Oscillating spin vortices in a two-sublattice uniaxial antiferromagnet

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A distribution of the antiferromagnetic vector in a uniaxial two-sublattice antiferromagnet is investigated. A new class of nonlinear solutions of the system of two well-known Landau–Lifshitz equations in the form of socalled nonlinear sigma-model is obtained and a new type of topological magnetic configuration in the investigated antiferromagnet is described. Examples of solutions of the found class are presented. These examples include vortex-like structures, both moving and static. It is assumed that such vortices have an oscillating nature, so that the angle between the antiferromagnetic vector and the magnetic symmetry axis oscillates with descending amplitude and tends to $\pi/2$ when the distance to the vortex axis increases.

Keywords: antiferromagnet, topological magnetic configuration, spin vortex, uniaxial magnetic anisotropy, Dzyaloshinskii–Moriya interaction.

Introduction

Antiferromagnets are a prospective materials for spinwave electronics with a variety of possible technical applications (mostly in data storage, transmission and processing devices). The generation and detection of spin waves in antiferromagnets in recent years $[1, 2]$ $[1, 2]$ have made these technical applications possible. These facts made studying of static magnetic configurations in antiferromagnets an actual topic of research (for direct applications in magnetic storage devices, as a basis for studying spin waves that can be exited in these static configurations, etc.). Unlike the ferromagnetic materials, antiferromagnets don't possess strong macroscopic intrinsic magnetic field. Applications of antiferromagnets allow the use of semiconductor materials and, most importantly, higher working frequencies, thus enabling ultrafast information processing — in comparison with ferromagnets $[3-5]$ $[3-5]$.

In recent years, during investigation of spin waves in antiferromagnets, nonlinear spin waves (with unique properties that are not inherent to linear spin waves) were also generated and investigated [\[6,](#page-5-4) [7\]](#page-5-5). Therefore, investigations of new types of nonlinear spin waves, that can be excited in antiferromagnets, and corresponding topological magnetic excitations are an actual topic of research.

It is known that topological magnetic excitations in magnetically ordered materials (solitons, skyrmion, vortices, etc.) play a key role in constructing novel spintronic devices (see, e.g., [\[8–](#page-5-6)[10\]](#page-5-7)). In antiferromagnets, a vortex-like excita-

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tion appears as a composed pattern of two anti-aligned sublattices and is rigid compared to vortices in ferromagnets [\[11\]](#page-5-8). Such vortices were recently observed in [\[12,](#page-5-9) [13\]](#page-5-10) and remain poorly researched. Static and moving [\[11,](#page-5-8) [14\]](#page-5-11) vortices in an antiferromagnet represent an interest for experimental and theoretical studies. This includes analysis of vortices behavior in both large systems and small confined samples (in which they are mostly observed). However, studies of AFM vortex-like excitations in confined geometries are still scarce $[10, 11]$ $[10, 11]$, despite being the most practically actual case for such excitations [\[11\]](#page-5-8).

It has been shown that the factor leading to magnetic vortex generation in an antiferromagnet can be, in particular, Dzyaloshinskii–Moriya (DM) anisotropy in a uniaxial antiferromagnet. Specifically, the Dzyaloshinskii–Moriya interaction (DMI) can cause a net unbalancing between adjacent AFM sublattices due to dipole canting [\[11,](#page-5-8) [15\]](#page-5-12). Considerations of the DM interaction and magnetic anisotropy is a logical step in creating a theory of a particular case of such vortices. Note that DMI becomes more essential at low temperatures [\[16\]](#page-5-13), thus, considered effects are more pronounced at low temperatures.

In the paper, a new type of solution of the Landau–Lifshitz equation (in the form of so-called nonlinear sigma model) for the antiferromagnetic vector in a two-sublattice uniaxial antiferromagnet is studied. The above-mentioned solution (generally speaking, nonlinear) describes both static distributions of the antiferromagnetic vector and dynamic ones

(e.g., moving vortices akin to moving domain wall). The existence of oscillating vortices is predicted: vortices in which the angle between the antiferromagnetic (Néel) vector and the magnetic symmetry axis oscillates with descending amplitude and tends to $\pi/2$ when the distance to the vortex axis increases. The predicted vortex configuration includes, as a particular case, an intermediate state between Néel and Bloch vortices.

System of equations for the azimuthal and polar angles of the antiferromagnetic vector

Consider a uniaxial two-sublattice antiferromagnet. Let us assume that the magnetization densities of the antiferromagnet sublattices (M_1 and M_2 , respectively) are equal in magnitude and opposite in direction $(M_1 = -M_2)$. Let us also assume that they are constant in magnitude $(|\mathbf{M}_1| = |\mathbf{M}_2| = M_0 = \text{const})$. Thus, the total magnetization vector $M = 0$. [In the some sections of this paper we also consider a nonzero magnetization vector. However, the considered magnetization vector is small in magnitude — $M^2 \ll L^2$, where L is the antiferromagnetic (Néel) vector and is neglected afterwards]. The antiferromagnetic vector is also constant in magnitude: $|\mathbf{L}| = L_0 = \text{const.}$ Let us denote the antiferromagnet parameters as follows: the uniaxial

anisotropy constants of the sublattices β_1 and β_2 , the nonuniform exchange constants of the sublattices α_1 and α_2 (here $\alpha_1 > 0$), the uniform exchange constant *A*. Consider also the Dzyaloshinskii–Moriya interaction in the antiferromagnet and introduce the Dzyaloshinskii–Moriya vector **d**.

Then we consider an antiferromagnetic (Néel) vector configuration in the above-described antiferromagnet (a static configuration or a configuration moving along the *Oz* axis that coincides in direction with the magnetic symmetry axis of the antiferromagnet). We will use the spherical coordinates (r, θ, φ) and denote the polar and azimuthal angles of the vector **L** as θ_L and φ_L , respectively. Therefore,

$$
\mathbf{L} = L_0 \left(\mathbf{e}_x \sin \theta_L \cos \varphi_L + \mathbf{e}_y \sin \theta_L \sin \varphi_L + \mathbf{e}_z \cos \theta_L \right),\tag{1}
$$

where \mathbf{e}_x , \mathbf{e}_y , and \mathbf{e}_z are the unit vectors of the *Ox*, *Oy*, and *Oz* axes, correspondingly. The absolute value of the polar angle $θ$ _{*L*} cannot exceed π. The azimuthal angle $φ$ _{*L*} can be considered unlimited after noticing that it possesses a periodicity (with the period equal to 2π).

The antiferromagnetic vector of the considered antiferromagnet should satisfy the Landau–Lifshitz equation with the energy functional written as follows (see, e.g., [\[17\]](#page-5-14)):

$$
W = \int dV \left(\frac{1}{2} A \mathbf{M}^2 + \frac{1}{2} \alpha_1 \sum_i \left(\frac{\partial \mathbf{L}}{\partial x_i} \right)^2 + \frac{1}{2} \alpha_2 \sum_i \left(\frac{\partial \mathbf{M}}{\partial x_i} \right)^2 - \frac{1}{2} \beta_1 L_z^2 - \frac{1}{2} \beta_1 M_z^2 + \mathbf{d} \left[\mathbf{M} \times \mathbf{L} \right] - \mathbf{M} \mathbf{H}_0 \right)
$$
(2)

(here we still consider a nonzero magnetization: $M^2 \ll L^2$). Here H_0 is the external magnetic field, and the integration is performed over the entire antiferromagnet volume.

Using the Landau–Lifshitz equation for a two-sublattice antiferromagnet in the form (see, e.g., $[17-19]$ $[17-19]$)

$$
\begin{cases}\n\frac{2}{gM_0} \frac{\partial \mathbf{m}}{\partial t} = \left[\mathbf{m} \times \mathbf{H}_{\mathbf{m}}^{(\text{ef})} \right] + \left[\mathbf{I} \times \mathbf{H}_{1}^{(\text{ef})} \right] \\
\frac{2}{gM_0} \frac{\partial \mathbf{l}}{\partial t} = \left[\mathbf{I} \times \mathbf{H}_{\mathbf{m}}^{(\text{ef})} \right] + \left[\mathbf{m} \times \mathbf{H}_{1}^{(\text{ef})} \right]\n\end{cases}
$$
\n(3)

where
$$
\mathbf{H}_{m}^{(ef)} = -\frac{1}{M_0^2} \frac{\delta W}{\delta \mathbf{m}}
$$
, $\mathbf{H}_{1}^{(ef)} = -\frac{1}{M_0^2} \frac{\delta W}{\delta \mathbf{l}}$ and the vec-

tors $I = \frac{M_1 - M_2}{2 M_1}$ $2M_{0}$ $\mathbf{l} = \frac{\mathbf{M}_1 - \mathbf{M}_2}{2 \mathbf{M}_1}, \ \mathbf{m} = \frac{\mathbf{M}_1 + \mathbf{M}_2}{2 \mathbf{M}_2}$ $2M_{0}$ $m = \frac{M_1 + M_2}{2M_1}$ with the energy func-

tional (2), one (after substituting zero magnetization) can obtain the following system of equations for the antiferromagnetic vector azimuthal and polar angles (the nonlinear sigma-model — see, e.g., [\[20\]](#page-5-16) — because we deal, in fact, with the normalized Néel vector):

$$
\begin{cases}\nc^2 \operatorname{div} \left(\sin^2 \theta_L \nabla \varphi_L\right) - \frac{\partial}{\partial t} \left(\sin^2 \theta_L \left(\frac{\partial \varphi_L}{\partial t} - \omega_H\right)\right) + \frac{\partial \omega_H}{\partial t} = 0 \\
c^2 \Delta \theta_L - \frac{\partial^2 \theta_L}{\partial t^2} - \left(c^2 \left(\nabla \varphi_L\right)^2 - \left(\frac{\partial \varphi_L}{\partial t} - \omega_H\right)^2 + \omega_0^2 \operatorname{sgn}\left(\beta_1 - \frac{d^2}{\delta}\right)\right) \sin \theta_L \cos \theta_L = 0\n\end{cases} (4)
$$

where $c = \frac{4\mu_0 M_0}{\hbar} \sqrt{A\alpha_1}$ is a characteristic parameter of the antiferromagnet with the dimension of speed, $\omega_0 = -\frac{4\mu_0 M_0}{\hbar} \sqrt{A|\beta_1|}$ and $\omega_H = \gamma H_0$ (for a constant in time external magnetic field value H_0 ; for a time-dependent H_0 the addend $\omega_H t$ should be replaced with $\int \omega_H dt$). An analogous starting system of equations for a uniaxial antiferromagnet was used, for example, by Bar'yakhtar and Ivanov [\[19\]](#page-5-15) (however, they consider a different solution of this system) and by one of the authors in earlier paper [\[21\]](#page-5-17). The characteristic speed *c* for $H_0 = 0$ is equal to the minimum phase velocity of spin waves in the linear theory (see, e.g., [\[19\]](#page-5-15)).

An equation for the polar angle of the antiferromagnetic vector

Consider a self-similar and, generally saying, nonstationary solution of the system of equations (4), analogous to the one presented by the authors in [\[17,](#page-5-14) [21,](#page-5-17) [22\]](#page-5-18). A self-similar solution for a spin wave propagating in the above-described antiferromagnet along the Oz axis with the velocity v can be written as follows:

$$
\begin{cases} \n\tan\left(\frac{\theta_L}{2}\right) = H\left(P(x, y, z - vt)\right) \\
\varphi_L = Q(x, y, z - vt) + \tilde{\varphi}_L(t)\n\end{cases}
$$
\n(5)

where $\omega_H = gH_0$, $\tilde{\varphi}_L(t) = \int \omega_H(t) dt$ (for a time-dependent external magnetic field; in the stationary case one have to replace $\int \omega_H dt$ with $\omega_H t$; *H*, *P* and *Q* are admissible functions. After substituting a self-similar solution (5) into the system (4), one can obtain for a stationary external magnetic field

$$
c^{2}\left(\pm\frac{8H(1-H^{2})H'}{\left(1+H^{2}\right)^{3}}\nabla P\nabla Q+\frac{4H^{2}}{\left(1+H^{2}\right)^{2}}\Delta Q\right)-\frac{4H}{\left(1+H^{2}\right)^{2}}\left(\frac{\partial Q}{\partial t}\left(\pm\frac{2\left(1-H^{2}\right)H'}{1+H^{2}}\frac{\partial P}{\partial t}\right)+\frac{\partial^{2} Q}{\partial t^{2}}H\right)=0
$$
\n
$$
c^{2}\frac{2}{1+H^{2}}\left(H'\Delta P+\left(H''-\frac{2H\left(H'\right)^{2}}{1+H^{2}}\right)\left(\nabla P\right)^{2}\right)-\frac{2}{1+H^{2}}\left(H'\frac{\partial^{2} P}{\partial t^{2}}+\left(H''-\frac{2H\left(H'\right)^{2}}{1+H^{2}}\right)\left(\frac{\partial P}{\partial t}\right)^{2}\right)+\frac{2H\left(1-H^{2}\right)}{\left(1+H^{2}\right)^{2}}\left(c^{2}\left(\nabla Q\right)^{2}-\left(\frac{\partial Q}{\partial t}\right)^{2}+\omega_{0}^{2}\operatorname{sgn}\left(\beta_{1}-\frac{d^{2}}{\delta}\right)\right)=0,
$$
\n(6)

where the sign " \pm " origins from the expression $\sin \theta_L = \pm \frac{2H}{1+H^2}$ $L - \frac{1}{1}$ *H H* $\theta_L = \pm$ $\frac{2H}{\epsilon + H^2}$ that implies from (5), or after the replacement $\frac{d}{dt} \rightarrow -v \frac{\delta}{\delta z}$ $\frac{\partial}{\partial t} \rightarrow -v \frac{\partial}{\partial x}$

$$
\frac{\pm \frac{8H(1-H^2)H'}{(1+H^2)^3} \nabla_c P \nabla_c Q + \frac{4H^2}{(1+H^2)^2} \Delta_c Q = 0}{\left(1+H^2\right)^3} \tag{7}
$$
\n
$$
\frac{2}{1+H^2} \left(H' \Delta_c P + \left(H'' - \frac{2H(H')^2}{1+H^2} \right) \left(\nabla_c P \right)^2 \right) \pm \frac{2H(1-H^2)}{\left(1+H^2\right)^2} \left(\nabla_c Q \right)^2 + \frac{\omega_0^2}{c^2} \text{sgn} \left(\beta_1 - \frac{d^2}{\delta} \right) = 0,
$$

where the operators

$$
\nabla_c = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{1}{\sqrt{1 - \frac{v^2}{c^2}} \frac{\partial}{\partial z} \end{pmatrix},
$$
(8)

$$
\Delta_c = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \left(1 - \frac{v^2}{c^2}\right) \frac{\partial^2}{\partial z^2}.
$$
(9)

For the sake of convenience, let us choose the "+" sign in the relation $\sin \theta_L = \pm \frac{2H}{1+H^2}$ $L - \frac{1}{1}$ *H H* $\theta_L = \pm$ $\frac{2H}{+H^2}$ (analogous procedures can be performed for the "-" sign). Then the system of equations (7) is satisfied, in particular, under the following set of conditions:

$$
\nabla_c P \nabla_c Q = 0,
$$

\n
$$
\Delta_c Q = 0,
$$

\n
$$
\Delta_c P = 0,
$$

\n
$$
(\nabla_c P)^2 - \frac{(\nabla_c Q)^2}{F(P)} - \frac{\omega_0^2}{c^2} \frac{\text{sgn}(\beta_1 - d^2/\delta)}{F(P)} = 0,
$$
 (10)
\n
$$
H'' - \frac{2H(H')^2}{1 + H^2} = \frac{H(1 - H^2)}{1 + H^2} F(P)
$$

 $_$. The contribution of the contribution of $\mathcal{L}_\mathcal{A}$

with the arbitrary function $F(P)$ [not equal to zero $F(P) \neq 0$ at any point].

Consider the case when the characteristic velocity *c* exceeds the wave velocity $v : v < c$. For that case, let us apply the following Lorentz-like coordinates' transformation (analogous to the ones used by the authors, e.g., in [\[22\]](#page-5-18)):

$$
X = \frac{x}{l_0}, Y = \frac{y}{l_0}, Z = \frac{z - vt}{l_0 \sqrt{1 - v^2/c^2}},
$$
(11)

where $l_0 = c/\omega_0$. After applying the transformation (11) to the system of equations (10) for the case $\beta_1 - d^2 / \delta > 0$, the latter transforms into the following system of equations:

$$
\nabla P \nabla Q = 0,
$$

\n
$$
\Delta Q = 0,
$$

\n
$$
\Delta P = 0,
$$

\n
$$
(\nabla P)^2 - \frac{(\nabla Q)^2}{F(P)} - \frac{1}{F(P)} = 0,
$$

\n
$$
H'' - \frac{2H(H')^2}{1 + H^2} = \frac{H(1 - H^2)}{1 + H^2}F(P)
$$
\n(12)

with the gradient and the Laplace operators taken in the coordinates (*X*, *Y*, *Z*). A solution of this system of equations can be sought, in particular, in the form $P(X, Y, Z) = P_0 \ln r$, $Q(X, Y, Z) = n\alpha + \eta Z + \alpha_0$, where *X* $\mathbf{r} = \begin{pmatrix} X \\ Y \end{pmatrix}$ is the radius vector of a point in the *XY* plane, α is

its azimuthal angle [so that $\alpha = \arctg(Y/X)$], *n* is an arbitrary integer, and η is the characteristic dimensionless length of the investigated magnetic structure (moving or static) along the *Z* axis. Then it turns out that for such solu-

tion the function
$$
F(P) = \frac{n^2 + (1 + \eta^2) \exp(2P/P_0)}{P_0^2}
$$
 and

the following equation for the polar angle of the antiferromagnetic vector θ_L can be written:

$$
r^{2} \theta_{L}'' + r \theta_{L}' - \frac{1}{2} \sin 2\theta_{L} \left(n^{2} + \left(1 + \eta^{2} \right) r^{2} \right) = 0. \quad (13)
$$

The last equation describes the considered type of static and dynamic distributions of antiferromagnetic vector. As it can be seen from the form of the equation, it includes nonlinear solutions. Let's analyze this equation.

Analysis of the equations for the polar angle of the antiferromagnetic vector

The Eq. (13) should be complemented with the boundary values: $\theta_L(0)$ and θ_L value on the boundary of the considered nanosystem $[\theta_L(r \rightarrow \infty)$ for an infinite antiferromagnetic film]. Let us investigate the case of vortex, either static or moving, in a nanodisc (nanocylinder): $\theta_L(r = R_0) = \pi/2$, where $R_0 l_0$ is the radius of the considered nanodisc (nanocylinder). Such configuration corresponds to a magnetic vortex and, therefore, the condition $\theta_L(0) = 0$ or $\theta_L(0) = \pi$ is fulfilled. Numerical simulations show that for the case $n = 1$ an additional condition [e.g., a value of θ' [(0)] is required in order to define the solution of (13). For solutions that satisfy the condition $\theta_L (r = R_0) = \pi / 2$, θ' _L(0) should have a negative value $[\theta'$ _L(0) = 0 is also permitted]. Therefore, for $n = 1$ a solution that satisfies the

condition $\theta_L (r = R_0) = \pi / 2$ can be found for an arbitrary (admissible) η and R_0 . For instance, for $n = 1$, $\eta = 1$, R_0 = 30 (that roughly correspond to the nanodisc radius \sim 10² nm for typical antiferromagnets), the condition $\theta_L(r = R_0) = \pi/2$ can be satisfied when $\theta_L'(0) \approx -0.48$, θ' _{*L*} (0) ≈ −4.48, etc. Numerical simulations show that the following dependences imply from the Eq. (13) for the case $\theta_L(0) = \pi$, $n = 1$ (see Fig. 1).

The dependence $\theta_L(r,\eta)$ for $n=1$ before applying the condition $\theta_L(r = R_0) = \pi/2$ [can be used either for a large sample or for a considered nanodisc or nanocylinder; in the latter case the radius of the considered nanosystem should satisfy the condition $\theta_L(r = R_0) = \pi/2$] has the form shown in Fig. 2.

Analysis shows that for all the cases $n \neq 1$ the condition θ' _{*L*} (0) = 0 θ' _{*L*} (0) = 0 if θ _{*L*}(*r* = *R*₀) = π /2, to be exact] fulfills, thus reducing the number of degrees of freedom for the obtained solution. As a result, a solution that satisfies

Fig. l. Dependences $\theta_L(r)$ for $n = 1$, $\eta = 1$, $\theta_L(0) = \pi$, $R_0 = 30$: $\theta'_{L}(0) \approx -0.48$ *(a)*; $\theta'_{L}(0) \approx -4.48$ *(b)*; $\theta'_{L}(0) \approx -1.76$ *(c)*.

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Fig. 2. Dependence $\theta_L(r,\eta)$ for $n = 1$, $\theta_L(0) = \pi$, $\theta'_L(0) = -1$.

the condition $\theta_L(r = R_0) = \pi/2$ exists only for a discrete set of values η for a given R_0 . The corresponding graphical representations are also obtained via computer simulations (see Fig. 3).

Discussion

Note that the obtained results can be applied for both the static antiferromagnetic vector distribution (in the case $v = 0$) and a distribution that moves with the velocity *v* along the *Oz* axis (that coincides with the magnetic anisotropy axis). In the first case, the obtained solution defines a static spin vortex in the considered antiferromagnet [which is determined by the boundary condition $\theta_L(r = R_0) = \pi/2$, in the second case, an object akin to the moving antiferromagnetic domain wall with a single vortex. Note that movement of AFM vortices on a domain wall has been observed, e.g., by Chetkin *et al.* [\[23](#page-5-19)[–25\]](#page-5-20).

Note also that the standard boundary conditions for the Néel vector in the considered model are obtained by integrating of the equation

$$
\left[\mathbf{1} \times \left(\alpha_1 \Delta \mathbf{l} - \frac{4}{\delta (gM_0)^2} \frac{\partial^2 \mathbf{l}}{\partial t^2} \right)\right] + \frac{4}{\delta gM_0} \frac{\partial \mathbf{l}}{\partial t} (\mathbf{l} \cdot \mathbf{h}_0) - \frac{2}{\delta gM_0} \frac{\partial \mathbf{h}_0}{\partial t} - \beta_1 (\mathbf{l} \cdot \mathbf{e}_z) [\mathbf{l} \times \mathbf{e}_z] +
$$

+
$$
\frac{1}{\delta} (\mathbf{d} \cdot \mathbf{l}) [\mathbf{d} \times \mathbf{l}] + \frac{1}{\delta} [\mathbf{l} \times [\mathbf{h}_0 \times \mathbf{d}]] - \frac{1}{\delta} [\mathbf{l} \times \mathbf{h}_0] (\mathbf{l} \cdot \mathbf{h}_0) = 0
$$
(14)

Fig. 3. Dependences $\theta_L(r)$ for $n = 2$, $\theta_L(0) = \pi$ ($n \approx 0.97$) (a) and $n = 0$, $\theta_L(0) = \pi$ ($\eta \approx 1.01$) (b).

tions (3) with respect to a thin layer near the antiferromagnet boundary, has the form $\int \alpha_1 [1 \times \Delta I] dV = 0$, where Ω_δ is a $\overline{\Omega}_{\delta}$ region with a thickness δ neighboring the antiferromagnet

 $_$. The contribution of the contribution of $\mathcal{L}_\mathcal{A}$ (here $h_0 = 2H_0 / M_0$) that implies from the system of equa-

boundary. The corresponding boundary condition $\left[1 \times \frac{\partial I}{\partial n}\right] = 0$ (where **n** is the unit vector normal to the boundary and directed outwards), see, e.g., [\[26\]](#page-5-21), can be

satisfied by the means of choosing the correct value of α_0 in the expression $Q(X, Y, Z) = n\alpha + \eta Z + \alpha_0$ (for $\eta = 0$). The resulting vortex is an intermediate state between the Néel vortex $(\alpha_0 = 0, \alpha_0 = \pi)$ and the Bloch vortex $(\alpha_0 = \pi/2, \pi)$ $\alpha_0 = -\pi/2$). Moreover, for the case of a surface anisotropy or surface DMI, the boundary conditions can be modified (see [\[27,](#page-5-22) [28\]](#page-5-23) for analogous considerations) thus making possible using of the boundary conditions with $\left[\mathbf{1} \times \frac{\partial \mathbf{l}}{\partial \mathbf{n}}\right] \neq 0$

and, therefore, expanding the area of application of the obtained result.

As an analogous solution of the Landau–Lifshitz equation can exist for a magnetization vector in a ferromagnet, the above-found solution can be used, in particular, for imprinted vortices in antiferromagnets (that emerge on a contact of a ferromagnet with a magnetization vortex with an antiferromagnet due to a strong exchange, see, e.g., [\[12,](#page-5-9) [13\]](#page-5-10)). Such imprinted vortices are an especially actual topic of research and represent a special interest for study (see, e.g., $[13]$).

Conclusions

We have investigated a new type of antiferromagnetic (Néel) vector distribution in a two-sublattice uniaxial antiferromagnet. The model used in the paper considers the magnetic dipole-dipole interaction, the magnetic anisotropy, the exchange and the Dzyaloshinskii–Moriya interaction. As a result, an explicit expression for the azimuthal angle and a differential equation for the polar angle of the antiferromagnetic vector of such a topological configuration were obtained. The considered distribution describes both a static magnetic configuration and a configuration moving along the magnetic anisotropy axis and includes nonlinear solutions of the starting equations.

The results are analyzed by numerical methods and it is shown that the obtained configuration describes, in particular, spin vortices (static or dynamic) in a nanodisc, nanocylinder or large antiferromagnetic sample. It is shown that these vortices possess an oscillating nature, so the angle between the antiferromagnetic (Néel) vector and the magnetic symmetry axis is oscillating with descending amplitude and tends to $\pi/2$ when the distance to the vortex axis increases. An intermediate state between Néel and Bloch vortices is a particular case for the obtained magnetic configurations.

The effects considered in the presented paper are more essential at the low temperatures (as the Dzyaloshinskii– Moriya interaction, which is essential for these effects, becomes more pronounced).

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Осцилюючі спинові вихори у двопідгратковому одновісному антиферомагнетику

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Досліджено розподіл вектора антиферомагнетизму в одновісному двопідгратковому антиферомагнетику. Отримано новий клас нелінійних розв'язків системи двох відомих рівнянь Ландау-Ліфшиця у формі так званої нелінійної сигмамоделі та описано новий тип топологічної магнітної конфігурації у розглянутому антиферомагнетику. Представлено приклади розв'язків знайденого класу. Ці приклади включають вихрові структури, як рухомі, так і статичні. Передбачається, що такі вихори мають осцилюючий характер, так що кут між вектором антиферомагнетизму та віссю магнітної симетрії коливається із амплітудою, яка спадає та прямує до π/2 при зростанні відстані до осі вихору.

Ключові слова: антиферомагнетик, топологічна магнітна конфігурація, спіновий вихор, одновісна магнітна анізотропія, взаємодія Дзялошинського–Морії.