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Persistent currents in the two-chain correlated electron model

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For the recently proposed exactly solvable two-chain correlated electron model with the anisotropic spin-spin interaction between electrons and the spin-orbit interaction the ground state persistent currents are calculated. The model describes the quasi-one-dimensional type II superconductor. It is shown, that the spin-orbit coupling determines the initial phase of oscillations of charge and spin persistent currents, related to unbound electron states. On the other hand, Cooper-like singlet pairs define oscillations of only charge persistent currents with the period, characteristic to pairs. Depending on the value of the external magnetic field and the band filling, the system can reveal the complicated picture of the interference of several kinds of oscillations.

Keywords: integrable models, anisotropy, persistent currents.

1. Introduction

Persistent currents are caused by the Aharonov–Bohm (AB) phase shift. The AB effect [1] appears when charges move along a loop, pierced by a magnetic flux. The AB effect manifests the force-free topological influence of electromagnetic interaction on quantum dynamics of charged particles in a non-simply connected geometry. For any closed path around a solenoid the phase change is equal to $\phi_{AB} = 2\pi\Phi / \Phi_0$, where Φ is the magnetic flux within the solenoid, and $\Phi_0 = 2\pi\hbar c / e$, where c is the speed of light in vacuum, and e is the charge of a particle. In classics the AB effect does not occur for any value of the flux, because the motion of particles takes place in the region outside the solenoid, where electric and magnetic fields are zero. In condensed matter the Aharonov–Bohm effect manifests itself as magnetic oscillations of kinetic and thermodynamic characteristics of samples in extremely weak magnetic fields, when field-induced forces can be disregarded. Aharonov–Bohm oscillations can serve as a testing ground for investigations of electron-electron correlations in conducting loops via the period, the initial phase shift and the magnitude of oscillations. For example, for superconductors the Aharonov–Bohm effect manifests oscillations with the period twice smaller than the one for a normal metal, describing that way the doubled charge of the Cooper pair, with respect to the charge of an electron. An external magnetic flux produces nonzero momentum of charged particles. The persistent current is the derivative of the energy of a system in equilibrium with respect to the applied magnetic flux: $J(\Phi) = -c(\partial\mathcal{F}(\Phi) / \partial\Phi$, where $\mathcal{F}$ is the Helmholtz free energy of the total ring; in the ground state it reduces to the derivative of the ground state energy E_0. The Aharonov–Casher (AC) effect [2] is dual to the AB one. It is related to the movement of a particle with a magnetic moment around an electric flux, e.g., the electric flux $F = 4\pi\tau$ generated by a string passing through the center of a ring with linear charge density τ. It produces the phase shift of a particle with spin $\phi_{AC} = 2\pi F / F_0$, where $F_0 = 2\pi\hbar c / \mu_e$ is the unit electric flux, μ_e is effective magneton related to the (quasi)particle. Then the spin persistent current is $J_s(F) = -c\partial\mathcal{F}(F) / \partial F$. Electro-magnetic fluxes Φ and F can be included in the Hamiltonian via the Peierls factors $\Phi_\uparrow = \pi[(\Phi / \Phi_0) + (F / F_0)]$ and $\Phi_\downarrow = \pi[(\Phi / \Phi_0) - (F / F_0)]$ for electrons with spins up and down, respectively. Then a gauge transformation can move those phase shifts into twisted boundary conditions. Obviously, spin and charge persistent current can serve as a useful tool to measure the charge and spin of elementary excitations of the studied system.

Unlike higher-dimensional correlated electron models, their low-dimensional quantum counterparts permit to obtain exact (non-perturbative) results [3]. The necessity of

non-perturbative studies in one-dimensional quantum systems is caused by the enhanced quantum and thermal fluctuations there, due to the features in the density of states [4]. It is known that the one-dimensional quantum models can describe not only real one-dimensional correlated electron systems, but also topological superconductors [5], and ultracold atoms in one-dimensional optical traps [6]. Low-dimensional quantum correlated electron models often reveal properties of quantum spin liquids. In the latter the order is suppressed down to the lowest temperatures due to the frustration of spin-spin interactions and/or enhanced quantum fluctuations in low-dimensional systems [7]. In many quantum spin liquids emergent magnetic excitations are fermions (as a rule they carry fractionalized charge or spin), instead of, e.g., magnons (bosons, which carry spin 1) for ordered magnetic systems. Namely because of that the investigation of the periodicity of AB and AC oscillations can provide information, whether emergent excitations carry fractional charges or spins.

In this work, we propose to study persistent currents in the recently proposed exactly solvable correlated electron model, which describes electrons living in two coupled chains, which interact at nearest and next nearest neighboring sites via the anisotropic exchange interaction and possessing the spin-orbit interaction (SOI) [8]. The model reveals the properties reminiscent of the type II superconductor, because its low-energy excitations are Cooper-like spin singlet pairs and unbound electron excitations [9]. Persistent charge [10] and spin [11] currents were first studied in exactly solvable correlated electron models like the Hubbard chain with the attraction of electrons. For the system with local pairs (the Hubbard model with the attraction between electrons sitting at the same site) the features of persistent currents were studied in Ref. 12. The goal of the work is to understand how the magnitudes and periods of the oscillations of charge and spin persistent currents can reveal the features of the ground state structure of the two-chain correlated electron system with superconducting correlations caused by the anisotropic spin-spin interaction of electrons on neighboring sites. Using the opportunity, we describe how the Hamiltonian and the exact solution of the stationary Schrödinger equation was obtained for the model [8].

2. The Hamiltonian and the exact integrability

The Hamiltonian of the considered model can be written as [8]

$$H = t\sum_j \left\{ H_{j,j+2} + \frac{\sin\mathrm{h}^2(\eta)}{\sin^2(\theta)+\sin\mathrm{h}^2(\eta)} \times \right.$$
$$\times \left[-H_{j,j+2} + \hat{B}\left(H_{j,j+1} + H_{j+1,j+2}\right) - i\frac{\tan(\theta)}{\sin\mathrm{h}(\eta)} \times \right.$$
$$\left. \left. \times \left(H_{j,j+1}, H_{j+1,j+2}\right) \right] \right\}, \qquad (1)$$

where [.,.] denotes the commutator, and the operator $\hat{B}$ modifies the hopping and the transverse interaction amplitudes by the factor $\cos(\theta)$. Here we denote

$$H_{j,j+1} = -\sum_\sigma P_j \left(a^\dagger_{j+1,\sigma} a_{j,\sigma} + \mathrm{H.c.}\right) P_j +$$
$$+\left(a^\dagger_{j,\downarrow} a_{j+1,\uparrow} a^\dagger_{j+1,\uparrow} a_{j,\downarrow} + a^\dagger_{j,\uparrow} a_{j+1,\downarrow} a^\dagger_{j+1,\downarrow} a_{j,\uparrow}\right) -$$
$$-\left(\mathrm{e}^{\eta} n_{j,\uparrow} n_{j+1,\downarrow} + \mathrm{e}^{-\eta} n_{j,\downarrow} n_{j+1,\uparrow}\right). \qquad (2)$$

In this expression $a^\dagger_{j,\sigma}$ $(a_{j,\sigma})$ creates (destroys) the electron with the spin $\sigma = \uparrow, \downarrow$ at the site j, $n_{j,\sigma} = a^\dagger_{j,\sigma} a_{j,\sigma}$, $P_j = (1-n_{j,-\sigma})(1-n_{j+1,-\sigma})$ projects out states with the double occupation of each site, the parameter η defines the anisotropy of the interactions, and the parameter θ defines the inter-chain coupling. We can also include the spin-orbit interaction, which distinguishes the same axis, as the anisotropy of the exchange interaction. For that we change $t \to t'$, where $t' = \sqrt{t^2 + g^2/4}\exp(i2\pi\sigma\phi)$, (let us denote $t'' = \sqrt{t^2 + g^2/4}$), and the phase factor $\phi = (1/\pi)\arctan(g/2t)$ is caused by the spin orbit interaction, SOI (with the SOI coefficient g) [8]. That phase factor can be transferred to the twisted boundary conditions [8]. We see that the Hamiltonian contains three groups of terms. The first group describes coupling and hopping between nearest neighbors (NN). The second group defines the coupling and the hopping between the next nearest neighbors (NNN). Obviously the model of the chain with interactions and hoppings for NN and NNN is totally equivalent to the two-chain zigzag model. Finally, there are three-site terms. The analysis of the effect of the latter is given in [8]. The presence of three-site terms is the necessary condition for the exact integrability of the model, see below.

2.1. Algebraic Bethe Ansatz and exact integrability

The method to solve the Schrödinger equation for the considered in [8] model and related class of models was pioneered by H. Bethe [13]. It is known as the Bethe Ansatz. It can be applied to models, in which the scattering between (quasi)particles is only elastic and non-dispersive. The condition of elasticity was formulated in the form of the well-known Yang–Baxter equations (YBE). Consider, e.g., several interacting (quasi)particles. Then the YBE for their two-particle scattering matrices can be written in the form [14]

$$\hat{S}_{12}(u_1,u_2)\hat{S}_{13}(u_1,u_3)\hat{S}_{23}(u_2,u_3) =$$
$$= \hat{S}_{23}(u_2,u_3)\hat{S}_{13}(u_1,u_3)\hat{S}_{12}(u_1,u_2), \qquad (3)$$

where indexes $j = 1,2,3$ numerate (quasi) particles with the quantum numbers (called rapidities) u_j, which parametrize

eigenfunctions and eigenvalues of the Schrödinger equation for the considered model. According to the YBE each scattering process produces only a phase shift (no reflection), and the order of scatterings is not essential. As a result of the YBE fulfillment, any multi-scattering process can be considered as the product of two-scattering processes. Rapidities satisfy the so-called Bethe Ansatz equations (BAE) [13]. The algebraic version of the Bethe Ansatz is known as the quantum inverse scattering method (QISM) [15,16]. This method is based on the mapping of a one-dimensional quantum problem onto the associated two-dimensional statistical problem. Within the QISM, for a large class of integrable models the R-matrices (their construction is related to one of the two-particle scattering matrices) satisfy the YBE

$$R_{12}(u-v)R_{13}(u)R_{23}(v) = R_{23}(v)R_{13}(u)R_{12}(u-v)\,, \quad (4)$$

where the indexes denote the Hilbert spaces in which the R matrix acts, and $u(v)$ is the spectral parameter. Define an L-operator acting on the tensor product between the "matrix-space" V_0 (it is often called the auxiliary subspace) and the quantum space V_n, which is identified with the Hilbert space over the nth site of the lattice. The following YBE for L-operators (they are often called intertwining relations) hold:

$$R(u-v)\big(L_n(u)\otimes L_n(v)\big) = \big(L_n(v)\otimes L_n(u)\big)R(u-v)\,, \quad (5)$$

where the tensor product is between quantum spaces, i.e., these YBE are nontrivial over the space $V_0\otimes V_0\otimes V_n$. The intertwining relations determine the structure of L-operators, if one already knows R-matrices. Then we can introduce the monodromy matrix acting in the space $V_0\otimes V_1\otimes\cdots\otimes V_L$ as

$$T^{\sigma_1',\ldots,\sigma_L',\tau'}_{\sigma_1,\ldots,\sigma_L,\tau}(u) = L_1(u)\cdots L_L(u) =$$

$$= S^{\tau,\gamma_1}_{\sigma_1,\sigma_1'}(u)\cdots S^{\gamma_{j-1}\gamma_j}_{\sigma_j\sigma_j'}(u)\cdots S^{\gamma_{L-1},\tau'}_{\sigma_L,\sigma_L'}(u)\,, \quad (6)$$

where L is the length of the chain. Due to the definition and intertwining relations for L-operators the monodromy matrices also satisfy YBE

$$R(u-v)\big(T(u)\otimes T(v)\big) = \big(T(v)\otimes T(u)\big)R(u-v)\,. \quad (7)$$

Tracing the monodromy matrix over the auxiliary space V_0 we obtain the transfer matrix

$$\hat{\tau}^{\sigma_1',\ldots,\sigma_L'}_{\sigma_1,\ldots,\sigma_L}(u) = \mathrm{Tr}\,L_1(\lambda)\cdots L_L(u) =$$

$$S^{\tau,\gamma_1}_{\sigma_1,\sigma_1'}(u)\cdots S^{\gamma_{j-1}\gamma_j}_{\sigma_j\sigma_j'}(\lambda)\cdots S^{\gamma_{L-1},\tau}_{\sigma_L,\sigma_L'}(u)\,. \quad (8)$$

From Eqs. (7) and (8) it follows that $[\hat{\tau}(u),\hat{\tau}(v)] = 0$, i.e., transfer matrices with different spectral parameters commute. This property is the fundamental property, which means the exact integrability of a system: There exist infinitely many (for a system with infinitely many degrees of freedom) integrals of motion, which commute mutually, and, hence, have the common set of eigenfunctions. Then one can construct any function of the transfer matrix, and due to the commutation of transfer matrices with different spectral parameters, all such functions commute mutually, and with the transfer matrix. The following series is used for the determination of integrals of motion in Bethe Ansatz-solvable models [17] (the Hamiltonian from this series was first introduced in [18])

$$\hat{Q}_n(u) = A_n\frac{\partial^{n-1}\ln\hat{\tau}(u)}{\partial u^{n-1}}\Big|_{u=0}\,, \quad (9)$$

where finally, after taking derivatives, the spectral parameter is taken to be equal to its value, at which the R-matrix is unity (or the two-particle scattering matrix is the permutation operator), and A_n are constants. This series is chosen because of the locality property: The integral of motion $\hat{Q}_n$ acts nontrivially only on n sites of a chain.

We summarize the idea of the QISM: Starting with the solution of the YBE for R-matrices, then using that solution for the construction of L-operators and monodromy operators of a Bethe Ansatz-integrable system, we can obtain the trace of the monodromy operator over the auxiliary subspace, the transfer matrix. As the consequence of the YBE for L-operators and monodromies (intertwining relations), transfer matrices with different spectral parameters commute, which constitutes the exact integrability of the model. Finally, the integrals of motion (including the operator of the energy, the Hamiltonian) can be constructed from the expression for the transfer matrix.

2.2. Integrable two-chain models

Consider now the two-chain $\tilde{L}$-operator for the site j of the form [19]

$$\tilde{L}_j(u) = L_{\sigma_{1,a},\sigma_{2,j}}(u+\theta)\otimes L_{\sigma_{1,a},\sigma_{1,j}}(u)\otimes$$

$$\otimes L_{\sigma_{2,a},\sigma_{2,j}}(u)\otimes L_{\sigma_{2,a},\sigma_{1,j}}(u-\theta)\,, \quad (10)$$

where the index j denotes the quantum space, and a denotes the auxiliary space. It is the product of four L-operators with the structure of L-operators of a single Bethe Ansatz-solvable chain, each of which satisfies the YBE. The parameter θ determines the strength of coupling between the chains. By construction $\tilde{L}$-operators also satisfy the YBE. The structure of $\tilde{L}$-operator is very clear. There are two chains, hence, we have to consider two quantum spaces and two auxiliary spaces (one set for each chain). The $\tilde{R}$-operator for the two-chain problem can be constructed similarly to a single chain case. The normalization for $\tilde{R}$-matrices implies $\tilde{R}(u)\hat{P}\tilde{R}(-u)\hat{P} = 1$. The monodromy for this two-chain construction is the ordered product of L $\tilde{L}$-operators

$$\tilde{T}_L(u,\theta) = \tilde{L}_L(u,\theta)\tilde{L}_{L-1}(u,\theta)\cdots\tilde{L}_1(u,\theta)\,. \quad (11)$$

The monodromy shows the result of scatterings of two auxiliary particles off two sets of physical particles in each of chains. Such a monodromy also satisfies the YBE by construction. The transfer matrix $\tilde{\tau}(u,\theta) = \mathrm{Tr}[\tilde{T}_L(u,\theta)]$ is

the trace over two auxiliary spaces of the monodromy. The associated two-dimensional statistical vertex problem is, naturally, the two-layered one. Defined that way transfer matrices with different u and θ commute (with θ and u being fixed, respectively). Hence $\tilde{\tau}(u,\theta)$ can be used as a generating functional for an infinite number of conservation laws. Hence, the problem is also integrable. It is interesting to point out [19] that the transfer matrix of the two-chain problem can be written as $\tilde{\tau}(u,\theta) = \tau(u) \otimes \tau(u+\theta)$. The Hamiltonian for the two-chain problem can also be determined as the logarithmic derivative of the transfer matrix $\tilde{\tau}(u,\theta)$ taken at $u = 0$.

2.3. Supersymmetric electron model with anisotropic interactions

To study the correlated supersymmetric electron model within the QISM it is convenient to use the graded version of the QISM [20]. Consider the graded linear space $V^{(n|m)} = V^{(n)} \oplus V^{(m)}$, where n and m denote the dimensions of the parts of this space and $\oplus$ denotes the direct sum. Let $\{e_1,\ldots,e_{n+m}\}$ be a basis of $V^{(n|m)}$, such that $\{e_1,\ldots,e_n\}$ is a basis of $V^{(n)}$ and $\{e_{n+1},\ldots,e_{n+m}\}$ is a basis of $V^{(m)}$. The Grassmann parities of the basis vectors can be given by $\epsilon_1 = \cdots = \epsilon_n = 0$ and $\epsilon_{n+1} = \cdots = \epsilon_{n+m} = 1$. Any linear operator on $V^{(n|m)}$ can be represented in a block form as

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \varepsilon \begin{pmatrix} A & 0 \\ 0 & D \end{pmatrix} = 0, \varepsilon \begin{pmatrix} 0 & B \\ C & 0 \end{pmatrix} = 1, \quad (12)$$

and the supertrace (the graded trace) of this matrix is defined as

$$\mathrm{str}\, M = \mathrm{tr}\, A - \mathrm{tr}\, D, \quad (13)$$

where the traces on the right hand side are the usual operator traces in $V^{(n)}$ and $V^{(m)}$, respectively. The graded tensor product $V^{(n|m)} \otimes V^{(n|m)}$ in terms of its basis vectors $\{e_a \otimes e_b\}$ (where $a,b = 1,\ldots,m+n$) can be defined as

$$v \otimes w = (e_a v_a) \otimes (e_b w_b) = (e_a \otimes e_b) v_a w_b (-1)^{\epsilon_{v_a}\epsilon_b}, \quad (14)$$

i.e., the additional factor $(-1)^{\epsilon_{v_a}\epsilon_b}$ occurs comparing to the standard tensor product. This factor originates from passing v_a past e_b. The action of the right linear operator $F \otimes G$ on the vector $v \otimes w$ in $V^{(n|m)} \otimes V^{(n|m)}$ has the form $(F \otimes G)(v \otimes w) = F(v) \otimes G(w)$ with its matrix elements

$$\left(F \otimes G\right)_{cd}^{ab} = F_{ab} G_{cd} (-1)^{\epsilon_c(\epsilon_a+\epsilon_b)}. \quad (15)$$

The unity operator in $V^{(n|m)} \otimes V^{(n|m)}$ is $\hat{I}_{cd}^{ab} = \delta_{a,b}\delta_{c,d}$ and the permutation operator is $\hat{P}_{cd}^{ab} = \delta_{a,d}\delta_{c,b}(-1)^{\epsilon_b\epsilon_d}$. One can show that the operator $R(u) = b(u)\hat{I} + c(u)\hat{P}$, with $c(u) = u/(u+i\eta)$ and $b(u) = i\eta/(u+i\eta)$ satisfies the YBE for R-matrices Eq. (4) acting in $V^{(n|m)} \otimes V^{(n|m)}$. Grading is used when one considers a system of n species of bosons and m species of fermions. In such a case $V_j^{(n|m)}$ denotes the quantum Hilbert space of configurations at every site of the lattice. For the supersymmetric $t-J$ model we have one boson (an empty state) and two fermions (electrons with spins directed upward and downward) at each site. Then the quantum space for each site for such a model can be considered as $V_j^{(1|2)}$. For such a model nine operators at each site j exists. The first of these operators is the unity operator $J_j^1 = I_j$, and the second one is related to the operator of the number of electrons in site j, $n_j = n_{j\uparrow} + n_{j\downarrow}$, as $J_j^2 = N_j \equiv 1-(1/2)n_j$. Three other operators, $J_j^3 = S_j^z \equiv \equiv (1/2)(n_{j\uparrow} - n_{j\downarrow})$, $J_j^4 = S_j^+ \equiv a_{j\uparrow}^\dagger a_{j\downarrow}$, and $J_j^5 = S_j^- \equiv \equiv a_{j\downarrow}^\dagger a_{j\uparrow}$, form the SU(2) sub-algebra. Here $a_{j\sigma}^\dagger (a_{j\sigma})$ creates (destroys) the electron with spin $\sigma = \uparrow,\downarrow$ at the site j. Four more operators are $J_j^6 = Q_{j\uparrow} \equiv (1-n_{j\downarrow})a_{j\uparrow}^\dagger$, $J_j^7 = Q_{j\downarrow} \equiv (1-n_{j\uparrow})a_{j\downarrow}^\dagger$, $J_j^8 = Q_{j\uparrow}^\dagger \equiv (1-n_{j\downarrow})a_{j\uparrow}$ and $J_j^9 = Q_{j\downarrow}^\dagger \equiv (1-n_{j\uparrow})a_{j\downarrow}$. These operators are the generators of the algebra gl(1|2) which can be written in the form

$$J_j^\alpha J_j^\beta - (-1)^{\epsilon_\alpha\epsilon_\beta} J_j^\beta J_j^\alpha = f_{\alpha\beta}^\gamma J_j^\gamma, \quad (16)$$

where $f_{\alpha\beta}^\gamma$ are the structure constants of gl(1|2), and $\epsilon_\alpha = 0$ for the first five generators (i.e., they are bosonic operators), and $\epsilon_\alpha = 1$ for the last four (fermionic) operators. The fundamental matrix representation of the generators is in the basis, in which the fermionic states are $e_{j1} = (1\,0\,0)_j^T$ for the electron with spin down, $e_{j2} = (0\,1\,0)_j^T$ for the electron with spin up, and the bosonic state is $e_{j3} = (0\,0\,1)_j^T$ (empty state) is:

$$J_j^2 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad J_j^3 = \frac{1}{2}\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$J_j^4 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad J_j^5 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$J_j^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad J_j^7 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$J_j^8 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad J_j^9 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \quad (17)$$

We can introduce the invariant nondegenerate bilinear form $K_{\alpha\beta}$, given as the supertrace over two generators $K_{\alpha\beta} = (K^{\alpha\beta})^{-1} = \mathrm{str}\, J_j^\alpha J_j^\beta$. It is easy to show that using

these operators we can re-write the Hamiltonian of the supersymmetric $t-J$ chain [21]

$$H_{tJ} = -t\sum_{j=1}^{L-1}\left[\sum_{\sigma} P\left(a^{\dagger}_{j,\sigma}a_{j+1,\sigma} + \text{H.c.}\right)P + \right.$$

$$\left. +2t\,\mathbf{S}_j\mathbf{S}_{j+1} - (1/2)\sum_{\sigma} n_{j,\sigma}n_{j+1,\sigma}\right], \qquad (18)$$

as

$$H_{tJ} = -t\sum_{j=1}^{L}\left(\sum_{\alpha,\beta=1}^{9} K^{\alpha\beta}J^{\alpha}_{j}J^{\beta}_{j+1} - 2n_j + 1\right) =$$

$$= -t\sum_{j=1}^{L}\left[\Pi_{j,j+1} - 2n_j + 1\right], \qquad (19)$$

where the graded operator $\Pi_{j,j+1}$ permutes the three possible configurations (empty state and states with electrons with spins up or down) between sites j and $j+1$, picking up a minus sign if both of the permuted configurations are fermionic. This Hamiltonian is obviously supersymmetric, because it is the quadratic form of the generators of the gl(1|2) algebra with the coefficients being the invariant nondegenerate bilinear form of those generators. Notice that the sums of all nine generators over all sites of the lattice commute with the Hamiltonian of the supersymmetric $t-J$ model, $\left[H_{tJ}, \sum_{j=1}^{L} J^{\alpha}_{j}\right] = 0$ for $\alpha = 1,\ldots,9$.

Let us now consider the algebraic Bethe Ansatz for the $\text{gl}_q(n\,|\,m)$-symmetric (i.e., the q-deformed gl($n\,|\,m$), uniaxial case) correlated electron chain. For the gl_q (1|2)-symmetric chain the Hilbert space at each site is isomorphic to C^3 and is spanned by the above mentioned three basis vectors. In the FFB grading, i.e., in which e_{j1} and e_{j2} are fermionic (the Grassmann parities are $\varepsilon_{1,2} = 1$) and e_{j3} is bosonic (the Grassmann parity is $\varepsilon_3 = 0$) we can start from the mathematical vacuum state $|\,0\rangle = \prod_{j=1}^{L} e_{j3}$. This choice of the grading implies that R-matrix for the gl_q(1|2)-symmetric chain has the form

$$R(u) = bI + $$

$$+\begin{pmatrix} x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -c_- & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & c_- & 0 & 0\\ 0 & -c_+ & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & x & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_- & 0\\ 0 & 0 & c_+ & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & c_+ & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & y \end{pmatrix}, \qquad (20)$$

where $b = \sin(i\eta)$, $x = \sin(u+i\eta) - \sin(i\eta)$, $y = \sin(-u+i\eta) - -\sin(i\eta)$, and $c_{\pm} = \exp(\pm i\eta)\sin(i\eta)$. Also the function $c(u) = \sin(u)/\sin(u+i\eta)$ will be useful. It is possible to check that such R-matrices satisfy the YBE. Performing then the procedure of the QISM described above, we obtain the Hamiltonian of the anisotropic gl_q(1|2)-symmetric correlated electron chain as the logarithmic derivative of the transfer matrix taken at zero value of the spectral parameter (see above), namely $H_{an} = t'\sum_j H_{j,j+1}$,

$$H_{j,j+1} = -\sum_{\sigma} P_j\left(a^{\dagger}_{j+1,\sigma}a_{j,\sigma} + \text{H.c.}\right)P_j +$$

$$+\left(a^{\dagger}_{j,\downarrow}a_{j+1,\uparrow}a^{\dagger}_{j+1,\uparrow}a_{j,\downarrow} + a^{\dagger}_{j,\uparrow}a_{j+1,\downarrow}a^{\dagger}_{j+1,\downarrow}a_{j,\uparrow}\right) -$$

$$-\left(e^{\eta}n_{j,\uparrow}n_{j+1,\downarrow} + e^{-\eta}n_{j,\downarrow}n_{j+1,\uparrow}\right). \qquad (21)$$

For small $\eta \to 0$ we obtain the R-matrix of the gl(1|2)-symmetric chain (i.e. the supersymmetric $t-J$ model). Then, according to the above mentioned procedure, we construct the Hamiltonian Eq. (1) of the two-chain anisotropic gl_q(1|2)-symmetric chain as the logarithmic derivative of the two-chain transfer matrix, see above. One can check that at half-filling (one electron per site) the model reduces to the two-chain spin-1/2 model with the uniaxial anisotropy [22].

2.4. Eigenvalues and eigenfunctions for the supersymmetric correlated electron model

To find eigenvalues and eigenfunctions of the Hamiltonian (1) let us consider the monodromy matrix $T(u)$ for the $\text{gl}_q(n\,|\,m)$ symmetric model as the $(n+m)\times(n+m)$ matrix in the auxiliary subspace

$$T(u) = \begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix}, \qquad (22)$$

where $\hat{A}$ is $n\times n$ matrix, $\hat{D}$ is $m\times m$ matrix, $\hat{B}$ is $n\times m$ matrix and $\hat{C}$ is $m\times n$ matrix. For the gl_q(1|2)-symmetric chain we have

$$\hat{A}(u) = \begin{pmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{pmatrix}, \qquad (23)$$

and

$$\hat{B} = (\hat{B}_1\ \hat{B}_2)^T, \quad \hat{C} = (\hat{C}_1\ \hat{C}_2). \qquad (24)$$

Hence, the transfer matrix $\hat{\tau}(u)$ is

$$\hat{\tau}(u) = \text{str}\,T(u) = -\text{tr}\,\hat{A} + \text{tr}\,\hat{D} \qquad (25)$$

where for the gl_q(1|2)-symmetric chain we have

$$\hat{\tau}(u) = -\hat{A}_{11} - \hat{A}_{22} + \hat{D}. \qquad (26)$$

Determine the action of the monodromy matrix on the mathematical vacuum so that the action of diagonal matrix elements $T_{\alpha\alpha}$ produces c-numbers, i.e., $T_{\alpha\alpha}(u)\,|\,0\rangle = a_\alpha(u)\,|\,0\rangle$ and the mathematical vacuum is the eigenstate for these diagonal components, and the action of all upper elements $T_{\alpha\beta}$ with $\alpha < \beta$ is zero, i.e., $T_{\alpha\beta}(u)\,|\,0\rangle = 0$ for $\alpha < \beta$. Then the monodromy matrix has the triangular form. Such a monodromy matrix satisfies the intertwining relation for monodromy matrices with our R-matrix in the graded space. Then one can show that transfer matrices with different spectral parameters commute, which constitutes the exact integrability of the problem. The action of the L-operator of the considered model on our mathematical vacuum $|\,0\rangle$ is

$$L_j(u)\,|\,0\rangle = \begin{pmatrix} c(u) & 0 & 0 \\ 0 & c(u) & 0 \\ b(u)Q_{j\uparrow}^{\dagger} & b(u)Q_{j\downarrow}^{\dagger} & 1 \end{pmatrix}, \qquad (27)$$

i.e., it has the triangular form, which implies

$$T(u)\,|\,0\rangle = \begin{pmatrix} c^L(u) & 0 & 0 \\ 0 & c^L(u) & 0 \\ C_1(u) & C_2(u) & 1 \end{pmatrix}. \qquad (28)$$

Equation (28) means the triangular action of the monodromy matrix on the mathematical vacuum. This choice of the L-operator can be used for the above described scheme for the two-chain model, yielding $a_1(u) = a_2(u) = c^L(u)c^L(u-\theta)$ and $a_3(u) = 1$. Applying the machinery of the QISM for the anisotropic supersymmetric $t-J$ model we obtain the eigenvalue of the transfer matrix

$$\Lambda(u) = \prod_{j=1}^{N} c^{-1}\left(\lambda_j^0 - u\right) - c^L(u)\prod_{j=1}^{N} c^{-1}\left(\lambda_j^0 - u\right)\times$$

$$\times\prod_{\gamma=1}^{M} c^{-1}\left(u - \Lambda_\gamma\right) - c^L(u)\prod_{\gamma=1}^{M} c^{-1}\left(\Lambda_\gamma - u\right). \qquad (29)$$

Here the quantum numbers $\{\lambda_j^0\}_{j=1}^N$ and $\{\Lambda_\gamma\}_{\gamma=1}^M$ satisfy the set of equations, known as the BAE

$$\prod_{j=1}^{N} c^{-1}\left(\lambda_j^0 - \Lambda_\gamma\right) = \prod_{\substack{\beta=1 \\ \beta\neq\gamma}}^{M} \frac{c(\Lambda_\gamma - \Lambda_\beta)}{c(\Lambda_\beta - \Lambda_\gamma)}, \quad \gamma = 1,\ldots,M\,, \quad (30)$$

and

$$c^{-L}\left(\lambda_j^0\right) = \prod_{\gamma=1}^{M} c^{-1}\left(\lambda_j^0 - \Lambda_\gamma\right), \quad j = 1,\ldots,N\,. \quad (31)$$

The numbers N and M (the number of electrons, and the number of electrons with down spins) are related to the values of the chemical potential μ and the magnetic field H, directed along the anisotropy axis.

One can obtain the BAE for periodic correlated electron integrable models under the non-force action of external electro-magnetic fluxes by introducing the operator

$$T = \mathrm{e}^{i\Phi\uparrow}\frac{1}{2}\left(I_0 + \sigma_0^z\right) + \mathrm{e}^{i\Phi\downarrow}\frac{1}{2}\left(I_0 - \sigma_0^z\right) \qquad (32)$$

into the monodromy operator of the associated problem (recall, subscript 0 denotes the auxiliary subspace).

2.5. Bethe Ansätze for rapidities

Changing the variables $\lambda_j^0 - i\eta/2 \to v_j$ we obtain the set of the BAE for the two-chain anisotropic supersymmetric $t-J$ model

$$\left[\frac{\sin(v_j + i\eta/2)}{\sin(v_j - i\eta/2)}\frac{\sin(v_j - \theta + i\eta/2)}{\sin(v_j - \theta - i\eta/2)}\right]^L \times$$

$$\times\exp\left(-i2\pi[(\Phi/\Phi_0) + (F/F_0) + \phi]\right) =$$

$$= \prod_{\alpha=1}^{M}\frac{\sin(v_j - \Lambda_\alpha + i\eta/2)}{\sin(v_j - \Lambda_\alpha - i\eta/2)}, \quad j = 1,\ldots,N\,,$$

$$\prod_{j=1}^{N}\frac{\sin(\Lambda_\alpha - v_j + i\eta/2)}{\sin(\Lambda_\alpha - v_j - i\eta/2)}\exp\left(-i4\pi[(F/F_0) + \phi]\right) =$$

$$= -\prod_{\substack{\beta=1 \\ \beta\neq\alpha}}^{M}\frac{\sin(\Lambda_\alpha - \Lambda_\beta + i\eta)}{\sin(\Lambda_\alpha - \Lambda_\beta - i\eta)}, \quad \alpha = 1,\ldots M\,. \qquad (33)$$

The energy of the state, characterized by the set of rapidities v_j and Λ_α can be obtained as the logarithmic derivative of the transfer matrix with respect to the spectral parameter, then taking $u = 0$; we obtain

$$E = -\mu N - 2t''\sum_{j=1}^{N}\left[\frac{1 - \cos(v_j)\cosh(\eta)}{\cosh(\eta) - \cosh(2v_j)} + \right.$$

$$\left. + \frac{1 - \cos(v_j - \theta)\cosh(\eta)}{\cosh(\eta) - \cosh(2v_j - 2\theta)}\right] - \frac{\mu_e H(N - 2M)}{2}. \quad (34)$$

We see that the BAE are periodic in θ, i.e., it is enough to consider θ in the domain $-\pi \le \theta \le \pi$.

3. The ground state of the model

It can be shown that the ground state is described by $N - 2M$ unbound electron states with the real rapidities v_j and M spin-singlet Cooper-like pairs (bound states) for which charge rapidities are complex conjugated pairs. To the exponential accuracy $\exp(-L)$, the rapidities of pairs can be written as $v_\alpha = \Lambda_\alpha \pm i\eta/2$, and for unbound electron states and Cooper-like pairs the BAE (up to the phase shifts, related to the AB and AC effects) and the expression of the energy coincide with the ones written in [8]. Taking the logarithm of the obtained that way BAE we get

$$\Theta(v_j,\eta/2)+\Theta(v_j-\theta,\eta/2)=$$

$$=L^{-1}\left[2\pi\left(I_j+\frac{\Phi}{\Phi_0}+\frac{F}{F_0}+\phi\right)+\right.$$

$$\left.+\sum_{\alpha=1}^{M}\Theta\left(v_j-\Lambda_\alpha,\eta/2\right)\right],\quad j=1,...N,$$

$$\Theta(\Lambda_\alpha,\eta)+\Theta(\Lambda_\alpha-\theta)=L^{-1}\left[2\pi\left(J_\alpha+\frac{2\Phi}{\Phi_0}\right)+\right.$$

$$\left.+\sum_{j=1}^{N-2M}\Theta(\Lambda_\alpha-v_j,\eta/2)+\sum_{\beta=1}^{M}\Theta(\Lambda_\alpha-\Lambda_\beta,\eta)\right],$$

$$\alpha=1,...,M, \qquad (35)$$

where numbers I_j and J_α arise, because the logarithm is a multivalued function, and $\Theta(x,y)=2\tan^{-1}[\tan(x)\coth(y)]$. Equations (35) are periodic in F with the period F_0 and in Φ with the periods Φ_0 and $\Phi_0/2$. Hence, they remain invariant under the replacements $(F/F_0)\to\{\{F/F_0\}\}$, $(\Phi/\Phi_0)\to\{\{\Phi/\Phi_0\}\}$ and $(2\Phi/\Phi_0)\to\{\{2\Phi/\Phi_0\}\}$, where $\{\{x\}\}$ denotes the fractional part of x to the nearest (half) integer (i.e., to the nearest I_j and J_α). Spin and charge rapidities parametrize each eigenvalue and eigenfunction of the Schrödinger equation, and, therefore, all characteristics of the model in the ground state reveal those periodicities also. In particular, the ground state charge and spin persistent currents manifest such periodicities. The phase ϕ, caused by the SOI, can be considered as the initial phase for those periodic persistent currents. The energy the state is given by

$$E=-2t''\sum_{j=1}^{N-2M}\left(\frac{1-\cos(2v_j)\cosh(\eta)}{\cosh(\eta)-\cos(2v_j)}+\right.$$

$$\left.+\frac{1-\cos(2v_j-2\theta)\cosh(\eta)}{\cosh(\eta)-\cos(2v_j-2\theta)}\right)-\frac{\mu_e H}{2}(N-2M)-$$

$$-2t''\cosh(\eta)\sum_{\alpha=1}^{M}\left(4-\frac{\sinh^2(\eta)}{\sin^2(\lambda_\alpha)+\sinh^2(\eta)}-\right.$$

$$\left.-\frac{\sinh^2(\eta)}{\sin^2(\lambda_\alpha-\theta)+\sinh^2(\eta)}\right)-\mu N. \qquad (36)$$

Then we can use the fact that the ground state energy of a one-dimensional metallic system (i.e., the system which has gapless low-lying excitations, the considered model belongs to this class) can be presented as the series $E_0=L\epsilon_\infty+E_1+L^{-1}E_2+\ldots$, where ϵ_∞ determines thermodynamic properties of the host, E_1 describe the thermodynamic behavior of an impurity (or edges of an open chain), and E_2 describes the behavior of excitations, etc. Notice that for systems with gl(2|1) (or SU(N)) symmerties logarithmic corrections of order of $(L\ln L)^{-1}$, etc. exist. The ground state of the studied model is organized by the filling of Fermi seas (i.e., states with negative energies) with $N-2M$ unbound electron states with real v_j rapidities, and M Cooper-like pairs with complex conjugated Λ_α rapidities. In the thermodynamic limit $L,N,M\to\infty$ with fixed ratios N/L and M/L the ground state is given by the solution of two Fredholm integral equations for the density of unbound electron states $\rho(v)$ ($\rho_h(v)$ is the density of the holes) and density of pairs $\sigma(\Lambda)$ ($\sigma_h(\Lambda)$ is the density of holes for pairs)

$$2\pi\left[\rho_h(v)+\rho(v)\right]=f(v,\eta/2)+f(v-\theta,\eta/2)-$$

$$-\int d\Lambda f(v-\Lambda,\eta/2)\sigma(\Lambda),$$

$$2\pi\left[\sigma(\Lambda)+\sigma_h(\Lambda)\right]=f(\Lambda,\eta)+f(\Lambda-\theta,\eta)-$$

$$-\int dz f(\Lambda-z,\eta)\sigma(z)-\int dv f(\Lambda-v,\eta/2)\rho(v), \qquad (37)$$

where $f(x,y)=2\sinh(2y)/[\cosh(2y)-\cos(2x)]$. The integrations in Eqs. (37) is over the values of Λ, v, and z, for which energies of states are negative. In the thermodynamic the energy of the system is

$$\epsilon_\infty=-2t''\int dv\rho(v)\left(\frac{1-\cos(2v)\cosh(\eta)}{\cosh(\eta)-\cos(2v)}+\right.$$

$$\left.+\frac{1-\cos(2v-2\theta)\cosh(\eta)}{\cosh(\eta)-\cos(2v-2\theta)}\right)-\mu_e H(N-2M)2L-$$

$$-2t''\cosh(\eta)\int d\Lambda\sigma(\Lambda)\left(4-\frac{\sinh^2(\eta)}{\sin^2(\Lambda)+\sinh^2(\eta)}-\right.$$

$$\left.-\frac{\sinh^2(\eta)}{\sin^2(\Lambda-\theta)+\sinh^2(\eta)}\right)-\frac{\mu N}{L}. \qquad (38)$$

4. Finite size corrections and persistent currents

One can clearly see that these equations do not depend on Φ and F explicitly. It respects the fact that ϵ_∞ does not depend on external electro-magnetic fluxes and the AB and AC quantum topological effects reveal themselves in the (highest) corrections of order of L^{-1} etc. The calculations of those corrections can be performed in the framework of the method, based on the use of the Euler–MacLaurin formula [23]. The finite size (mesoscopic) correction to the energy, E_2, for the simplest case of only two Fermi seas for unbound electron states and spin-singlet pairs (i.e., for the spatially homogeneous distributions of charge and spin densities in the ground state; due to nonzero θ there can be more than one Fermi sea for each of eigenstates [8] for some ranges of the chemical potential and the external magnetic field, see below), are given by [24]

$$E_2 = \pi \sum_{l=u,p} \left\{ 2v_l \left[\left(\sum_{q=u,p} z_{q,l} (D_q - \phi_q) \right)^2 + \right. \right.$$

$$\left. \left. + n_l^+ + n_l^- - \frac{1}{12} \right] + \frac{v_l}{2} \left(\sum_{q=u,p} (\hat{z}^{-1})_{l,q} \Delta N_q \right)^2 \right\}, \quad (39)$$

where v_l are the Fermi velocities

$$v_u = \frac{d\varepsilon(v)/dv_{v=v_0}}{2\pi\rho(v_0)}, \quad v_p = \frac{d\Psi(\Lambda)/d\Lambda_{\Lambda=\Lambda_0}}{2\pi\sigma(\Lambda_0)}, \quad (40)$$

of the unpaired electron states and pairs, respectively,

$$\Delta N_u = \Delta(N - 2M), \quad \Delta N_p = \Delta M,$$

$$D_u = \frac{1}{2}(\Delta N_u + \Delta N_p) \mod 1,$$

$$D_p = \frac{1}{2}\Delta N_p \mod 1 . \quad (41)$$

and $\phi_u = (\Phi/\Phi_0) + (F/F_0) + \phi$, $\phi_p = 2\Phi/\Phi_0$. Here we have introduced the "dressed" energies for unpaired electron states $\varepsilon(v)$ and pairs $\Psi(\Lambda)$, which are determined from the following set of integral equations,

$$\varepsilon(k) = t'' f(v, \eta/2) + t'' f(v - \theta, \eta/2) - \mu - \frac{\mu_e H}{2} -$$

$$- \frac{t''}{2\pi} \int d\Lambda \Psi(\Lambda) f(v - \Lambda, \eta/2),$$

$$\Psi(\Lambda) = t'' f(\Lambda, \eta) + t'' f(\Lambda - \theta, \eta) - 2\mu -$$

$$- \frac{t''}{2\pi} \int dz f(\Lambda - z, \eta) \Psi(z) - \frac{t''}{2\pi} \int dv f(\Lambda - v, \eta/2) \varepsilon(v) . \quad (42)$$

The expression for the momentum is

$$\Delta P = \sum_{l=u,p} \left(\frac{2\pi}{L} \left[(D_l - \phi_l) \Delta N_l + n_l^+ - n_l^- \right] + \right.$$

$$\left. + 2 p_l^F (D_l - \phi_l) \right). \quad (43)$$

The quantum numbers $\Delta N_{u,p}$ refer to the change in the number of states in each Fermi sea by the low-energy excitation and $D_{u,p}$ are the corresponding backscattering quantum numbers (transfer from the left Fermi point to the right Fermi point). The number of particle-hole excitations about each Fermi point is denoted by $n_{u,p}^{\pm}$. The backscattering quantum numbers are shifted by the phases related to the SOI (internal phase) and the AB and AC effect (external phases). The Fermi momenta corresponding to the Fermi seas are $p_u^F = \pi(N - 2M)/L$ and $p_p^F = \pi M/L$. The generalized dressed charges, $z_{lq} = \xi_{lq}(B_q)$, where $B_q = v_0$ and $B_p = \Lambda_0$, are obtained from the solution of the following set of integral equations ($l = u, p$)

$$\xi_{lu}(v) = \delta_{l,u} - \int d\Lambda f(v - \Lambda, \eta/2) \xi_{lp}(\Lambda),$$

$$\xi_{lp}(\Lambda) = \delta_{l,p} - \int dz f(z - \Lambda, \eta) \xi_{lp}(z) -$$

$$- \int dv f(v - \Lambda, \eta/2) \xi_{lu}(v) . \quad (44)$$

Equations for the components of the dressed charge depend on θ via the limits of integration. In the ground state there are no excitations, and we can use the expression (39) with $\Delta N_l = D_l = n_l = 0$. Persistent currents are related to the first term in Eq. (39), and their magnitudes are proportional to the Fermi velocities of the low-energy states (unbound electrons and Cooper-like pairs), and dressed charge components.

5. Analysis

Depending on the values of the chemical potential μ and the external magnetic field H there can be two, four, or no Fermi seas. Respectively, persistent spin and charge currents manifest the behavior, connected with the states, which form Fermi seas. The magnetic field behavior of the persistent currents is reminiscent of the type II superconductor. There exist two critical fields, H_c, below which only Cooper-like singlet pairs exist in the ground state, and H_s, above which only unbound electron states have the Fermi sea [8]. On the other hand, the magnitudes of persistent currents strongly depend on the band filling. Magnitudes are zero for the empty chain, and increase with the growth of the band filling, being maximal at the half-filling. Depending on the value of θ there can exist additional Fermi seas [8], which are responsible for the formation of the CDW and/or SDW (charge and spin density wave) like incommensurate states. Notice that additional Fermi seas are related to the same kind of low-energy states (either unbound electrons, or pairs). At the critical values of θ, μ, and H, the magnitudes of persistent currents manifest features of their behavior: Fermi velocities of related low-energy states become zero.

The oscillations of persistent currents have the saw-tooth like form, which is usual for any system with a large number of particles in it. Fermi velocities of low-lying excitations v_l, and dressed charge matrix (which determine magnitudes of persistent currents) explicitly depend on the parameter of the inter-chain interaction.

It is not so for ϕ, caused by the SOI: Only initial shifts of persistent currents are determined by the SOI phase in the main ($\sim L^{-1}$) approximation. Initial phases of oscillations of persistent currents also depend on the parity of the number of electrons N and down-spin electrons M mod (4). The nonzero θ determines the nonzero ground state momentum of the system (due to topological charge and spin currents, i.e., three-site terms, caused by the inter-chain coupling), which is nothing else than the nonzero charge and spin chirality (or Noethers topological currents).

From the above obtained finite size corrections we see that the spin persistent current reveals oscillations, caused

by the external electric flux F, with the period of oscillations F_0 and the initial phase ϕ. The period of oscillations manifests that unbound electron excitations carry fractional spin 1/2, i.e., the model belongs to the class of quantum spin liquids. That current is connected with unbound electron states. Hence, for $H < H_c$ such oscillations are absent. For $H > H_s$ the spin persistent current manifest properties of the one for the non-interacting one-dimensional system (notice that for this gapped state the magnitude of the spin persistent current is exponentially small with L). On the other hand, for $H_c \le H \le H_s$ spin persistent current exists, however, its magnitude depends on the value of the magnetic field and the band filling. The magnitude is maximal for the half-filled band, reminiscent to the one of the zig-zag spin chain.

The charge persistent current in general situation manifests the interference of oscillations caused by the external magnetic flux Φ with two periods: Φ_0 and $\Phi_0 / 2$, related to unbound electron states, and Cooper-like pairs, respectively. For $H < H_c$ it only oscillates with the period $\Phi_0 / 2$, characteristic for Cooper-like pairs. For $H > H_s$ only oscillations with the period Φ_0 survive, with the properties, characteristic for the non-interacting electrons, however with the exponentially small with L magnitude. For $H_c \le H \le H_s$ oscillations with two periods exist.

In the zero-anisotropy limit we have $H_c \to 0$. For the half-filling at $H = 0$ the magnitudes of oscillations of persistent currents with the periods Φ_0 and $\Phi_0 / 2$ become equal to each other, however, the initial phases of those oscillations are different due to the SOI. However, for any nonzero H and non-half filled case the magnitudes of oscillations with periods Φ_0 and $\Phi_0 / 2$ become different from each other, and the interference of those two periodic functions has to manifest itself in the ground state.

Oscillations of charge and spin persistent currents are maximal in the ground state, their main effect is of order of L^{-1}. However, for $T \ne 0$ persistent currents become exponentially small with L [25]. The saw-tooth like form of oscillations is related to only the ground state. Any nonzero temperature $T > \hbar v_l / L$ strongly reduces the magnitudes of the most of harmonics, and sinusoidal-like oscillations result.

6. Summary

In summary, we have considered persistent currents in the integrable two-chain correlated electron ring with the anisotropy of electron-electron interactions. We have shown that the model belongs to the class of quantum spin liquids, because it does not have any ordering even in the ground state, and has emergent excitations, which carry fractional spin 1/2. We have shown how and at which conditions such fractional spin can be measured in the periodicity of spin persistent currents. On the other hand, the considered correlated electron model in the ground state manifests features, characteristic for type II superconductors (notice that in one space dimension quantum and thermal fluctuations destroy the superconducting ordering). The periodicity of the charge persistent current can manifest the interference of oscillations, related to the charges of Cooper-like pairs and unbound electron states, which form the ground state Fermi seas of the model.

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1. Y. Aharonov and D. Bohm, *Phys. Rev.* **115**, 485 (1959).
2. Y. Aharonov and A. Casher, *Phys. Rev. Lett.* **53**, 319 (1984).
3. See, e.g., A.A. Zvyagin, *Quantum Theory of One-Dimensional Spin Systems*, Cambridge Scientific Publishers, Cambridge (2010).
4. P.C. Hohenberg, *Phys. Rev.* **158**, 383 (1967).
5. For the review, see, e.g., J. Alicea, *Rep. Progr. Phys.* **75**, 076501 (2012); C.W.J. Beenakker, *Annu. Rev. Condens. Matter Phys.* **4**, 113 (2011); T.D. Stanescu and S. Tewari, *J. Phys.: Condens. Matter* **25**, 233201 (2013).
6. A. Friedenauer, H. Schmitz, J.T. Glueckert, D. Porras, and T. Schaetz, *Nat. Phys.* **4**, 747 (2008); K. Kim, M.-S. Chang, S. Korenblit, R. Islam, E.E. Edwards, J.K. Freericks, G.-D. Lin, L.-M. Duan, and C. Monroe, *Nature* **465**, 590 (2010); B.P. Lanyon, C. Hempel, D. Nigg, M. Müller, R. Gerritsma, F. Zähringer, P. Schindler, J.T. Barreiro, M. Rambach, G. Kirchmair, M. Hennrich, P. Zoller, R. Blatt, and C.F. Roos, *Science* **334**, 57 (2011); J.W. Britton, B.C. Sawyer, A.C. Keith, C.-C.J. Wang, J.K. Freericks, H. Uys, M.J. Biercuk, and J.J. Bollinger, *Nature* **484**, 489 (2012); R. Islam, C. Senko, W.C. Campbell, S. Korenblit, J. Smith, A. Lee, E.E. Edwards, C.-C.J. Wang, J.K. Freericks, and C. Monroe, *Science* **340**, 583 (2013).
7. P.A. Lee, *Science* **321**, 1306 (2008); L. Savary and L. Balents, *Rep. Progr. Phys.* **80**, 016502 (2017).
8. A.A. Zvyagin, *Fiz. Nizk. Temp.* **44**, 1588 (2018) [*Low Temp. Phys.* **44**, 1237 (2018)].
9. See, e.g., M. Tinkham, *Introduction to Superconductivity*, McGrow-Hill (1995).
10. A.A. Zvyagin, *Fiz. Tv. Tela* **32**, 1546 (1990) [*Sov. Phys. Sol. St.* **32**, 905 (1990)].
11. I.V. Krive and A.A. Zvyahin, *Mod. Phys. Lett. B* **6**, 871 (1992).
12. A.A. Zvyagin, *Zh. Eksp. Teor. Fiz.* **103**, 307 (1992) [*JETP* **76**, 167 (1993)].
13. H. Bethe, *Z. Phys.* **71**, 205 (1931).
14. R.J. Baxter, *Exactly Solved Models in Statistical Mechanics*, Academic Orlando (1982).
15. L.D. Faddeev, E.K. Sklyanin, and L.A. Takhtajian, *Theor. Math. Phys.* **40**, 194 (1979).
16. L.A. Takhtajan and L.D. Faddeev, *Russ. Math. Surveys* **34**, 511 (1979).
17. M. Lüsher, *Nucl. Phys. B* **135**, 1 (1978).
18. B. Sutherland, *Phys. Rev B* **12**, 3795 (1975).
19. A.A. Zvyagin, *Phys. Rev. B* **52**, 15050 (1995); A.A. Zvyagin, *Phys. Rev. B* **57**, 1035 (1998).
20. F.H.S. Essler and V.E. Korepin, *Phys. Rev. B* **46**, 9147 (1992).

21. C.K. Lai, *J. Math. Phys.* **15**, 167 (1974); B. Sutherland, *Phys. Rev. B* **12**, 3795 (1975); P. Schlottmann, *Phys. Rev. B* **36**, 5177 (1987).
22. V.Yu. Popkov and A.A. Zvyagin, *Phys. Lett. A* **175**, 295 (1993).
23. A.A. Zvyagin, *Finite Size Effects in Correlated Electron Systems: Exact results*, Imperial College Press, London (2005).
24. V.E. Korepin, N.M. Bogoliubov, and A.G. Izergin, *Quantum Inverse Scattering Method and Correlation Functions*, Cambridge University Press, Cambridge (1993).
25. For the review, see, e.g., A.A. Zvyagin and I.V. Krive, *Fiz. Nizk. Temp.* **21**, 687 (1995) [*Low Temp. Phys.* **21**, 533 (1995)].

Персистентні струми у двохланцюжковій моделі корельованих електронів

А.А. Звягін

Розраховано персистентні струми в основному стані для нещодавно запропонованої точно розв'язуваної двохланцюжкової моделі корельованих електронів з анізотропією спін-спінової взаємодії між електронами та спін-орбітальною взаємодією. Модель описує квазіодновимірний надпровідник II роду. Показано, що спін-орбітальна взаємодія визначає початкові фази осциляцій зарядових та спінових персистентних струмів, які визначаються непов'язаними електронними станами. З іншого боку, спін-синглетні пари типу Купера визначають осциляції тільки зарядових персистентних струмів з періодом, характерним для пар. Залежно від величин зовнішнього магнітного поля та заповнення зони, в системі може проявлятися складна картина інтерференції декількох типів осциляцій.

Ключові слова: моделі, що інтегруються, анізотропія, персистентні струми.

Персистентные токи в двухцепочечной модели коррелированных электронов

А.А. Звягин

Рассчитаны персистентные токи в основном состоянии для недавно предложенной точно решаемой двухцепочечной модели коррелированных электронов с анизотропией спин-спинового взаимодействия между электронами и спин-орбитальным взаимодействием. Модель описывает квазиодномерный сверхпроводник II рода. Показано, что спин-орбитальное взаимодействие определяет начальные фазы осцилляций зарядовых и спиновых персистентных токов, которые определяются несвязанными электронными состояниями. С другой стороны, спин-синглетные пары типа Купера определяют осцилляции только зарядовых персистентных токов с периодом, характерным для пар. В зависимости от величин внешнего магнитного поля и заполнения зоны, в системе может проявляться сложная картина интерференции нескольких типов осцилляций.

Ключевые слова: интегрируемые модели, анизотропия, персистентные токи.