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Shear elastic surface waves and system symmetry

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The effect of the anisotropy of an elastic medium and a crystal on the properties of surface waves is considered. It leads to symmetry breaking of a semi-bounded space with respect to reflection. This fact affects the possibility of the existence and properties of surface elastic shear waves. It is shown that, while maintaining the crystallographic symmetry in crystals with the surface orientation (110), the anisotropy breaks the indicated symmetry, but retains the possibility of propagation of surface waves with their strong modification. In the case of an anisotropic half-space with a thin film coating, which allows the propagation of the Love waves in an isotropic medium, anisotropy leads to the absence of such stationary waves.

Keywords: surface waves, shear waves, Love waves, crystallographic symmetry.

1. Introduction

Investigations of surface waves (SW), despite its more than a century history since their theoretical prediction by Rayleigh (see, for example, [1]), have not lost their relevance until now.

This is due to the importance of such waves both from a fundamental point of view and in connection with their technological applications (in seismology, magnetically ordered systems, piezoelectric crystals, in nondestructive testing methods, etc.). In the currently developing nanophysics, the contribution of surface waves to the thermodynamic characteristics (for example, heat capacity) of finely dispersed media is the main one at low temperatures [2]. In an isotropic medium, in which two transverse sound velocities coincide, the Rayleigh surface wave is two-component — the displacement vector lies in the sagittal plane specified by the normal to the surface and the direction of wave propagation. In anisotropic systems (in a crystal lattice), the surface wave in the general case can also be three-component [3]. The conditions imposed on the symmetry of the lattice and the direction of wave propagation were investigated in [4] under which a three-component surface wave is split into a two-component wave polarized in the sagittal plane (Rayleigh polarization) and a one-component wave polarized perpendicular to the sagittal plane (SH wave). Such purely shear waves in the long-wavelength limit penetrate deep into the crystal much deeper than the Rayleigh-type waves and therefore much more strongly depend on the surface properties [5]. The high sensitivity to the surface properties makes it possible to use the surface SH waves as highly sensitive sensors for measuring such physical characteristics as, for example, the parameters of a superfluid helium film or the atomic mass of an impurity surface layer [6]. In addition, these one-component waves are described by scalar dynamic equations, which simplifies their theoretical study. As noted earlier, in the case of a sufficiently symmetric geometry of the problem, pure shear surface waves are split off from the waves of Rayleigh polarization. Therefore, when studying such one-component surface waves, without loss of generality, one can consider systems of reduced dimension (for example, two-dimensional), in which the propagation of Rayleigh polarization waves is impossible. As well as for the waves of Rayleigh polarization, it is possible to consider the transition from “ordinary” SWs, in which the decrease in the amplitude of oscillations with distance from the surface occurs monotonically, to “generalized” SWs, in which a decrease in the amplitude of the wave is accompanied by its oscillations [7, 8].

In the case of an ideal surface, the existence of the Rayleigh SWs is due to the presence of two wave components with polarizations in the sagittal plane. Moreover, in the framework of the local and linear theory of elasticity, purely shear surface waves are absent. Their existence in the case of a continuous medium is possible only when the properties of the medium bulk differ from those of the surface

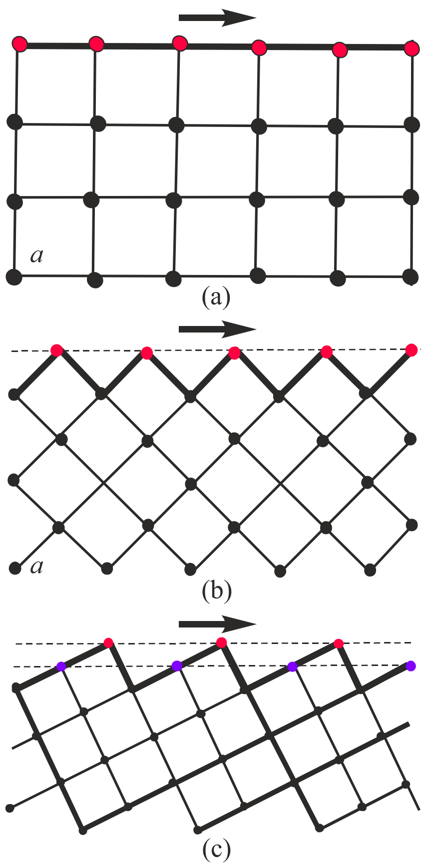

Fig. 1. Two-dimensional square lattice with various ideal surface orientations.

layer or when nonlocality (spatial dispersion) is taken into account. The presence of a surface covering of an elastic half-space leads to the appearance of purely shear so-called the Love waves [9]. The situation can change when passing from a continuous elastic medium to a crystal lattice. It was shown in [5, 10] that for a certain orientation of the lattice axes with respect to the surface, pure shear SWs become possible even in the case of an ideal surface. The considered crystal orientation (110) is shown in Fig. 1(b). Surface atoms are highlighted in the figure, but it was assumed that in their properties (masses and energy of interaction with neighbors) they did not differ from bulk atoms. At the same time, it was shown that with the orientation of the (100) surface shown in Fig. 1(a), surface shear waves are absent when interaction with only the nearest neighbors is taken into account. In [10], the question was formulated about the existence of surface waves when the orientation of an ideal surface is intermediate between these two orientations, i.e., for the orientations of the form (1*n*0). A particular case of this orientation (120) is shown in Fig. 1(c).

The last example differs from the two previous ones because due to the surface geometry there is no symmetry $x \to -x$ or $n \to -n$, where x is the coordinate along the direction of the surface and n are the numbers of atoms in this direction. In this paper, we consider the question of the influence of symmetry on the properties and conditions for the existence of surface waves. In this case, we restrict ourselves to systems in which the symmetry of the medium satisfies the condition of the indicated symmetry, but the elastic properties do not possess such symmetry — it is broken by the anisotropy of elastic interactions. Two models will be considered: an anisotropic crystal cubic lattice with surface orientation (110) and an anisotropic continuous elastic medium with a surface coating.

2. Surface shear waves in a semi-bounded crystal with a square lattice and anisotropy of elastic properties

Consider the modification of the model proposed earlier in [10] and shown in Fig. 2.

The Fig. 2 shows the sagittal plane of the crystal, which is a half-space of a planar square lattice with a lattice constant a and with a surface oriented perpendicular to the axis y. We assume that the nearest neighboring atoms interact and this interaction is different in the directions of the crystallographic axes. In the figure, the surface atoms are highlighted and atoms in the volume are shown separately. Due to the orientation of the surface, it is convenient to separate atoms in layers with even numbers from the surface and with displacements v_{nm} and in layers with odd numbers and displacements u_{nm}, where the first index indicates the number of the atom in the layer, and the second indicates the number of the layer. The elastic moduli are equal to μ and μ' along the two crystallographic directions (see figure). [Since shear waves with displacements perpendicular to the plane (*XY*) are considered, these are two shear moduli.] Due to the difference in elastic moduli, the system does not have symmetry with respect to reflection $x \to -x$ or $n \to -n$.

The equations of motion for the bulk atoms for the layers highlighted in Fig. 2 have the form (for simplicity, we set the masses of atoms equal to unity)

$$\ddot{u}_{n+1,m} + \mu'\left(2u_{n+1,m} - v_{n+2,m} - v_{n,m+1}\right) + \\ + \mu\left(2u_{n+1,m} - v_{n,m} - v_{n+2,m+1}\right) = 0, \quad (1)$$

$$\ddot{v}_{n,m} + \mu'\left(2v_{n,m} - u_{n+1,m-1} - u_{n-1,m}\right) + \\ + \mu\left(2v_{n,m} - u_{n-1,m-1} - u_{n+1,m}\right) = 0. \quad (2)$$

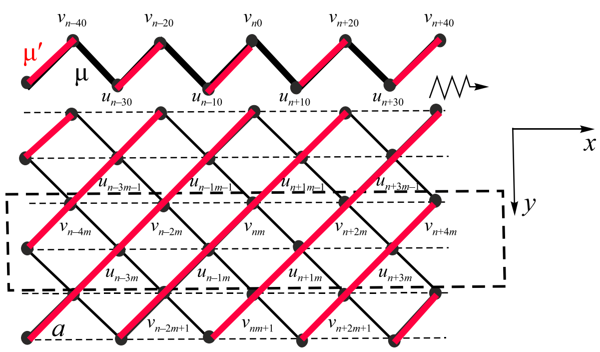

Fig. 2. Anisotropic half-space model for the square lattice crystal.

In a bulk wave propagating along the axis x, the displacements do not depend on the layer number m and have the form

$$u_{n,m}=u\exp(-i\omega t+ikan),\ v_{n,m}=u\exp(-i\omega t+ikan). \quad (3)$$

Substituting the expressions (3) into Eqs. (2), we obtain the dispersion law, which can be written for two branches of the in-phase ($u=v$) and antiphase ($u=-v$) oscillations of the nearest layers:

$$\omega_1=\sqrt{2}\omega_0\sin(ka/2),\ u=v, \quad (4)$$

$$\omega_2=\sqrt{2}\omega_0\cos(ka/2),\ u=-v, \quad (5)$$

where $\omega_0=\sqrt{2(\mu+\mu')}$. These dependences are shown in Fig. 3 in the form of curves $u=v$ and $u=v$.

In the long-wavelength limit $ak<<1$, the bulk waves have the usual spectrum of elastic waves with negative dispersion ($d^2\omega/dk^2>0$)

$$\omega_v=\sqrt{2}\omega_0\sin(ka/2)\approx\frac{\omega_0}{\sqrt{2}}\left(ka-\frac{k^3a^3}{24}\right). \quad (6)$$

Let us consider the possibility for the existence of surface waves. For them, the system of equations for the dynamics (1), (2) is supplemented by the boundary condition for the surface atoms

$$\ddot{v}_{n,0}+\mu'(v_{n,0}-u_{n-1,0})+\mu(v_{n,0}-u_{n+1,0})=0. \quad (7)$$

This equation can be rewritten as

$$\ddot{v}_{n,0}+\mu'(2v_{n,0}-u_{n+1,-1}-u_{n-1,0})+\mu(2v_{n,0}-u_{n,-1}-u_{n+1,0})-$$
$$-(\mu+\mu')v_{n,0}+\mu' u_{n+1,-1}+\mu u_{n,-1}=0, \quad (8)$$

formally introducing the displacements of the absent atoms outside the boundary $u_{n,-1}$.

We consider the solution for surface waves in the form

$$u_{n,m}=u\exp(-i\omega t+ikan-\kappa am),$$
$$v_{n,m}=v\exp(-i\omega t+ikan-\kappa am). \quad (9)$$

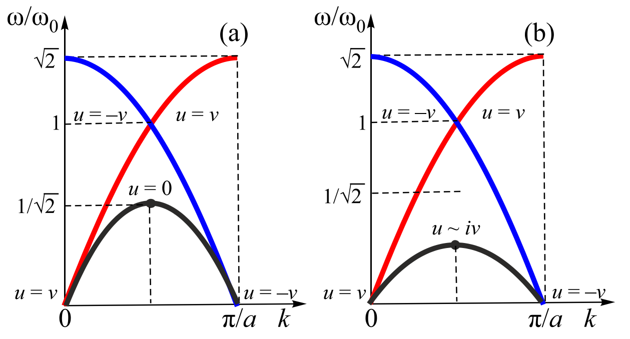

Fig. 3. Dispersion laws for the bulk and surface waves in cases $\mu=\mu'$ (a) and $\mu\neq\mu'$ (b). (The values of the parameter ω_0 in the two figures are different.)

Stationary surface oscillations correspond to the solutions with purely real values of frequency ω and wave vector k in the direction of the axis x. As for the parameter κ, it can be both purely real in ordinary surface waves and complex in generalized surface waves, in which the decrease in the amplitude inside the crystal is accompanied by its oscillations. Substituting the expressions (9) into Eqs. (1), (2), we obtain the following system of algebraic equations:

$$(\omega^2-\omega_0^2)v+\mu' u(e^{iak}e^{a\kappa}+e^{-iak})+\mu u(e^{-iak}e^{a\kappa}+e^{iak})=0, \quad (10)$$

$$(\omega^2-\omega_0^2)u+\mu' v(e^{iak}+e^{-iak}e^{-a\kappa})+\mu v(e^{-iak}+e^{iak}e^{-a\kappa})=0. \quad (11)$$

We can use the transformed above Eq. (8) for the displacements, which now have the same form (9) as in all other layers. Therefore, in this expression, the first and second lines are nullified separately. In this case, the displacements in the "layer" with $m=-1$ have an amplitude $\sim\exp(\kappa a)$. Since in this approach the double layer with $m=0$ does not differ from all other pairs of layers, the amplitude ratio u/v is preserved in it. Equating the first and second lines in (8) to zero, we obtain the system of equations (10), (11) for all layers, including the surface and the boundary conditions in the form

$$(2\omega^2-\omega_0^2)v+2u(\mu e^{iak}+\mu' e^{-iak})=0. \quad (12)$$

In the isotropic case with $\mu=\mu'=\omega_0^2/4$ considered in [10], these equations are simplified and have a simple solution:

$$\omega=\frac{\omega_0}{\sqrt{2}}\sin ak,\quad \cosh\kappa=\frac{1+\cos^4 ak}{2\cos^2 ak}. \quad (13)$$

Thus, in the case of an isotropic crystal, the parameter κ is purely real, and the surface wave has the ordinary form, and its amplitude decreases monotonically with distance into the interior of the crystal. In this case, it follows from (13) that at $k=0$ and π/a the parameter κ vanishes and the surface wave is delocalized. Maximum localization occurs at a value $k=\pi/2a$ at which $\kappa=\infty$. At this point of the spectrum we have $u=0$, i.e., only surface atoms oscillate in antiphase, and all other atoms are motionless. In addition, it follows from boundary condition (12) that

$$u=v\cos ak. \quad (14)$$

Therefore, the layers oscillate relative to each other with different amplitudes. This difference in the vibration amplitudes of even and odd layers, even in the depth of the crystal, is a "memory" of the difference in oscillations of the surface and near-surface layers in which atoms are in different conditions. In the limit $k\to 0$, they oscillate in phase ($u=v$), and in the limit $k\to\pi/a$, in antiphase ($u=-v$). In the long-wavelength limit for the surface and bulk waves, we have the asymptotics of the oscillations spectra:

$$\omega_s = \frac{\omega_0}{\sqrt{2}}\sin ak \approx \frac{\omega_0}{\sqrt{2}}\left(ak - \frac{a^3k^3}{6}\right),\ \omega_v \approx \frac{\omega_0}{\sqrt{2}}\left(ak - \frac{a^3k^3}{24}\right). \tag{15}$$

So, in the isotropic case with $\mu' = \mu$, the splitting off for the frequency ω_s of the surface waves from the frequency ω_v of the bulk waves occurs only in the second order in the amplitude of the wave vector $\Delta\omega \sim \omega_0 a^3 k^3$. In Fig. 3(a), the lower branch corresponds to surface waves.

In the case of an anisotropic crystal with $\mu \neq \mu'$, the situation is more complicated. As shown below, the surface wave becomes a generalized SW. In this case, it is convenient to introduce the complex quantity

$$G = \mu' e^{ika} + \mu\, e^{-ika} = (\mu' + \mu)\cos ak + i(\mu' - \mu)\sin ak, \tag{16}$$

depending only on the parameter k. In terms of this parameter, the oscillation equations and boundary conditions (10)–(12) take a simple form

$$\left(\omega^2 - \omega_0^2\right)v + \left(e^{a\kappa}G + \bar{G}\right)u = 0, \tag{16}$$

$$\left(\omega^2 - \omega_0^2\right)u + \left(G + e^{-a\kappa}\bar{G}\right)v = 0, \tag{17}$$

$$\left(2\omega^2 - \omega_0^2\right)v + 2u\bar{G} = 0. \tag{18}$$

Note that in this case the amplitudes of the oscillations u and v become complex. Eliminating the parameter κ from the system of equations (16), (17) and using the relation u/v from the boundary condition (18), we find the dispersion law of surface waves

$$\omega_s = \sqrt{\frac{4\mu\mu'}{\mu + \mu'}}\sin ak = \frac{\omega_0}{\sqrt{2}}\sqrt{1 - \left(\frac{\mu - \mu'}{\mu + \mu'}\right)^2}\sin ak. \tag{19}$$

Thus, the asymptotics of the dispersion law for small wave vectors has the form $\omega_s^2 \approx a^2k^2 4\mu\mu'/(\mu + \mu')$. For comparison: the asymptotics of the dispersion law for bulk waves in this limit is equal to $\omega_v^2 \approx a^2k^2(\mu + \mu')$. The splitting off for the frequencies of the surface wave from the frequencies of the bulk waves occurs already in the first order in the amplitude of the wave vector:

$$\Delta\omega \approx ak\frac{(\mu - \mu')^2}{2(\mu + \mu')^{3/2}}. \tag{20}$$

For $ak = \pi/2$, the local mode frequency reaches its maximum value $\omega_m/\omega_0 = \sqrt{1 - (\mu - \mu')^2/(\mu + \mu')^2}/\sqrt{2} < 1/\sqrt{2}$. At this point, $u = iv(\mu' - \mu)/(\mu' + \mu)$. Note that, in the case of an anisotropic crystal, the displacements of different atoms are no longer in-phase or antiphase: a phase shift arises between them. At the point of the maximum frequency, this phase shift is equal to $\pi/2$. In the general case of arbitrary values of the parameter k, the ratio of the amplitudes of oscillations for the layers has the form

$$\frac{u}{v} = \cos ak + i\frac{\mu' - \mu}{\mu' + \mu}\sin ak = \frac{G}{\mu' + \mu}. \tag{21}$$

The frequency dependence for the local mode is shown as the bottom curve in Fig. 3(b).

Taking into account the dependence of the frequency on the wave number (19) and the ratio of the amplitudes (21) from equation (17), it is easy to find an expression for the parameter κ of the wave attenuation with a distance from the surface:

$$\exp\left(-\frac{a\kappa}{2}\right) = \frac{G}{\mu' + \mu}. \tag{22}$$

This expression shows that in the case of a stationary localized surface wave with a real frequency and wave vector, the parameter κ cannot be purely real, and the wave cannot be a simple surface wave. Let the parameter κ be complex, $\kappa = g + ip$, and equate to zero the real and imaginary parts of relation (22). In this case, we obtain the dependences of the smoothly decreasing and oscillating parts of the surface wave on the wave vector k:

$$\tanh\frac{ag}{2} = \frac{2\mu'\mu\tan^2 ak}{(\mu + \mu')^2 + \left(\mu^2 + \mu'^2\right)\tan^2 ak}, \tag{23}$$

$$\tan\frac{ap}{2} = \frac{\mu - \mu'}{\mu + \mu'}\tan ak. \tag{24}$$

Since the oscillations of the amplitude are determined by a factor $\exp(iamp)$, the period of the oscillations is equal to $L = 2\pi\sqrt{2}/p$ and is determined by expression (24). In the limit $k \to 0$, the oscillation period becomes infinite, and at $k = \pi/2a$, the oscillation period is equal to $L = 2\sqrt{2}a$. That is, the layers through one oscillate in antiphase. If these are v layers, then the u layers also oscillate in antiphase, but with a different amplitude $|u| = |v|(\mu' - \mu)/(\mu' + \mu)$ and with a phase shift of the oscillation by $\pi/2$.

Formula (23) determines the dependence of the characteristics of the exponential decay of the SW amplitude on the wave vector. From this expression it follows that in the limit $k \to 0$ the surface wave is delocalized ($g \to 0$), and at $k = \pi/2a$ we have $\exp(-ag/2) = (\mu - \mu')/(\mu + \mu')$. Consequently, in this limit with weak anisotropy $\mu' - \mu << \mu$, the parameter g increases, and the wave penetration depth tends to zero. Note that the considered system admits a state with a frequency $\omega/\omega_0 = 1/\sqrt{2}$ localized exactly on the surface atoms, which oscillate in antiphase with the amplitudes $v_{n,0} = -(\mu'/\mu)v_{n+2,0}$.

Thus, in the proposed model, which does not possess symmetry $x \to -x$, $n \to -n$, it remains possible for the existence of specific surface waves, which differ significantly from the SW in the isotropic case and are characterized by an oscillating decrease in amplitude with distance into the crystal.

However, this relation between the characteristics of surface waves and the properties of symmetry is not general. In the next section, a model is considered in which symmetry breaking leads to the impossibility of the propagation of stationary surface waves.

3. Dynamics of an anisotropic elastic medium with a thin surface coating

As another model, consider the previous crystal lattice, which has different shear moduli along two orthogonal directions, but within of a continuous medium. In this case, consider the configuration of a half-space, which also does not have mirror symmetry of reflection relative to a plane perpendicular to the plane of the surface. In the case of a continuous medium, pure shear waves do not localize near an ideal surface. Therefore, in order to compare situations with the presence and absence of this symmetry, let us consider purely shear Love waves [10], which can exist in an isotropic medium with a half-space covered with a thin layer of another substance, and which are also scalar. The Love waves exist in the case when, for example, a half-space with a shear modulus μ and density ρ_0 is covered with a layer of thickness l of another substance with the same modulus, but with a higher density $\rho/\rho_0 = s > 1$. At small coating thicknesses, the frequency spectrum of the surface wave of the type $\exp(-i\omega t + ikx - \kappa y)$ has the form $\omega^2 \approx \mu k^2 - \mu(s-1)^2 l^2 k^4$ and is split off from the frequency spectrum of the bulk waves $\omega^2 = \eta k^2$. (The coordinate x is directed along the wave propagation in the plane of the surface, and the coordinate y is perpendicular to the surface into the bulk of the medium.) The decrement of the wave amplitude with distance from the surface is determined by the parameter $\kappa \approx l k^2 (s-1)$. This result can be obtained in a simpler model by replacing the layer with changed properties with a δ function. In this case, the effective equation will have the obvious form:

$$\ddot{u} - \mu u_{xx} - \mu\, u_{yy} = -(s-1)\delta(y/l)\ddot{u}, \tag{25}$$

where u are the transverse displacements of the medium in the direction perpendicular to the sagittal plane (x, y). This simplified model exactly retains the results presented above for the spectrum of surface waves and the amplitude decrement.

Let us consider an anisotropic medium with a thin coating, and assume that in two orthogonal directions the shear moduli have different values μ and μ'. Keeping the designations chosen above for the coordinates (x, y) associated with the geometry of the problem, we introduce a coordinate system (ξ, η) associated with orthogonal axes along which the elastic bonds are characterized by shear moduli μ and μ', respectively. These two coordinate systems can be rotated relative to each other in the sagittal plane at an arbitrary angle φ. The geometry of the problem is shown in Fig. 4.

In the coordinate system associated with the axes (ξ, η), the dynamical equation in the bulk of the half-space has the form

$$\ddot{u} - \mu' u_{\xi\xi} - \mu\, u_{\eta\eta} = 0. \tag{26}$$

The Fig. 4 shows the connection between the two coordinate systems:

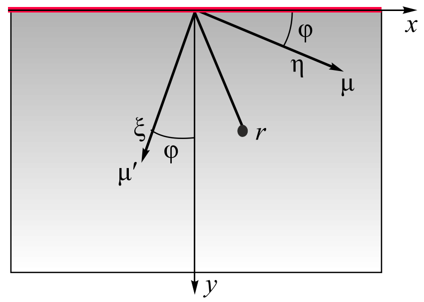

Fig. 4. Anisotropic elastic half-space with the surface coating with thin film.

$$y = \xi\cos\varphi + \eta\sin\varphi, \quad x = -\xi\sin\varphi + \eta\cos\varphi. \tag{27}$$

Using these relations, we rewrite Eq. (26) in a new coordinate system, taking into account the presence of a coating on the surface $y = 0$, similar to the case of an isotropic medium and Eq. (25):

$$\ddot{u} - \left(\mu\sin^2\varphi + \mu'\cos^2\varphi\right)u_{yy} - \left(\mu\cos^2\varphi + \mu'\sin^2\varphi\right)u_{xx} + $$
$$+ 2\left(\mu' - \mu\right)\sin\varphi\cos\varphi\, u_{yx} = -\left(s-1\right)\delta\left(y/l\right)\ddot{u}. \tag{28}$$

For a bulk wave of the form $u = u_0\exp(-i\omega t + ikx)$ propagating along the surface in the direction of the axis x, the dispersion relation has the form

$$\omega^2 = \left(\mu\cos^2\varphi + \mu'\sin^2\varphi\right)k^2. \tag{29}$$

To obtain the boundary condition, we supplement the considered half-space in the region $y < 0$ with a similar half-space and use the solutions that are symmetric with respect to the plane $y = 0$. Then we integrate Eq. (28) over a narrow region near the interface, tending the region of integration to zero. It is easy to see that in this case a boundary condition of the following form arises:

$$\left(\mu\sin^2\varphi + \mu'\cos^2\varphi\right)u_y\Big|_{y=0} = -l\omega^2\left(s-1\right)u\Big|_{y=0}. \tag{30}$$

Let us consider the question of the possibility for the existence of surface waves in this system. It is natural to look for a solution for such waves in the form $u = u_0\exp(-i\omega t + ikx - \kappa y)$. A normal surface wave should correspond to a solution of this type with real frequency ω, real wave vector k, and real parameter κ. The generalized surface wave corresponds to the complex parameter κ. Substituting the solution of the indicated form into the equation in the bulk and into the boundary condition, we obtain the relations

$$\omega^2 = k^2\left(\mu\cos^2\varphi + \mu'\sin^2\varphi\right) - \kappa^2\left(\mu\sin^2\varphi + \mu'\cos^2\varphi\right) -$$
$$-2ik\kappa\left(\mu' - \mu\right)\sin\varphi\cos\varphi, \tag{31}$$

$$\kappa\left(\mu\sin^2\varphi + \mu'\cos^2\varphi\right) = l\omega^2\left(s-1\right). \tag{32}$$

It follows from relation (32) that the parameter κ at a real frequency (i.e., for stationary waves) can take only the real values. However, it follows from the first equation in (31) that the wave vector k cannot be real. So, for an arbitrary angle φ, when the system does not have symmetry of reflection with respect to the coordinate $x \to -x$, there are no stationary surface waves. As can be seen from (31), they are possible only in an isotropic medium with $\mu = \mu'$ or in an anisotropic medium with $\mu \neq \mu'$, but only at angles $\varphi = 0$ and $\pi/2$ at which the indicated symmetry is restored even in a nonisotropic medium. In the long-wavelength limit, in the case $\varphi = 0$, the surface wave has a frequency spectrum $\omega^2 \approx \mu k^2 - \mu'\mu l^2 k^4 (s-1)^2$ and its localization is characterized by a parameter $\kappa = (\mu/\mu') l k^2 (s-1)$. In the case $\varphi = \pi/2$, the corresponding characteristics have the form $\omega^2 \approx \mu k^2 - \mu'\mu l^2 k^4 (s-1)^2$ and $\kappa = (\mu'/\mu) l k^2 (s-1)$.

In the absence of dissipation in the system when a surface wave is excited, its frequency is a real quantity, but in this case the values of the parameters κ and k become complex. Excluding the quantity κ, it is easy to relate the frequency ω to the complex value of the wave vector along the axis x:

$$\omega^2 = \omega_v^2(k)/P, \tag{33}$$

where the dependence $\omega_v(k)$ corresponds to the dispersion law of bulk waves in an unbounded medium (29), but with a complex vector k and

$$P = 1 + l(s-1)\left(1 + ik\frac{2(\mu'-\mu)\sin\varphi\cos\varphi}{\mu\sin^2\varphi + \mu'\cos^2\varphi}\right). \tag{34}$$

These expressions are simplified in the case of a weak anisotropy of the medium with $\mu' - \mu << \mu$ and with a weak effect of the surface coating: $l(s-1) << 1$. When introducing a complex vector $k = K + i\Gamma$, we can calculate the depth of penetration of the surface wave into the bulk in the direction of the axis x: $L_x = 1/\Gamma$ and compare it with the localization region in the near-surface layer in the direction of the y axis: L_y. In the indicated approximation, these parameters have the values

$$L_x \approx \frac{\mu}{l(s-1)\omega^2}\frac{1}{\varepsilon}, \quad L_y \approx \frac{\mu}{l(s-1)\omega^2}, \tag{35}$$

where the parameter ε is small and equal to

$$\varepsilon = (\mu' - \mu)\sin\varphi\cos\varphi << 1. \tag{36}$$

Consequently, the depth of wave penetration along the surface significantly exceeds its localization near the surface.

Conclusion

It is shown that the nature and the conditions for the existence of shear surface elastic waves substantially depend on the symmetry of the properties of the elastic medium or the crystal lattice with respect to the symmetry of the crystal surface. Two models of an anisotropic square lattice and an anisotropic continuous medium with a coating surface are considered, in which the anisotropy of elastic properties breaks the reflection symmetry in a plane perpendicular to the surface plane. In the case of a crystal lattice with a surface orientation (110), the existence of a surface wave remains possible, but it transforms into a generalized surface wave with amplitude modulation, which accompanies its monotonic decrease into the bulk of the crystal. In addition, the splitting off for the spectrum of surface waves from the spectrum of the bulk waves increases: the corresponding frequencies differ already in the first order in the amplitude of the wave vector in the long-wavelength limit. In the model for an anisotropic continuous medium bounded by a surface with a “heavy” surface coating, the anisotropy of elastic moduli leads to the absence of stationary surface waves, but they penetrate into the bulk of the material along the surface over long distances, inversely proportional to the anisotropy of the medium.

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Зсувні поверхневі хвилі та симетрія системи

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Розглянуто вплив анізотропії пружного середовища та кристалічної гратки, яка призводить до порушення симетрії напівобмеженого простору відносно відбиття, на можливість існування та властивості поверхневих пружних зсувних хвиль. Показано, що при збереженні кристалографічної симетрії при орієнтації поверхні (110) анізотропія усуває вказану симетрію, але зберігає можливість розповсюдження сильномодифікованих поверхневих хвиль. У випадку анізотропного напівобмеженого простору з тонким покриттям, який допускає розповсюдження хвиль Лява в ізотропному середовищі, анізотропія призводить до відсутності таких стаціонарних хвиль, що не згасають вздовж поверхні середовища.

Ключові слова: поверхневі хвилі, зсувні хвилі, хвилі Лява, кристалографічна симетрія.