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Catastrophe theory in the phenomenological description of the avalanche effect in dc-biased microwave HTSC transmission lines

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Within the framework of the proposed generalization of the phenomenological model of a microwave nonlinear HTSC transmission line, the effects of direct current on the transmission line are studied. Taking into account the next term in the expansion of the nonlinear dependence of the resistance on current allows us to explain the anomalous dependence of the insertion loss on the input power level observed in the experiment at subcritical temperatures. An additional direct current through the microwave waveguide can lead to the appearance of a bifurcation region in the space of control parameters. This is manifested in the possibility of an abrupt change in the properties of the waveguide at the boundaries of this region and the transition of the HTSC waveguide to a strongly dissipative state. The qualitative correspondence of the properties of the generalized phenomenological model to the data of experimental studies of the HTSC coplanar waveguide is obtained.

Keywords: high-temperature superconductor, nonlinear phenomena, HTSC-transmission line, direct current, avalanche junction, phenomenological model, catastrophe theory.

In the physical world, only the measurable is real,
in fact, only that which is measurable twice at least.
Matthias Hein
University of Wuppertal, Germany, 1999

1. Introduction

As a rule, the study of new substances and new phenomena begins with the determination of linear relationships between external factors acting on a physical object, and the reaction of the latter to external influences. However, nature insists that it is nonlinear. This, on the one hand, complicates research, and on the other hand, stimulates researchers' desire for a deeper understanding of the world. Nonlinear effects in the microwave physics of condensed matter, exposed to electromagnetic fields are of great interest not only for fundamental science, but also for applications. This is due to the fact that the nonlinear properties of substances determine the nonlinear characteristics of devices, and this nonlinearity can have both positive and negative significance. For example, nonlinearity determines the operating range of devices created based on certain materials. On the other hand, nonlinear properties allow you to create many useful and necessary devices. It is appropriate to note here that superconductors allow the creation of quantum nonlinear devices, for example, microwave quantum electromagnetic radiation detectors with unique characteristics [1].

Microwave nonlinear phenomena in superconductors, especially in high-temperature (HTSC) ones, have been the subject of many works (see, for example, [2–5]). In this case, HTSC samples in various electrodynamic structures were studied. Moreover, the indicated structures in the form of microwave resonators or transmission lines were performed, as a rule, on the basis of the studied HTSC epitaxial films. By the nonlinear characteristic of a transmission line we mean the nonlinear dependence of the signal power at the output of the transmission line on the signal power at its input or, in other words, the dependence of the losses in the line on the signal level at the input of the line.

Earlier, we conducted experimental studies of a microwave transmission line of the coplanar waveguide (CPW) type based on HTSC. For certain values of the input power Pin and direct current I_{dc}, the effect of a strong (abrupt) change in microwave losses was detected at the waveguide temperature $T < T_c$, where T_c is the critical temperature [6,7].

Note that the first experimental studies of HTSC CPW were carried out without direct current ($I_{dc} = 0$), when the observed avalanche-like effect was absent [4,8–10]. In this regard, we should also point out several works in which the transition of the microwave HTSC structure to the dissipative state occurred under the influence of direct current, however, the microwave signal was obviously weak and was used only to monitor the state of the HTSC structure (see, for example, [11,12]). In all cases, there is no strict electrodynamic model for describing the features of a nonlinear HTSC transmission line. Moreover, the well-known phenomenological model of the passage of a microwave signal in a transmission line [4,9,10] was applied without taking into account the direct current component. It should also be emphasized the lack of an unambiguous idea of the nature of the avalanche effect observed in [6,7] despite a number of notable results obtained with allowance for the motion of magnetic vortices in HTSC structures (see, for example, [13–16]). In addition, it is worth mentioning some works in which the response of superconductors to combined direct and alternating currents drive was also studied experimentally and theoretically (see, e.g., [17,18] accordingly).

In this regard, it becomes necessary to generalize the well-known phenomenological model [4,9,10], based on telegraph equations, also to the case of displacement of the microwave transmission line offset via direct current, to quantify the observed effect. On the one hand, such a model would give hope to more deeply investigate the nature of the discovered effect. On the other hand, it makes it possible to find out the practically important possibility of accurately measuring the characteristics of HTSC transmission lines in a highly dissipative state, in which the object under investigation can be for a very limited time (of the order of seconds or less) due to the possibility of its thermal destruction.

In [19], it was suggested that the nature of the studied dependence allows us to consider the abrupt change in the properties of a nonlinear transmission line as a "catastrophe", for example, of a "fold" type [20] and use the methodological and mathematical apparatus of the theory of "catastrophes" to predict new results. Preliminary theoretical studies have been carried out, which fundamentally confirmed this possibility. This paper presents the results of further research in this direction. One of them is the hysteresis in the temperature dependence of microwave losses, which can be confirmed in subsequent experiments.

It is also possible to predict the high accuracy of reproducing the critical temperature of the "jump" at a given constant current or contrary, the reproduction of the critical current at a constant temperature, etc. These effects, in principle, can allow the creation of high-precision sensors of microwave power, temperature, and direct current based on microwave HTSC transmission lines, in particular, CPWs.

2. Notes on experiment details and study design

The technical details of the experiment are given in [6,7]. Here we only note that the transmission line was a coplanar waveguide made by photolithography based on an epitaxial film $YBa_2Cu_3O_{7-\delta}$ with a thickness $d = 0.075$ μm on a single crystal substrate MgO (T_c = 86.5 K, I_c = $= 3.6\cdot10^6$ A/cm^2 at T = 77 K) manufactured by THEVA (Germany). A microwave signal was supplied and received through the waveguide and direct current was transmitted using special microwave planar tees.

The control parameters in the work were the input microwave power P_{in}, sample temperature T and the magnitude of the direct current I_{dc}. The dependence of microwave losses $IL = 10\lg\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right)$ on one of the parameters given the other two was measured. Measurements at a constant temperature were carried out only at 77 K.

The experiment was carried out only with increasing temperature (slow enough to maintain thermodynamic equilibrium in the transmission line). In this case, the exposure time of microwave radiation (current) to the HTSC structure was determined by a pulse duration of 5 мs with a pulse duty cycle of $2.5\cdot10^4$ Hz, which makes it possible to neglect the general heating of the HTSC film during measurements. The direct current value was always below the critical current I_c.

It was shown that at certain values of the controlled parameters and in the absence of direct current, the effect of an avalanche-like transition of the transmission line to a strongly dissipative state may not be observed and, conversely, occurs when the current is turned on [6]. It is important to note that in the absence of direct current in the temperature dependence of IL losses, an increase in steepness was observed in a certain temperature range. As a working hypothesis, we can assume that the presence of a direct current leads to an even steeper dependence of IL on temperature, and at a certain "critical" value of the current, to an "avalanche-like" transition of the structure to a strongly dissipative state, when the losses increase by 2–3 orders of magnitude.

The abrupt change in the properties of the observed system with smooth change of control (external) parameter is called "catastrophe" [20]. In the experiments under consideration, the electrodynamic state of the waveguide is characterized by one state parameter, IL, measured for the main harmonic of the microwave current. Its value depends on the values of three control parameters: direct current I_{dc}, input microwave power P_{in} and film temperature T. Therefore, in the general "catastrophe" model, the response surface is a three-dimensional surface defined by the function $IL(I_{dc}, P_{\text{in}}, T)$ in four-dimensional space. In this case, four types of "catastrophe" are possible, which are included in the generally accepted classification: "butterfly", "swallowtail", elliptical and hyperbolic umbilicals.

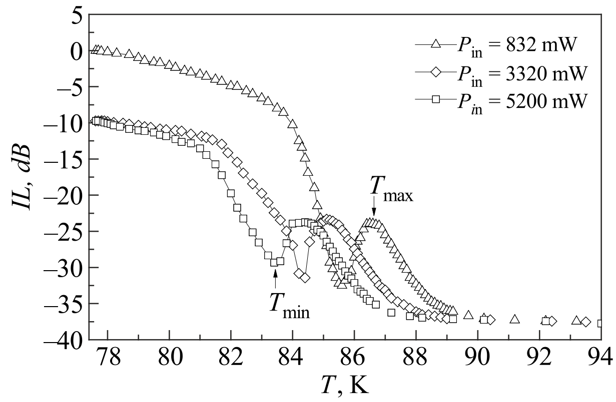

Fig. 1. The curvature of the 2-dimensional response surface $IL(P_{in}, T)$ of the states of the coplanar waveguide in the absence of a direct current component.

However, the technical limitations on the simultaneous change of all three control parameters in the experiment do not allow a sufficiently complete study of the response surface in this general case. Instead, as a rule, one of these parameters smoothly changes with a constant value of the other two. Therefore, we will further consider particular manifestations of a "catastrophe" of the "fold" or "cusp" type in experiments with one state parameter and one or two control parameters.

For example, the experimental dependences $IL(T)$, obtained for a given P_{in} and the absence of a direct current (Fig. 1), exhibit distortions in the temperature range 82–90 K. This could lead to the effect of a catastrophe in the case when the film temperature T is a state parameter and the insertion loss IL is a control parameter. But we do not yet see the possibilities for the physical realization of this effect.

Instead, the effect of the direct current component on the nature of the dependence $IL(T)$ was investigated in experiments. The abrupt change in IL at some (critical) values of T or I_{dc} suggests that the shape of the $IL(T)$ dependence is distorted, as shown in Fig. 2.

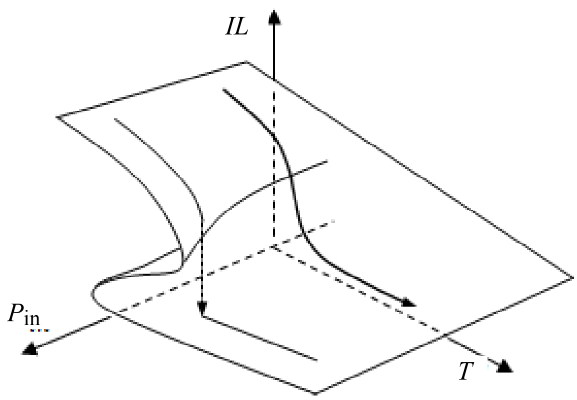

Fig. 2. The state-space of the system can take the form of an "assembly" catastrophe that arose on the two-dimensional surface $IL(P_{in}, T)$ when a direct current component is added.

For some values I_{dc} and P_{in} a bifurcation region appears on the axis of the control parameter T, in which the $IL(T)$ dependence becomes three-valued. This leads to a "catastrophe" of the "fold" type. The family of $IL(T)$ dependencies obtained for different P_{in} values forms a two-dimensional "cusp" type catastrophe surface. The experimental data suggest that a similar "assembly" surface is also formed for the $IL(I_{dc}, P_{in})$ dependence in experiments with stabilized temperature T.

To analyze this assumption, we generalized the existing phenomenological model of microwave nonlinear HTSC transmission lines [4]. In particular, the influence of the direct current component I_{dc} and the second nonzero term in the expansion of the nonlinear dependence of the resistance on the total current was studied [19].

3. An analysis of the phenomenological model of current flow along a waveguide based on HTSC film

A number of studies investigated the physical causes of the nonlinearity of the physical properties of the HTSC epitaxial film. The main ones may be the following:

— current dependence of the density of a superconducting charge carriers according to the Ginzburg–Landau equations [3];

— excitation and motion of the Abrikosov vortices [12].

However, obtaining the exact form of the dependence of the film resistance on temperature and the current flowing through it on the basis of such models is a complex and still unsolved problem. In this regard, a number of phenomenological models of the nonlinear properties of HTSCs have been proposed to explain the experimental data.

The simplest model of this kind is the approximation of the nonlinear dependence of the linear resistance on current by expanding this function in a power series, taking into account only the first nonzero (quadratic) term [4]:

$$R_1(t) = \sum_{n=0}^{\infty} a_n \left[I(t)\right]^{2n} \approx a_0 + a_1 \left[I(t)\right]^2 = R_{1.1}\left[1 + \frac{I(t)^2}{I_{0R}^2}\right]. \quad (1)$$

Here $R_{1.1}$ is the initial (in the absence of current) linear resistance of the transmission line, which depends on the effective surface resistance of the superconductor R_s and the geometric dimensions of the waveguide in the cross-section, I_{0R} is a phenomenological parameter characterizing the dependence of resistance on current.

It is shown that this model gives good agreement with experimental data at low current values. In [9], the nonlinearity of the inductive component of the impedance was additionally taken into account.

Due to the isotropic properties of the HTSC film, in these models, the linear component of the dependence is considered equal to zero. In this case, however, there remains the possibility of the presence in the power dependence of the term proportional to the current modulus.

Also note that Eq. (1) does not contain inertial components. This assumption can be considered true if the relaxation time of the superconducting state of the film is much shorter than the period of microwave vibrations.

When analyzing the model, the telegraph equations for the waveguide line are solved by the harmonic balance method [4]. Nonlinearity (1) leads to an anharmonic solution, which can be represented as the sum of harmonics. For the first harmonic of the current amplitude $I_m(x)$ a simple analytical expression was obtained. The experimental results [4,9,10] confirm the adequacy of this model for temperatures noticeably lower than T_c and at low currents. However, in the general case, the results of our studies [6] contradict them. In particular, at temperatures close to critical, IL can decrease with increasing P_{in} or with decreasing T.

Several physical mechanisms have been proposed to explain this effect. They can be associated with both microscopic phenomena in high-temperature superconductors [21] and macroscopic properties of the waveguide [9]. However, when considering the phenomenological model, we will be interested not in the physical nature of nonlinearity, but in its mathematical description.

To obtain a more accurate model of nonlinearity, one of the two approximations accepted in [4,9,10] should be refused. This is either an assumption about the inertia-free properties of an HTSC film at frequencies of no more than 10 GHz, or an assumption about the possibility of restricting oneself to the first nonzero term of the expansion in powers of the current in (1). In this paper, we consider the second of them.

4. Generalized model of nonlinearity of resistance of HTSC waveguide

Note that the absence of a linear term in the power-law dependence of the HTSC linear resistance on current is related to the symmetry (isotropy) of the film properties. However, this property is possessed not only by the terms of the power expansion with even powers, but also by the term proportional to the current modulus. Such a generalization of the model can also lead to an explanation of the effects observed in the experiment, however, it requires a more fundamental analysis of the physical causes of nonlinearity.

In this paper, we consider a generalization of model (1) related to taking into account the next term in the expansion of the dependence in even powers of the current. This leads to the following approximation of the linear resistance:

$$R_1(t) \approx R_{1.1}\left[1 - \frac{I(t)^2}{I_{0R}^2} + \frac{I(t)^4}{I_{1R}^4}\right]. \tag{2}$$

The signs of the terms in this approximation are selected in such a way as to ensure the possibility of a nonmonotonic dependence of the linear resistance on the input microwave power and the requirement of its positivity at any current value. In (2), at low powers, the function turns out to be decreasing. It can be assumed that this effect (decrease in resistance with increasing current) is associated with the specific structure of the field in the cross-section of the superconducting CPW as a planar structure. In general, the ac response of superconductor film is not only inertial, but also nonlocal [22,23]. So, the approach based on the telegraph equations with a subsequent usage of Eqs. (1), (2) it should be considered as a phenomenological description of the phenomenon in which nonlocality effects are taken into account in the dependence of the resistance on current averaged over the C cross-section (2).

After substituting (2) into the system of telegraph equations and solving them by the harmonic balance method for the first (main) harmonic of the signal, we obtain a first-order differential equation

$$-\frac{1}{U}\frac{dU}{dx'} = 1 - \frac{3}{4}U + \frac{5}{8}\vartheta U^2; \tag{3}$$

where $U = \dfrac{I_m^2(x)}{I_{0R}^2}$; $x' = 2\alpha x$; $\vartheta = \left(\dfrac{I_{0R}}{I_{1R}}\right)^4$.

Here $\alpha = \dfrac{R_{1.1}}{2Z_0}$ and $Z_0 = \sqrt{\dfrac{L_1}{C_1}}$ are respectively — propagation constant and wave line impedance (for the considered waveguides $Z_0 = 50$ Ohms). Note that to ensure the positiveness of R_1 at any current value, the condition $\vartheta > 0.25$ must be satisfied.

The analytical solution (3) can be written as:

$$U + \frac{3}{4\sqrt{\frac{5}{2}\vartheta - \frac{9}{16}}}\arctan\left(\frac{\frac{5}{4}\vartheta U - \frac{3}{4}}{\sqrt{\frac{5}{2}\vartheta - \frac{9}{16}}}\right) - \frac{1}{2}\ln\left(1 - \frac{3}{4}U + \frac{5}{8}\vartheta U^2\right) + C = -2\alpha x. \tag{4}$$

A typical form of the dependence $U(x)$ is shown in Fig. 3.

The dimensionless quantity U introduced above is proportional to the power

$$U(x) = \frac{2P(x)}{I_{0R}^2 Z_0}. \tag{5}$$

This allows us to determine the integration constant in (4)

$$C = -f(U_{\text{in}}) = -f\left(\frac{2P_{\text{in}}}{I_{0R}^2 Z_0}\right) \tag{6}$$

and get $IL(P_{\text{in}})$, addiction as

$$IL(P_{\text{in}}) = 4.34\ln\frac{U_{\text{in}}}{U_{\text{out}}}. \tag{7}$$

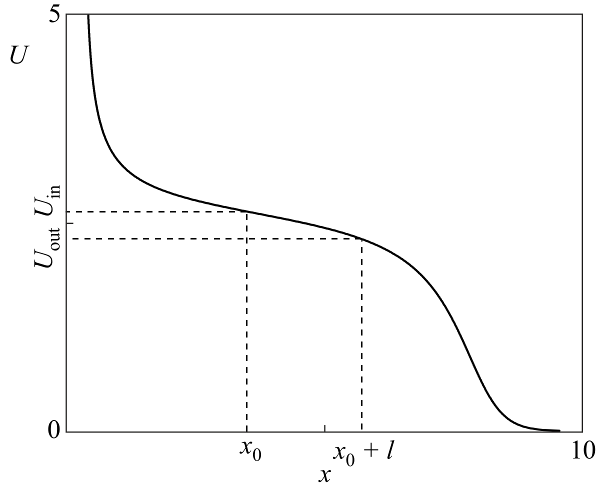

Fig. 3. A characteristic form of the dependence $U(x)$ from Eq. (4).

Equation (4) does not have an analytical solution with respect to U. Therefore, the value of U_{out} and the corresponding $IL(P_{in})$ are determined graphically. To do this, first find the value U_{in} and the corresponding (x_0) coordinate (for an arbitrary value of C) on the graph $U(x)$, and then the point $(x_0 + l)$ and the corresponding value U_{out}. Then calculate IL. The «*S*»-shaped curve of $U(x)$ leads to the appearance of a maximum in the dependence $IL(P_{in})$ (Fig. 4).

Note that in Fig. 1, the shape of the region of the nonlinear $IL(T)$ anomaly is almost the same for different values of the input power P_{in}. With increasing P_{in} this characteristic region shifts to lower T and slightly decreases in amplitude. This allows the total dependence of the insertion loss IL to be approximated by the dependence on some fractional rational function of P_{in} and T. In a first approximation of such a function, we can write:

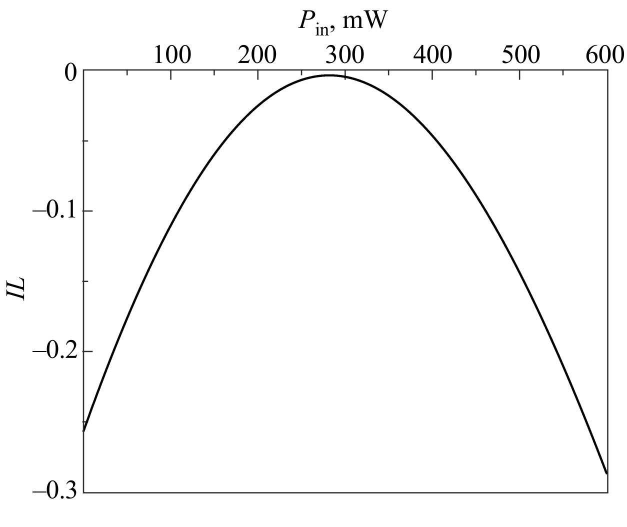

Fig. 4. Dependencies of $IL(P_{in})$ obtained on the basis of (4), (8) in the framework of model (2), (3).

$$IL(P_{in};T) = IL(P_{in} + \gamma(T - T_0)),$$

$$\text{where } \gamma \approx 1900\frac{\text{W}}{\text{K}};\ T_0 \approx 115\ \text{K}. \quad (8)$$

This approximation allows us to restore the theoretical $IL(T)$ dependence (at a constant P_{in} value) based on the already obtained $IL(P_{in})$ dependence (at a constant T value). For this, it is necessary to stretch and shift the $IL(P_{in})$ graph relative to the horizontal axis, taking into account the coefficients (8). The shape of these two dependencies is identical when condition (8) is satisfied. It qualitatively coincides with the experimental dependences (Fig. 1). Quantitative correspondence can be achieved by selecting the phenomenological parameters of the model (3): α, I_{0R} and ϑ. To determine them, a number of additional experiments are necessary. Therefore, in this work, we limited ourselves to explaining the qualitative correspondence of the proposed model to the experimental results (Fig. 1). A small change in the amplitude of the nonlinear dependence in the characteristic range of temperatures and powers can be explained by the weak (linear in a first approximation) of the temperature dependence of these parameters in formula (2).

Thus, the generalized phenomenological model (2) of the dependence of the linear resistance on current allows us to explain the anomalies in the experimental dependences of $IL(T)$ and $IL(P_{in})$, obtained at high power and temperatures close to critical.

5. Accounting for the DC component of the current in a generalized nonlinearity model

The presence of a nonzero component of the direct current leads to the fact that for the first harmonic of the current we seek the solution to the telegraph equations in the form:

$$I(x,t) = I_m(x)\cos(wt - \beta x) + I_{dc}. \quad (9)$$

Earlier, we showed [17] that for the coordinate dependence of the amplitude of the first harmonic of the current $I_m(x)$ instead of (3), we obtain the equation:

$$-\frac{1}{U}\frac{\partial U}{\partial x} = 2\alpha\left[\left(1 - 3\frac{I_{dc}^2}{I_{0R}^2} + 5\vartheta\frac{I_{dc}^4}{I_{0R}^4}\right) - \right.$$

$$\left. -\frac{3}{4}U\left(1 - 10\vartheta\frac{I_{dc}^2}{I_{0R}^2}\right) + \frac{5}{8}\vartheta U^2\right]. \quad (10)$$

Replacing the variable U and the parameters $(\alpha;\vartheta)$ allows us to reduce it to equation (3) and use the analytical solution obtained for it (4).

$$-\frac{1}{U^*}\frac{\partial U^*}{\partial(2\alpha^* x)} = 1 - \frac{3}{4}U^* + \frac{5}{8}\vartheta^* U^{*2}. \quad (11)$$

However, in contrast to the solution (4), in the experiment, the effect of a jump-like change in the properties of

HTSCs is observed when some T, P_{in}, or I_{dc} values are exceeded. Therefore, we study in more detail the obtained equation (11) and show that for some values of the input parameters the effect of "catastrophe" occurs (Fig. 2). In equation (11), in contrast to (3), the coefficients of the right side of the equation depend on the magnitude of the DC component of the current. While in (3) the right-hand side was positive for any admissible $\vartheta > 0.25$, in (10) it can vanish (the corresponding quadratic equation has roots) in a wide range of admissible values of I_{dc} and ϑ. In this case, the left-hand side also becomes 0, which corresponds to the trivial solution $U^*(x) = \text{const}$.

Denote $U_{dc} = I_{dc}^2 / I_{0R}^2$. Under condition

$$\frac{9}{16}(1-10U_{dc})^2 > \frac{5}{2}\vartheta(1-3U_{dc}+5\vartheta U_{dc}^2) \qquad (12)$$

the right-hand side of (10), considered as a square trinomial with respect to U, can have real roots $U_{1,2}$. The fulfillment of this condition and the signs of the roots depend on the ratio of the parameters ϑ and U_{dc}.

In Fig. 5, the darkened region corresponds to the values $\sqrt{U_{dc}} = I_{dc}/I_{0R}$ (horizontal axis) and ϑ (vertical axis), at which U has roots, and at least one of them is positive. Upon leaving this region, the asymptotes of the function $\left[-\frac{1}{U}\frac{\partial U}{\partial x}\right]$ become negative, or equal to 0. This leads to the effect of an abrupt change in *IL* (the catastrophe effect).

The boundary of the domain of existence of solutions is divided into two sections. With a smooth change of one or more control parameters and the intersection of the first section (dash-dotted border), the real positive roots of the corresponding equation disappear. Solution (10) $U(x)$ abruptly changes the asymptote (from the largest of the roots to 0). This leads to an abrupt change in *IL* at $x = l$.

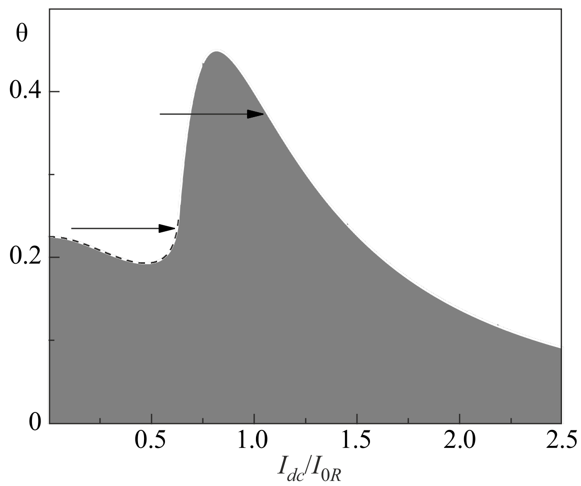

Fig. 5. Parameter area, on the border of which there are conditions for an abrupt change in the properties of HTSC ("catastrophe").

The second section (solid border) corresponds to the transition of both roots from the positive to the negative region. In this case, the asymptote $U(x) \to 0$ coincides with the point of a qualitative change in the nature of the dependence $U(x)$. Therefore, a "catastrophe" formally occurs only in the asymptotic limit $\alpha x >> 1$, and for finite waveguide lengths ($\alpha l \geq 1$) it turns out to be smoothed.

The numerical solution of Eq. (10) confirms the conclusions made. Figure 6 shows illustrations of both of these possibilities of manifestation of a "catastrophe" with a smooth increase in the dc component of the current.

Comparison of the results of numerical simulation with the experimental dependencies of $IL(I_{dc})$ (Fig. 7) shows a good agreement between the latter and the first type of catastrophe (Fig. 6(a)). With the appropriate selection of model parameters, one can achieve not only a qualitative analogy, but also a numerical coincidence of the values IL_{crit} and $(I_{dc})_{\text{crit}}$.

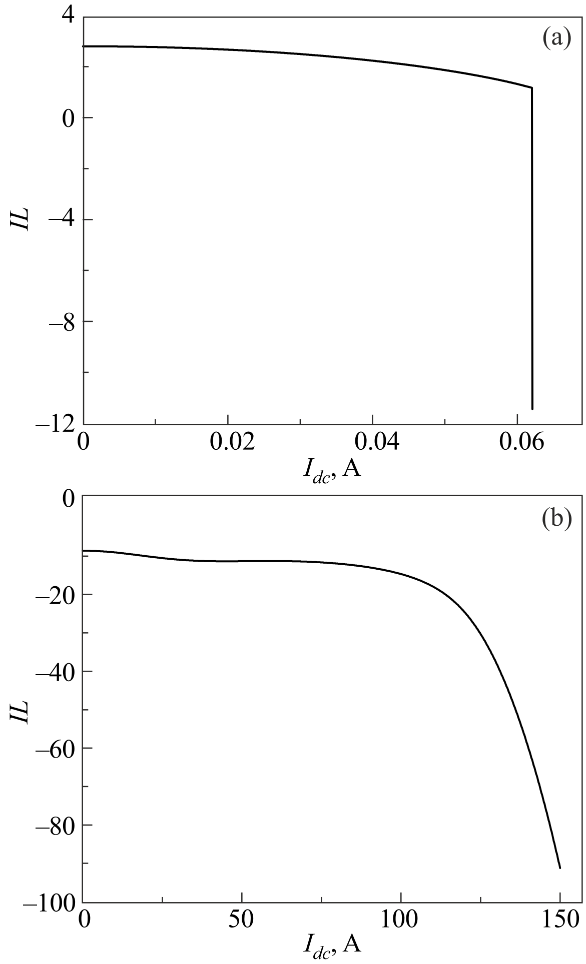

Fig. 6. (a) Illustration of a "catastrophe" at the boundary of the region of disappearance of real asymptotes in the phenomenological model of the waveguide ($\vartheta = 0.2$) (b) Illustration of a "catastrophe" at the boundary of the sign-changing region of real asymptotes in the phenomenological model of the waveguide ($\vartheta = 0.3$).

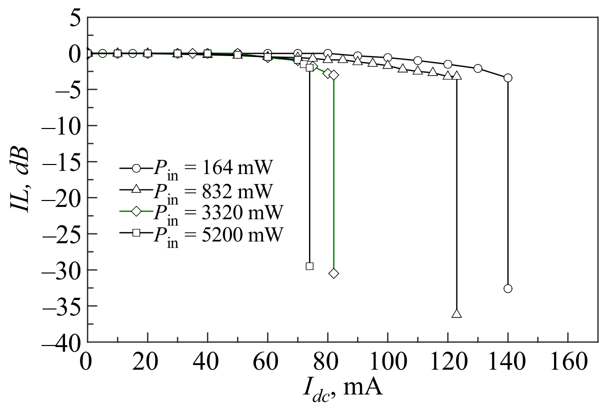

Fig. 7. The dependence of the insertion loss on direct current for a coplanar waveguide based on an HTSC film of 75 nm for different values of the input power $P_{\rm in}$.

However, some objections of a physical nature do not allow us to consider model (2) as the final version of the approximation of the nonlinear dependence $R(I)$. So, for example, at values $\vartheta < 0.25$ the linear resistance, in accordance with (2), becomes negative, which is not physical. Therefore, the value of R calculated from (2) in the framework of this model should be considered only as some effective value of the resistance included in the telegraph equation of propagation of an electromagnetic wave.

Thus, we can conclude that the proposed model, in principle, explains the effect of the occurrence of a "catastrophe" — an abrupt increase in insertion loss with a smooth increase in the DC component of the current. But to obtain numerical correspondence, the model requires further refinement of the approximation (2). For example, by adding a term proportional to the current modulus. The results of the analysis of such a refined model will be presented in subsequent publications.

Conclusions

In the work, a recently proposed model of a nonlinear HTSC transmission line is developed, generalizing the well-known phenomenological model to the case of direct current influence on the transmission line. The work is stimulated by the recently discovered effect of an avalanche-like transition of an HTSC coplanar waveguide with the direct current into a strongly dissipative state. In general, the ac response of superconductor film is nonlocal [22,23]. So, our approach should be considered as a phenomenological description of the phenomenon, in which dependence of the resistance on current is averaged over the waveguide cross-section (2). A theoretical analysis of the possibility of this generalization allows us to draw the following conclusions:

— Taking into account only the quadratic term in the nonlinear dependence of the linear resistance on current does not allow us to explain the experimental data obtained for various values of the input microwave signal power.

— Possible generalizations of the model, regardless of the physical nature of the nonlinearity, can be related to taking into account the inertia of the HTSC properties and (or) taking into account the next term in the expansion of the nonlinear dependence of the properties in a power series.

— Taking into account the next term of the expansion (fourth-order) of the nonlinear dependence of the resistance on current leads to the appearance of an additional (quadratic) term in the equation for calculating the coordinate dependence of the amplitude of the microwave signal. This, however, does not lead to the possibility of a "catastrophe" — an abrupt change in the properties of the waveguide with a smooth change in the control parameters.

— An additional direct current through the waveguide can lead to the appearance of a bifurcation region in the space of control parameters. This is manifested in the possibility of an abrupt change in the properties of the waveguide at the boundaries of this region.

— A certain correspondence of the properties of the generalized phenomenological model to the data of experimental studies is obtained.

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Теорія катастроф у феноменологічному описі лавиноподібного ефекту в мікрохвильовій ВТНП лініях передач постійного струму

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В рамках запропонованого нами узагальнення феноменологічної моделі мікрохвильової нелінійної ВТНП лінії передачі досліджено ефекти впливу на лінію передачі постійного струму. Врахування наступного члена розкладання нелінійної залежності опору від струму дозволяє пояснити аномальний характер внесених втрат від рівня вхідної потужності, що спостерігається в експерименті при докритичних температурах. Додатковий постійний струм через мікрохвильовий хвилевід може привести до виникнення області біфуркації в просторі керівних параметрів. Це проявляється у можливості різкої зміни властивостей хвилеводу на межах цієї області та переходу ВТНП копланарного хвилеводу в сильно дисипативний стан. Отримано якісну відповідність властивостей узагальненої феноменологічної моделі результатам експериментальних досліджень ВТНП копланарного хвилеводу.

Ключові слова: високотемпературний надпровідник, нелінійні явища, ВТНП лінія передачі, постійний струм, лавиноподібний перехід, феноменологічна модель, теорія «катастроф».

Теория катастроф в феноменологическом описании лавинообразного эффекта в микроволновых ВТСП линиях передач постоянного тока

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В рамках предложенного обобщения феноменологической модели микроволновой нелинейной ВТСП линии передачи изучено влияние постоянного тока на линию передачи. Учет следующего слагаемого в разложении нелинейной зависимости сопротивления от тока позволяет объяснить аномальную зависимость вносимых потерь от уровня входной мощности, наблюдаемую в эксперименте при докритических температурах. Дополнительный постоянный ток через микроволновой волновод может привести к появлению бифуркационной области в пространстве параметров управления. Это проявляется в возможности резкого изменения свойств волновода на границах этой области и перехода волновода ВТСП в сильно диссипативное состояние. Получено качественное соответствие свойств обобщенной феноменологической модели данным экспериментальных исследований ВТСП компланарного волновода.

Ключевые слова: высокотемпературный сверхпроводник, нелинейные явления, ВТСП линия передач, постоянный ток, лавиноподобный переход, феноменологическая модель, теория «катастроф».