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Effect of long-range 1/*r* interaction on thermal and quantum depletion of a dipolar quasi-two-dimensional Bose gas

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The depletion of a quasi-two-dimensional (quasi-2D) dipolar Bose condensed gas in the presence of both contact and long-range $1/r$ interactions is investigated in the framework of Hartree–Fock–Bogoliubov (HFB) approximation. When the characteristic wavelength of a mode is much larger than the trap size, the dipole-dipole (DD) interaction can be treated as a contact interaction and in the low momentum limit the long-range nature of the $1/r$ interaction has the dominant contribution and leads to the nonlinear (nonphononic) dispersion relation. We will show that quantum depletion is temperature independent and is determined by the contact, DD and long-range $1/r$ coupling constants ($\varepsilon_{dd} = g_d/g$ and C). The small momentum behavior of the quantum depletion is affected by long-range $1/r$ interaction and at large momentum limits the momentum dependence of quantum depletion unaffected by $1/r$ interaction.

Keywords: dipolar Bose condensed, long-range interactions, quantum and thermal depletion.

1. Introduction

Due to the fact that the physics of low-dimensional systems is fundamentally different from those in three dimensions, low-dimensional ultracold dipolar atoms have attracted attention both theoretically and experimentally [1–3]. At finite temperature strong quantum phase fluctuations prevent to occur Bose–Einstein condensation (BEC) in a homogeneous two-dimension system but when temperature is lowered enough the Berezinskii–Kosterlitz–Thouless (BKT) transition can be occur in these systems [4,5].

Novel quantum phases and many-body phenomena can be observed in bosonic systems with the dipole-dipole (DD) interaction [6–8]. Compared to a Bose gas with short-range interactions, a dipolar Bose gas qualitatively has different physics due to the anisotropic and nonlocality nature of the long-range, DD interaction. Especially the long-range nature of the dipolar interaction may be relating the BKT transition and phase coherence. Interplay between the nonlocal DD interaction and the usual local short-range contact interaction, leads to the possibility of experimental realization of highly controllable and stable solitary structures in BEC [9]. The anisotropic nature of the DD interaction introduces novel phenomena such as geometry-dependent mechanical stability [10], *d*-wave collapse [11] and a roton-maxon dispersion relation in quasi-2D systems [12].

Another impressive long-range interaction is $1/r$ potential which depends on its sign could be a repulsive Coulomb interaction of charged atoms or an attractive gravitational interaction. The former exists in charged boson systems but the latter is only possible in nature on stellar scales and it is too weak in dilute atom gases that cannot be detected experimentally.

The attractive form of this potential which can be created by certain laser configuration [13], is similar to gravitation but up to 17 orders of magnitude stronger. Because the attractive $1/r$ interaction balances both the kinetic energy and the contact interaction a self-binding situation can occur. Such enormous attractive gravitational forces are only possible in nature on stellar scales. Therefore, it provides conditions to study gravitational effects on the stellar scale which customarily only important in the laboratory. Such ultracold quantum gases can also be used to investigate the possibility of a Bose star which, so far, has only been discussed theoretically [14,15].

By means of the Kubo formalism, low-temperature shear viscosity of a spin polarized two-component quasi-2D dipolar Fermi gas with long-range $1/r$ interaction in the BEC limit, was calculated [16]. By using the time-dependent mean-field approach based on the Popov approximation, the Landau damping in a Bose–Fermi superfluid mixture in the presence of a long-range $1/r$ interaction between bosons for three and two dimensions at finite temperature was also studied [17].

A theory of damping of low energy, collective excitations in a quasi-2D, homogenous, dipolar Bose gas at zero temperature, via processes whereby an excitation decays into two excitations with lower energy was developed [18]. In the Bogoliubov–de Gennes (BDG) theory beyond mean-field approximation, properties of both homogeneous and harmonically trapped dipolar Bose gases, focusing on the low-lying excitations have been theoretically investigated [19].

Thermal and quantum fluctuations of confined Bose–Einstein condensate beyond the Bogoliubov approximation were considered [20]. Using Bogoliubov theory beyond mean field, correction to the equation of state of a weakly interacting Bose gas in the presence of a tight 2D optical lattice was calculated [21].

To the best of our knowledge thermal and quantum depletion of a dipolar quasi-2D Bose gas in the presence of long-range $1/r$ interaction have not yet been considered. In this paper by generalizing the Bogoliubov approximation which assumes that most of the atoms are in the Bose condensate which is applicable near zero temperature to the finite temperatures where the condensate is strongly depleted, the effect of the noncondensate atoms in a self-consistent manner is included. From the resulting correlation functions the thermal and quantum depletions of homogeneous quasi-2D dipolar Bose gas in the presence of both contact and long-range $1/r$ interactions are obtained.

2. Formalism

For an ultracold, dilute gas of interacting bosons, the many body Hamiltonian in terms of the Bose field operators $(\psi, \psi^\dagger)$, can be written as

$$H = \int d\mathbf{r}\psi^\dagger(\mathbf{r})\left(-\frac{\hbar^2}{2M}\nabla^2 + U(\mathbf{r})\right)\psi(\mathbf{r}) + \\ + \frac{1}{2}\int d\mathbf{r}\int d\mathbf{r}'\psi^\dagger(\mathbf{r})\psi^\dagger(\mathbf{r}')V_{2D}(\mathbf{r}-\mathbf{r}')\psi(\mathbf{r}')\psi(\mathbf{r}), \quad (1)$$

where M is the mass of a single boson, $U(\mathbf{r})$ is the external, or trapping potential, and $V_{2D}(\mathbf{r}-\mathbf{r}')$ is the two-body interaction potential. The factor of 1/2 in the interaction term corrects for a double counting that is inherent in the integration.

The two-body interaction potential includes contact, DD and long-range $1/r$ interactions in momentum space is

$$V_{2D}(k) = g_{q2D}(k) + U_{q2D}^{dd}(k) + V_{2D}^{\mathrm{Coul}}(k), \quad (2)$$

where

$$V_{2D}^{\mathrm{Coul}}(k) = C\frac{2\pi}{k}. \quad (3)$$

Here C is the coupling constant ($C > 0$ and $C < 0$ corresponding to Coulomb interaction for charged bosons and gravitational interaction for neutral particles, respectively).

In the regime in which we refer to the condensate as being quasi-2D, the harmonic oscillator trap size $l_z = \sqrt{\hbar / m\omega_z}$ in the tightly confined direction is much larger than the three dimension scattering length a_{3D} of the system, i.e., the scattering processes still take place in three dimensions. In this regime the contact potential has the form [22]

$$g_{q2D} = \frac{2\sqrt{2}\,\hbar^2 a_s}{M\, l_z}. \quad (4)$$

Within the 2D mean field regime the effective quasi-2D, DD interaction can be written as [23,24]

$$U_{2D}^{dd}(k) = \frac{g_d}{\sqrt{2\pi}l_z}\left[F_\perp\left(\frac{kl_z}{\sqrt{2}}\right)\cos^2\alpha + F_\parallel\left(\frac{kl_z}{\sqrt{2}}\right)\sin^2\alpha\right], \quad (5)$$

where $g_d = 4\pi\hbar^2 a_{dd}/M$ with the dipole lengths $a_{dd} = Md^2/3\hbar^2$ (d is the dipole moment), $q = \sqrt{q_x^2 + q_y^2}$ and the dimensionless functions $F_\perp$ and $F_\parallel$, respectively, are

$$F_\perp(k) = 2 - 3\sqrt{\pi}ke^{k^2}\mathrm{erfc}(k), \quad (6)$$

$$F_\parallel(k) = -1 + 3\sqrt{\pi}\frac{k_x^2}{k}e^{k^2}\mathrm{erfc}(k). \quad (7)$$

We assume the DD interaction is isotropic($\alpha = 0$) in the xy plane and only depends on the magnitude of momentum in the momentum-space. In this case Eq. (5) becomes

$$U_{2D}^{dd}(k) = \frac{g_d}{\sqrt{2\pi}l_z}F_\perp\left(\frac{kl_z}{\sqrt{2}}\right). \quad (8)$$

Including the above interactions the full quasi-2D momentum-space interaction potential takes the form

$$V_{2D}(k) = \frac{2\sqrt{2\pi}\,\hbar^2 a_s}{Ml_z} + \frac{g_d}{\sqrt{2\pi}l_z}F_\perp\left(\frac{kl_z}{\sqrt{2}}\right) + V_{2D}^{\mathrm{Coul}}(k) = \\ = \frac{g}{\sqrt{2\pi}l_z}\left(1 + \varepsilon_{dd}F_\perp\left(\frac{kl_z}{\sqrt{2}}\right)\right) + C\frac{2\pi}{k}, \quad (9)$$

where $g = 4\pi a_s\hbar^2/M$ and $\varepsilon_{dd} = g_d/g$.

In the ultracold regime ($T << T_c$), where T_c is the critical temperature for BEC, the number of bosons occupying the condensed state is macroscopic to a good approximation, we can treat the condensate part of the field operator as a c-number and write the field operator as

$$\psi(\mathbf{r}) = \langle\psi(\mathbf{r})\rangle + \varphi(\mathbf{r}) = \phi_0(\mathbf{r}) + \varphi(\mathbf{r}), \quad (10)$$

where $\phi_0(\mathbf{r})$ is a classical wave function that describes the macroscopic state of condensed atoms, and $\varphi(\mathbf{r})$ corresponds to the excited, non-condensed states, or the so-called quantum fluctuations. An important consequence of the Bogoliubov approximation is that due to replacement of the operators by a number the particle number is no longer conserved. Thus we work in the grand-canonical ensemble by introducing the chemical potential μ as a Lagrange multiplier to conserve particle number [25].

By using equation (10) the grand-canonical Hamiltonian $K = H - \mu N$ in the HFB approximation can be expressed perturbatively in orders of the condensate occupation as $K = K_0 + K_1 + K_2 + ...$, where K_i contains the terms that are ith order in the quantum fluctuations $\varphi(\mathbf{r})$. The first term K_0 which contains only the condensate wave function is given by

$$K_0 = \int d\mathbf{r}\, \phi_0^*(\mathbf{r})\left(-\frac{\hbar^2}{2M}\nabla^2 + U(\mathbf{r}) - \mu\right)\phi_0(\mathbf{r}) + \frac{1}{2}\int d\mathbf{r}\int d\mathbf{r}'\phi_0^*(\mathbf{r})\phi_0^*(\mathbf{r}')V_{2D}(\mathbf{r}-\mathbf{r}')\phi_0(\mathbf{r}')\phi_0(\mathbf{r}). \tag{11}$$

Minimized K_0 with respect to small variations in the condensate field ensures that K_1 vanishes which leads to the non-local Gross–Pitaevskii equation (GPE) [26]:

$$\mu\phi_0(\mathbf{r}) = \left\{-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{r}) + \int d\mathbf{r}'\phi_0^*(\mathbf{r}')V_{2D}(\mathbf{r}-\mathbf{r}')\phi_0(\mathbf{r}')\right\}\phi_0(\mathbf{r}). \tag{12}$$

Assuming the interaction potential has the even symmetry $V_{2D}(\mathbf{r}-\mathbf{r}') = V_{2D}(\mathbf{r}'-\mathbf{r})$, the second order term in the expansion of K can be written as

$$K_2 = \int d\mathbf{r}\,\varphi^\dagger(\mathbf{r})\left(-\frac{\hbar^2}{2M}\nabla^2 + U(\mathbf{r}) - \mu\right)\varphi(\mathbf{r}) + \frac{1}{2}\int d\mathbf{r}\int d\mathbf{r}' V_{2D}(\mathbf{r}-\mathbf{r}')\times$$

$$\times\left\{\varphi^\dagger(\mathbf{r})\varphi^\dagger(\mathbf{r}')\,\phi_0(\mathbf{r}')\phi_0(\mathbf{r}) + 2\varphi^\dagger(\mathbf{r})\varphi(\mathbf{r}')\phi_0^*(\mathbf{r}')\phi_0(\mathbf{r}) + \varphi(\mathbf{r})\,\varphi(\mathbf{r}')\phi_0^*(\mathbf{r}')\phi_0^*(\mathbf{r})\right\}. \tag{13}$$

Expression (13) can be diagonalized by means of the Bogoliubov linear transformation for the non-condensed operators

$$\varphi(\mathbf{r}) = \sum_j \left(u_j(\mathbf{r})\,\alpha_j + \upsilon_j^*(\mathbf{r})\,\alpha_j^\dagger\right), \tag{14}$$

$$\varphi^\dagger(\mathbf{r}) = \sum_j \left(u_j^*(\mathbf{r})\,\alpha_j^\dagger + \upsilon_j(\mathbf{r})\,\alpha_j\right), \tag{15}$$

where α_j and $\alpha_j^\dagger$ are quasiparticle annihilation and creation operators which satisfy the usual Bose commutation relations from which the normalization condition for the $u_i(\mathbf{r})$ and $\upsilon_i(\mathbf{r})$ wave functions can be obtained as

$$\int d\mathbf{r}\left[u_i^*(\mathbf{r})\,u_j(\mathbf{r}) + \upsilon_i^*(\mathbf{r})\,\upsilon_j(\mathbf{r})\right] = \delta_{ij}. \tag{16}$$

Inserting the transformations (14) and (15) into Eq. (13) in the Popov mean-field approximation [27], the quasiparticle energies are obtained from the solutions of the coupled BDG equations

$$\hbar\omega_i u_i(\mathbf{r}) = \left(-\frac{\hbar^2}{2M}\nabla^2 + U(\mathbf{r}) - \mu\right)u_i(\mathbf{r}) + \int d\mathbf{r}' V_{2D}(\mathbf{r}-\mathbf{r}')\left\{\phi_0^*(\mathbf{r}')\,u_i(\mathbf{r}') + \phi_0(\mathbf{r}')\,\upsilon_i(\mathbf{r}')\right\}\phi_0(\mathbf{r}), \tag{17}$$

$$-\hbar\omega_i\upsilon_i(\mathbf{r}) = \left(-\frac{\hbar^2}{2M}\nabla^2 + U(\mathbf{r}) - \mu\right)\upsilon_i(\mathbf{r}) + \int d\mathbf{r}' V_{2D}(\mathbf{r}-\mathbf{r}')\left\{\phi_0^*(\mathbf{r}')\,\upsilon_i(\mathbf{r}') + \phi_0(\mathbf{r}')\,u_i(\mathbf{r}')\right\}\phi_0(\mathbf{r}). \tag{18}$$

The coupled Eqs. (17) and (18) account for quasiparticle-condensate interactions while neglecting quasiparticle-quasiparticle. In a dilute gas at approximately zero temperature, the quasiparticle-quasiparticle interactions can be neglected since depletion is always smaller than one percent of the total number of atoms in the condensate [28]. On the other hand, the quasiparticle-condensate interactions cannot be negligible and they are characterized by direct and exchange terms which respectively describes a quasiparticle scattering off of the condensate and a quasiparticle scattering into or out of the condensate.

At finite temperatures the quasiparticle wave functions are required to construct the thermal and anomalous correlation functions. At equilibrium the coupled BDG equation can be written as

$$\hbar\omega_i u_i(\mathbf{r}) = \left(-\frac{\hbar^2}{2M}\nabla^2 + U(\mathbf{r}) - \mu + \int d\mathbf{r}' n(\mathbf{r}')V_{2D}(\mathbf{r}-\mathbf{r}')\right)u_i(\mathbf{r}) +$$

$$+\int d\mathbf{r}'\left[n_0(\mathbf{r}-\mathbf{r}') + \hat{n}(\mathbf{r}-\mathbf{r}')\right]V_{2D}(\mathbf{r}-\mathbf{r}')u_i(\mathbf{r}') - \int d\mathbf{r}'\, m_0(\mathbf{r}-\mathbf{r}')V_{2D}(\mathbf{r}-\mathbf{r}')\upsilon_i(\mathbf{r}'), \tag{19}$$

$$-\hbar\omega_i\upsilon_i(\mathbf{r}) = \left(-\frac{\hbar^2}{2M}\nabla^2 + U(\mathbf{r}) - \mu + \int d\mathbf{r}'\, n(\mathbf{r}')V_{2D}(\mathbf{r}-\mathbf{r}')\right)\upsilon_i(\mathbf{r}) +$$

$$+\int d\mathbf{r}'\left[n_0(\mathbf{r}-\mathbf{r}') + \hat{n}(\mathbf{r}-\mathbf{r}')\right]V_{2D}(\mathbf{r}-\mathbf{r}')\upsilon_i(\mathbf{r}') - \int d\mathbf{r}'\, m_0^*(\mathbf{r}-\mathbf{r}')V_{2D}(\mathbf{r}-\mathbf{r}')\, u_i(\mathbf{r}'), \quad (20)$$

where $n(\mathbf{r}) = n_0(\mathbf{r}) + \hat{n}(\mathbf{r})$ is the total density with $n_0(\mathbf{r},\mathbf{r}') = \phi_0^*(\mathbf{r})\phi_0(\mathbf{r}')$ and $\hat{n}(\mathbf{r},\mathbf{r}') = \left\langle \varphi^\dagger(\mathbf{r})\varphi(\mathbf{r}') \right\rangle$ is the thermal correlation function; $n_0(\mathbf{r}) = n_0(\mathbf{r},\mathbf{r})$ and $\hat{n}(\mathbf{r}) = \hat{n}(\mathbf{r},\mathbf{r})$, respectively, are the local condensed and thermal densities; $m_0(\mathbf{r}-\mathbf{r}') = \left\langle \phi_0(\mathbf{r})\phi_0(\mathbf{r}')\right\rangle$ is the anomalous condensate correlation function.

Form Eqs. (14) and (15), the thermal correlation functions can be written as

$$\hat{n}(\mathbf{r},\mathbf{r}') = $$

$$= \sum_i \left\{\left[u_i^*(\mathbf{r}')\, u_i(\mathbf{r}) + \upsilon_i(\mathbf{r}')\, \upsilon_i^*(\mathbf{r})\right] f_{BE}^i + \upsilon_i(\mathbf{r}')\, \upsilon_i^*(\mathbf{r})\right\}, \quad (21)$$

where $f_{BE}^i = \left[\exp(\hbar\omega_i / k_B T) - 1\right]^{-1}$ with k_B is Boltzmann's constant.

The thermal and quantum depletions are describes, respectively, by the first and second terms of Eq. (21). At zero temperature the Bose distribution function becomes zero($f_{BE}^i = 0$) and depletion of the condensed is determined only by quantum fluctuations.

The total number of particles satisfies

$$N = \int d\mathbf{r}\left[n_0(\mathbf{r}) + \hat{n}(\mathbf{r})\right]. \quad (22)$$

Here we consider a quasi-2D geometry, which can be performed in harmonically trapped gases with a trapping potential of the form

$$U(r) = \frac{1}{2}m\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right), \quad (23)$$

where $\omega_z >> \omega_x, \omega_y$.

For the homogeneous case ($\omega_x = \omega_y = 0$), the field operators may be expanded in a plain wave basis

$$\psi = \sum_k e^{i\mathbf{k}.\mathbf{r}} a_k$$

and Bogoliubov transformation

$$a_k = u_k b_k + \upsilon_{-k}^* b_{-k}^\dagger, \quad (24)$$

$$a_k^\dagger = u_k^* b_k^\dagger + \upsilon_{-k} b_{-k} \quad (25)$$

leads to the diagonal form of the Hamiltonian as

$$H = E_0 + \sum_k \hbar\omega(k) b_k^\dagger b_k, \quad (26)$$

where E_0 is the ground state (pure condensate) energy and the Bogoliubov, or quasiparticle dispersion relation for quasi-2D Bose gas is given by

$$\hbar\omega(k) = $$

$$= \sqrt{\frac{\hbar^2 k^2}{2M}\left[\frac{\hbar^2 k^2}{2M} + n_{2D}\frac{2g}{\sqrt{2\pi} l_z}\left(1 + \varepsilon_{dd} F_\perp\left(\frac{k l_z}{\sqrt{2}}\right)\right) + n_{2D} C \frac{4\pi}{k}\right]}. \quad (27)$$

The quasiparticle amplitudes u_k and υ_k satisfying the relations

$$u_k = $$

$$= \sqrt{\frac{1}{2}\left(\frac{\frac{\hbar^2 k^2}{2M} + n_{2D}\frac{g}{\sqrt{2\pi} l_z}\left(1 + \varepsilon_{dd} F_\perp\left(\frac{k l_z}{\sqrt{2}}\right)\right) + n_{2D} C \frac{2\pi}{k}}{\hbar\omega(k)} + 1\right)}, \quad (28)$$

$$\upsilon_k = $$

$$= \sqrt{\frac{1}{2}\left(\frac{\frac{\hbar^2 k^2}{2M} + n_{2D}\frac{g}{\sqrt{2\pi} l_z}\left(1 + \varepsilon_{dd} F_\perp\left(\frac{k l_z}{\sqrt{2}}\right)\right) + n_{2D} C \frac{2\pi}{k}}{\hbar\omega(k)} - 1\right)}. \quad (29)$$

In this case the density of the excited particles is

$$\hat{n} = \int d^2 k \left\{\left[u_k^2 + \upsilon_k^2\right] f_{BE}(k) + \upsilon_k^2\right\}. \quad (30)$$

From Eq. (30) we can separate the thermal and quantum depletion respectively as

$$\hat{n} = \hat{n}_T + \hat{n}_Q: \quad (31)$$

$$\hat{n}_T = $$

$$= \int d^2 k \frac{\frac{\hbar^2 k^2}{2M} + n_{2D}\frac{g}{\sqrt{2\pi} l_z}\left(1 + \varepsilon_{dd} F_\perp\left(\frac{k l_z}{\sqrt{2}}\right)\right) + n_{2D} C \frac{2\pi}{k}}{\hbar\omega(k)} f_{BE}(k), \quad (32)$$

$$\hat{n}_Q = $$

$$= \int d^2 k \frac{1}{2}\left(\frac{\frac{\hbar^2 k^2}{2M} + n_{2D}\frac{g}{\sqrt{2\pi} l_z}\left(1 + \varepsilon_{dd} F_\perp\left(\frac{k l_z}{\sqrt{2}}\right)\right) + n_{2D} C \frac{2\pi}{k}}{\hbar\omega(k)} - 1\right). \quad (33)$$

When the characteristic wavelength of a mode is much larger than the trap size, we have $\lim_{q l_z \to 0} F_\perp\left(q l_z / \sqrt{2}\right) = 2$, and Eq. (9) becomes

$$V_{2D}(k) = \frac{g}{\sqrt{2\pi} l_z}(1 + 2\varepsilon_{dd}) + C\frac{2\pi}{k}. \quad (34)$$

In this limit the DD interaction can be treated as a contact interaction with coupling $2\varepsilon_{dd}$. From Eq. (34) in the absence of long-rang interaction we see that for the quasi-2D BEC to be energetically stable the interaction couplings must be such that $\varepsilon_{dd} > -1/2$, i.e., quasi-2D dipolar BEC is energetically stabilized for $a_s > -2a_{dd}$.

The number of particles in the condensate can be obtained as

$$n_0 = n - \hat{n} = n - \int d^2k \left\{ \left[u_k^2 + \upsilon_k^2 \right] f_{BE}(k) + \upsilon_k^2 \right\}. \quad (35)$$

In the small momentum limit, $k << 1/\xi$, where $\xi = \hbar/(\sqrt{2}MC_{2D})$ is the healing length of the condensate, the quasiparticle dispersion relation (Eq. (27)) becomes a nonphononic dispersion:

$$\hbar\omega(k \to 0) = \hbar\sqrt{\frac{2\pi C n_{2D}}{M}k} = \hbar\gamma\sqrt{k}. \quad (36)$$

By using Eqs. (27), (29), (30) and (36) we can see that the contribution of quantum depletion to the particle number changes as $k^{-3/2}$ ($\upsilon_k^2 \propto k^{-3/2}$).

In the absence of long-range 1/*r* interaction the quasiparticle dispersion relation (Eq. (27)) becomes a phononic dispersion:

$$\hbar\omega(k \to 0) = \hbar C_{2D} k, \quad (37)$$

where

$$C_{2D} = \sqrt{n_{2D}\frac{g}{M\sqrt{2\pi}l_z}(1+2\varepsilon_{dd})}$$

is the speed of sound in 2D dipolar Bose gas.

In this case Eqs. (27), (28), (30) and (37) lead to the real particle occupation number at small momentum regime which varies as $k^{-1}(\upsilon_k^2 \propto k^{-1})$ due to quantum depletion.

On the other hand, in the large momentum limit $k > 1/\xi$ the quantum depletion disappears according to the law of k^{-4} in the presence and absence of long-range 1/*r* interaction.

Thus only the small momentum behavior of the quantum depletion is affected by long-range 1/*r* interaction and at large momentum limit the momentum dependence of quantum depletion unaffected by 1/*r* interaction.

Quantum depletion is temperature-independent and is determined by the coupling constants of interactions ($\varepsilon_{dd} = g_d/g$ and C). Clearly in the absence of 1/*r* interaction depletion the quantum fluctuations proportional to sound velocity C_{2D} which means that the quantum depletion increases as the contact and DD coupling constants increase. In this case the same result for the quantum depletion was obtained in the regime of dipole interaction dominated condensates [24].

In order to calculate the real particle occupation number due to thermal depletion in the low-momentum regime, we can use Eqs. (27)–(30), (36), and (37) and the low-energy expansion of the Bose–Einstein distribution $f_{BE}(k) \approx \approx k_B T/\hbar\omega(k)$ which in both the presence and absence of the long-range 1/*r* interaction lead to the k^{-2} variation of the occupation number due to thermal depletion. At large momentum limit the thermal depletion disappears exponentially with temperature.

At temperature $T << \mu$ the main contribution to the integral in Eq. (32) comes from the long wavelength excitations and in the presence and absence of 1/*r* interaction, Eq. (32) respectively leads to

$$\hat{n}_T(k \to 0) = \int_0^\infty d^2k \frac{n_{2D}C2\pi/k}{\hbar\gamma\sqrt{k}}\left(\frac{1}{e^{\beta\hbar\gamma\sqrt{k}}-1}\right) = \frac{4\pi k_B TM}{\hbar^2}\ln\left(1-e^{-\beta\hbar\gamma\sqrt{k}}\right), \quad (38)$$

$$\hat{n}_T(k \to 0) = \int_0^\infty d^2k \frac{n_{2D}g/\sqrt{2\pi}l_z(1+2\varepsilon_{dd})}{\hbar k C_{2D}}\left(\frac{1}{e^{\beta\hbar k C_{2D}}-1}\right) = \frac{2\pi M k_B T}{\hbar^2}\ln\left(1-e^{-\beta\hbar k C_{2D}}\right). \quad (39)$$

In both cases the divergent of the integral implies that a true BEC is not present in a quasi-2D Bose gas. The origin of this infrared divergence is related to long-wave fluctuations of the phase. In the low-temperatures limit, similar to a quasi-2D Bose gas with contact interaction the long-wave fluctuations of the phase destroy the long-range order and true BEC and this behavior unaffected by long-range 1/*r* interaction.

3. Conclusion

In this paper we have calculated the contributions of thermal and quantum fluctuations to the depletion of a dipolar quasi-2D trapped Bose condensed gas in the presence of long-range 1/*r* interaction. In the HFB approximation we expressed the grand-canonical Hamiltonian perturbatively in orders of the condensate occupation which leads to the nonlocal GPE. In the Popov mean-field approximation from the solutions of the coupled BDG equations we obtained the quasiparticle dispersion relation. The long-range nature of the 1/*r* interaction leads to significant modifications (nonphononic) of dispersion. To construct the thermal and anomalous correlation functions the quasiparticle wave functions at finite temperatures were also obtained. From these correlation functions the thermal and quantum depletions of homogeneous quasi-2D dipolar Bose gas were calculated.

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Вплив далекодіючої 1/*r*-взаємодії на теплове та квантове виснаження диполярного квазідвовимірного бозе-газу

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У рамках наближення Хартрі–Фока–Боголюбова (ХФБ) досліджено виснаження квазідвовимірного (квазі-2D) диполярного бозе-конденсованого газу при наявності як контактної, так і далекодіючої 1/*r*-взаємодії. Коли характерна довжина хвилі моди набагато більше розміру пастки, диполь-дипольну (DD) взаємодію можна розглядати як контактну взаємодію, і в межі малого імпульсу внесок, що домінує, вносить далекодіюча 1/*r*-взаємодія, що призводить до нелінійного (нефононного) дисперсійного співвідношення. Показано, що квантове виснаження не залежить від температури і визначається константами контакту, DD та далекодіючого 1/*r*-спарювання ($\varepsilon_{dd} = g_d/g$ та C). На поведінку квантового виснаження при малих імпульсах впливає далекодіюча 1/*r*-взаємодія, а в межах великих імпульсів вплив імпульсу на квантове виснаження обмежений та 1/*r*-взаємодія на нього не впливає.

Ключові слова: диполярний бозе-конденсат, далекодіюча взаємодія, квантове та теплове виснаження.

Влияние дальнодействующего 1/*r*-взаимодействия на тепловое и квантовое истощение диполярного квазидвумерного бозе-газа

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В рамках приближения Хартри–Фока–Боголюбова (ХФБ) исследовано истощение квазидвумерного (квази-2D) диполярного бозе-конденсированного газа при наличии как контактного, так и дальнодействующего 1/*r*-взаимодействия. Когда характерная длина волны моды намного больше размера ловушки, диполь-дипольное (DD) взаимодействие можно рассматривать как контактное взаимодействие, и в пределе малого импульса доминирующий вклад вносит дальнодействующее 1/r-взаимодействие, что приводит к нелинейному (нефононному) дисперсионному соотношению. Показано, что квантовое истощение не зависит от температуры и определяется константами контакта, DD и дальнодействующего 1/*r*-спаривания ($\varepsilon_{dd} = g_d/g$ и C). На поведение квантового истощения при малых импульсах влияет дальнодействующее 1/*r*-взаимодействие, а в пределе больших импульсов влияние импульса на квантовое истощение ограничено и 1/*r*-взаимодействие на него не влияет.

Ключевые слова: диполярный бозе-конденсат, дальнодействующее взаимодействие, квантовое и тепловое истощение.