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The turbulent drag force in superfluid ^{3}He–^{4}He mixtures under oscillations of a quartz tuning fork

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We have studied the resonance curves of a quartz tuning fork of the fundamental frequency 32 kHz immersed in superfluid ^{4}He and mixtures ^{3}He–^{4}He with ^{3}He concentration of 5 and 15 % in the temperature range of 0.35–2.5 K at saturated vapor pressure. Two types of experiments have been carried out, with a tuning fork both not covered by a bulb and coated by a bulb, i.e., in the restricted geometry. In both cases, the velocity-force dependences for the tuning fork showed a linear damping force at low peak velocities and extra drag due to the appearance of vortex lines accompanying the transition to turbulence under increasing peak velocity. These dependencies are mainly determined by the density of the normal helium component. There is a pronounced difference between superfluid ^{4}He and mixtures of ^{3}He in ^{4}He, where ^{3}He impurity particles provide a constant temperature-independent contribution to the normal component of the mixture. The extra contribution to the damping force, so called “turbulent drag force”, decreases with concentration increase at the same peak velocity of the tuning fork that can be explained by the extension of the range of laminar flow with an increase in the concentration of ^{3}He. We found that the drag coefficient in superfluid ^{4}He and mixtures ^{3}He–^{4}He reaches a plateau at different peak velocities and different exciting forces and explained this fact by different conditions for vortex formation depending on the different thickness of the near-wall viscous layer. The comparison between the data obtained in restricted and unrestricted geometries shows that there is an excessive dissipation of the tuning fork motion associated with the emission of the first sound wave in unrestricted geometry.

Keywords: superfluid helium, superfluid mixtures ^{3}He–^{4}He, turbulence and laminar flows, quartz tuning fork.

1. Introduction

Last decades there is a great interest in the problem of the onset and development of classical and quantum turbulence in a superfluid liquid. The main tools for generating the laminar and turbulent flows are oscillating bodies immersed in a liquid: wires, grids, spheres, and quartz tuning forks in liquid ^{3}He, ^{4}He, and their mixtures. The characteristics of vibration processes allow obtaining an extensive information about the dynamics of superfluids and dissipative processes accompanying the liquid flow. For example, measurements of the resonant frequency provide information on the inertial backflow around the vibrating body while the damping of oscillations characterizes the dissipation that accompanies the motion of the superfluid and normal components of the liquid.

Earlier there was shown [1–5] that there is a change in the dependence of the peak oscillator vibration rate on the excitation force at a certain critical velocity and a quadratic dependence appears instead of a linear one in superfluid ^{4}He and ^{3}He–^{4}He mixtures. The temperature dependence of the critical transition rate was also obtained. This change is attributed to the transformation of a laminar oscillating flow into a turbulent one [6]. Another feature of the transition from a laminar flow to a turbulent one is the change in the dependence of the drag coefficient to the movement of the tuning fork on the oscillation peak velocity. With an increase in the peak velocity, but at its low value, the drag

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coefficient decreases, and at higher value of the peak velocity, in the turbulent region, the drag coefficient value reaches a plateau.

At sufficiently low temperature where one can neglect the presence of the normal component of liquid helium, quantum turbulence can be generated in the liquid above the critical velocity [2, 3, 7, 8], associated with an appearance of the vortices with quantized circulation (quantum vortices) and subsequent evolution of their tangle. However, at a temperature above ≈ 1 K, where the presence of a normal component in superfluid helium is essential ("two-liquid" state), different scenarios of a transition to the turbulence are possible. It depends on whether the normal or superfluid component becomes unstable first and as well as on the intensity of interaction between the components [9]. Therefore, for being clearer in the behavior of flows in helium and the onset of turbulence, one should study not only the superfluid liquid at low temperatures but also in the "two-liquid" state of helium both near the superfluid transition temperature and above it, in a completely classical liquid. Previously, such studies were carried out in pure ^{4}He [1, 10, 11]. At the same time, ^{3}He–^{4}He mixtures are still little studied, whereas the presence of ^{3}He impurities ensures the existence of a normal component up to zero temperature and the structure of quantized vortices strongly differs from that in pure ^{4}He due to the condensation of ^{3}He impurities in the vortex core [12, 13].

In the present work, we study the features of the transition from laminar to turbulent flow in superfluid ^{3}He–^{4}He mixtures and the effect of ^{3}He impurities on the stability of the normal component in liquid solution under oscillation of a quartz tuning fork in the temperature range 0.4–4.2 K. Also the experimental dependences are obtained of the oscillation peak velocity of the tuning fork prong top on the excitation force, as well as the dependence of the turbulent drag force of the tuning fork and the drag coefficient on the peak velocity of the tuning fork in the region of transition from laminar to turbulent flow. The features of tuning fork oscillations in a mixture in the presence of a protective bulb surrounding the tuning fork, restricting the propagation of acoustic waves, are also investigated. The dependences obtained in ^{3}He–^{4}He mixtures are compared with those in pure ^{4}He.

2. Experimental procedure

The method of a piezoelectric quartz tuning fork vibrating in the liquid was used for the experimental study of turbulent drag force in concentrated mixtures of ^{3}He in ^{4}He. Tuning forks of industrial production were used with a frequency f = 32768 Hz and geometric dimensions: prong height L = 3.79 mm, thickness H = 0.59 mm, width W = = 0.3 mm and the distance between prongs D = 0.3 mm. There were used both open tuning forks with a completely removed surrounding bulb and closed ones in the manufacturer's bulb of the inner diameter of 2.8 mm which has a small orifice for filling by the liquid. The tuning fork was placed in a cylindrical copper cell with an inner diameter of 11.8 mm and a height of 1.5 cm, which had a threaded thermal contact with the cold plate of the ^{3}He evaporation cryostat. The temperature of the liquid was measured by the calibrated RuO_2 resistance thermometer located inside the cell. The measuring cell was attached to the cold plate of the ^{3}He evaporation chamber so that the prongs of the tuning fork were facing downward to reduce possible contamination. The cell filling capillary was thermally short-circuited on the cold plate. The measurements were carried out at the fundamental frequency of the tuning fork at a constant cell temperature. The standard technique of full frequency sweeping near the tuning fork resonance was used in the experiments. The electrical circuit that used for measurements is similar to that commonly used in experiments with similar tuning forks and is described in detail in [5].

The tuning forks piezoelectric constant a was determined in the beginning of each experiment by measuring the amplitude-frequency characteristics of the tuning fork in vacuum at temperature about ≈ 1.5 K and the peak values of the excitation voltage of the tuning fork U_0 = 0.01–0.1 V:

$$a = \sqrt{\frac{4\pi m_{\text{eff}} \Delta f_{\text{vac}} I_0}{U_0}}\ , \qquad (1)$$

where I_0 and Δf_{vac} are the measured values of the current amplitude of the measuring signal and the width of the resonance line at resonance in vacuum, respectively, $m_{\text{eff}} = (\rho_q LWH)/4$ is the effective mass of the tuning fork, and ρ_q is the density of quartz. To establish a connection between the measured electrical characteristics of the tuning fork with its mechanical properties, we used the relations given in Refs. 2 and 14. The peak velocity v of the top of the prongs is related to the amplitude of the current I, excited by the movement of the tuning fork, by the ratio $v = I/a$, and the driving force F and the amplitude of the applied voltage U as $F = aU/2$. In practice, the tuning fork constant depends on its geometry and surface quality; therefore, before each experiment the cell and all connecting lines were thoroughly washed with dry gaseous nitrogen and pumped out by mechanical and adsorption pumps.

3. Results

The study of superfluid mixtures of ^{3}He in ^{4}He of different concentrations makes it possible to extend the possibilities of studying the flows excited by the quartz tuning fork vibrations. In actual work, the main interest is in determining the influence of impurity ^{3}He particles on the turbulent drag force arising during the transition to turbulence and the drag coefficient, as well as the influence of the confining geometry, which suppresses the emission of the first sound. It should be noted that, in the present work, we consider only the initial stage of the transition to a turbulent flow. Developed turbulence requires higher flow velocities. The temperature dependences of the critical velocity of transition to a turbulent flow regime in ^{4}He and concentrated mixtures of 5 and 15 % ^{3}He in ^{4}He were obtained in [5].

3.1. Dependences of the tuning fork peak velocity on the excitation force

The dependences of the vibration peak velocity v of the tuning fork leg tip on the peak driving force F, within the fundamental resonance for different temperatures in ^{4}He and the mixtures with ^{3}He concentration of 5 and 15 % in superfluid and normal state at saturated vapor pressure are shown in Fig. 1. At the low excitation force of the tuning fork, the oscillation peak velocity of the prong is proportional to the driving force, $v \sim F$. The linear section corresponds to the potential flow of the superfluid component and the laminar flow of the normal component of helium. The ideal classical fluid behaves in the same way [6]. The ratio F/v is proportional to the width of the resonance line Δf and determines the damping coefficient of the tuning fork in a laminar flow regime. The tuning fork linear damping in concentrated mixtures of ^{3}He–^{4}He was treated in Ref. 15 as a combination of internal damping of a tuning fork, first and second sounds which make a noticeable contribution at the lowest temperatures, and hydrodynamic viscous damping, which dominates at higher temperatures.

Since the driving force increases above certain peak velocities, the shape of the tuning forks resonance curve changes. The resonant frequency decreases and the line width increases whereas the dependence $v(F)$ becomes $v^2 \sim F$. As it was shown in Ref. 6, this dependence appears as a result of the formation of vortices and the transition to a turbulent regime of fluid flow. Such behavior was previously observed for tuning forks oscillating in superfluid ^{4}He [2, 3], as well as for vibrating wires [16, 17] and spheres [7]. In addition, the transition to turbulence exhibits hysteresis [2] at low temperatures. In the present work, we used frequency sweeps near the resonance of a tuning fork that was taken at a constant temperature and with an increase of the exciting force.

In Fig. 1(a), one can see that the transition between laminar and turbulent regimes is rather smooth at high temperature in ^{4}He but becomes steep with decreasing temperature. In this case, the dependence of $v(F)$ has an obvious shift (the value of the excitation force increases) with temperature increase which is also preserved during the transition of ^{4}He to the normal state.

Dependences of $v(F)$ in a mixture ^{3}He in ^{4}He with a concentration of 5 % ^{3}He, Fig. 1(b), also demonstrate both laminar and turbulent regions in the entire temperature range (0.35–2.6 K), but in contrast to pure ^{4}He we did not observe the shift of the dependence under a temperature change in superfluid mixture. The shift occurs only in a normal mixture.

In the mixture with ^{3}He concentration of 15 %, Fig. 1(c), the dependences $v(F)$ are very close to each other in the entire temperature range in the superfluid state of the mixture and at the same time still have the dependences of $v(F)$ corresponding to the laminar and turbulent flow regimes. In the normal state of a 15 % solution at T = 2.25 K the dependence $v(F)$ shifts slightly. Note that in this case, one could not obtain the dependence of $v(F)$ at high values of the tuning fork excitation force, since the resonance curves cease to be a Lorentzian.

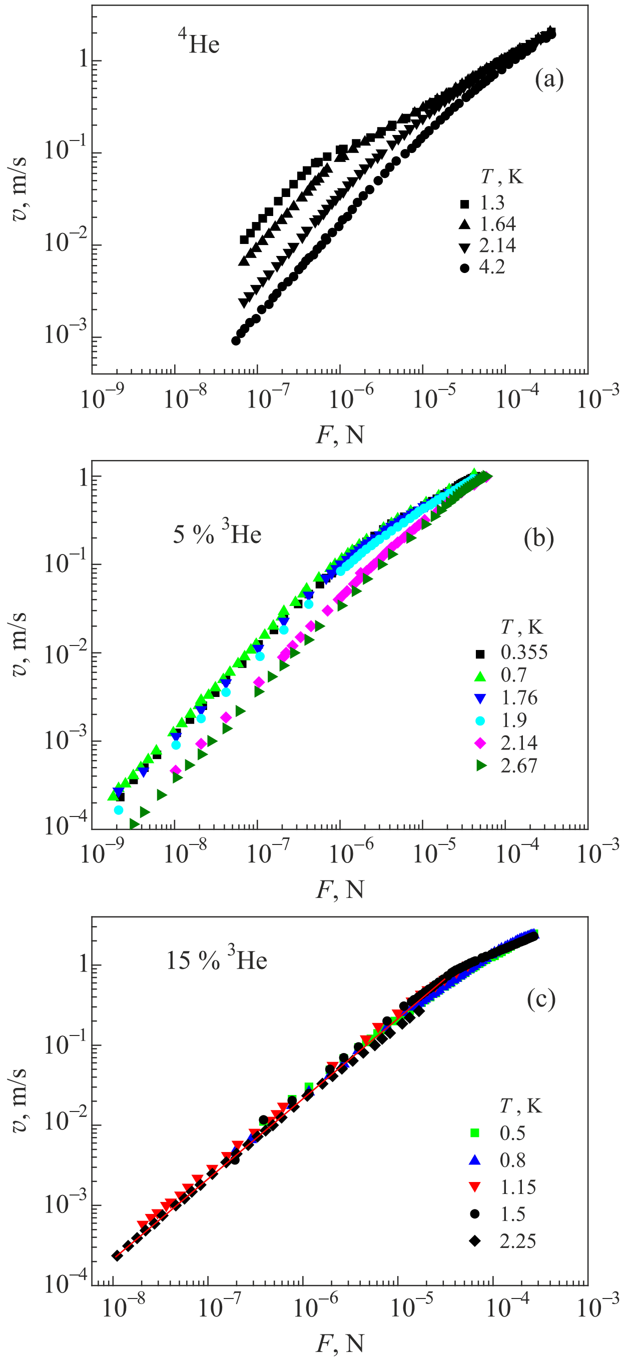

Fig. 1. (Color online) Dependences of the peak velocity of the tuning fork on the peak driving force within the fundamental resonance for different temperatures in ^{4}He (a), mixtures with a concentration of 5 % (b) and 15 % (c) ^{3}He in ^{4}He.

For a more detailed study of the differences in the behavior of the obtained dependences $v(F)$ in pure ^{4}He and concentrated mixtures of ^{3}He in ^{4}He, the turbulent drag force and drag coefficient of the studied liquids were estimated.

3.2. A turbulent drag force

The turbulent drag contribution to the damping force can be determined by subtracting, from the applied force, the force proportional to velocity, which corresponds to the laminar regime [2]:

$$F_{\text{turb}}(v) = F(v) - \gamma v\,, \tag{2}$$

where γ is determined from the ratio $\gamma = F/v$ at low peak velocities. Thus, we remove the intrinsic damping of the tuning fork and the contributions to the damping associated with the environment viscosity and the emission of the first and second sounds. Consequently, only the contribution to the tuning fork damping, associated with the onset and development of turbulence, remains in Eq. (2).

The dependences of the turbulent drag force on the tuning fork peak velocity $F_{\text{turb}}(v)$, obtained at constant temperatures for ^{4}He, are shown in Fig. 2(a). At low values of the peak velocity ($v < 0.2$ m/s), all points, both in superfluid and normal helium, almost fit in one-line, $\Delta F \sim v^2$. This result is consistent with that of Refs. 2 and 4. When the peak velocity increases, the dependences $F_{\text{turb}}(v)$ start to diverge, and one observes a tendency to a F_{turb} decrease when velocity v being constant and temperature increases and, consequently, the density of the normal component of helium also increases. Only the dependence $F_{\text{turb}}(v)$ at $T = 2.14$ K is slightly higher than the data obtained at $T = 2.14$ K near the λ transition.

A similar result for the dependence of the turbulent drag force on the peak velocity of the tuning fork is observed in a mixture with ^{3}He concentration of 5 %, Fig. 2(b). Here, as in pure ^{4}He, at low velocities, the dependencies merge into one line and diverge slightly only at $v > 0.4$ m/s. For these dependences of $F_{\text{turb}}(v)$, it seems difficult to determine the trend of change under increasing temperature. The weakest dependence $F_{\text{turb}}(v)$ is observed at $T = 2.67$ K, in a normal mixture in the absence of a superfluid component.

The most pronounced changes in the dependences $F_{\text{turb}}(v)$ under temperature increase are observed in the mixture with ^{3}He concentration of 15 %, Fig. 2(c). One also observes a region of low peak velocities, where the dependences of $F_{\text{turb}}(v)$ are almost the same, but they diverge sharply at higher velocities, while a decrease of the turbulent drag force is observed at a constant peak velocity and under increasing the temperature. One should also note that, in 15 % mixture, the least excess of the excitation force relative to the laminar regime is required to excite the turbulence. Another feature of this mixture is that at $T = 2.25$ K, in the normal mixture, one can see only the initial part of the $F_{\text{turb}}(v)$ curve, up to velocities $v \approx 0.3$ m/s. At higher peak velocities the resonance curves are strongly distorted compared to the Lorentzian.

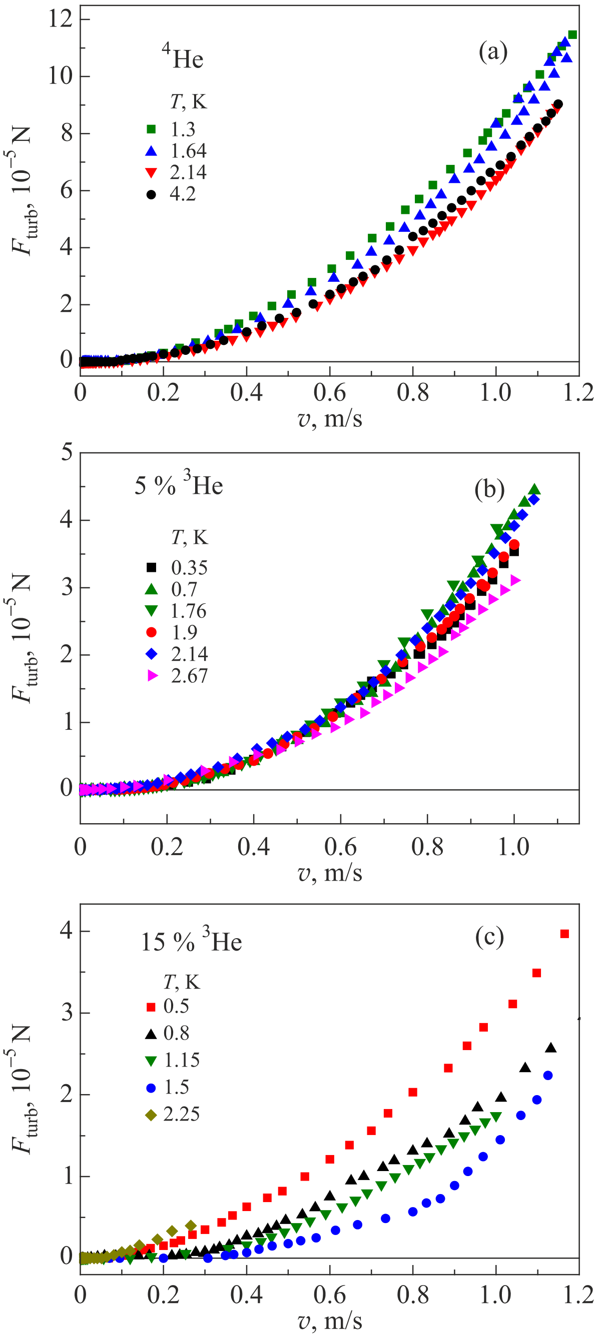

Fig. 2. (Color online) Dependences of the turbulent drag force $F_{\text{turb}}(v)$ on the peak vibration velocity of the tuning fork, obtained at fixed temperatures for ^{4}He (a), ^{3}He–^{4}He mixtures with ^{3}He concentrations 5 % (b), and 15 % (c).

3.3. The drag coefficient

The drag coefficient of the oscillating tuning fork from the liquid was determined using the expression obtained for the resistance force of the environment [2, 14]:

$$F = 1/2 C_D \rho A v^2\,, \tag{3}$$

where C_D is the drag coefficient, ρ is the density of helium, $A = LW$ is the cross-sectional area of the prong that perpendicular to the movement direction, where L and W are the length and width of the tuning fork prong, respectively. In the case of a tuning fork, drag force refers to the amplitude of the driving force at resonance which compensates for the drag force. Usually, instead of the drag force F the drag coefficient C_D is considered. According to Eq. (3) C_D is

$$C_D = 2F / (\rho A v^2). \tag{4}$$

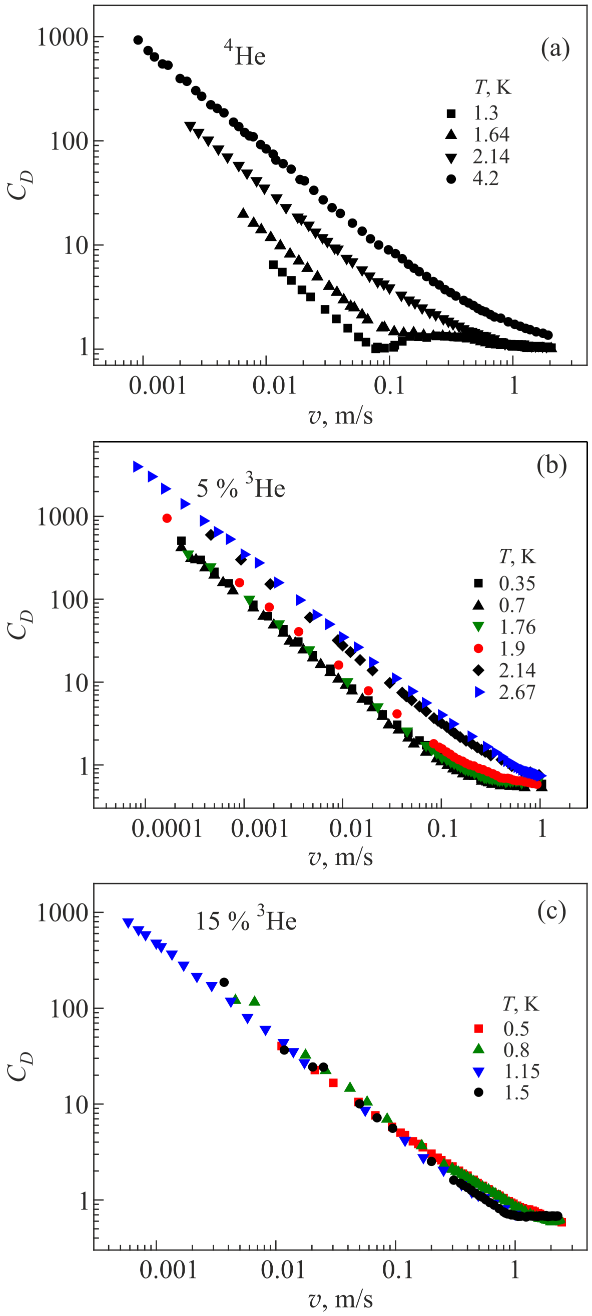

Fig. 3. (Color online) Drag coefficient C_D as a function of the peak vibration velocity of the tuning fork at fixed temperatures: ^{4}He (a), mixtures with a concentration of 5 % (b), and 15 % (c) ^{3}He in ^{4}He.

In Fig. 3, the dependences are shown of the drag coefficient C_D of the tuning fork on the peak velocity of the tuning fork tip at fixed temperatures in ^{4}He and mixtures with 5 and 15 % concentrations of ^{3}He in ^{4}He and corresponding to the data shown in Fig. 1. At small peak velocity amplitudes, the drag force is proportional to the velocity, therefore the drag coefficient changes as $C_D \sim 1/v$. Under velocity increase, due to the onset of turbulence, additional resistance appears and the dependence $F \sim v^2$ arises, and the drag coefficient smoothly passes to a constant value of the order of unity.

In superfluid ^{4}He, the drag coefficient decreases over the entire velocity range with decreasing temperature. The normal component density decreases according to the T^4 law, which leads to the decrease of the dissipation of the vibration energy due to the viscous friction of the oscillating tuning fork prongs. As a result the dependence $C_D \sim 1/v$ shifts to the lower velocities. In normal helium at $T = 4.2$ K, the peak vibration velocity of the tuning fork (≈ 2 m/s) is sufficient only for a slight deviation from the linear dependence $v(F)$ within the initial stage of turbulence.

The drag coefficient of ^{3}He–^{4}He mixtures at low velocity amplitudes is determined by the laminar flow of the viscous normal component of the liquid and is given by the dependence $C_D \sim 1/v$. At high-velocity amplitudes, turbulent forces contribute to the drag coefficient, and it approaches a constant value near unity. The temperature dependence of the drag coefficient is not observed in this velocity range at a given mixture concentration. It should be noted that there is an essential difference in the drag coefficients of 5 and 15 % mixtures (about 4 times at $v = 0.01$ m/s). This difference is explained by the higher resistance force of 15 % mixture (see Fig. 1) and a noticeably lower density.

3.4. Tuning fork in restricted geometry

The study of tuning forks in restricted geometry seems rather interesting because one excludes the influence of the first sound emission on the turbulence in superfluid helium. In present experiments, we investigate tuning forks in the manufacturer's bulb of an inner diameter of 2.8 mm. The bulb has a small orifice for its filling by liquid helium. The tuning fork vibrates in restricted geometry and in the temperature range studied the wavelength of the first sound at a frequency of 32 kHz (≈ 7 mm) exceeded the distance from the tuning fork to the wall of the confining bulb. The sound wave process did not develop, and the effect of acoustic emission of the first sound was negligible [15, 18]. This was experimentally verified in [19, 20], where measurements were made at different tuning fork frequencies. At high frequencies, when the wavelength of the first sound decreased noticeably and became comparable to the size of the bulb, a resonance was observed.

In actual work, the experiments with a tuning fork in restricted geometry were carried out in the mixture with ^{3}He

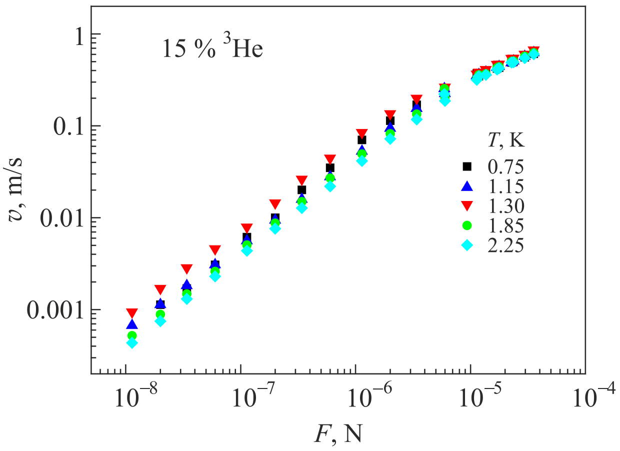

Fig. 4. (Color online) The peak vibration velocity of a tuning fork in a restricted geometry as a function of the driving force at few temperatures. ^{3}He in ^{4}He mixture with ^{3}He concentration of 15 %.

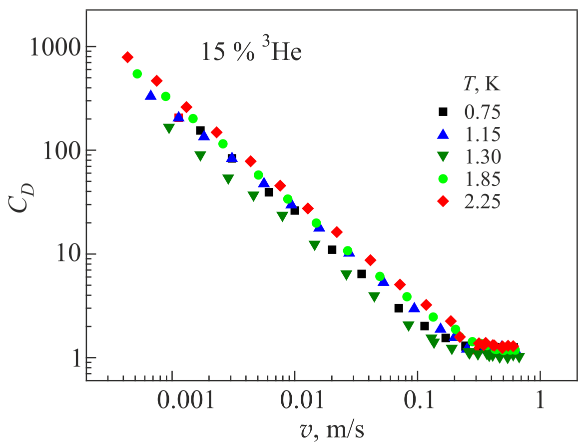

Fig. 6. (Color online) Drag coefficient as a function of the peak vibration velocity of the tuning fork (conditions are the same as in Fig. 5).

concentration of 15 %. In Fig. 4 we depicted the dependences of the peak vibration velocity of the tuning fork prongs on the driving force at fundamental resonance at few fixed temperatures. As for an unrestricted tuning fork, at low excitation forces, a linear section is observed in Fig. 4, where the peak oscillation velocity of the prong is proportional to the driving force. Also one observes section where the dependence $v(F)$ becomes $v^2 \sim F$. However, now the peak velocity, at same excitation force, is 2 to 3 times higher than that for an unrestricted tuning fork. This indicates a decrease in the vibration energy loss resulting to the onset of a turbulent state at lower excitation forces of the tuning fork.

Figure 5 shows the dependence of the turbulent drag force on the peak vibration velocity of the tuning fork prongs in restricted geometry at fixed temperatures. One observes, in the entire temperature range of the study, both in normal and superfluid state of the mixture with 15 % ^{3}He concentration, the dependences $F_{\rm turb}(v)$ are very close to each other. This result is very different from the dependences of Fig. 2(c) and indicates a noticeable contribution of the first sound to the energy dissipation of the tuning fork in restricted geometry. The turbulent drag force of the tuning fork in restricted geometry is 2–2.5 times higher at the same peak velocity and the mixture enters the turbulent zone at lower peak velocities.

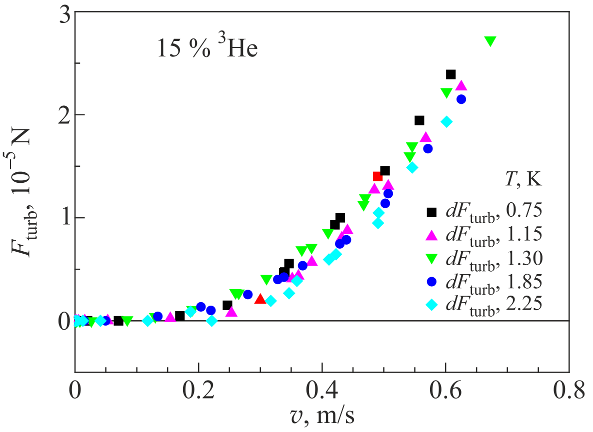

Fig. 5. (Color online) Dependence of turbulent drag force on the peak vibration velocity of a tuning fork in 15 % mixture of ^{3}He in ^{4}He in restricted geometry at fixed temperatures.

Figure 6 shows the dependences of the drag coefficient on the peak vibration velocity of the tuning fork prongs in restricted geometry in the 15 % mixture for few fixed temperatures. There are also dependences $C_D(v)$ run very close to each other with a slight shift towards the growth of the peak velocity with increasing temperature in the entire temperature range studied in both normal and superfluid state of the mixture. Comparing the behavior of the tuning forks in unrestricted and restricted geometry, one can note that in the latter case, the drag coefficient is approximately 2–2.5 times lower at the same peak velocity. Since the drag coefficient is a value equal to the ratio of the dissipated energy of the tuning fork to the kinetic energy of the fluid flow, it means that the dissipation energy decreases. This result is in agreement with the change in the dependence of the turbulent drag force on the peak vibration velocity of the tuning fork observed in restricted geometry.

Conclusions

The results of the experiments show that the dependences of the peak velocity of the oscillation of the tuning fork prongs tip on the peak driving force are, to a large extent, determined by the density of the normal helium component. There is a pronounced difference between superfluid ^{4}He and mixtures of ^{3}He in ^{4}He, where ^{3}He impurity particles provide a constant, temperature-independent contribution to the normal component of the mixture and where the transition to turbulence is accompanied by the condensation of ^{3}He into the vortex core under conditions of phonon and roton scattering on these impurities. It should also be noted that the depth of the near-wall layer (the depth of penetration of a viscous wave) is the lowest in 15 % solution compared to 5 % mixture of ^{3}He in ^{4}He and pure ^{4}He. Vortices are formed in this layer. Apparently, vortex formation is most difficult in 15 % mixture. This can explain that in this mixture one should apply the highest excitation force for the transition to a turbulent flow.

If the peak velocity in pure ^{4}He and the two studied mixtures is the same, then the turbulent drag force decreases with concentration increasing. Thus the range of laminar flow with a linear dependence $v(F)$ is “dragged out” with an increase in the concentration of ^{3}He and the difference $F(v) - \gamma v$ decreases (see Figs. 1 and 2).

The drag coefficient reaches a plateau at different peak velocities and different exciting forces. In 15 % mixture, the velocity to reach a plateau (which means a transition to a turbulent regime) is the highest in comparison with 5 % solution and pure ^{4}He. This can also be explained by the difficulty of vortex formation in such a mixture due to the lowest thickness of the near-wall viscous layer.

When comparing the results obtained in restricted and unrestricted geometries, it should be kept in mind that, in the case of unrestricted geometry, the dissipation is available of tuning fork motion, associated with the emission of the first sound wave. There is no such a dissipation in the restricted geometry, in which the acoustic wave is not formed. It means that, in a restricted geometry, one can apply a lower excitation force to the tuning fork to provoke the transition to a turbulent flow regime, in comparison with an unrestricted geometry. Thus, with the same exciting force in an unrestricted geometry, a laminar flow region with a linear force dependence on velocity exists at higher velocities than in a restricted one. As a result, the turbulent force for a tuning fork in restricted geometry is lower (Figs. 2 and 5). For the same reason the drag coefficient should be lower which is consistent with the data shown in Fig. 6.

Acknowledgments

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Турбулентна сила опору в надплинних сумішах ^{3}He–^{4}He при коливанні кварцового камертона

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Вивчено резонансні криві кварцового камертона основної частоти 32 кГц, який занурено в надплинний ^{4}He та суміші ^{3}He–^{4}He з концентрацією ^{3}He 5 та 15 % в діапазоні температур 0,35–2,5 K при тиску насиченої пари. Проведено два типи експериментів: камертон, не покритий колбою, та покритий колбою, тобто в обмеженій геометрії. В обох випадках залежності сила–швидкість для камертона демонстрували лінійну силу демпфування при малих пікових швидкостях та додатковий опір через появу вихрових ліній, що супроводжують перехід до турбулентності при зростанні пікової швидкості. Ці залежності головним чином визначаються щільністю нормального компонента гелію. Існує добре виражена різниця між надплинним ^{4}He та сумішами ^{3}He в ^{4}He, де частинки домішок ^{3}He забезпечують постійний незалежний від температури внесок у нормальний компонент суміші. Додатковий внесок у силу демпфування, так звана «турбулентна сила опору», зменшується зі збільшенням концентрації при однаковій піковій швидкості камертона, що можна пояснити розширенням діапазону ламінарного потоку зі збільшенням концентрації ^{3}He. Встановлено, що коефіцієнт опору в надплинному ^{4}He та суміші ^{3}He–^{4}He досягає плато з різними піковими швидкостями та різними збуджуючими силами, цей факт пояснено різними умовами утворення вихрів залежно від різної товщини пристінного в’язкого шару. Порівняння даних, які отримані в обмеженій та необмеженій геометрії, показує, що спостерігається надмірне розсіювання руху камертона, що пов’язано з випромінюванням хвилі першого звуку в необмеженій геометрії.

Ключові слова: надплинний гелій, надплинні суміші ^{3}He–^{4}He, турбулентний та ламінарний потоки, кварцовий камертон.