Critical phenomenon of vortex motion in superconductors: vortex instability and flux pinning

A. Leo^{2,1}, A. Nigro^{2,1}, and G. Grimaldi¹

¹CNR, SPIN Institute, via Giovanni Paolo II, 132 Fisciano (SA), I-84084, Italy ²University of Salerno, Physics Department "E. R. Caianello", Fisciano (SA), I-84084, Italy E-mail: gaia.grimaldi@spin.cnr.it

Received November 25, 2019, published online February 28, 2020

We have studied vortex dynamics in superconducting materials at very high vortex velocities as a function of the applied magnetic field. High velocity vortex dynamics can become critical, so that an instability occurs, leading the system to quench abruptly to the normal state. The presence of pinning mechanisms in all superconductors not only is able to foster high critical currents but it can strongly in-fluence vortex flow, thus determining a different behavior of the critical vortex velocity v^* . The magnetic field dependence of v^* is extremely sensitive to the type of material pinning, and this is crucial for an applicative point of view, since vortex motion approaching v^* means a dissipative flux flow state which will probably end with a flux flow instability. If it is possible to predict these critical parameters, than it will be easier to control those critical phenomena. Although a fully theoretical model of flux flow instability in the presence of pinning is still lacking, a phenomenological approach has been recently proposed for the hot-electron vortex flow instability. Here we present a successful example of perfect correspondence between experiment and theoretical approach in the case of Mo₃Ge thin films with and without geometrical pinning barriers.

Keywords: superconducting films, critical currents, flux pinning, flux flow instability.

The main characteristic of superconductors is the lossless ability of electrical current transport, a common property that is held in their critical current value, which sets the limits of the critical state in the material. Then, in order to push the performance of a superconductor, material pinning became a huge area of speculation and research, both from a fundamental point of view and for applications [1]. High Temperature Superconductors (HTS) have been deeply studied for so long because, for instance, overcoming their anisotropy would boost coated conductor technology [2]. The Iron Based Superconductors (IBS) as well, after ten years of research from the discovery, are becoming very attractive technical materials for applied research, due to their larger ξ and smaller anisotropy factor γ [3]. However, Low Temperature Superconductors (LTS) still are mainly used for electronic based devices in nanostructured photon detectors [4] or for emerging quantum technology [5]. In particular, among the amorphous superconductors, it has been employed the most homogeneous MoGe alloy in its optimized composition to obtain microstructures in view of attaining continuous nanowires fabrication easier than using pure metals, such as aluminum or gold, whose granularity can prevent the realization of continuous, thin nanowires [6]. However, whatever superconducting applications you have in mind, an unstable state of vortex matter would be detrimental since any instability allows for electric power dissipations, with a consequent sudden jump to the normal state.

Firstly, the high velocity vortex motion has been investigated for fundamental aspects related to non-equilibrium phenomena arising from the changes of the quasiparticle distribution in the vortex core during vortex motion at temperatures close to T_c , and measured by a finite inelastic scattering time τ_E of the quasiparticles [7–10]. A direct access to this microscopic parameter has been provided by the theory of Larkin and Ovchinnikov (LO) so far, which predicted the non-equilibrium distribution of quasiparticles, thus resulting in a magnetic field-independent critical vortex velocity v^* , due to the vortex core shrinking [11]. Another subsequent theoretical approach has been implemented for a different temperature regime, that is $T \ll T_c$, based on the raise of the electronic temperature due to the electric field induced by the high velocity vortex motion, which adds quasiparticles rather than removing them from the vortex core, thus causing mainly the expansion of the vortex core rather than its shrinking [12-14]. For both

mentioned electronic models of flux flow instability (FFI) an S-shaped current-voltage characteristics should be observed in a current bias mode, as it was in both LTS and HTS materials [15–20], and IBS as well [21–23]. Some other theoretical refinements have been proposed to take into account self-heating effects [24,13] in addition to electronic effects [11], so that a hot electron vortex instability was established in the absence of any pinning mechanism. The exploration of the electronic instability in regime of low magnetic fields let to discover that a field-dependent $v^*(B) \approx B^{-1/2}$ is necessary to recover a basic assumption of LO model, that is a non-equilibrium guasiparticles distribution uniformly distributed in the whole superconductor [25]. It is impressive how many experiments have been devoted to confirm this phenomenological prediction, indeed, very well verified in many superconducting materials [26–30]. Anyway, no flux pinning has been theoretically included till the recently proposed model of pinning effect on the hot-electron vortex flow instability [31], which account for many experimental evidences that such a pinning influence is crucial to interpret several experimental findings on different superconductors [32-35]. In fact, in the last years, a systematic study has been conducted in order to distinguish which is the role of flux pinning beside flux flow instability [36–40]. Very recently, for example, a periodic pinning landscape has been used to guide fast vortex dynamics till FFI occurs [41], whereas a local FFI has been established in films with strong point disorder [42]. We also discovered peculiar effects of pinning mechanism in determining the magnetic field dependence of v^* , which shows unexpected behavior at low fields, depending on the pinning strength in the case of weaker or stronger pinning materials [30,32-36], as well as on the pinning distribution for different irradiation processes on superconducting materials [33,37,38], or on a simple geometry acting as a pinning barrier [35,39], and finally on isotropic pinning materials which compete with their intrinsically layered structure [17,21,22]. In all these cases, regardless of the electronic nature of FFI, material pinning plays a role, whose fingerprint can be observed in the $v^*(B)$ behavior. Therefore a fully theoretical description of FFI in the presence of pinning is claimed. Nevertheless, the phenomenological approach, recently proposed to take into account pinning effects on the hot-electron FFI, is a theoretical study able to address the non-monotonicity of $v^*(B)$ that is systematically observed in experiments. This model can figure out the anomalous $v^*(B)$ behavior starting from the measured magnetic field dependence of the critical current density $J_c(B)$ and the current-voltage characteristics of the superconductor under investigation. By the way, this approach may be limited by the type of pinning potential chosen for the calculation that is a washboard pinning potential, even if it is able to reproduce all the previous results concerning the hot-electron instability in the absence of pinning [12]. In our experiment we successfully used this model to fit the data of $v^*(B)$ in the case of two Mo₃Ge superconducting bridges realized with different widths, thus obtaining a mesoscopic geometry that acts as a surface pinning barrier. Finally, this data analysis works out as an alternative to the timedependent Ginzburg–Landau (TDGL) modelling of FFI previously reported on this Mo₃Ge superconductor [39], again in perfect agreement with the experiment.

Samples are microbridges obtained from Mo₃Ge thin film by electron-beam lithography with thickness d = 50 nm, length $L = 160 \,\mu\text{m}$, and two widths of w = 5 and 100 μm . These two samples are named S1 and S100, respectively. Their superconducting properties have been previously investigated and it results a critical temperature $T_c = 5.5 \,\text{K}$ for bridge S1 and 6.5 K for bridge S100 [39]. A London penetration depth $\lambda = 500 \,\text{nm}$ is estimated for both bridges. Thus, in the case of S1 the Pearl length is $\Lambda = 2\lambda^2/d =$ $= 10 \,\mu\text{m} > w$, while the Ginzburg–Landau coherence length is $\xi_{GL} = 5 \,\text{nm} \ll w$ as well as $d \ll w$, so that a mesoscopic behavior is expected [39].

For the critical current density $J_c(B)$, we suppose a power law dependence on magnetic field of the type

$$J_c\left(B\right) = \frac{J_0}{B^m}.$$
 (1)

For the vortex critical velocity $v^*(B)$ behaviors, we refer to Eq. (26) in Ref. 31:

$$v^*(B) = \frac{c\alpha\gamma}{\sqrt{B}\sqrt{\sqrt{1+\mu^2} + \mu}},$$
 (2)

with α , γ and μ defined as:

$$\alpha = \frac{3\pi^3}{2em_e cp_F\left(kT_c\right)},\tag{3}$$

$$\gamma = P^* / \alpha, \tag{4}$$

$$2\mu = \left(\frac{J_c}{J_0^*}\right)^2,\tag{5}$$

where *e* is the electron charge, m_e is the electron mass, *c* is the speed of light, p_F is the Fermi impulse, *k* is the Boltzmann constant, $P^* = E^* J^*$ is the critical power and J_0^* is the instability current density in the absence of pinning [13].

In Fig. 1 (a) and (b), we report the model curves of $v^*(B)$ and $J_c(B)$ for m < 1 and m > 1, respectively. As it is shown, for m < 1 and m > 3 there are only few differences among the curves at different m values. Thus, in Fig. 2, we considered two cases in the theoretical model, by showing curves at half m = 0.5 (case A) and four times m = 4 this value (case B). In the first case A a continuous monotonic decrease of $J_c(B)$ can be observed, corresponding to an equally monotonic decrease in the $v^*(B)$ curve. In the case B (m = 4), even if we still observe a continuous monotonic decrease of $J_c(B)$, it has to be remarked that the corre-



Fig. 1. (Color online) Model critical vortex velocity (main panels) and critical current behaviors (insets) as obtained by equations (2) and (1), respectively, reported in the text, for different *m* values: m < 1 (a), m > 1 (b). The units in the figure are arbitrary.



Fig. 2. (Color online) Theoretical curves for the critical current density J_c and the vortex critical velocity v^* behaviors as a function of the applied magnetic field for different values of the power law exponent in Eq. (1). The units in the figure are arbitrary.

sponding $v^*(B)$ curve shows a peak. Then, two other more complex, are considered where a change in the *m* value is set at an arbitrarily value B = 2 combining the *m* values of case A and B, as shown in Fig. 2. In case C, we set

 $m_1 = 0.5$ for B < 2 and $m_2 = 4$ for $B \ge 2$, while in case D we set $m_1 = 4$ for B < 2 and $m_2 = 0.5$ for $B \ge 2$. In both C and D cases, the change of the exponent of the power law can be easily recognized in the $J_c(B)$, and the related $v^*(B)$ are quite different. Indeed, in case C, the $v^*(B)$ shows and highly non-monotonic behavior, with at least a minimum and a peak. On the contrary, in case D, the $v^*(B)$ curve follow almost exactly the curve of case B.

In the upper panels of Fig. 3 and Fig. 4, the J_c data as a function of the applied magnetic field $\mu_0 H$ for bridge S100 and S1, respectively, are shown together with the fitting curves from Eq. (1), where J_0 and *m* are assumed as fitting parameters. The resulting value of *m* for S100 is 0.79, while in the case of S1 it has been necessary to assume a change in the power law exponent for $\mu_0 H = 0.21$ T, obtaining $m_1 = 1.5$ and $m_2 = 0.6$. Accordingly to the theoretical prediction, the $v^*(\mu_0 H)$ behavior for sample S100 shows a monotonic decrease, while the $v^*(\mu_0 H)$ of S1 presents a peak at about 0.25 T (see lower panels of Fig. 3 and Fig. 4). On both sets of data, a fitting procedure has been performed on the base of Eq. (2), which can be written in the form:

$$v^{*}(B) = \frac{c\alpha\gamma}{B^{-1/2} \left[\left(1 + \frac{J_{0}^{4}}{4\gamma^{4}B^{4m-2}} \right)^{1/2} + \frac{J_{0}^{2}}{2\gamma^{2}B^{2m-1}} \right]}, \quad (6)$$

where α and γ are fitting parameters.

Summarizing, only based on the intrinsic pinning of the Mo₃Ge superconducting material, we have demonstrated that the phenomenological approach to the FFI works satisfactorily. This is the case of S1 sample, for which a perfect agreement between theory and experiment is found for the anomalous non-monotonic trend of $v^*(B)$ that increases instead of decreases at low fields. In the usual macroscopic case of S100, instead, the standard power law dependence of $v^*(B) \approx B^{-1/2}$ is observed and perfectly fitted as well.



Fig. 3. (Color online) Experimental $J_c(\mu_0 H)$ curve (upper panel) and $v^*(\mu_0 H)$ curve (lower panel), together with fitting curves, for bridge S100 at the reduced temperature $t = T/T_c = 0.25$.



Fig. 4. (Color online) Experimental $J_c(\mu_0 H)$ curve (upper panel) and $v^*(\mu_0 H)$ curve (lower panel), together with fitting curves, for bridge S1 at the reduced temperature $t = T/T_c = 0.45$.

Thus, this means that in the absence of the surface pinning caused by the mesoscopic geometry [39], the experimental results follow the usual expectation of the hot-electron FFI that is the power law monotonic dependence.

We conclude that the Mo₃Ge superconductor, although a weak pinning material in comparison with the other LTS, due to the narrow microbridge geometry chosen to be in an appropriate mesoscopic limit, shows a clear dependence of the critical vortex velocity behavior $v^*(B)$ on the pinning barrier. Such a pinning effect has been descripted within the recent phenomenological Shklovskij's model, which offers the possibility to address the non-monotonicity of $v^*(B)$, so that to provide a different $v^*(B)$ behavior with respect to LO as well as Kunchur's predictions of a constant v^* in the former case, and a power law dependence in the latter one. Finally, we confirm that starting from the knowledge of two experimental curves that are the currentvoltage characteristic and the $J_{c}(B)$ dependence, the developed Shklovskij's theory well reproduces our experimental findings on the pinning influence of a surface barrier on a weak pinning superconducting material, in both cases of Mo₃Ge thin films with and without the geometrical pinning barriers. Thus, this approach can be promising to overcome the missing theory in order to implement flux pinning in vortex instability, and it offers the way to test any other pinning materials.

Acknowledgments

The research leading to this work has received funding from the PON Research and Competitiveness 2007-2013 under grant agreement PON NAFASSY, PONa3_00007. G.G. acknowledges support from the Short Term Mobility program – STM 2015 by CNR. The authors acknowledge Prof. A.V. Silhanek for providing the samples.

- 1. R. Wordenweber, *Superconductors at the Nanoscale: From Basic Research to Applications*, De Gruyter Ed. (2017), Ch. 7.
- X. Obradors and T. Puig, *Supercond. Sci. Technol.* 27, 044003 (2014).
- H. Hosono, A. Yamamoto, H. Hiramatsu, and Y. Ma, *Mater. Today* 21, 278 (2018).
- C.M. Natarajan, M.G. Tanner, and R.H. Hadfield, Supercond. Sci. Technol. 25, 063001 (2012).
- A. Acin, I. Bloch, H. Buhrman, T. Calarco, C. Eichler, J. Eisert, D. Esteve, N. Gisin, S.J. Glaser, and F. Jelezko, *New J. Phys.* 20, 080201 (2018).
- 6. A. Bezryadin, *Superconductivity in Nanowires: Fabrication* and *Quantum Transport*, Wiley-VCH (2012), p. 215.
- C. Attanasio and C. Cirillo, J. Phys.: Condens. Matter 24, 083201 (2012).
- A. Leo, G. Grimaldi, R. Citro, A. Nigro, S. Pace, and R.P. Huebener, *Phys. Rev. B* 84, 014536 (2011).
- G. Grimaldi, A. Leo, A. Nigro, S. Pace, and R.P. Huebener, *Phys. Rev. B* 80, 144521 (2009).
- 10. M. Liang and M. Kunchur, Phys. Rev. B 82, 144517 (2010).
- 11. A.I. Larkin and Y.N. Ovchinnikov, *Nonequilibrium Superconductivity*, Elsevier-Amsterdam (1986), Ch. 11.
- 12. M.N. Kunchur, Phys. Rev. Lett. 89, 137005 (2002).
- J.M. Knight and M.N. Kunchur, *Phys. Rev. B* 74, 064512 (2006).
- 14. D.Y. Vodolazov and F.M. Peeters, *Phys. Rev. B* **76**, 014521 (2007).
- D. Babic, J. Bentner, C. Surgers, and C. Strunk, *Phys. Rev. B* 69, 092510 (2004).
- 16. C. Peroz and C. Villard, Phys. Rev. B 72, 014515 (2005).
- G. Grimaldi, A. Leo, F. Avitabile, N. Martucciello, A. Galluzzi, M. Polichetti, S. Pace, and A. Nigro, *Nanotechnology* 30, 424001 (2019).
- S.G. Doettinger, R.P. Huebener, R. Gerdemann, A. Kuhle, S. Anders, T.G. Trauble, and J.C. Villégier, *Phys. Rev. Lett.* 73, 1691 (1994).
- Z.L. Xiao, P. Voss-de Haan, G. Jakob, and H. Adrian, *Phys. Rev. B* 57, R736 (1998).
- B. Kalisky, P. Aronov, G. Koren, A. Shaulov, Y. Yeshurun, and R.P. Huebener, *Phys. Rev. Lett.* 97, 067003 (2006).
- A. Leo, P. Marra, G. Grimaldi, R. Citro, S. Kawale, E. Bellingeri, C. Ferdeghini, S. Pace, and A. Nigro, *Phys. Rev.* B 93, 054503 (2016).
- G. Grimaldi, A. Leo, A. Nigro, S. Pace, V. Braccini, E. Bellingeri, and C. Ferdeghini, *Sci. Rep.* 8, 4150 (2018).
- 23. A. Leo et al., IEEE Trans. Appl. Supercond. 27, 730045 (2017).
- 24. A.I. Bezuglyj and V.A. Shklovskij, *Physica C* 202, 234 (1992).
- S.G. Doettinger, R.P. Huebener, and A. Kuhle, *Physica C* 251, 285 (1995).
- G. Grimaldi, A. Leo, C. Cirillo, A. Casaburi, R. Cristiano, C. Attanasio, A. Nigro, S. Pace, and R.P. Huebener, *J. Supercond. Nov. Magn.* 24, 81 (2011).
- G. Grimaldi, A. Leo, A. Nigro, S. Pace, C. Cirillo, and C. Attanasio, *Physica C* 468, 765 (2008).

- O.V. Dobrovolskiy, V.A. Shklovskij, M. Hanefeld, M. Zorb, L. Kohs, and M. Huth, *Supercond. Sci. Technol.* 30, 085002 (2017).
- V. Rouco, D. Massarotti, D. Stornaiuolo, G.P. Papari, X. Obradors, T. Puig, F. Tafuri, and A. Palau, *Materials* 11, 211 (2018).
- C. Cirillo, G. Carapella, M. Salvato, R. Arpaia, M. Caputo, and C. Attanasio, *Phys. Rev. B* 94, 104512 (2016).
- 31. V.A. Shklovskij, *Physica C* 538, 20 (2017).
- 32. G. Grimaldi, A. Leo, D. Zola, A. Nigro, S. Pace, F. Laviano, and E. Mezzetti, *Phys. Rev. B* 82, 024512 (2010).
- G. Grimaldi, A. Leo, A. Nigro, A.V. Silhanek, N. Verellen, V.V. Moshchalkov, M.V. Milosevic, A. Casaburi, R. Cristiano, and S. Pace, *Appl. Phys. Lett.* **100**, 202601 (2012).
- C. Peroz, C. Villard, A. Sulpice, and P. Butaud, *Physica C* 369, 222 (2002).
- A. Leo, F. Avitabile, N. Martucciello, J.C. Villegier, S. Pace, A. Nigro, and G. Grimaldi, *IEEE Trans. Appl. Supercond.* 28, 2200404 (2018).
- A.V. Silhanek, A. Leo, G. Grimaldi, G.R. Berdiyorov, M.V. Milošević, A. Nigro, S. Pace, N. Verellen, W. Gillijns, and V. Metlushko, *New J. Phys.* 14, 053006 (2012).
- A. Leo, G. Grimaldi, A. Nigro, E. Bruno, F. Priolo, and S. Pace, *Physica C* 503, 140 (2014).
- G. Grimaldi, A. Leo, A. Nigro, E. Bruno, F. Priolo, and S. Pace, *IEEE Trans. Appl. Supercond.* 23, 8200704 (2013).
- G. Grimaldi, A. Leo, P. Sabatino, G. Carapella, A. Nigro, S. Pace, V.V. Moshchalkov, and A.V. Silhanek, *Phys. Rev. B* 92, 024513 (2015).
- V.A. Shklovskij, A.P. Nazipova, and O.V. Dobrovolskiy, *Phys. Rev. B* 95, 184517 (2017).
- 41. O.V. Dobrovolskiy, V.M. Bevz, E. Begun, R. Sachser, R.V. Vovk, and M. Huth, *Phys. Rev. Appl.* **11**, 054064 (2019).
- A.I. Bezuglyj, V.A. Shklovskij, R.V. Vovk, V.M. Bevz, M. Huth, and O.V. Dobrovolskiy, *Phys. Rev. B* 99, 174518 (2019).

Критичні явища руху вихорів в надпровідниках: вихрова нестійкість та пінінг потоку

A. Leo, A. Nigro, G. Grimaldi

Вивчено динаміку вихорів у надпровідних матеріалах при дуже високих швидкостях вихорів у залежності від прикладеного магнітного поля. Високошвидкісна вихрова динаміка може стати критичною, внаслідок чого виникне нестабільність, яка веде до раптового зриву системи в нормальний стан. Наявність різних механізмів пінінгу у всіх надпровідниках не тільки здатна сприяти виникненню високих критичних струмів, але й може сильно вплинути на вихровий потік, визначаючи тим самим іншу поведінку критичної швидкості вихору v^{*}. Залежність v^{*} від магнітного поля надзвичайно чутлива до типу пінінга в матеріалі. Це важливо з прикладної точки зору, оскільки вихровий рух, що наближається до критичного значення v*, означає такий стан дисипативного потоку, який, певно, закінчиться нестабільністю. Якщо можливо передбачити ці критичні параметри, то буде легше контролювати ці критичні явища. Незважаючи на те, що повна теоретична модель для опису нестійкості потоку в присутності пінінгу досі відсутня, нещодавно був запропонований феноменологічний підхід опису для нестійкості вихрового потоку з «гарячими» електронами. Наведено вдалий приклад такої ідеальної відповідності між експериментом та теоретичним підходом для тонких плівок Mo₃Ge з й без геометричних бар'єрів з пінінгом.

Ключові слова: надпровідні плівки, критичний струм, пінінг потоку, нестійкість потоку.

Критические явления движения вихрей в сверхпроводниках: вихревая неустойчивость и пиннинг потока

A. Leo, A. Nigro, G. Grimaldi

Изучена динамика вихрей в сверхпроводящих материалах при очень высоких скоростях вихрей в зависимости от приложенного магнитного поля. Высокоскоростная вихревая динамика может стать критической, вследствие чего возникнет нестабильность, приводящая к внезапному срыву системы в нормальное состояние. Наличие различных механизмов пиннинга во всех сверхпроводниках не только способствует возникновению высоких критических токов, но и может сильно повлиять на вихревой поток, определяя тем самым другое поведение критической скорости вихря v*. Зависимость v* от магнитного поля чрезвычайно чувствительна к типу пиннинга в материале. Это важно с прикладной точки зрения, поскольку вихревое движение, приближающееся к критической v*, означает такое состояние диссипативного потока, которое, по-видимому, закончится нестабильностью. Если возможно предсказать эти критические параметры, то тогда будет легче контролировать эти критические явления. Несмотря на то, что полная теоретическая модель для описания неустойчивости потока в присутствии пиннинга до сих пор отсутствует, недавно был предложен феноменологический подход описания для неустойчивости вихревого потока с «горячими» электронами. Представлен удачный пример такого идеального соответствия между экспериментом и теоретическим подходом для тонких пленок Mo3Ge с и без геометрических барьеров с пиннингом.

Ключевые слова: сверхпроводящие пленки, критический ток, пиннинг потока, неустойчивость потока.