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Aharonov–Casher effect and electric field control of magnetization dynamics

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A spin wave (SW) propagating in an external electric field acquires an extra phase, the so-called Aharonov–Casher phase. To linear order, that is equivalent the SW frequency shift linear in both the applied electric field and the wave vector of SW's and can be described by adding effective Dzyaloshinskii–Moriya-like interaction between spins. This effect is a promising way to control and manipulate magnetization precession dynamics by electric field and opens up a new way of SWs steering in magnonic devices. The goal of the report is to emphasize a fundamental physical difference in the nature of external electric field effect through the Aharonov–Casher phase shift and of a voltage control of magnetic anisotropy (VCMA). In the context of an experimental performance, we deal with almost identical experimental conditions. Yet, from the quantum physics point of view, we deal with different effects and this is important to understand the experimental results. In the case of the Aharonov–Casher phase shift it is a global nonlocal (topological) effect and in the case of VCMA it is the influence on a system's local (Landau) magnetic parameters (e.g., magnetic anisotropy).

Keywords: magnonics, the Aharonov–Casher phase, magnetic topological effects.

Spin waves (SWs) — the collective oscillations of the magnetization in magnetic materials — have attracted a great attention in recent years as information carriers for magnetoelectronic devices. In contrast to electronic currents, SWs propagate with very little dissipation of energy and therefore hold promise for substantial reducing the energy consumption in the next generation of spintronic devices. One of the fundamental challenges that motivate the research activities in SWs electronics, magnonics, is to seek mechanisms enable to control and effectively manipulate SWs power flow. Although SWs can be manipulated via a magnetic field, this type of waveguides usually has a large spatial footprint. Among alternative and efficient solutions is SWs steering via static electric (**E**) field, i.e., **E**-field controlled spin dynamics. Among the first steps in this direction was the paper by V.G. Bar'yakhtar, V.A. L'vov, and D.A. Yablonskii [1] where the authors revealed that electric field can interact with magnetization spatial inhomogeneity. As it has been noted by Bar'yakhtar *et al.*, while the electric field cannot couple to the uniform magnetization in ferromagnets having inversion symmetry, this field can interact with spatially-non-uniform magnetization. To linear order of the **E** field, this interaction can be taken into account by adding a Dzyaloshinskii–Moriya-like (DM-like) interaction between neighboring spins [2]. This results in an induced spin wave frequency shift, that is linear in both the applied **E** field and the wave vector **k** of a SW. While a magnetic field shifts the SW dispersion vertically by increasing or decreasing the frequency at a fixed wave vector, the electric field shifts the dispersion horizontally by increasing or decreasing the wave vector at fixed frequency.

In 1984, around the same time as the **E**-field induced DM-like interaction has been discussed in magnetism, the topological effects associated with adiabatic quantum evolution were discovered in condensed matter physics [3,4]. With the introduction of topology, the perspective is changed from describing complex systems in terms of local (Landau) order parameters to a characterization by global nonlocal quantities. Different phase states of the system from the point of view of a local order parameter can be the same state from the point of view a topological state, and vice versa, different system's phase states from the concept of a topological picture can be the same state from the point of view a local order parameter. In magnetism, the mainstream of topological concepts can be conditionally divided on two different scenarios: (i) a nontrivial topology of a static magnetic order in real space, e.g., a magnetic structure such as skyrmions (Refs. 5, 6 and references therein), and (ii) topological effects on magnetization dynamics finding its realization in topological protection of SWs (Ref. 7 and references therein). In particular, the case under question, an extra geometric phase, the so-called Aharonov–Casher (AC) phase [4,8], acquired by the quantum orbital motion of neutral magnetic moments in mesoscopic rings in electric field is an example of different effects in topological magnonics.

In this report, we briefly review the basis of the AC effect in ferromagnetic insulators and stress on the difference in the nature of a very similar external exposure on a magnetic system static and dynamic properties from a perspective of a local (Landau) order parameter state and a global topological state. Namely, we discuss a magnetization precession dynamic under an effect of external **E** field with the acquisition by magnons of the AC phase and a voltage control of magnetic anisotropy (VCMA). From the point of an experimental performance, we deal with the almost identical action/realization. Yet, in the context of concepts arising from topology the perspective is changed. We deal with two different phenomena, and this is important to deep our understanding and to interpret the experimental results.

Aharonov–Casher phase. The AC effect [4], an extra geometric phase, in addition to a standard dynamic phase, acquired by the wave function's cyclic adiabatic evolution in an external parameter space, is a topological quantum phenomenon predicted for neutral particles with magnetic moment traveling in static electric field. The AC phase is in fact the spin-dependent case of the Aharonov–Bohm phase [9] spread on the case when a spin-orbit coupling generalizes spin-dependent magnetic vector potential. The most studies on the concept of AC effect are focused on fermions ([10] and references therein). The total number of fermions is a constant and, at low temperature, they occupy levels from lower to the Fermi one. As was shown by Cao *et al.* [8] this topological effect can be extended to a boson system as well. Specifics of the spin-waves system are: (i) the SW is a boson excitation; (ii) at low temperature most of them belong to low-energy excitations; (iii) the total particle number is not conserved; (iv) the SW is a kind of collective excitations and arises at a finite temperature, i.e., its number is dependent on the temperature. To elucidate the **E**-field effect on a SWs current, we will follow [8,11].

Consider a one-dimensional ferromagnetic chain of spins $S = 1/2$. We will suppose that the system is in an ordered state and all spins in site $\mathbf{S}_l \parallel \mathbf{z}$ where axis $\mathbf{z}$ is perpendicular to the chain. Within the Heisenberg model, the system's Hamiltonian is $\hat{H} = -J\Sigma_l \mathbf{S}_l \mathbf{S}_{l+1}$, where $\mathbf{S}_l$ is a spin operator at the lth site, $J > 0$ is an exchange interaction. When an external electric field is applied, besides a conventional dynamic phase, the inversing spin obtains an additional phase — a quantum phase φ (or –φ) when it hops from the lth site to the (l±1)th site. In this case, using the conventional S^z, $S^{\pm}$ representation the system's Hamiltonian can be rewritten as

$$\hat{H} = -J\sum_l \left[S_l^z S_{l+1}^z + \frac{1}{2}\left(e^{i\varphi} S_l^+ S_{l+1}^- + e^{-i\varphi} S_l^- S_{l+1}^+ \right) \right]. \quad (1)$$

Here the phase shift φ is

$$\varphi = \frac{1}{\hbar c^2} \int_l^{l+1} [\mathbf{m} \times \mathbf{E}]\, d\,\mathbf{l}, \quad (2)$$

where $\hbar$ is the reduced Planck constant, **m** is the magnetic moment of the inversing spin, and c stands for the light velocity.

Using operators of spin waves in the momentum space, b_k and b_k^+, which satisfy the Boson's commutation relation, a diagonalized Hamiltonian of the system can be written as $\hat{H} = \sum_k E_k(\varphi) b_k^+ b_k$, where the energy of SW reads $E_k(\varphi) = 2J[1 - \cos(ak - \varphi)]$. Within a linear order of the electric field, the energy shift of SW due to topological quantum phenomena can be expressed as

$$E_k(\varphi) = E_k(\varphi = 0) - 2J\varphi \sin(ak) \approx$$
$$\approx E_k(\varphi = 0) - C\mathbf{k}[\mathbf{m} \times \mathbf{E}]. \quad (3)$$

Here $C \approx J/\hbar c^2$ characterizes spin-orbit interaction in vacuum, and the last term in Eq. (3) is the energy of interaction of a moving magnetic moment with the electric field [8].

Thus, neutral magnetic dipoles, i.e., magnons, traveling in vacuum under an electric field effect display (are under an exposure of) a topological quantum phenomenon — a Berry phase shift [3,10,12] that is determined by the spin-orbit interaction in vacuum. Naturally, the question arises about adaptability of these results to magnetic materials, e.g., magnetic insulators.

AC effect in magnetic insulators. Recent microscopic calculation based on the super-exchange model predicted that in magnetic insulators the electric-field induced AC phase is sufficiently large and can be experimentally detected, e.g., in such classical magnetic insulator as YIG (yttrium iron garnet, $Y_3Fe_5O_{12}$) [13]. The physical reason why the **E** field effect in this case is much larger than in vacuum is that the spin carries–electrons–virtually hopping between the orbital of magnetic ions in a magnetic insulator have strong intrinsic spin-orbit coupling. As was shown in Ref. 13, within the Heisenberg model, the resulting spin Hamiltonian can be written in the form (compare with Eq. (1)):

$$\hat{H} = -J\sum_{i,j} \left[S_i^z S_j^z + \frac{1}{2}(e^{i\alpha_{ij}} S_i^+ S_j^- + e^{-i\alpha_{ij}} S_i^- S_j^+) \right]. \quad (4)$$

Here the phase shift α_{ij} is proportional to the distance between neighboring sites and independent of the direction of the localized magnetic moments: $\alpha_{ij} = \alpha(i - j)$, and $\alpha = \frac{eaE}{E_{SO}}$ (e stands for the absolute value of the electron charge, E_{SO} stands for an energy scale associated with the inverse of the spin-orbit coupling strength). I.e., the inclusion of intrinsic spin-orbit interaction is equivalent to the inclusion of a spin-dependent vector potential $\mathbf{A} \sim [\mathbf{E} \times \mathbf{m}]$, which modifies the electron hopping by a spin-dependent phase factor. Within a linear order of the electric field, the spin Hamiltonian (4) can be written as [13]:

$$\hat{H} \approx -J \sum_{i,j} \left\{ \mathbf{S}_i \mathbf{S}_j + \sin\alpha_{ij} \cdot [\mathbf{S}_i \times \mathbf{S}_j]_z \right\} \approx$$

$$\approx -\sum_{i,j} \left\{ J_{ij} \mathbf{S}_i \mathbf{S}_j + \mathbf{d}_{ij} [\mathbf{S}_i \times \mathbf{S}_j] \right\}, \qquad (5)$$

where the axis z is in the direction perpendicular to $\mathbf{E}$ and vector $\mathbf{e}_{ij}$ along the line connecting the magnetic ions. Therefore, to a linear order of the electric field, the effect of electric-field induced phase is equivalent to adding a DM-like interaction between the spins $\mathbf{S}_i$ and $\mathbf{S}_j$ of neighboring ions and can be written in a traditional form as $\mathbf{d}_{ij}[\mathbf{S}_i \times \mathbf{S}_j]$, where $\mathbf{d}_{ij} = J \cdot e[\mathbf{E} \times \mathbf{e}_{ij}]/E_{SO}$ is the vector perpendicular to both the electric field $\mathbf{E}$ and the unit vector $\mathbf{e}_{ij}$ along the line connecting the magnetic ions. (Here and below we will assume that the electric field and SW's wave vector are perpendicular to each other, i.e., the Doppler shift vanishes [14].)

Historically, a DM-like interaction due to the **E**-field and the spin Hamiltonian (5) were proposed in 1983 [1] and 2008 [2] in the reports where the authors tried to describe theoretically an electric field interaction with magnetic inhomogeneity in ferromagnets. The real possibility to control electrically SWs geometric phase in magnetic insulators was recognized, in fact, only recently (Refs. 13, 15, 16). The experimental data obtained for YIG [16] showed that, indeed, the SW phase electric tuning can be realized in thin ferromagnetic films with high efficiency. Such phase changes induced by different electric fields have been precisely detected in single-crystal YIG. At an applied electric field of $\sim 10^6$ V/m, the resulting phase (normalized to the propagation distance) was of the order of 10^{-5} rad/mm [16]. This value can be drastically enhanced by decreasing the wavelength. The phase shift signal has a clear dependence on the electric field, demonstrating the electric tuning origin. From the measurement data, the authors [16] estimate the magnitude of effective α_{ij}, spin-orbit coupling parameter. It was two orders of magnitude greater than previously estimated for YIG [2]. This finding provides opportunities for direct electric tuning of SWs in magnonic devices, that is essential ingredient for the topological magnonics.

AC effect to control SWs geometric phase. Naturally, the possibility to control the properties of propagating SWs in ultrathin ferromagnetic films using the electric-field-induced phase shift has attracted considerable attention. Indeed, after excitation, SWs need to be guided through a given channel to ensure that they reach the target position in variety of magnonic devices. In this section, we give a few examples substantiating the statement that the electric-field-induced AC phase shift can be used as an effective method to control SW power flow in magnonics devices.

It was shown theoretically [17] that at given frequencies, by applying an electric field one can create highly focused beams of energy — caustics. The focusing patterns are nonreciprocal, with the caustic beams appearing only on one side of a point source. Thus, by inducing the DM-like interaction there is also a possibility to create interference patterns. It was demonstrated that the degree of SW power flow asymmetry can be effectively tuned with the external **E** field. Thus, the external electric field gives a much more convenient opportunity to tune the dispersion relations and alter the focusing patterns in comparison with the film thickness changing.

In Refs. 18 and 19, it was examined how electric field influences on nonreciprocity of SW propagation in thin ferromagnetic film, where the transverse magnetization is enforced by an externally applied field (the Damon–Eshbach geometry). The combination of the magneto-dipole interaction and the electric field results in nonreciprocal unidirectional caustic beams of dipole-exchange SWs. It was demonstrated that it is possible to induce and/or control nonreciprocity of spin wave propagation along a stripe of ferromagnet by external static electric field. An important for practical application consequence is that the opposite shifts of the SW group velocity and focusing patterns can be achieved by simple changing the sign of applied electric field. This provides a new direction for spin-based device development by utilizing an electric field for spin current control.

Magnon spintronics, or magnonics, is the magnetic analog of photonics. This analogy motivates the authors [20] to analyze theoretically how the SW reflection and transmission in thin ferromagnetic film can be modified by an external electric field. It was shown that SW's scattering on a boundary between magnetic film's regions with and without **E**-field induced DM-like interaction follows a generalized Snell's law, where these two regions work as two different magnetic media.

The studies mentioned above are focused on effects of a homogeneous static external field. Recently it has been proposed to extend topological AC effects in magnetic systems under either the presence of an electric field gradient or an external time-dependent field. These topological aspects of magnons dynamics have been actively discussed on examples of antiferromagnetic insulator [21,22]. In Ref. 21, making use of the AC effect induced by electric field gradients, authors propose a magnonic analog of the quantum spin Hall effect for edge states that carry helical magnons in antiferromagnets. It was shown that, due to the electric field gradient-induced AC effect, up and down helical magnons perform cyclotron motion with the same frequency but in opposite directions giving rise to helical edge magnon states.

In Ref. 22, it has been proposed to extend topological effects on systems under the effect of external time-dependent field **E**(t). Such electric field can be induced by periodically changing a voltage difference between a back and top gate enclosing the magnetic sample, or due to the laser oscillating electric field, at frequency ω. Specifically, the important question is a time-periodic control of a magnon geometrical phase. Since the electric field **E**(ω) can be considered as a time-periodic perturbation, the authors [22] shown that in insulating antiferromagnets a circularly polarized laser can generate chiral edge magnon states and induce the magnonic thermal Hall effect. The direction of the magnon chiral edge modes and the resulting thermal Hall effect can be controlled by the chirality of the circularly polarized laser through the change from the left-circular to the right-circular polarization.

These results provide an opportunity to control magnon's topological properties in insulating anti- and ferromagnets.

Voltage-control of magnetism. Besides the electric field modification SWs dynamics by means of such scenario as the AC phase, which is the physical consequence of adiabatic quantum evolution and topological effects, several other mechanisms have been proposed to implement controlled SWs dynamics. In ferromagnet/oxide heterostructures there is the possibility to control interface magnetic characteristics by a gate voltage through magnetoelectric coupling mechanisms [23,24]. Magnetoelectric coupling allows influence on magnetism through a modulation of the interfacial magnetic anisotropy by electric field/voltage. Control of SW properties using VCMA is also considered at present as a direct possibility to control magnetization dynamics [25–29]. In general, when artificial ferromagnetic/ferroelectric film's heterostructure is subjected to a voltage applied between the gate electrodes, magnetostriction induced by the piezoelectric strain varies in the magnetic film's surface magnetic anisotropy. Typically, the strain-mediated magnetoelectric response is symmetric with respect to the sign of the applied voltage because the induced strain depends on the square of the ferroelectric polarization.

On a microscopic level, the effect on an electronic occupation state due to the applied voltage is supposed to be the main mechanism behind VCMA [26,29–31]. When electric field is applied at, for example, 3d transition magnetic metal film/nonmagnetic oxide insulator interface, the number of electrons in 3d orbitals of transition metal is changed (as was shown from first-principle calculations, voltage-driven oxygen migration may simultaneously occur as well [30,31]). This affects the bonding strength between 3d and 2p orbitals of oxygen resulting in a substantial change of interfacial magnetism. Since the penetration depth of electric field in a metal is only a few nanometers, the VCMA effect is limited to ultrathin magnetic films.

As was mentioned above, from the point of view of the experimental realization, we deal with almost identical conditions in the case of the AC phase shift and the VCMA effects. However, from the quantum mechanics paradigm, the AC phase shift and the VCMA effects are different phenomena. In the VCMA case, electric field modifies local (Landau) system's parameters, whereas taking into account the AC effect on phase we describe complex systems in terms of the global order parameters, which are measured nonlocally and which endow the system with a global stability to perturbations. The AC effect is independent on the electric field modification of local (Landau) magnetic parameters and manifests itself in different physical effects. Therefore, in our opinion, it is important to take the topological effects into account and critically reconsider the interpretation of the existing VCMA experimental data [23–29].

In conclusion. Conventional understanding of magnetic system's properties is largely based on two fundamental paradigms of physics: the Landau theory of phase transition and the Landau–Lifshitz–Gilbert equation of magnetization dynamics. The experimental demonstration that SWs propagating in an applied electric field acquire the Aharanov–Casher phase stimulated a new paradigm in field of magnetization dynamics, which can be generalized as topological effects in magnonics. The electric-field shift of the magnon energy due to the additional geometrical Aharanov–Casher phase is a striking example of such effects. By introducing quantum topological effects in magnonics, which were previously overlooked, we will get important consequences in deepening our understanding and interpretation of the experimental results.

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Ефект Ааронова–Кашера та керування динамікою намагніченості електричним полем

В.М. Криворучко

Спінова хвиля (SW), що поширюється у зовнішньому електричному полі, набуває додаткову фазу, так звану фазу Ааронова–Кашера. У лінійному наближенні зсув частоти є лінійним як від прикладеного електричного поля, так і від хвильового вектора SW, та може бути описаний додаванням ефективної взаємодії між спінами типу Дзялошинського–Морія. Цей ефект є перспективним способом контролю та керування динамікою прецесії намагніченості електричним полем і відкриває новий засіб управління SW в пристроях магноніки. Акцентовано фундаментальну фізичну відмінність у характері впливу зовнішнього електричного поля за допомогою фазового зсуву Ааронова–Кашера та контролю магнітної анізотропії різницею потенціалів (VCMA). З експериментальної точки зору — це майже однакові експериментальні умови, проте з позицій квантової фізики маємо справу з різними ефектами. У випадку фазового зсуву Ааронова–Кашера це глобальний нелокальний (топологічний) ефект, у випадку VCMA — вплив на локальні (відповідно Ландау) магнітні параметри системи (наприклад, магнітну анізотропію).

Ключові слова: магнони, фаза Ааронова–Кашера, магніто-топологічний ефект.