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Corrections to thermodynamics of the system of magnetically charged anyons

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In this paper, we calculate the thermodynamics of the system of anyons with magnetic charges in the magnetic field. We demonstrate how the contribution of the energy spectrum correction due to magnetic charges affects the second virial coefficient and the magnetic susceptibility. Dependences of the respective corrections as functions of temperature and the anyonic parameter are presented.

Keywords: anyons, magnetic charge, second virial coefficient, magnetic susceptibility.

1. Introduction

Problems in condensed matter physics often involve complicated mathematical apparatus and effective models are efficient tools for description of many phenomena. In the present paper, we join two exotic physical models aiming to demonstrate the calculation of thermodynamic functions in systems where planar geometry can induce non-standard types of excitations.

Back in 1977, Leinaas and Myrheim proved that the traditional division of particles into fermions and bosons blurs as one goes into a two-dimensional space [1]. In 1982, Wilczek proposed for such particles the term “anyons” because upon the permutation of two particles the phase of the wave function can change by any value, not only 0 or π [2].

Anyons are used in the description of the fractional quantum Hall effect, which is observed in two-dimensional systems of electrons at low temperatures and strong magnetic fields [3–6]. On the basis of anyons it was proposed to construct a topological quantum computer, which, due to its topological nature, should be much more tolerant to interference and errors than an “ordinary” quantum computer [7,8]. Note that some hints towards experimental observations of anyonic excitations were reported [9–11].

The second model we consider is inclusion of hypothetical magnetic charges into the problem, alongside the ordinary electric charges. The existence of magnetic charges, so-called magnetic monopoles, was theoretized yet in 1890s [12,13]. A consistent quantum-mechanical model was suggested by Dirac [14,15], which in particular explains the quantization of electric charge. Magnetic monopoles are perhaps the best known modifications of the electromagnetic field equations arising in several formulations [16–19], while some exotic approaches include also deformations of field equations due to coordinate non-commutativity and minimal length [20–23].

Quasiparticles imitating magnetic monopolies were found in solids (at distances much larger than the lattice constant). Examples include the spin ice of dysprosium titanate $Dy_2Ti_2O_7$ [24]. It was also shown that the magnetic monopole can appear in the momentum space of ferromagnetic crystal solids in the low-energy region (~ 0.1 to 1 eV) in the context of the anomalous Hall effect, e.g., in $SrRuO_3$ [25]. Theoretically, such formations can exist also in the Bose–Einstein condensate. Computer simulations allowed creation of vortices, which behaved very similarly to the Dirac magnetic monopoles [26].

We thus see than both anyons and magnetic monopoles arise as effective models in condensed matter physics, in particular at low temperatures and in constrained geometries. A simultaneous study of these two models envisages the coincidental consideration of relevant effects.

The paper is organized as follows. In Sec. 2, we briefly describe the approach used to introduce magnetic charges and summarize previous results about the spectrum of the respective two-anyon problem. Section 3 contains details of calculations of the second virial coefficient, with special attention on the correction due to magnetic charges. Calculations of thermodynamic functions is demonstrated in Sec. 4 for the magnetic susceptibility. Brief discussion in Sec. 5 concludes the paper.

2. Magnetic charges and anyonic spectrum

To describe the electromagnetic field in a system involving magnetic charges we will follow the Cabibbo–Ferrari approach [27,28]. The four-vector of the electromagnetic potential associated with electrical charges is denoted as $A_\xi^{(\mathrm{e})}$, while the respective magnetic contribution is $A_\xi^{(\mathrm{m})}$. For particle with mass m, electric charge e, and magnetic charge μ, the action is defined as

$$S = \int\left\{-mc^2\sqrt{\left(1-\frac{v^2}{c^2}\right)}dt - \frac{e}{c}A_\xi^{(\mathrm{e})}dx^\xi - \frac{\mu}{c}A_\xi^{(\mathrm{m})}dx^\xi\right\} =$$

$$= \int\left\{-mc^2\sqrt{1-\frac{v^2}{c^2}} - e\varphi^{(\mathrm{e})} - \mu\varphi^{(\mathrm{m})} + \frac{e}{c}\mathbf{A}^{(\mathrm{e})}\mathbf{v} + \frac{\mu}{c}\mathbf{A}^{(\mathrm{m})}\mathbf{v}\right\}dt =$$

$$= \int \mathcal{L}\,dt. \qquad (1)$$

Note that the Einstein summation over the repeating indices ξ is implied.

So, the Lagrangian is given by

$$\mathcal{L} = -mc^2\sqrt{1-\frac{v^2}{c^2}} - e\varphi^{(e)} - \mu\varphi^{(m)} + \frac{e}{c}\mathbf{A}^{(e)}\mathbf{v} + \frac{\mu}{c}\mathbf{A}^{(m)}\mathbf{v}. \qquad (2)$$

The generalized momentum $\mathbf{P} = \partial\mathcal{L}/\partial\mathbf{v}$ is linked with the mechanical momentum $\mathbf{p} = m\mathbf{v}(1-v^2/c^2)^{-1/2}$ by the following relation:

$$\mathbf{P} = \mathbf{p} - \frac{e}{c}\mathbf{A}^{(\mathrm{e})} - \frac{\mu}{c}\mathbf{A}^{(\mathrm{m})}. \qquad (3)$$

Note that magnetic field equal

$$\mathbf{B} = \mathrm{rot}\,\mathbf{A}^{(\mathrm{e})} - \frac{1}{c}\frac{\partial\mathbf{A}^{(\mathrm{m})}}{\partial t}. \qquad (4)$$

In a two-dimensional space, the permutation of two particles can yield any phase change in the wave function,

$$\hat{P}_{12}\,|12\rangle = \mathrm{e}^{i\pi\alpha}\,|12\rangle, \qquad (5)$$

where the real number $\alpha \in [0;1]$ is the so-called anyonic parameter. It interpolates between the Bose–Einstein statistics ($\alpha = 0$) and the Fermi–Dirac statistics ($\alpha = 1$). In the case of $0 < \alpha < 1$, we get anyons.

The solution of the two-anyon problem yields the following wave function corresponding to the relative motion [29]:

$$\Psi(r,\varphi) = \mathrm{e}^{i(l-\alpha)\varphi}R(r). \qquad (6)$$

The energy eigenvalues for two anyons in a constant magnetic field $\mathbf{B}_0$ are [29]

$$E_{n,l} = [2n + |l-\alpha| - (l-\alpha) + 1]\hbar\omega_c, \qquad (7)$$

where $n = 0, 1, 2, \ldots$, $l = 0, \pm2, \pm4, \ldots$, and the cyclotron frequency

$$\omega_c = \frac{eB_0}{2mc}. \qquad (8)$$

Note that these are of two types:

$$E_{n,l}^{(1)} = (2n+1)\hbar\omega_c \qquad \text{for} \quad l > 0, \qquad (9)$$

$$E_{n,l}^{(2)} = [2n + 2|l| + 2\alpha + 1]\hbar\omega_c \qquad \text{for} \quad l \leqslant 0. \qquad (10)$$

We consider next the system with magnetic charges placed in a constant magnetic field. This is ensured by choosing the vector potential terms in the form

$$\mathbf{A}^{(\mathrm{e})} = \frac{1}{2}[\mathbf{B}_0, \mathbf{r}], \qquad \mathbf{A}^{(\mathrm{m})} = \mathbf{a}t, \qquad (11)$$

where $\mathbf{B}_0$ and $\mathbf{a}$ are constant vectors. So,

$$\mathbf{B} = \mathbf{B}_0 + \frac{1}{c}\mathbf{a}. \qquad (12)$$

Let us consider that $a \equiv |\mathbf{a}|$ has a small magnitude, so that the time dependence is weak. That can make possible to discuss problem in adiabatic approximation.

In [30], the spectrum of two anyons was calculated for $\mathbf{a} = a\mathbf{e}_\varphi$. The expressions for the correction to energies (9), (10) can be shown to equal

$$\Delta E_{n,l}^{(1)} = \frac{\hbar^2}{m}\sqrt{\frac{2\hbar}{m\omega_c}}\frac{e\mu}{4\hbar^2c^2}\frac{Q(n,l,\alpha) - P(n,l,\alpha)}{\Gamma(n+1+l-\alpha)}B_0at \qquad (13)$$

$$\text{for } l = 2, 4, 6\ldots,$$

$$\Delta E_{n,l}^{(2)} = \frac{\hbar^2}{m}\sqrt{\frac{2\hbar}{m\omega_c}}\frac{e\mu}{4\hbar^2c^2}\frac{Q(n,|l|,-\alpha) - P(n,|l|,-\alpha)}{\Gamma(n+1+|l|+\alpha)}B_0at,$$

$$\text{for } l = 0, -2, -4, \ldots, \qquad (14)$$

where coefficients Q and P are expressed through the hypergeometric function:

$$Q(n,l,\alpha) = \frac{\Gamma(l-\alpha+3/2)\Gamma(n-1/2)\Gamma(n+l-\alpha+1)}{n!\Gamma(-1/2)\Gamma(l-\alpha+1)}\times$$

$$\times\,{}_3F_2\left(-n, l-\alpha+\frac{3}{2}, \frac{3}{2}; -n+\frac{3}{2}, l-\alpha+1; 1\right), \qquad (15)$$

$$P(n,l,\alpha) = (l-\alpha)\frac{\Gamma(l-\alpha+1/2)\Gamma(n+1/2)\Gamma(n+l-\alpha+1)}{n!\Gamma(1/2)\Gamma(l-\alpha+1)}\times$$

$$\times\,{}_3F_2\left(-n, l-\alpha+\frac{1}{2}, \frac{1}{2}; -n+\frac{1}{2}, l-\alpha+1; 1\right). \qquad (16)$$

It gives us the correction to energy at the coefficient

$$\gamma = \frac{\hbar^2}{m}\sqrt{\frac{2\hbar}{m\omega_c}}\frac{e\mu}{4\hbar^2c^2}B_0at. \qquad (17)$$

The respective numerical factors are of the orders of unity [30], for instance:

$n = 0, l = 0$: $0.886227 - 1.22857\alpha + 2.30937\alpha^2 + \ldots$,

$n = 1, l = 0$: $1.55090 - 1.04222\alpha + 1.39789\alpha^2 + \ldots$,

etc.

Let us estimate the values of a suitable for perturbative calculations, so that the energy corrections remain small enough:

$$\frac{\Delta E_{n,l}}{E_{n,l}} \sim \frac{1}{\hbar\omega_c}\frac{\hbar^2}{m}\sqrt{\frac{2\hbar}{m\omega_c}}\frac{e\mu}{4\hbar^2 c^2}B_0 at \lesssim 1. \quad (18)$$

Taking into account Dirac's quantization condition [14] for the magnetic charge,

$$\frac{2e\mu}{\hbar c} = k, \qquad \text{where} \quad k = 1, 2, 3, \ldots, \quad (19)$$

we get

$$\frac{\Delta E_{n,l}}{E_{n,l}} \sim \frac{k}{2}\frac{a}{c}t\sqrt{\frac{\hbar c}{e^2}\frac{2c}{m\omega_c}}, \quad (20)$$

where $e^2/(\hbar c) \simeq 1/137$ is the fine structure constant. Taking into account typical values for the cyclotron frequencies $\omega_c \sim 10^6\ \mathrm{s}^{-1}$ corresponding to magnetic fields $\sim 10^{-1}$ G and m as the mass of electron, we find

$$\frac{\Delta E_{n,l}}{E_{n,l}} \sim \frac{a}{c}t \cdot 10^{17} \lesssim 1, \quad (21)$$

so that

$$\frac{a}{c}t \lesssim 10^{-17}\ \mathrm{G\cdot s}, \quad (22)$$

and for $t \sim 1/\omega_c$

$$\frac{a}{c} \lesssim 10^{-11}\ \mathrm{G}. \quad (23)$$

Note that magnetic field of such low orders are detectable by modern magnetometers [31]. For fields $\sim 10^4$ G we obtain in the similar fashion the estimation

$$\frac{a}{c} \lesssim 10^{-6}\ \mathrm{G}. \quad (24)$$

3. Second virial coefficient

The equation of state describing the dependence between pressure p, temperature T, and concentration $\rho = N/A$, where N is the number of particles and A is the area (an analog of volume in a two-dimensional geometry) can be written as the virial expansion

$$p = \rho T\left[1 + b_2(\rho\lambda^2) + b_3(\rho\lambda^2)^2 + \ldots\right], \quad (25)$$

where the thermal de Broglie wavelength for a particle of mass m is

$$\lambda = \sqrt{\frac{2\pi\hbar^2}{mT}}. \quad (26)$$

Based on the cluster expansion, the expression for the second virial coefficient can be written as follows, cf. [29]:

$$b_2(\alpha) = -\frac{1}{4} - 2\frac{Z_2(\alpha) - Z_2(0)}{Z_1}, \quad (27)$$

where the partition function Z_1 is given by

$$Z_1 = \frac{1}{4\sinh^2 \beta\hbar\omega_c/2} \quad (28)$$

with $\beta = 1/T$ being the inverse temperature and $Z_2(\alpha)$ being the partition function of the two-anyon problem.

So, the correction to the second virial coefficient can be defined as

$$\Delta b_2(\alpha) = -2\frac{\Delta Z_2(\alpha) - \Delta Z_2(0)}{Z_1}. \quad (29)$$

The correction to the partition function consists of two terms (for $l > 0$ and for $l \leqslant 0$):

$$\Delta Z = \Delta Z^{(1)} + \Delta Z^{(2)}. \quad (30)$$

Taking into account that

$$Z = \sum_{n,l} \mathrm{e}^{-\beta(E_{n,l} + \Delta E_{n,l})} \approx \sum_{n} \mathrm{e}^{-\beta E_{n,l}}(1 - \beta\Delta E_{n,l}),$$

we obtain

$$\Delta Z = -\beta\sum_{n,l} \mathrm{e}^{-\beta E_{n,l}}\left[\Delta E^{(1)}_{n,l} + \Delta E^{(2)}_{n,l}\right]. \quad (31)$$

Note that positive ls yield divergent expressions when calculating the contribution from $\Delta E^{(1)}_{n,l}$ in $\Delta Z(\alpha)$. However, to obtain Δb_2 we only need to find the difference $\Delta Z(\alpha) - \Delta Z(0)$, which appears finite. The respective contribution exhibits rather slow convergence, $1/\sqrt{l_{\max}}$, where $l_{\max}$ is the upper limit of summation over l. This is demonstrated in Table 1. The accuracy of numerical fitting is of the order $10^{-6} - 10^{-5}$.

Table 1. Results for sums over l in $\Delta Z(\alpha) - \Delta Z(0)$ at some values of α and n depending on the upper limit $l_{\max}$. The values at $l_{\max} = \infty$ correspond to the results of numerical fitting

$l_{\max}$	$\alpha = 0.1$		$\alpha = 0.5$	
	$n = 0$	$n = 1$	$n = 0$	$n = 1$
100	0.01958	0.0465	0.1062	0.2460
1000	0.02079	0.0501	0.1122	0.2640
10000	0.02117	0.05126	0.1141	0.2698
20000	0.02121	0.05144	0.1144	0.2706
50000	0.02124	0.05161	0.1146	0.2712
100000	0.02133	0.05167	0.1147	0.2716
∞	0.02134	0.05182	0.1150	0.2724

Using (13)–(16), we can calculate $\Delta Z(\alpha) - \Delta Z(0)$ and obtain the corrections to the second virial coefficient at the factor $\beta\gamma$ [see Eq. (17)]. For convenience, we take $\hbar\omega_c = 1$ as the energy unit. The results are shown in Fig. 1 and Table 2 at different values of the inverse temperature β and the anyonic parameter α.

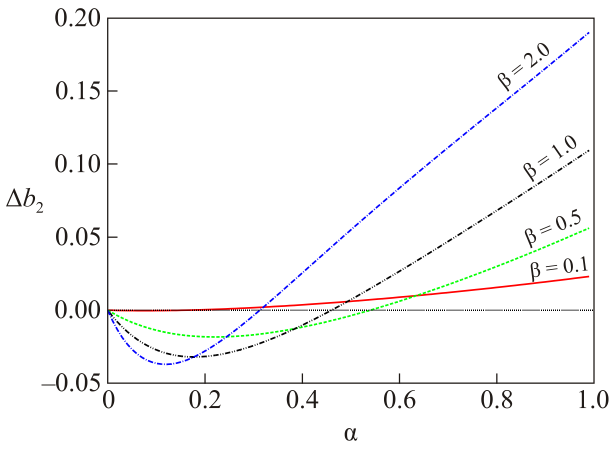

Fig. 1. The correction to the second virial coefficient at the factor of $\beta\gamma$.

Table 2. The correction to the second virial coefficient $\Delta b_2(\alpha)$ at the $\beta\gamma$ factor

α	β = 0.1	β = 0.5	β = 1.0	β = 2.0
0.0	0.0	0.0	0.0	0.0
0.01	–0.000078	–0.001948	–0.004238	–0.007197
0.05	–0.000255	–0.008358	–0.01745	–0.02686
0.1	–0.000217	–0.01377	–0.02722	–0.03645
0.2	0.000551	–0.01825	–0.03175	–0.02806
0.3	0.002038	–0.01690	–0.02428	–0.003644
0.4	0.004092	–0.01167	–0.01031	0.02545
0.5	0.006616	–0.003747	0.007202	0.05498
0.6	0.009545	0.006124	0.02663	0.08366
0.7	0.01040	0.01746	0.04710	0.1114
0.8	0.01646	0.02995	0.06819	0.1385
0.9	0.02039	0.04339	0.08971	0.1655
1.0	0.02461	0.05766	0.1117	0.1930

Figure 1 shows that near the bosonic side (parameter $\alpha \in (0; 0.5)$, depending on β) the correction Δb_2 has negative values. So, the nature of statistical interaction changes the contribution into the thermodynamics as α increases. Note that this statistical attraction / repulsion depends on the sign of the product electric and magnetic charges $e\mu$ which defines the parameter γ.

As α approaches the fermionic side, $\alpha \to 1$, the correction to the second virial coefficient becomes linear. This observation might help simplifying numerical calculations in this limit.

4. Magnetic susceptibility

Magnetic susceptibility can be written through the magnetization as a derivative with respect to the magnetic field:

$$\chi = \frac{\partial M}{\partial B}, \tag{32}$$

where M is defined as

$$M = -\frac{\partial F}{\partial B}. \tag{33}$$

The Helmholtz free energy F is expressed through the partition function Z of the system, $F = -T \ln Z$. Using (32) and (33), we obtain χ as

$$\chi = \frac{\partial M}{\partial B} = -\frac{\partial^2 F}{\partial B^2} = \frac{\partial^2 T \ln Z}{\partial B^2}. \tag{34}$$

In the principal approximation, considering N noninteracting particles, we can rewrite Z through the one-particle partition function Z_1 as $Z = Z_1^N$. In this way, F can be written as

$$F = -T \ln Z = -NT \ln Z_1 \tag{35}$$

and accordingly the magnetic susceptibility is

$$\chi = NT \frac{\partial^2 \ln Z_1}{\partial B^2}. \tag{36}$$

We will consider the specific magnetic susceptibility, that is, relative to one particle:

$$\chi_0 = \frac{\chi}{N} = T \frac{\partial^2 \ln Z_1}{\partial B^2}. \tag{37}$$

Using (28), we have the following expression for χ_0:

$$\chi_0 = \left(\frac{\hbar e}{4mcT}\right)^2 \frac{2T}{\sinh^2 \frac{\hbar\omega_c}{2T}}. \tag{38}$$

Obviously, taking into account (29), we also obtain that correction to magnetic susceptibility due to the anyonic contribution is proportional to correction to the second virial coefficient:

$$\Delta\chi \sim \frac{\partial^2}{\partial B^2} \Delta b_2(\alpha). \tag{39}$$

As analytical expressions are rather cumbersome, this correction was calculated numerically. The results are plotted in Figs. 2, 3 for different temperatures and values of the anyonic parameter.

The $\Delta\chi$ correction does not change sign at temperatures $T \gtrsim 0.7$. At lower temperatures, the bosonic and fermionic sides correspond to different signs of the correction, with the point of $\Delta\chi = 0$ shifting gradually to the bosonic side $\alpha = 0$ as the temperature lowers.

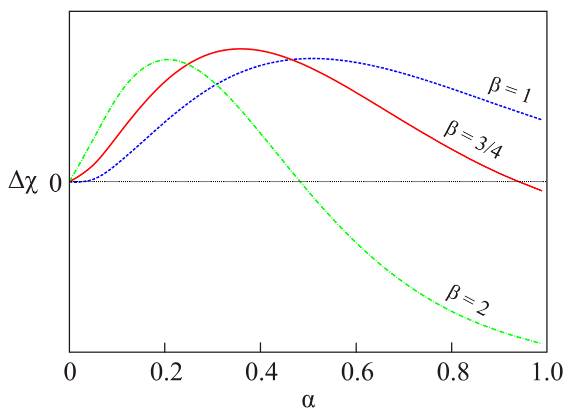

Fig. 2. The correction to the magnetic susceptibility.

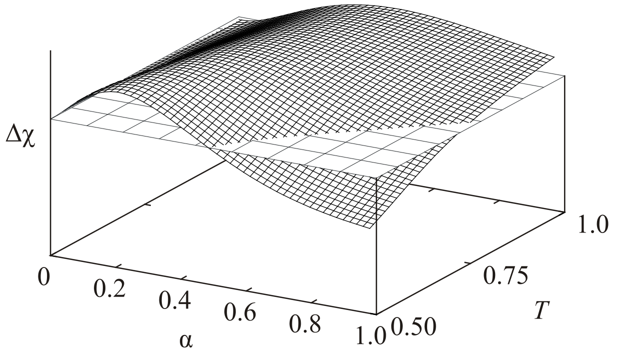

Fig. 3. Qualitative picture of the correction to the magnetic susceptibility $\Delta\chi$ as a function of the anyonic parameter α and temperature T. Horizontal plane corresponds to $\Delta\chi = 0$.

5. Conclusions

In this paper, we have considered the system of anyons in the magnetic field with magnetic charges and demonstrated approaches to calculate thermodynamic functions. Using results for the spectrum of a two-anyon problem accounting for the contribution from the magnetic charges, we have calculated the correction Δb_2 to the second virial coefficient. From the obtained dependence one can see that on the bosonic side (anyonic parameter $\alpha \lesssim 0.5$) this correction has the extremum (minimum or maximum depending on the sign of the product of the electric and magnetic charges $e\mu$).

The correction to the second virial coefficient allows for calculation of the thermodynamic functions, which has been demonstrated by defining the correction to magnetic susceptibility depending on temperature and the anyonic parameter α.

The presented approach can be used to calculate thermodynamic functions in condensed matter systems, where anyonic statistics and magnetic charges appear effectively. This applies also to calculations of magnetic properties in other fractional-statistics systems, cf. [32].

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Поправки до термодинаміки системи еніонів з магнітними зарядами

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Зроблено розрахунки термодинаміки системи еніонів з магнітними зарядами в електромагнітному полі. Показано, як внесок у поправку до енергетичного спектра, пов'язаний з магнітними зарядами, впливає на другий віріальний коефіцієнт та магнітну сприйнятливість. Наведено залежності відповідних поправок як функції температури й еніонного параметра.

Ключові слова: еніони, магнітний заряд, другий віріальний коефіцієнт, магнітна сприйнятливість.

Поправки к термодинамике системы энионов с магнитными зарядами

Богдана Собко, Андрей Ровенчак

Проведены расчеты термодинамики системы энионов с магнитными зарядами в электромагнитном поле. Показано, как вклад в поправку к энергетическому спектру, связанный с магнитными зарядами, влияет на второй вириальний коэффициент и магнитную восприимчивость. Приведены зависимости соответствующих поправок как функции температуры и энионного параметра.

Ключевые слова: энионы, магнитный заряд, второй вириальний коэффициент, магнитная восприимчивость.