low-temperature superconductors [9]. In the case of JJs on high-temperature superconductors, the CPR becomes anharmonic [11–13]:

$$I = I_{c0} f_\alpha(\varphi) = I_{c0}(\sin \varphi + \alpha \sin 2\varphi), \tag{2}$$

where anharmonicity parameter α depends on the junction preparation technology. In general, anharmonicity in the CPR for high-temperature and Fe-based superconductors based JJ are associated with the *d*-wave behavior of the order parameter and many band character of superconducting state in new superconducting compounds [14]. Dynamical properties of single JJs with an anharmonic CPR (2) were previously studied in [15–18].

In the case of JJs based on topological superconductors, the CPR include additional fractional term [19–21]

$$I = I_{c0} f_m(\varphi) = I_{c0}(\sin \varphi + m \sin(\varphi / 2)). \tag{3}$$

Second term in Eq. (3) related with Majorana quasiparticles and dynamical detection of this particles seems very challenging in condensed matter physics. Discovery of Majorana fermions seems very interesting from the point of fault-tolerant quantum computing [22]. The few number of papers devoted to dynamical properties of single JJs with Majorana term (3) [23,24]. In this study, we carried out the detailed analysis of the influence of unconventional current-phase relations (2), (3) of JJ on the escape rate in macroscopic quantum tunneling regime. The details of the changing of escape rate and related properties of JJ with unconventional CPR are detailed in the following section.

Basic equations

In consideration of the influence of the any fluctuations on the S $\rightarrow$ R (from superconducting state to resistive state) switching of JJ seems interesting from the point of applications. If the current growing rate r is small $r << \gamma^{3/2}$, where $\gamma = kT/E_J$, then the thermal fluctuation can initiate switching to resistive R state (Fig. 1(a)) over potential barrier in close to critical current I_{c0} [5,7]

$$\Delta U = \frac{4\sqrt{2}}{3} E_J (1-i_e)^{3/2}, \quad E_J = \frac{\hbar I_{c0}}{2e}, \tag{4}$$

where $i_e = I_e/I_{c0}$ is the normalized external current. For the probability of such switching [5] is true expression

$$p(t) = 1 - \exp\left[1 - \int_{-\infty}^{t} \Gamma_{TA}(I(t'))dt'\right], \tag{5}$$

where tunneling rate of thermal activation $\Gamma_{TA}(I)$ calculated as

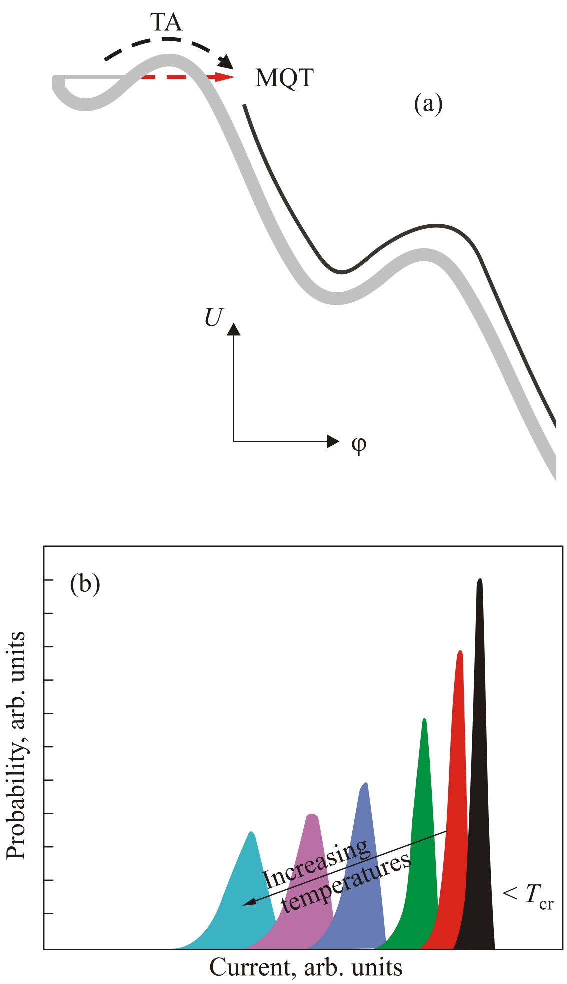

Fig. 1. (a) Potential profile of JJ under exrernal current in two regimes: thermal activation (TA), macroscopic quantum tunneling (MQT). (b) Critical current hystograms under fluctuations at different temperatures.

$$\Gamma_{TA}(I) = \frac{\Omega_{p0}}{2\pi} \exp\left[-\frac{\Delta U(I)}{kT}\right]. \tag{6}$$

In the Eq. (6), the plasma frequency Ω_{p0} of JJ calculated as [1–5]

$$\Omega_{p0} = \left(\frac{2eI_c}{\hbar}\right)^{1/2} (1-i_e)^{1/4}. \tag{7}$$

At low temperatures, the phase dynamics influenced by the quantum fluctuations and escape rate in macroscopic quantum tunneling limit can be calculated by the equation (see [1–3,10])

$$\Gamma_{MQT}(I) = \frac{\Omega_p}{2\pi} \exp\left[-\frac{\Delta U(I)}{\hbar \Omega_p}\right]. \tag{8}$$

In lowering of temperature, at $T_{\rm cr}$ there is a crossover between thermal activation and macroscopic tunneling regimes [4]:

$$T_{\rm cr} = \frac{\hbar\Omega_p}{2\pi k}\sqrt{1 + \frac{4}{Q^2}} - \frac{1}{2Q}, \tag{9}$$

where quality factor of JJ is determined by the following expression:

$$Q = \Omega_p RC. \tag{10}$$

In Eq. (10) R and C are the resistance and capacitance of JJ, correspondingly. Experimental observation of macroscopic quantum tunneling phenomena in low-temperature superconductors was conducted in [25,26] many years ago. There is experimental data recent years on macroscopic quantum tunneling in d-wave and many-band superconductors (see below and [7]). Histograms of critical current of JJ with decreasing of temperature presented in (Fig. 1(b)). It is clear that the dispersion of critical current will increase when the temperature increases.

Results and discussions

Presence of second harmonic in CPR in Eqs. (2), (3) leads to renormalization of critical current I_{c0}. For the calculations, we utilized analytical solution for the maximum point of the functions $f_{\alpha;m}(\varphi)$ (2), (3) similarly to study [27]. Calculation leads to expression for the renormalized critical current

$$\frac{I_c}{I_{c0}} = \begin{cases} 1 + 2\alpha^2, & \text{anharmonic term} \\ 1 + \dfrac{m}{\sqrt{2}} + \dfrac{7}{64} m^2, & \text{majorana term} \end{cases}. \tag{11}$$

The effective critical current I_c of a JJ with an anharmonic CPR as a function of amplitude of second term (α or m) (see Eqs. (2), (3)) presented in Fig. 2. As you can see, with the increasing of this amplitude, effective critical current I_c also increased. Quadratic behavior at small α (Fig. 2, lower line) converted to linear dependence at high values of anharmonicity parameter α. Inclusion of

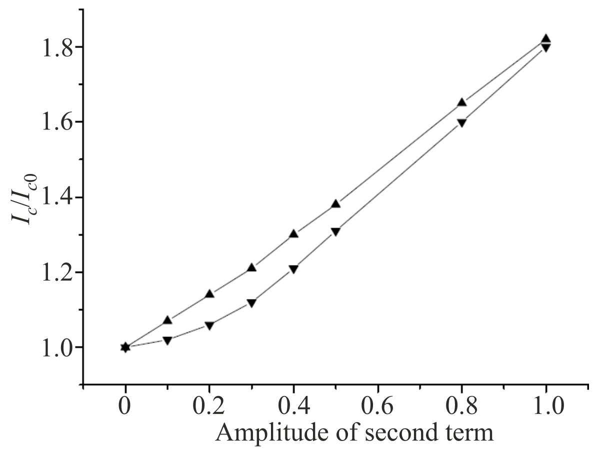

Fig. 2. Effective critical current of JJ with unconventional CPR: upper with Majorana term, Eq. (3); lower anharmonic case, Eq. (2).

Majorana term m leads to a linear increasing of critical current (upper line in Fig. 2).

Recalculated escape rate Γ_{MQT} (Eq. (8)), taking into account critical current renormalization (11) leads to given expression

$$\frac{\Gamma_{MQT}}{\Gamma_{MQT0}} = f_{\alpha,m}^{1/2} \exp\left[\frac{\Delta U(I)}{\hbar\Omega_p}(1-f_{\alpha,m}^{1/2})\right], \qquad (12)$$

where Γ_{MQT0} is the escape rate for the CPR (1). Result of calculations using Eq. (12) presented in Fig. 3, where denoted dependence of normalized quantity $\ln(\Gamma/\Gamma_0)/\delta u$; where $\delta u = \Delta U(I)/(\hbar\Omega_{p0})$ as a function of amplitude of second term (anharmonicity parameter α and Majorana term m). It is clear decreasing of the escape rate Γ_{MQT} with increasing anharmonicity parameter α and Majorana term m. Such result can be explaned by the increasing of the Josephson junction critical current I_c and as a result highest of the potential barrier (4) (Fig. 1(a)). Such conclusion in agreement with results of behavior of quantum particle in potential hole from quantum mechanics textbooks. Similar estimation of escape time in JJ between single order parameter and multiband superconductor was conducted in a previous study [28]. Within calculations, effective action renormalized by the Josephson–Leggett mode is considered [28]. Drastic enhancement of the escape rate was predicted based on the Josephson–Leggett mode.

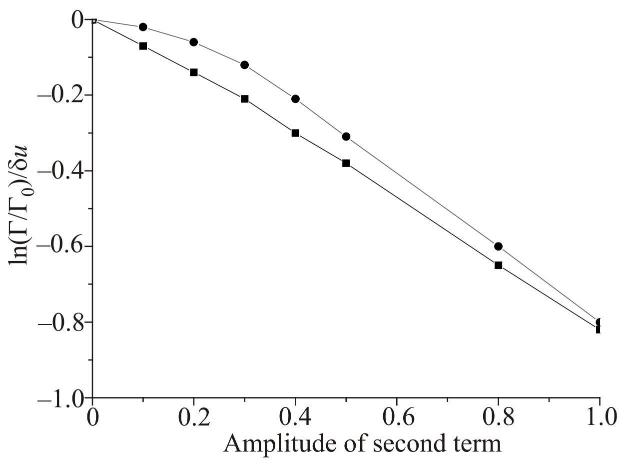

Fig. 3. Influence of the amplitude of second term on the escape rate in macroscopic quantum regime: upper line (anharmonic term); lower line (Majorana term).

Thus, in this study, the influence of quantum fluctuations on the escape rate of JJ with unconventional CPR was investigated. Renormalization of critical current in JJ with anharmonic and Majorana terms leads to decreasing of the escape rate with the increasing of amplitude of second term in CPR.

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Вплив нетрадиційного співвідношення струм–фаза джозефсонівського переходу на швидкість виходу в макроскопічному режимі квантового тунелювання

I.N. Askerzade

Проведено аналіз квантових флуктуацій в динаміці джозефсонівських переходів (ДП) з незвичайним співвідношенням струм–фаза (ССФ). Проаналізовано два випадки ССФ, що обумовлені ангармонічними та майоранівськими квазічастинками. Вирази отримано для швидкості виходу в макроскопічному тунельному режимі. Показано, що зміна швидкості виходу ДП з незвичайним ССФ визначається перенормуванням критичного струму ДП.

Ключові слова: квантові флуктуації, джозефсонівський перехід, майоранівський ферміон.

Влияние нетрадиционного соотношения ток–фаза джозефсоновского перехода на скорость выхода в макроскопическом режиме квантового туннелирования

I.N. Askerzade

Проведен анализ квантовых флуктуаций в динамике джозефсоновских переходов (ДП) с необычным соотношением ток–фаза (СТФ). Проанализированы два случая СТФ, обусловленные ангармоническими и майорановскими квазичастицами. Выражения получены для скорости выхода в макроскопическом туннельном режиме. Показано, что изменение скорости выхода ДП с необычным СТФ определяется перенормировкой критического тока ДП.

Ключевые слова: квантовые флуктуации, джозефсоновский переход, майорановский фермион.