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Magnetoresistance of electrons in diluted magnetic semiconductor Volcano ring

A.M. Babanlı

Department of Physics, University of Süleyman Demirel, 32260 Isparta, Turkey
E-mail: arifbabanli@sdu.edu.tr

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The influence of exchange interaction on the transport properties of a two-dimensional diluted magnetic semiconductor quantum ring with finite width has been investigated in the presence of a uniform perpendicular magnetic field. The dependence of magnetoresistance on the magnetic field, Mn concentration, and quantum ring width are investigated. In the absence of exchange interaction, a typical beating pattern with well-defined node-positions in the oscillating magnetoresistance is observed. It was shown that in the present exchange interactions the beating pattern is destroyed.

Keywords: diluted magnetic semiconductor, magnetoresistance, ballistic conductance.

The semiconductor quantum ring has been extensively studied from both theoretical and experimental points of view. Their atom-like properties together with a high flexibility in size and shape made them a very strong candidate for device applications such as far-infrared laser amplifiers and high-speed electro-optical modulators [1]. Interference effect in quantum ring under external magnetic field leads to the Aharanov–Bohm (AB) oscillations in physical quantities, such as conductance, orbital magnetism and persist currents. These quantum systems have many applications such as single electron and photon devices, spintronics. An important class of materials for spintronics forms diluted magnetic semiconductors (DMS). They are A_2B_6 or A_3B_5 solutions with a high density of magnetic impurities (usually, Mn). The presence of localized magnetic ions in DMSs leads to an exchange interaction between the *sp*-band electrons and the *d*-electrons associated with Mn, resulting in extremely large Zeeman splitting of electronic levels [2,3].

In the paper [4] was found that the amplitudes of the AB oscillation in conductance were usually dominated by random fluctuations of the order of e^2 / h. This led to the discovery of the universal conductance fluctuations. Liu showed that when four spin-degenerate subbands in the ring are populated, random signs dominate the AB interference patterns [5].

In the paper [6], exact energy spectra and wave functions analytically for a ring in the presence of both a uniform perpendicular magnetic field and a thin magnetic flux through the ring center were obtained. It was used as a model to study the Aharonov–Bohm effect in an ideal annular ring that is weakly coupled to both the emitter and the collector. The spinless electrons were considered. The effect of the quantum ring width on the resistance was not studied in [6]. In Ref. 7 the orbital magnetism of mesoscopic ring systems with finite width on the basis of the exactly solvable Tan model was investigated by calculating numerically magnetization and spatial dependence of persistent current in the presence of magnetic fields for the wide range of temperatures.

In the work [8], explicit analytical expressions for the magnetic moment and persistent current of the Volcano ring were derived. The magnetic moment was investigated as a function of the magnetic field strength and the temperature. An influence of magnetic ions in DMSs on coherent transport phenomena was studied in free-standing wires of (Cd,Mn)Te which is made by MBE grown thin films was studied in Ref. 9 and shown that weak field magnetoresistance and aperiodic but reproducible resistance fluctuations are clearly seen in $Cd_{0.99}Mn_{0.01}Te$.

At very low temperatures and in the strong-tunneling regime, the valley conductance that corresponds to an odd number of electrons can be enhanced due to the Kondo effect. Higher-order virtual tunneling processes that effectively flip the unpaired spin on the dot can lead to a coherent many-body resonance at the Fermi energy, known as the Kondo resonance. The Kondo effect in a quantum dot was only recently observed by Goldhaber–Gordon *et al.* [10]. The energy scale for observing the Kondo resonance a Kondo temperature T_K, which is essentially the binding energy of the resonance. In the Ref. 11 shown, that to bring T_K within the range of experimentally accessible temperatures it was necessary to fabricate much smaller dots ($L \sim 100$ nm). In our case, quantum ring radius was taken as 800 nm. Accordingly in this paper, we do not take into account the Kondo effect. We investigated nonmagnetic impurity states.

The purpose of this work is to generalize the results of paper [6] to the DMS quantum ring. We study the effect of exchange interactions and quantum ring width on the magnetoresistance of the quantum ring prepared from a diluted magnetic semiconductor with Volcano potential:

$$V(r) = \frac{a_1}{r^2} + a_2 r^2 - 2\sqrt{a_1 a_2}. \tag{1}$$

The effective width of the Volcano ring at a given Fermi energy E_F then is $\Delta r = \sqrt{8E_F / m_n \omega_0^2}$, and the average radius of quantum ring define as $r_0 = (a_1 / a_2)^{1/4}$, the potential has a minimum $V(r_0) = 0$. The Volcano potential model has been successfully used to explain the beats in the Aharonov–Bohm oscillations, which have been experimentally observed in a two-dimensional semiconductor ring [5]. Figure 1 shows a sketch of a top view of the whole quantum ring configuration envisioned here. The electron in one lead can reach the other one only by tunneling through the quasibound circular states in the ring.

The quantum ring is subjected to a uniform magnetic field along the z direction. The total Hamiltonian of the system is given by

$$H = \left[\frac{1}{2m_e}(\mathbf{p} + e\mathbf{A})^2 + V(r)\right] I + \frac{1}{2} g\sigma_z \mu_B H + H_{\text{ex}}, \tag{2}$$

where I is a 2×2 unit matrix, m_e is the effective electrons mass, $\mu_B = e\hbar / 2m_0$ is the Bohr magneton, m_0 is the free electron mass, $\mathbf{A}$ is the vector potential and σ_z is the z component of Pauli spin matrices and g is the Lande factor of electrons in the absence of the exchange interaction. In the mean-field approximation the exchange Hamiltonian term [12]

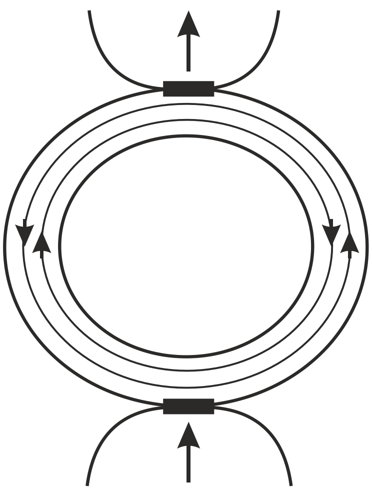

Fig. 1. The weakly coupled DMS quantum ring with finite width.

$$H_{\text{ex}} = \frac{1}{2} < S_z > N_0 x J_{s-d} \sigma_z = 3A\sigma_z, \tag{3}$$

where J_{s-d} is a constant which describes the exchange interaction, N_0 is the density of the unit cells. The thermodynamic average $< S_z >$ of the z component of the localized Mn spin is determined by the expression

$$< S_z > = -S_0 B_{5/2}\left(\frac{S g_{\text{Mn}} \mu_B H}{k_B (T + T_0)}\right), \tag{4}$$

where $B_{5/2}\left(\frac{S g_{\text{Mn}} \mu_B H}{k_B (T + T_0)}\right)$ is the Brillouin function, $g_{\text{Mn}} = 2$ is the g-factor of Mn ions, $S = 5/2$, and k_B is the Boltzmann constant. T_0 is the phenomenological parameter describing the antiferromagnetic superexchange between neighboring Mn ions. For uniform magnetic field, the vector potentials in cylindrical coordinates have the components $A_\phi = Hr / 2$, $A_r = 0$ and Schrödinger equation in polar coordinates is

$$\left[\left(-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial r^2} - \frac{\hbar^2}{2m_e}\frac{\partial}{\partial r} - \frac{\hbar^2}{2m_e r^2}\frac{\partial^2}{\partial \phi^2} + \frac{i\hbar e H}{2m_e}\frac{\partial}{\partial \phi} + \frac{e^2 H^2}{8m_e} r^2 + \right.\right.$$
$$\left. + V(r) - E\Big) I + 3A\sigma_z + \frac{1}{2} g\sigma_z \mu_B H\right]\Psi(r,\phi) = 0. \tag{5}$$

The solution of Eq. (5) have the form [6]

$$\Psi^\sigma_{n,m}(r,\phi) = \frac{1}{\lambda}\sqrt{\frac{\Gamma(n+M+1)}{2^{M+1} n! [\Gamma(M+1)]^2 \pi}}\, e^{im\phi} e^{-\frac{1}{4}\left(\frac{r}{\lambda}\right)^M} \times$$
$$\times {}_1F_1\left(-n, M+1, \frac{1}{2}\left(\frac{r}{\lambda}\right)^2\right)\chi_\sigma, \tag{6}$$

$$E_{n,m,\sigma} = \left(n + \frac{1}{2} + \frac{M}{2}\right)\hbar\omega - \frac{m}{2}\hbar\omega_c - \frac{m_e}{4}\omega_0^2 r_0^2 + \sigma\frac{\mu_B H}{2} g^*. \tag{7}$$

Here the following designations are used

$$M = \sqrt{m^2 + \frac{2a_1 m_e}{\hbar^2}}, \quad \lambda = \sqrt{\frac{\hbar}{m_e \omega}},$$
$$\omega = \sqrt{\omega_c^2 + \omega_0^2}, \quad g^* = g + \frac{6A}{\mu_B H}, \tag{8}$$

where ${}_1F_1$ is the confluent hypergeometric function, $\omega_c = eH / m_e$ is the cyclotron frequency, $\Gamma(x)$ is the Gamma function, $\omega_0 = \sqrt{8a_2 / m_e}$ is the confinement frequency, quantum number $n = 0, 1, 2, \ldots$ shows the order of the radial mode and $m = 0, \pm 1, \pm 2, \ldots$ gives the angular momentum, and $\sigma = \pm 1$ for $\sigma = \uparrow, \downarrow$, and χ_σ is the electron spin written as the column vector $\chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\chi_{-1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

For finding the magnetoresistance of the electrons in DMS rings, it is necessary to obtain the ballistic conductance of the electron gas in the quantum ring. The ballistic conductance of the electron gas can be determined of the Landauer formula [6]:

$$G(H) = \frac{e^2}{h}\sum_{n,\sigma} T_n^{\sigma\sigma}(H, E_F), \tag{9}$$

where $T_n^{\sigma\sigma}(H, E_F)$ is the magnetic-field-dependent transmission coefficient of the nth channel in the leads at the Fermi energy E_F. If we assume that the two leads are weakly coupled to the DMS ring, the electron in one lead can reach the other one only by tunneling through the quasibound circular states in the ring. In such a case, the conductance can be approximately expressed in the form [6]

$$G(H) = \frac{e^2}{h}\sum_{n,m,\sigma}\frac{\Gamma^e_{n,m,\sigma}\Gamma^c_{n,m,\sigma}}{\left[E_F - E_{n,m,\sigma}(H)\right]^2 + \frac{1}{4}\left(\Gamma^e_{n,m,\sigma}+\Gamma^c_{n,m,\sigma}+\Gamma^i_{n,m,\sigma}\right)^2}\times \frac{\Gamma^e_{n,m,\sigma}+\Gamma^c_{n,m,\sigma}+\Gamma^i_{n,m,\sigma}}{\Gamma^e_{n,m,\sigma}+\Gamma^c_{n,m,\sigma}}, \tag{10}$$

where $E_{n,m,\sigma}(H)$ is the energy of the (n,m,σ)th quasibound ring states. Furthermore, we can approximate the energies of these quasibound states with those of the isolated ring given in Eq. (7). $\Gamma^e_{n,m,\sigma}$, $\Gamma^c_{n,m,\sigma}$ and $\Gamma^i_{n,m,\sigma}$ are the broadening of the (n,m,σ)th ring state caused by leaking into the emitter (collector) and inelastic scattering, respectively. In order to determine the qualitative dependence of the conductance on the magnetic field, we assume that $\Gamma^e_{n,m,\sigma} = \Gamma^c_{n,m,\sigma} = 0.005\,\text{meV}$, $\Gamma^i_{n,m,\sigma} = 0.004\,\text{meV}$ [6].

For our calculation we consider the parameter corresponding to $Cd_{1-x}Mn_xTe$ materials: $m_e = 0.096\,m_0$, where m_0 is the free electron mass, and $g_e = -1.67$, $N_0 J_{s-d} = 0.22\,\text{eV}$ [13]. (The reason why the interaction constant is small is that the wave functions of the s band electrons and the d electrons belong to Mn ions overlap less.) $T = 0$, $T_0 = 3$ K, ring radius $r_0 = 800$ nm and a ring width $\Delta r = 300$ nm at $E_F = 9.7$ meV are taken from the literature [13].

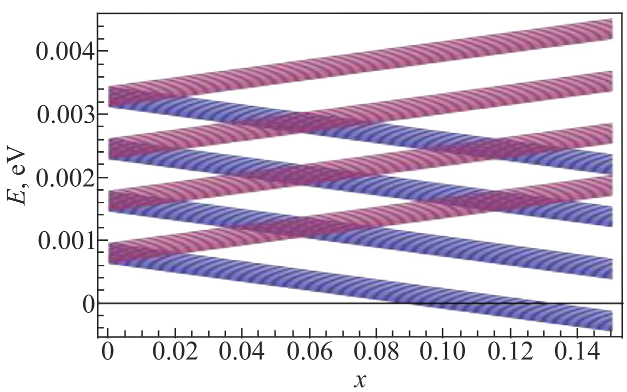

Fig. 2. The electron energy spectrum versus Mn concentration at fixed magnetic field $H = 0.05$ T for quantum number $n = 0, 1, 2, 3$, $|m| \le 5$, $\sigma = \pm 1$, in DMS quantum ring.

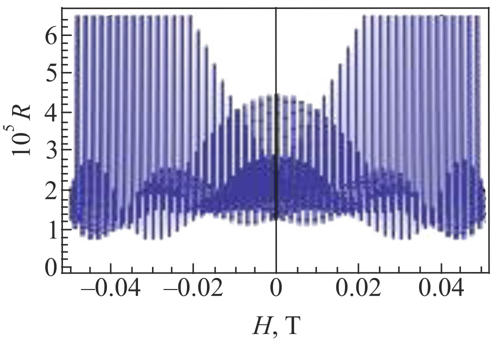

Fig. 3. The magnetic field dependence of resistance without Mn in DMS quantum ring.

In Fig. 2 we plot the electron energy spectrum versus Mn concentration at fixed magnetic field $H = 0.05$ T for quantum number $n = 0, 1, 2, 3$, $|m| \le 5$, $\sigma = \pm 1$. It can be seen from the figure that, with increasing Mn concentration, the energy of a DMS ring state (n, m, $\sigma = \pm 1$) is shifted upward or downward depending on the direction of the electron motion. The variation of relative resistivity in a magnetic field is

$$\frac{\Delta R}{R(0)} = \frac{R(H) - R(0)}{R(0)}. \tag{12}$$

The resistance ($R = G^{-1}$) versus magnetic field without Mn in the quantum ring is shown in Fig. 3 — calculated with four populated $n = 0, 1, 2, 3$, $|m| \le 450$, $\sigma = \pm 1$. According to Fig. 3 of the magnetoresistance of the quantum ring have no periodic oscillations of the beat type. At a certain magnetic field, the Fermi level in the emitter and collector aligns with an energy level in the quantum ring with corresponds to the peak observed in magnetoresistance. As beat is clearly a result of the interference of waves with different phase factors.

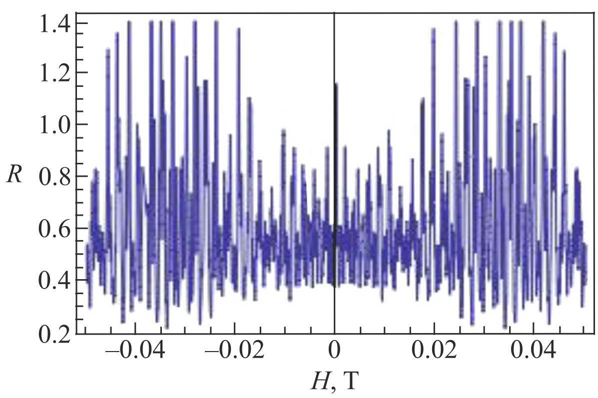

Fig. 4. The magnetic field dependence of magnetoresistance in the Mn concentration $x = 0.015$.

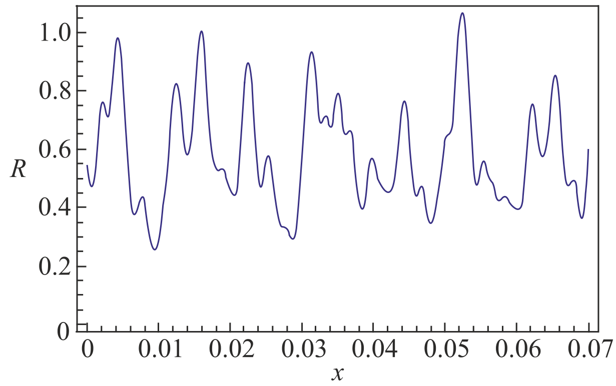

Fig. 5. Dependence of the magnetoresistance in terms of e^2/h on the Mn concentration for values magnetic field $H = 0.05$ T.

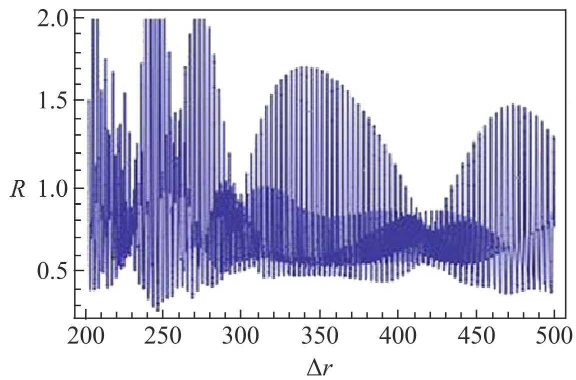

Fig. 7. Magnetoresistance as function of quantum ring width Δr for CdTe at $H = 1$ T.

The magnetic field dependence of magnetoresistance in the Mn concentration $x = 0.015$ is shown in Fig. 4. As seen in Fig. 4 the influence exchange interaction destroys beating in the field dependence of magnetoresistance. It follows from the general considerations that the destructive interference of the contributions made by different electron trajectories to the wave function phase should distort the strict periodicity of the Aharonov–Bohm oscillations. In Fig. 5 the magnetoresistance plotted as a function of the Mn concentration in DMS quantum ring at $H = 0.05$ T. As see from Fig. 5 the magnetoresistance undergoes oscillation as a function of Mn concentrations. The relative magnetoresistance as function of the magnetic field plotted in Fig. 6. We see the relative magnetoresistance oscillates between negative and positive values with the amplitude that random with the magnetic field. As seen from Fig. 6 relative negative magnetoresistance occur at a low magnetic field. The negative resistance in our calculation is due to the increasing of density of states at the Fermi energy with an increasing magnetic field. Magnetoresistance as a function of quantum ring width is presented in Fig. 7 at $x = 0$, $H = 1$ T. As seen from Fig. 7 the magnetoresistance oscillates with quantum ring width and has a beating.

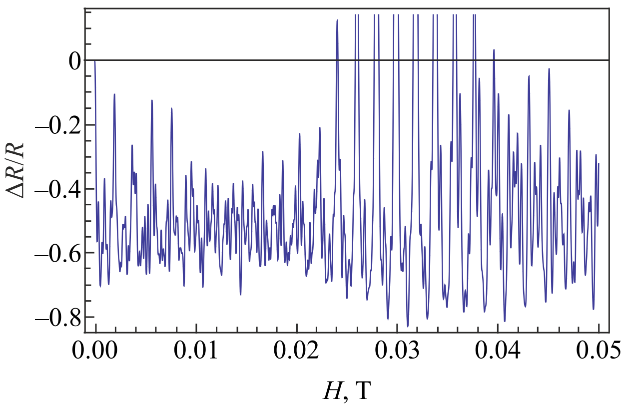

Fig. 6. The relative magnetoresistance as function of magnetic field in the Mn concentration $x = 0.015$.

To summarize, in the paper we consider 2D electron gas magnetoresistance in DMS quantum ring with Volcano potential profile. The energy spectrum and wave functions of electrons are calculated for a DMS quantum ring of finite width under the uniform perpendicular magnetic field and the exchange interactions. We show that 2D electron gas magnetoresistance depending on Mn concentrations and ring width oscillates with random amplitudes.

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Магнітоопір електронів в розведеному магнітному напівпровідниковому кільці з вулканоподібним потенціалом

A.M. Babanlı

Досліджено вплив обмінної взаємодії на транспортні властивості двовимірного розведеного магнітного напівпровідникового квантового кільця кінцевої ширини при наявності однорідного поперечного магнітного поля. Досліджено залежність магнітоопору від магнітного поля, концентрації Mn та ширини квантового кільця. За відсутності обмінних взаємодій спостерігається типова картина биття з чітко визначеними положеннями вузлів на магнітоопорі, що осцилює. Показано, що при наявності обмінних взаємодій картина биття руйнується.

Ключові слова: розведений магнітний напівпровідник, магнітоопір, балістична провідність.

Магнитосопротивление электронов в разбавленном магнитном полупроводниковом кольце с вулканоподобным потенциалом

A.M. Babanlı

Исследовано влияние обменного взаимодействия на транспортные свойства двумерного разбавленного магнитного полупроводникового квантового кольца конечной ширины при наличии однородного поперечного магнитного поля. Исследована зависимость магнитосопротивления от магнитного поля, концентрации Mn и ширины квантового кольца. В отсутствие обменных взаимодействий наблюдается типичная картина биений с четко определенными положениями узлов на осциллирующем магнитосопротивлении. Показано, что при наличии обменных взаимодействий картина биений разрушается.

Ключевые слова: разбавленный магнитный полупроводник, магнитосопротивление, баллистическая проводимость.