# Vortex generation in a superfluid gas of dipolar chains in crossed electric and magnetic fields

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Crossed electric and magnetic fields influence dipolar neutral particles in the same way as the magnetic field influences charged particles. The effect of crossed fields is proportional to the dipole moment of the particle (inherent or induced). We show that this effect is quite spectacular in a multilayer system of polar molecules. In this system molecules may bind in chains. At low temperature the gas of chains becomes the superfluid one. The crossed fields then induce vortices in the superfluid gas of chains. The density of vortices is proportional to the number of particles in the chain. The effect can be used for monitoring the formation and destruction of chains in multilayer dipolar gases.

Keywords: dipolar gases, multilayer systems, crossed fields, quantum vortices.

Neutral particles in crossed electric and magnetic fields behave as if they are charged and subjected to an effective magnetic field. The effective magnetic field can be expressed through an effective vector potential which is proportional to the vector product of the magnetic field **B** and the dipole moment **d** of the particle. The direction of the dipole moment can be fixed by the external electric field. To produce nonzero effective vector potential the electric and magnetic fields should be noncollinear.

The effective vector potential applied to a neutral superfluid can induce a superfluid current [\[1,](#page-3-0)[2\]](#page-3-1). At least one the fields (the electric or magnetic one) should be nonuniform. Otherwise, the effective vector potential can be eliminated by the gauge transformation. The effective vector potential and the effective magnetic field applied to a dipole particle can be described in terms of the Aharonov–Bohm phase [\[3,](#page-3-2)[4\]](#page-3-3). Two spatially separated positive and negative charges of a dipole particle feel slightly different vector potentials and acquire slightly different in module and opposite in sign phases under the motion of the dipole particle. The overall phase is equal to the flux of the magnetic field through the area covered by the vector  $\mathbf{r}_d = \mathbf{d}/e$  during its motion. One can introduce an effective vector potential and express the Aharonov–Bohm phase through the integral of this potential. The effective magnetic field is defined as the curl of the effective vector potential.

Generation of vortices in electrically neutral superfluids by a nonuniform magnetic field was considered in [\[5,](#page-3-4)[6\]](#page-3-5) (see also [\[7\]](#page-4-0) for a review) with reference to a superfluid gas of electron–hole pairs in a bilayer system. The bilayer system consists of two conducting layers with carriers of opposite signs separated by a dielectric layer. The dipole moment of an electron–hole pair in a bilayer is proportional to the distance between the layers and it can reach the value up to  $10<sup>3</sup>$  Debyes. Therefore moderate magnetic fields are required to generate quantum vortices. Generation of vortices in bilayers in a nonuniform electric field and uniform magnetic field was considered in [\[8\]](#page-4-1). Due to large polarizability of electron–hole pairs vortices are generated already in moderate electric fields.

The typical value of the dipole moment of a polar molecule does not exceed 5 Debyes [\[9–](#page-4-2)[11\]](#page-4-3) and much larger magnetic fields are required to generate vortices. Dipolar gases placed into a multilayer trap demonstrate a tendency to form chains  $[12-14]$  $[12-14]$ . The phenomenon is connected with that the dipole-dipole interaction between molecules located at different layers is attractive at small distances. The dipole moment of a chain is proportional to the number of molecules in a chain and it is comparable in value to the dipole moment of the electron–hole pair in the bilayer.

It is expected that increasing of the dipole strength results in a phase transition from a molecular superfluid to

a dipole chain superfluid [\[12\]](#page-4-4), and then in a transition from the chain superfluid to a dipole Wigner crystal  $[15]$ . In a system of Fermi polar molecules in a multilayer trap a transition from the dipole chain superfluid to a dimerized superfluid may take place  $[16]$ . The vortices generated by the crossed fields should disappear or their density should reduce considerably under the transitions from a chain superfluid to a molecular or dimerized superfluid. Vortices also disappear in the Wigner crystal state.

Let us derive the effective vector potential and the effective magnetic field using the conception of the Aharonov–Bohm phase. The motion of a dipole particle along a closed loop results in generation of two Aharonov–Bohm phases, one is for the positive charge and the other, for the negative one. The sum of two phases is

$$
\varphi_{A-B,d} = \frac{e}{\hbar c} \left( \oint\limits_{C_+} \mathbf{A} d\mathbf{r} - \oint\limits_{C_-} \mathbf{A} d\mathbf{r} \right) = \frac{e}{\hbar c} \int\limits_{\Delta S} \mathbf{B} d\mathbf{S}, \qquad (1)
$$

where  $C_+$  and  $C_-$  are the paths of the positive and negative charges, correspondingly, and ∆*S* is the area covered by the vector  $\mathbf{r}_d$  under such a motion. In a typical situation a variation of the magnetic field and the dipole moment is small at the scale of  $r_d$ . Then the phase (1) can be expressed through the integral of the effective vector potential  $A_{\text{eff}}$  along the center of mass path *C*:

$$
\varphi_{A-B,d} = \frac{e}{\hbar c} \oint_C \mathbf{A}_{eff} \cdot d\mathbf{R},\tag{2}
$$

where

$$
\mathbf{A}_{\text{eff}} = \frac{\mathbf{B} \times \mathbf{d}}{e}.
$$
 (3)

Introducing the effective magnetic field

$$
\mathbf{B}_{\rm eff} = \nabla \times \mathbf{A}_{\rm eff} \tag{4}
$$

one can rewrite the phase (2) through the integral over the area surrounded by the contour *C*:

$$
\varphi_{A-B,d} = \frac{e}{\hbar c} \int_{S_C} \mathbf{B}_{\text{eff}} \cdot d\mathbf{S}.
$$
 (5)

Taking into account that the dipole moment of the particle or its direction may depend on the coordinate we obtain the following expression for the effective magnetic field:

$$
\mathbf{B}_{\rm eff} = \frac{1}{e} [\mathbf{B}(\nabla \cdot \mathbf{d}) + (\mathbf{d} \cdot \nabla)\mathbf{B} - (\mathbf{B} \cdot \nabla)\mathbf{d}].
$$
 (6)

The effective field (6) influences neutral dipolar particles in the same way as the real magnetic field influences charged particles with the charge +*e*.

There is an essential difference between the Aharonov– Bohm effect for a charged particle and for a dipole particle. For a charged particle the Aharonov–Bohm phase can be nonzero even if the particle moves in the space where the magnetic field is zero. In contrast the real magnetic field should be nonzero along the path *C* to produce the Aharonov–Bohm phase (5).

If the dipole moment does not depend on **R** the component of the effective field parallel to **d** can be expressed though the two-dimensional divergence of the magnetic field:

$$
B_{z,\text{eff}} = \frac{d}{e} \partial_z B_z = -\frac{d}{e} \left( \partial_x B_x + \partial_y B_y \right). \tag{7}
$$

Here the *z* axis is directed along **d**.

The superfluid chain phase emerges in a strong electric field directed perpendicular to a stack of two-dimensional (2D) traps. The dipole moment of a chain is directed along the electric field and does not depend on the coordinate. To produce the effective magnetic field the real magnetic field should have nonzero component in the plane parallel to 2D traps and be nonuniform.

Here we consider two configurations of the magnetic field. The first one emerges at the face end of a solenoid. The second is generated by a flat coil. The magnetic field of a solenoid is circularly symmetric. Near the face end it has nonzero radial component. The projection of the magnetic field to the plane of the face end of a long solenoid is equal to  $\mathbf{B}_{\text{pl}} \approx B_s \mathbf{r} / 4R_s$ , where  $B_s$  is the magnetic field deep inside the solenoid,  $R<sub>s</sub>$  is the solenoid radius, and *r* is counted from the solenoid axis (the inequality  $r < R<sub>s</sub>$  is implied). The normal to the face end plane component of the effective magnetic field is uniform:

$$
B_{z,\text{eff}} = -\frac{d}{2e} \frac{B_s}{R_s}.
$$
 (8)

A flat coil induces the radial magnetic field in the coil plane. For the distance *a* from the coil much smaller than the coil radius  $R_{\text{coil}}$ , and for  $a < r < R_{\text{coil}}$  (*r* is counted from the coil axis) the magnetic field can be approximated as  $\mathbf{B}_{\text{pl}} \approx H_{\text{coil}} \mathbf{r} / r$ , where  $H_{\text{coil}} = 2\pi I n_{\text{coil}}$ , *I* is the electrical current in the coil and  $n_{\text{coil}}$  is the density of turns of the coil. The effective magnetic field normal to the coil plane is nonuniform:

$$
B_{z,\text{eff}}(r) = -\frac{d}{e} \frac{H_{\text{coil}}}{r}.
$$
 (9)

The crossed fields generate vortices if the effective flux  $\Phi_{\text{eff}}$  (the flux of the effective magnetic field) through the Bose cloud exceeds the critical value Φ*<sup>c</sup>* . This critical value depends on the particle density profile in the trap and the dependence of  $B_{z \text{eff}}$  on coordinate. We specify the case of an axially symmetric multilayer harmonic trap centered at  $r = 0$  with layers parallel to the face end plane or the coil plane. Then

$$
\Phi_c = f \Phi_0 \left( \ln \frac{R_{TF}}{\xi_0} - \frac{1}{2} \right),\tag{10}
$$

where  $\Phi_0 = 2\pi \hbar c / e$  is the flux quantum,  $R_{TF}$  is the Tomas–Fermi radius of a Bose cloud,  $\xi_0$  is the vortex core radius ( $\xi_0 \ll R_{TF}$ ), and f is the numerical factor equal to  $f = 2$  for the case of the uniform effective field (8), and to  $f = 3/4$  for the field (9). The effective flux  $\Phi_{\text{eff}}$  can be expressed through the real magnetic field *B* at  $r = R_{TF}$ :

$$
\Phi_{\rm eff} = \frac{2\pi R_{TF} d}{e} B_{\rm pl}(R_{TF}) = \frac{dR_{TF}}{e\ell^2} \Phi_0, \tag{11}
$$

where  $\ell = \sqrt{\hbar c / eB_{\text{pl}}(R_{TF})}$  is the magnetic length. Taking  $B_{\text{pl}}(R_{TF}) = 0.1 \text{ T}$ ,  $R_{TF} = 500 \text{ }\mu\text{m}$  and  $d = 3.5 \text{ Debye}$  we obtain  $\Phi_{\text{eff}} \approx 6\Phi_0$ . It corresponds to a state with one or few vortices.

Let us now consider the conditions of emergence of a superfluid chain phase in a stack of 2D traps. We imply the same density of particles in each trap and equal distances *b* between next neighbour 2D traps. The electric field aligns the dipole moments normally to the 2D traps. The interaction between dipoles located in the same  $(n = 0)$ or different ( $n \neq 0$ ) traps is given by equation

$$
V_n(r) = \frac{d^2[r^2 - 2(nb)^2]}{(r^2 + (nb)^2)^{5/2}},
$$
\n(12)

where  $r$  is the 2D radius vector, and  $n$  is the distance between the traps in units of *b*. Since the dipole-dipole interaction is attractive for the molecules located in different layers not far from each other it may cause binding of molecules from different layers.

The interaction strength is characterized by the dimensionless parameter:

$$
U_0 = \frac{d^2 m}{\hbar^2 b},\tag{13}
$$

where *m* is the mass of the molecule. In two dimensions a particle in the potential  $\lambda V(x)$  that satisfies the conditions  $\int d^2 x V(x) = 0$  and  $V(\infty) = 0$  has a bound state at any  $\lambda$  [\[17\]](#page-4-8) (see, also [\[18\]](#page-4-9)). The potential (12) is of that form. Two polar molecules from the adjacent layers bind in a pair at any  $U_0$ , but at small  $U_0$  the binding energy is exponentially small:  $E_b \sim (\hbar^2 / mb^2) \exp{(-8/U_0^2)}$  [\[19\]](#page-4-10). At large  $U_0$ the formation of chains can be described analytically.

At large  $U_0$  one can use the harmonic approximation for the potential (12):

$$
V_n(r) \approx -\frac{2d^2}{\left(nb\right)^3} + \frac{6d^2r^2}{\left(nb\right)^5}.
$$
 (14)

The energy of the bound state of two particles in the potential (14) with  $n = 1$  is equal to

$$
E_{b,2} = -\frac{2d^2}{b^3} \left( 1 - \sqrt{\frac{6}{U_0}} \right). \tag{15}
$$

The energy (15) is the sum of the classical binding energy and the zero-point energy of quantum fluctuations. The binding energy per molecule is  $E_{b,2}$  / 2.

Considering a long chain  $N \gg 1$  and neglecting the edge effects we obtain the following classical binding energy for the dipole chain:

$$
E_{\text{cl},N} = -\frac{2Nd^2}{b^3} \sum_{n=1}^{\infty} \frac{1}{n^3} = -\frac{2Nd^2}{b^3} \zeta(3),\tag{16}
$$

where  $\zeta(s)$  is the zeta-function ( $\zeta(3) \approx 1.2$ ), and *N* is the number of molecules in the chain. The spectrum of low energy excitations of the chain contains two degenerate transverse modes with the energies

$$
\Omega(q) = \frac{2d^2}{b^3} \sqrt{\frac{12}{U_0}} \sqrt{\sum_{n=1}^{\infty} \frac{\sin^2\left(\frac{qnb}{2}\right)}{n^5}}.
$$
 (17)

The zero-point energy is  $E_{zp} = \sum_{q=0}^{2\pi/b} \Omega(q)$ . The sum  $E_{\text{cl},N} + E_{zp}$  yields the binding energy

$$
E_{b,N} \approx -\frac{2Nd^2}{b^3} \left( 1.2 - 0.92 \sqrt{\frac{6}{U_0}} \right). \tag{18}
$$

The binding energy should be negative. Therefore the harmonic approximation (14) is justified only at large  $U_0$  $(U_0 > 6)$ . For LiK molecules with  $d = 3.5$  Debye and  $m = 7.6 \cdot 10^{-23}$  g for the stack with  $b = 250$  nm the interaction strength parameter is equal to  $U_0 \approx 30$ .

One can see from (15) and (18) that for a long chain the binding energy per molecule  $E_{h,N}$  / *N* is more than in two times larger than of the same energy for a dimer  $(E_{h,2}/2)$ . The edge effect reduces the binding energy, and the binding energy per molecule decreases under decrease of *N*. Therefore it is energetically preferable for the molecules to bind in the longest chains (*N*-segment chains for the *N*-layer system).

At large density the chains overlap due to transverse vibrations. Overlapping may cause destruction of the chains. To evaluate the effect of vibration we calculate the average square transverse displacement of molecules in the chain:

$$
\langle \zeta^2 \rangle = 2 \sum_{q \neq 0} \frac{\hbar^2}{2mN\Omega(q)} \big( 1 + 2N_B[\Omega(q)] \big), \qquad (19)
$$

where  $N_B(\Omega) = (e^{\Omega/T} - 1)^{-1}$  is the Bose distribution function. Calculation of the integral over *q* in (19) yields

$$
\langle \zeta^2 \rangle \approx b^2 \left( \frac{1}{\pi \sqrt{12 \zeta(3) U_0}} \ln N + \frac{1}{12 \pi^2 U_0 \zeta(3)} \frac{T}{T_d} n_{\rm ch} b^2 \right),
$$
\n(20)

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where  $n_{ch}$  is the density of the chains,  $T_d = \hbar^2 n_{ch} / M$  is the temperature of degeneracy, and  $M = Nm$  is the mass of the chain. One can see that for reasonable *N* ( $N \le 100$ ),  $U_0 \gg 1$ ,  $T < T_d$  and  $n_{ch}b^2 \lesssim 1$  the average displacement satisfies inequality  $\langle \zeta^2 \rangle \ll b^2$ . Thus for the densities of chains of order of  $b^{-2}$  or smaller the condition  $n_{ch} \zeta^2 \ll 1$ is fulfilled and overlapping of the chains is small.

If polar molecules are bosons, the chains also satisfy the Bose statistics. The 2D gas of such chains goes into the superfluid state. The transition into the superfluid state is of the Berezinskii–Kosterlitz–Thouless type. The transition temperature  $T_c$  is given by equation

$$
T_c = \frac{\pi}{2} \frac{\hbar^2}{M} n_s(T_c),
$$
\n(21)

where  $n_{\rm s}(T)$  is the superfluid density at finite temperature.

For the contact interaction between particles the superfluid density is obtained from the equation

$$
\frac{n_{\rm ch} - n_s}{n_{\rm ch}} = \frac{3\zeta(3)}{2\pi} \left(\frac{T}{T_d}\right)^3 \left(\frac{\hbar^2}{M\gamma}\right)^2,\tag{22}
$$

where  $\gamma$  is the constant of the contact interaction. For the chains the constant  $\gamma$  is evaluated as  $\gamma \sim Nd^2/W$  [\[20\]](#page-4-11), where *W* is the width of the individual trap. Since  $\gamma \gg \hbar^2 / M$ , the difference between the superfluid density  $n_s$  and the total density  $n_{ch}$  is small at  $T < T_d$ . Thus, with a good accuracy the critical temperature is given by the expression  $T_c = \pi \hbar^2 n_{\text{ch}} / 2M$ .

Taking  $n_{ch} = b^{-2}$ ,  $b = 250$  nm and  $N = 100$ , we obtain the critical temperature  $T_c \approx 3 \text{ nK}$  for LiK molecules. For  $B_{\text{pl}}(R_{TF}) = 0.1$  T and  $R_{TF} = 500$  µm the effective flux reaches the value of  $\Phi_{\text{eff}} \approx 6.10^2 \Phi_0$ . It corresponds to a multivortex state with the average density of vortices  $n_v \approx 8.10^4$  cm<sup>-2</sup>. For smaller *N* the critical temperature is larger, but the vortex density is smaller. For instance, for  $N = 10$  we obtain  $T_c \approx 30$  nK and  $n_v \approx 8.10^3$  cm<sup>-2</sup>.

The vortex density is proportional to the dipole moment of the compound particle and independent of its mass. Therefore the transition from the superfluid gas of chains to *N* uncoupled superfluid 2D gases should be accompanied with a strong decrease in the vortex density. Such a transition can take place if one increases the distance between the traps in the stack or decreases the electric field that aligns dipoles along the *z* axis.

At low density the statistics of individual molecules is not important and Fermi molecules at even *N* binds into Bose chains as well. At larger densities Fermi systems may demonstrate some specific features. It was shown in [\[16\]](#page-4-7) that the ground state of a Fermi gas of dipolar particles in the multilayer system is a dimerized superfluid, with the Cooper pairing only between every other layer. The dimerized superfluid [\[16\]](#page-4-7) is a variant of the Bardin–Cooper– Schrieffer (BCS) state [\[21\]](#page-4-12). BCS state corresponds to the weak coupling limit. The weak coupling limit can be also

understood as the high density limit in a sence that the size of the Cooper pair is much larger than the average distance between the pairs. In the strong coupling limit the self-consistence equation for the BCS order parameter is reduced to the Schroedinger equation for the pair of particles [\[22\]](#page-4-13).

The situation in multilayers is more complicate. The BCS state is the state with only the two-body coupling. The 4-body, 6-body etc. coupling is out of the BCS approximation and superfluid state of chains cannot be described within the BCS approach. The pairing in multilayers is similar to one in many-particle barionic systems, where the transition from the BCS paired superfluid to a quartet Bose–Einstein condensation (BEC) occurs under decrease in density [\[23\]](#page-4-14). By analogy with [\[23\]](#page-4-14) in multilayer dipolar Fermi gases a transition from the dimerized superfluid to the chain superfluid is expected. In nonuniform magnetic field that induces vortices such a transition should be accompanied with a strong increase of the vortex density.

We would mention that multilayer ultracold polar gases are accessible now experimentally. In particular, in [\[24\]](#page-4-15) a multilayer ( $N > 20$ ) stack with polar <sup>40</sup>K<sup>87</sup>Rb molecules with the centre layer having more than 2000 molecules and a peak density of  $3.4 \cdot 10^7$  cm<sup>-2</sup> at  $T = 500$  nK was realized. A few-layer stack with polar  $^{23}Na^{40}K$  molecules at  $T = 300$  nK was realized in [\[25\]](#page-4-16). Also it was reported recently [\[26\]](#page-4-17) on a realization of a degenerate three-dimensional Fermi gas of <sup>40</sup>K<sup>87</sup>Rb molecules at  $T = 50$  nK with density  $n \approx 2.10^{12}$  cm<sup>-3</sup> and total number of molecules  $3 \cdot 10^4$ . Therefore one can hope that degenerate multilayer dipole gases will be obtained soon. Thus, current state of art in creating, cooling and trapping of dipole gases give expectations that systems with required parameters will be realized in the nearest future.

In conclusion, we have shown that crossed electric and magnetic fields can be considered as a tool for the study of the phase transitions in multilayer dipolar gases. In such systems subjected to the electric field that aligns the dipole moments perpendicular to the layers polar molecules bind in long chains. At low temperature at which the gas of chains becomes the superfluid one nonuniform magnetic field with nonzero two-dimensional divergence may generate quantum vortices in a gas of chains. The disappearance of the vortex pattern under variation of the parameters of the system can be a signature of the initial presence of chains which then undergo dissociation.

<span id="page-3-0"></span>1. S.I. Shevchenko, *JETP Lett.* **28**, 103 (1978).

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- <span id="page-3-1"></span>2. S.I. Shevchenko, *[Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.75.3312)* **75**, 3312 (1995).
- <span id="page-3-2"></span>3. M. Wilkens, *[Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.72.5)* **72**, 5 (1994).
- <span id="page-3-3"></span>4. H. Wei, R. Han, and X. Wei, *[Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.75.2071)* **75**, 2071 (1995).
- <span id="page-3-4"></span>5. S.I. Shevchenko, *[Phys. Rev. B](https://doi.org/10.1103/PhysRevB.56.10355)* **56**, 10355 (1997).
- <span id="page-3-5"></span>6. E.B. Sonin, *[Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.102.106407)* **102**, 106407 (2009).
- <span id="page-4-0"></span>7. D.V. Fil and S.I. Shevchenko, *Fiz. Nizk. Temp*. **44**, 1111 (2018) [*[Low. Temp. Phys](https://doi.org/10.1063/1.5052674)*. **44**, 867 (2018)].
- <span id="page-4-1"></span>8. S.I. Shevchenko, *Fiz. Nizk. Temp*. **39**, 411 (2013) [*[Low](https://doi.org/10.1063/1.4801987)  [Temp. Phys.](https://doi.org/10.1063/1.4801987)* **39**, 319 (2013).
- <span id="page-4-2"></span>9. T. Lahaye, C. Menotti, L. Santos, M. Lewenstein, and T. Pfau, *[Rep. Prog. Phys.](https://doi.org/10.1088/0034-4885/72/12/126401)* **72**, 12401 (2009).
- 10. M.A. Baranov, *[Phys. Rep.](https://doi.org/10.1016/j.physrep.2008.04.007)* **464**, 71 (2008).
- <span id="page-4-3"></span>11. B. Gadway and B. Yan, *[J. Phys.](https://doi.org/10.1088/0953-4075/49/15/152002) B* **49**, 152002 (2016).
- <span id="page-4-4"></span>12. D.W. Wang, M.D. Lukin, and E. Demler, *[Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.97.180413)* **97**, 180413 (2006).
- 13. B. Capogrosso-Sansone and A.B. Kuklov, *[J. Low Temp.](https://doi.org/10.1007/s10909-011-0386-5)  [Phys.](https://doi.org/10.1007/s10909-011-0386-5)* **165**, 213 (2011).
- <span id="page-4-5"></span>14. J.R. Armstrong, N.T. Zinner, D.V. Fedorov, and A.S. Jensen, *[Eur. Phys. J. D](https://doi.org/10.1140/epjd/e2012-20611-x)* **66**, 85 (2012).
- <span id="page-4-6"></span>15. J.-S. You and D.-W. Wang, *New [J. Phys.](https://doi.org/10.1088/1367-2630/16/7/073041)* **16**, 073041 (2014).
- <span id="page-4-7"></span>16. A.C. Potter, E. Berg, D.W. Wang, B.I. Halperin, and E. Demler, *[Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.105.220406)* **105**, 220406 (2010).
- <span id="page-4-8"></span>17. B. Simon, *[Ann. Phys.](https://doi.org/10.1016/0003-4916(76)90038-5)* (*N.Y.*) **97**, 279 (1976).
- <span id="page-4-9"></span>18. A. Pikovski, M. Klawunn, G.V. Shlyapnikov, and L. Santos, *[Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.105.215302)* **105**, 215302 (2010).
- <span id="page-4-10"></span>19. M. Klawunn, A. Pikovski, and L. Santos, *[Phys. Rev. A](https://doi.org/10.1103/PhysRevA.82.044701)* **82**, 044701 (2010).
- <span id="page-4-11"></span>20. N.T. Zinner, B. Wunsch, D. Pekker, and D.W. Wang, *[Phys.](https://doi.org/10.1103/PhysRevA.85.013603)  [Rev. A](https://doi.org/10.1103/PhysRevA.85.013603)* **85**, 013603 (2012).
- <span id="page-4-12"></span>21. J.R. Schrieffer, *Theory [of Superconductivity](https://doi.org/10.1007/BF%2000683774)*, Benjamin, New York (1964).
- <span id="page-4-13"></span>22. P. Nozieres and S. Schmitt-Rink, *[J. Low Temp. Phys.](https://doi.org/10.1007/BF%2000683774)* **59**, 195 (1985).
- <span id="page-4-14"></span>23. G. Ropke, A. Schnell, P. Schuck, and P. Nozieres, *[Phys. Rev.](https://doi.org/10.1103/PhysRevLett.80.3177)  [Lett.](https://doi.org/10.1103/PhysRevLett.80.3177)* **80**, 3177 (1998).
- <span id="page-4-15"></span>24. M.H.G. de Miranda, A. Chotia, B. Neyenhuis, D. Wang, G. Quéméner, S. Ospelkaus, J.L. Bohn, J. Ye, and D.S. Jin, *[Nature Phys](https://doi:10.1038/nphys1939)*. **7**, 502 (2011).
- <span id="page-4-16"></span>25. F. Seeßelberg, X.-Y. Luo, M. Li, R. Bause, S. Kotochigova, I. Bloch, and C. Gohle, *[Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.121.253401)* **121**, 253401 (2018).
- <span id="page-4-17"></span>26. L. De Marco, G. Valtolina, K. Matsuda, W.G. Tobias, J.P. Covey, and J. Ye, *[Science](https://doi.org/10.1126/science.aau7230)* **363**, 853 (2019).

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### Генерування вихорів у надплинному газі дипольних ланцюжків у схрещених електричному та магнітному полях

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Схрещені електричне та магнітне поля впливають на дипольні нейтральні частинки таким самим чином, як магнітне поле впливає на заряджені частинки. Дія схрещених полів пропорційна дипольному моменту частинки (власному або індукованому). Показано, що такий вплив достатньо ефективно проявиться у багатошаровій системі полярних молекул. У такій системі полярні молекули можуть зв'язуватися у ланцюжки. При низькій температурі газ ланцюжків стає надплинним, тоді схрещені поля індукують у ньому вихори. Щільність вихорів пропорційна числу молекул у ланцюжку. Ефект може бути застосований для спостереження за формуванням та розпадом ланцюжків у багатошарових дипольних газах.

Ключові слова: дипольні гази, багатошарові системи, схрещені поля, квантові вихори.

## Генерирование вихрей в сверхтекучем газе дипольных цепочек в скрещенных электрическом и магнитном полях

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Скрещенные электрическое и магнитное поля воздействуют на нейтральные дипольные частицы таким же образом, как магнитное поле воздействует на заряженные частицы. Воздействие скрещенных полей пропорционально дипольному моменту частицы (собственному либо индуцированному). Показано, что такое воздействие достаточно эффективно проявится в многослойных системах полярных молекул. В этих системах молекулы могут связываться в цепочки. При низкой температуре газ цепочек становится сверхтекучим, тогда скрещенные поля индуцируют в нем вихри. Плотность вихрей пропорциональна числу частиц в цепочке. Эффект может быть использован для наблюдения за формированием и распадом цепочек в многослойных дипольных газах.

Ключевые слова: дипольные газы, многослойные системы, скрещенные поля, квантовые вихри.