

# Flux flow instability in type II superconducting strips: Spatially uniform versus nonuniform transition

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We discuss two possible mechanisms of the flux flow instability (FFI) in type II superconducting strips. While the nature of nonequilibrium effects leading to this instability is widely accepted (Joule heating and finite relaxation time of the superconducting order parameter) still there is a question how FFI develops in space. According to one scenario instability occurs simultaneously in the whole sample and superconductor jumps to the normal or resistive state with no change in the structure of moving vortex array. Another scenario predicts appearance of the vortex rivers at the instability point and jump of the superconductor to the normal or the resistive state but with strongly modified structure of the moving vortices.

Keywords: flux flow instability, Joule heating, order parameter relaxation time.

The vortex motion in current-carrying superconductor results in nonequilibrium distribution of electrons over the energy  $f(\varepsilon)$  and due to sensitivity of superconducting order parameter  $\Delta = |\Delta| \exp(i\phi)$  to the shape and value of  $f(\varepsilon)$  it affects vortex motion itself. One source of disequilibrium originates from the Joule heating when moving vortices create electric field  $\mathbf{E}$  (voltage  $V$ ), related with time-dependent phase of the superconducting order parameter  $\dot{\phi} \sim V$  and the superconductor is heated up. Second, less obvious source of disequilibrium, comes from the time-dependent magnitude of the order parameter  $|\dot{\Delta}|$  around the moving vortex. First it was noticed by Larkin and Ovchinnikov (LO) [1] who showed that it also may affect  $f(\varepsilon)$  and viscosity of the vortex motion decreases with increasing of its velocity which also leads to instability of flux flow.

Both mechanisms of disequilibrium lead to nonlinear current-voltage ( $I$ - $V$ ) characteristics of superconducting strip and existence of quench or instability current  $I_q$  at which the superconductor jumps to the normal or more resistive state in the current driven regime. Such a behavior was observed in many experiments and it was interpreted as an effect of Joule heating [2–4], time variation of  $\Delta$  [2,5–9] or both of them [10].

Note, that in the majority of the theoretical papers the spatially averaged Joule heating [2–4,10] and electron cooling due to  $|\dot{\Delta}|$  was assumed [1]. For example, Larkin and Ovchinnikov considered the moving periodical vortex lattice in Wigner–Zits approximation which implies existence of the circular vortex cell instead of triangular one

and which cannot take into account effects, connected with spatial transformation of the moving single vortex [11,12] and corresponding change of vortex lattice (see Fig. 1(a)).

First theoretical attempt to study spatial transformations in the moving vortex array was made in Ref. 12. Using numerical modeling and generalized time-dependent Ginzburg–Landau (TDGL) equation [13] it was demonstrated that with increasing the current and, hence, velocity of the vortices the vortex lattice exhibits the set of consequent transformations which are ended up by appearance of the quasi-phase slip lines or vortex rivers (see Fig. 1(b)) and jump in the voltage. The found effect originates from finite relaxation time of  $|\Delta|$  when behind the moving vortex there is a wake with partially suppressed superconductivity [11] and it favors motion of other vortices along this wake. Qualitatively similar result was later found using ordinary TDGL equation [14] and generalized TDGL in nanostructured [15] and plain [16] bridges.

We have to note that generalized TDGL equation is valid only when time change of  $|\Delta|$  is much larger than inelastic electron-phonon scattering time  $\tau_{ep}$  because only at this condition one can neglect the term with time derivative  $\partial f / \partial t$  in the kinetic equation (so-called local equilibrium approximation [13]). The ordinary TDGL equation is valid for still slower time variation of  $|\Delta|$  when one may neglect change in  $f(\varepsilon)$  due to finite  $|\dot{\Delta}|$ . Recently, very similar vortex array transformation was found using TDGL equation coupled with heat conductance equation [17]. That model could be used for studying relatively fast

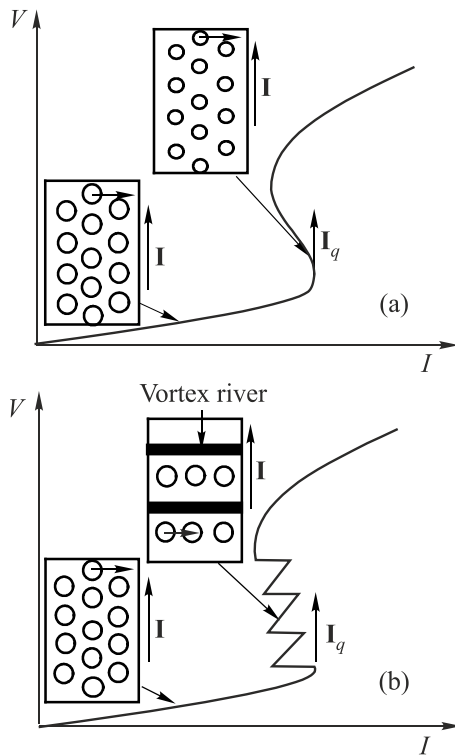


Fig. 1. Sketch of current-voltage characteristics and vortex distributions at different voltages in two scenarios. (a) Spatially uniform transition is connected with shrinking of vortex core without change of vortex lattice — LO scenario [1]. (b) Transition is connected with change of vortex lattice and appearance of vortex rivers [12,14–16]. The stair-like structure is expected in voltage driven regime in this case.

processes (faster than  $\tau_{ep}$ ) but with an assumption about fast thermalization in the electron system (limit of short inelastic electron-electron scattering time  $\tau_{ee} \ll \tau_{ep}$ ). Due to qualitative similarity of the results found in Refs. 12, 14–17 one may conclude that to have spatially nonuniform FFI one only needs finite relaxation time of  $|\Delta|$ . This time could be rather different in different theoretical models, and it brings quantitative difference to the results (shape of  $I$ - $V$  curves, dependence of instability current/voltage on temperature,  $\tau_{ep}$  etc.).

Current-induced vortex lattice transformation was observed in different superconducting materials. In patterned Al strip it was seen with help of “frozen” vortices [18]. Direct visualization of the vortex rivers was done in [16] using nano-SQUID magnetometry and Pb bridge. The justification could be also found from Ref. 19 where stair-like current-voltage characteristics were experimentally observed in the voltage driven regime and each stair could be associated with an appearance of the vortex river which then is expanded to the normal domain. The vortex rivers were also observed (at zero magnetic field) in BSSCO whiskers [20], where one cannot expect uniform current distribution across the whisker and therefore steps on  $I$ - $V$  characteristics

cannot be interpreted simply as traces of phase slip lines as in Ref. 21. The reorientation of the moving vortex lattice was observed in MoGe film [22] which also says in favor of spatially nonuniform FFI.

From another side LO model (with or without modifications made by Bezuglyj and Shklovskij [10]) was confirmed in many experiments and sometimes even  $I$ - $V$  curves were satisfactorily fitted by theoretical expressions (see, for example, Refs. 2, 7). The most close to the model of LO is the model considered in Ref. 17 which predicts spatially nonuniform FFI. In both models the relatively fast (on time scale  $\lesssim \tau_{ep}$ ) variations of  $f(\varepsilon)$  and  $|\Delta|$  are taken into account. The main differences between these models are i) in the assumption that  $\tau_{ee} \ll \tau_{ep}$  and ii) consideration of the Joule heating in Ref. 17. Calculated  $I$ - $V$  curves, their evolution with increase of  $H$  and dependences  $I_q(H)$  and  $V_q(H)$  from Ref. 17 qualitatively resembles many experimental results on FFI. Therefore it would be interesting to see if spatially nonuniform FFI exists in superconductors those  $I$ - $V$  curves are well described by LO model. For this purpose one can use STM to see possible changes in the moving vortex lattice at currents close but below  $I_q$ . As it was discussed in Ref. 17 the tip of STM measures time averaged density of states  $\langle N(\varepsilon) \rangle$  and far from the edge of the superconductor  $\langle N(\varepsilon) \rangle$  weakly depends on coordinate due to absence of strongly correlated vortex motion, while near the edge vortices enter and move along the same paths (nucleus of vortex rivers), which leads to strong coordinate-dependent  $\langle N(\varepsilon) \rangle$ .

Another way to check it is to use both current and voltage driven regimes. In the voltage driven regime in case of spatially nonuniform FFI the steps, resembling phase slip steps, should appear on  $I$ - $V$  curve above the instability voltage (see Fig. 1(b)). Each of these steps corresponds to appearance of one vortex river or quasi-phase slip line. In case of spatially uniform FFI the steps should be absent and the branch with negative differential resistance should exist on  $I$ - $V$  curve at  $V > V_q$ , as it predicts LO model (see Fig. 1(a)).

It is important to note that not only finite relaxation time of  $|\Delta|$  but also pinning of the vortices may lead to spatially nonuniform FFI. Indeed, when vortex pins are distributed nonuniformly across the superconductor there is finite probability for existence of “channels” with weak pins (having small pinning strength) across whole width of the strip. In this case vortices will have larger velocity along these “channels” and instability first starts there, leading to spatially nonuniform FFI [23–25] (similar effect is expected in case of periodic pinning array [26]). Therefore for the experiment with STM or in voltage driven regime it is better to use low pinning superconductors where LO-like behavior was demonstrated (for example, MoGe or NbGe [2,7]).

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### Нестабільність руху вихорів в смужках надпровідника II роду: просторово однорідний та неоднорідний переходи

Д.Ю. Водолазов

Обговорюються два можливих механізми нестабільності руху вихорів (FFI) в смужках надпровідника II роду. Хоча природа нерівноважних ефектів, що призводять до нестабільності, широко відома (джоулів нагрів і кінцевий час релаксації параметра порядку надпровідника), проте залишається питання, як FFI розвивається в об'ємі зразка. Згідно з одним сценарієм, нестабільність виникає одночасно у всьому зразку і надпровідник переходить в нормальний або резистивний стан без зміни структури рухомого вихрового масиву. Інший сценарій передбачає появу вихрових річок в точці нестабільності та перехід в нормальний або резистивний стан надпровідника зі стану з сильно зміненою структурою вихрової ґратки.

Ключові слова: нестабільність потоку, джоулів нагрів, час релаксації параметра порядку.

### Нестабільність движения вихрей в полосках сверхпроводника II рода: пространственно однородный и неоднородный переходы

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Обсуждаются два возможных механизма нестабильности движения вихрей (flux flow instability (FFI)) в полосках сверхпроводника II рода. Хотя природа неравновесных эффектов, приводящих к нестабильности, широко известна (джоулев нагрев и конечное время релаксации параметра порядка сверхпроводника), тем не менее остается вопрос, как FFI развивается в объеме образца. Согласно одному сценарию, нестабильность возникает одновременно во всем образце и сверхпроводник переходит в нормальное или резистивное состояние без изменения структуры движущегося вихревого массива. Другой сценарий предсказывает появление вихревых рек в точке нестабильности и переход в нормальное или резистивное состояние сверхпроводника из состояния с сильно измененной структурой вихревой решетки.

Ключевые слова: нестабильность потока, джоулев нагрев, время релаксации параметра порядка.