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Magnetoresistance of electrons in quantum ring with Rashba spin-orbit interaction

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The influence of Rashba spin-orbit interaction on the transport properties of the two-dimensional quantum ring with finite width has been investigated in the presence of the uniform perpendicular magnetic field. The dependence of magnetoresistance on the magnetic field and Rashba spin-orbit coupling parameter in quantum ring with finite width are investigated. It was shown that in the presence Rashba spin-orbit interaction that the beating pattern is destroyed.

Keywords: magnetoresistance, ballistic conductance.

1. Introduction

The semiconductor quantum ring is a mesoscopic system for studying topological effects in semiconductors [1]. The interference effect in a quantum ring under external magnetic field leads to the Aharanov–Bohm (AB) oscillations in physical quantities, such as conductance, orbital magnetism, and persist currents.

The conductance of a quantum ring was been calculated in the work [2] on the basis of the tunneling Hamiltonian in the quasiballistic regime of the motion of electrons with allowance for the spin-orbit interaction.

The linear and nonlinear optical absorption in a disk-shaped quantum dot with parabolic potentials plus an inverse squared potential in the presence of a static magnetic field was theoretically investigated in the paper [3].

In the paper [4] was found that the amplitudes of the AB oscillation in conductance were usually dominated by random fluctuations of the order of e^2/h. This led to the discovery of universal conductance fluctuations. Liu [5] showed that when four spin-degenerate subbands in the ring are populated, random signs dominate the AB interference patterns.

The influence of Rashba spin-orbit interaction on the optical properties of two-dimensional mesoscopic ring was been investigated in work [6] in the presence of uniform perpendicular magnetic field.

In the paper [7] in the quantum system of pseudodots was studying the direct interband transitions under the influence of an external magnetic field. In Ref. 8 was investigated the influence of a screw dislocation on the energy levels and the wave functions of an electron in a two-dimensional pseudoharmonic quantum dot under the influence of an external magnetic field inside a dot and Aharonov–Bohm field inside a pseudo dot.

In the paper [9] exact energy spectra and wave functions analytically for a ring in the presence of both a uniform perpendicular magnetic field and a thin magnetic flux through the ring center was obtained. It was used as a model to study the Aharonov–Bohm effect in an ideal annular ring that is weakly coupled to both the emitter and the collector. The spinless electrons were considered. The influence of exchange interaction on the transport properties of two-dimensional diluted magnetic semiconductor quantum ring with finite width was been investigated in the presence of uniform perpendicular magnetic field in the paper [10].

In the work [11] explicit analytical expressions for the magnetic moment and persistent current of the Volcano ring was derived. The magnetic moment was investigated as a function of the magnetic field strength and the temperature. In the paper [12] was investigated the energy spectrum and corresponding wave functions of an electron confined by a pseudoharmonic potential both including harmonic dot and antidot potentials in the presence of a strong magnetic field together with an Aharonov–Bohm flux field. The energies of the electron and hole weak-coupling polarons was determined in quantum rings of finite width in a uniform magnetic field. It was shown that polaron correction exhibit oscillatory behavior as a function in a magnetic field [13].

In the paper [14] was studied the effects of magnetic field and size on internal energy and entropy of a quantum pseudodot.

2. Theory

The purpose of this work is to generalize the results of paper [9] to quantum ring with Rashba spin-orbit interactions. We study the effect of Rashba spin-orbit interactions on the magnetoresistance of quantum ring prepared from a semiconductor with radial potential:

$$V(r)=\frac{a_1}{r^2}+a_2r^2-2\sqrt{a_1a_2}\,. \tag{1}$$

The expansion of (1) near the minimum yields

$$V(r)=\frac{m_n\omega_0^2}{2}(r-r_0)^2+\text{const}\,. \tag{2}$$

Where one can obtain a relation between a_1 and a_2 and experimentally determined parameters r_0 and ω_0:

$$\omega_0=\sqrt{\frac{8a_2}{m_n}},\quad r_0=\left(\frac{a_1}{a_2}\right)^{\frac{1}{4}}. \tag{3}$$

It is worth noting that the confinement model is very powerful. This potential represents a quantum ring of radius and width determined by the choice of the parameters a_1 and a_2. The (1) potential admits an analytic solution of the Schrödinger equation in a magnetic field. The effective width of the Volcano ring at a given Fermi energy E_F then is $\Delta r=\sqrt{8E_F/m_n\omega_0^2}$. The (1) potential model has been successfully used to explain the beats in the Aharonov–Bohm oscillations, which have been experimentally observed in a two-dimensional semiconductor ring [5]. Figure 1 shown the profile $U(r)$ in terms of the Cartesian coordinates.

The electron in one lead can reach the other one only by tunneling through the quasi-bound circular states in the ring. The quantum ring is subjected to a uniform magnetic field along the z direction. The total Hamiltonian of the system is given by:

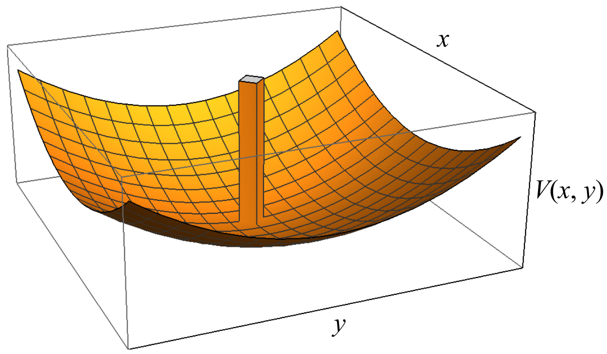

Fig. 1. The profile $V(r)$ in terms of the Cartesian coordinates.

$$H=\left(\frac{1}{2m_n}(\mathbf{p}+e\mathbf{A})^2+V(r)\right)I+\frac{1}{2}g\sigma_z\mu_BH+H_R, \tag{4}$$

where I is a 2×2 unit matrix, m_n is the effective mass electrons, $\mu_B=e\hbar/2m_0$ is the Bohr magneton, m_0 is the free electron mass, $\mathbf{A}$ is the vector potential, and σ_z is the z component of Pauli spin matrices and g is the Lande factor of electrons.

$$H_R=i\alpha\left(-\sin\phi\cdot\sigma_x+\cos\phi\cdot\sigma_y\right)\frac{\partial}{\partial r}-$$
$$-\frac{\alpha}{r}\left(\cos\phi\cdot\sigma_x+\sin\phi\cdot\sigma_y\right)\left(i\frac{\partial}{\partial\phi}+\frac{\Phi}{\Phi_0}\right), \tag{5}$$

where $\Phi=\pi r_0^2H$ and $\Phi_0=ch/e$ are magnetic flux through the ring and a flux quantum. For uniform magnetic field, the vector potentials in cylindrical coordinates have the components $A_\phi=Hr/2$, $A_r=0$ and Schrödinger equation in polar coordinates is:

$$\left[\left(-\frac{\hbar^2}{2m_n}\frac{\partial^2}{\partial r^2}-\frac{\hbar^2}{2m_n}\frac{\partial}{\partial r}-\frac{\hbar^2}{2m_nr^2}\frac{\partial^2}{\partial\phi^2}+\frac{i\hbar eH}{2m_n}\frac{\partial}{\partial\phi}+\frac{e^2H^2}{8m_n}r^2+V(r)-E\right)I+H_R+\frac{1}{2}g\sigma_z\mu_BH\right]\Psi(r,\phi)=0\,. \tag{6}$$

The solution of Eq. (6) have the form [15]:

$$\Psi(r,\phi)=A_J\Phi_{n,l}\,\chi_1+B_J\Phi_{n,l+1}\,\chi_2, \tag{7}$$

where $\Phi_{n,l}$ corresponds solution Schrödinger equation of the case when $\alpha=0$,

$$\Phi_{n,l}(r,\phi)=Cl\mathrm{e}^{-il\phi}\mathrm{e}^{-1/4\left(\frac{r}{\lambda}\right)^2}\left(\frac{r}{\lambda}\right)^{M1}L_n^{M1}\left(\frac{1}{2}\left(\frac{r}{\lambda}\right)^2\right), \tag{8}$$

$$E_{n,l,\sigma}^0=\left(n+\frac{1}{2}+\frac{M1}{2}\right)\hbar\omega-\frac{l}{2}\hbar\omega_c-\frac{m_n}{4}\omega_0^2r_0^2+\sigma\frac{\mu_BH}{2}g, \tag{9}$$

here the following designations are used:

$$M1=\sqrt{l^2+\frac{2a_1m_n}{\hbar^2}},\quad M2=\sqrt{(l+1)^2+\frac{2a_1m_n}{\hbar^2}},$$
$$\lambda=\sqrt{\frac{\hbar}{m_n\omega}},\quad \omega=\sqrt{\omega_c^2+\omega_0^2}, \tag{10}$$

$$C1 = \frac{1}{\lambda}\sqrt{\frac{n!}{2^{M1+1}\Gamma(n+M1+1)\pi}}, \quad (11)$$

where $L_n^M\left[\frac{1}{2}\left(\frac{r}{\lambda}\right)^2\right]$ is the a associated Laguerre polynomials, $\omega_c = eH/m_n$ is the cyclotron frequency, $\Gamma(x)$ is the Gamma function, quantum number $n = 0, 1, 2, \ldots$ shows the order of the radial mode and $m = 0, \pm 1, \pm 2, \ldots$ gives the angular momentum, and $\sigma = \pm 1$ for $\sigma = \uparrow, \downarrow$, and χ_σ is the electron spin written as the column vector $\chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\chi_{-1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Substituting Eq. (7) into Eq. (6) we find for the coefficients A and B following equations:

$$\left(E_{n,l}^0 + \frac{1}{2}g\mu_B H - E\right)A_J\Phi_{n,l} + \left[-\frac{\alpha}{r}e^{-i\phi}\left(i\frac{\partial}{\partial\phi} - \frac{r^2\Phi}{r_0^2\Phi_0}\right) + \alpha e^{-i\phi}\frac{\partial}{\partial r}\right]B_J\Phi_{n,l+1} = 0, \quad (12)$$

$$\left[-\frac{\alpha}{r}e^{i\phi}\left(i\frac{\partial}{\partial\phi} - \frac{r^2\Phi}{r_0^2\Phi_0}\right) - \alpha e^{i\phi}\frac{\partial}{\partial r}\right]A_J\Phi_{n,l} + \left(E_{n,l+1}^0 - \frac{1}{2}g\mu_B H - E\right)B_J\Phi_{n,l+1} = 0. \quad (13)$$

We multiply Eq. (12) with $\Phi_{n,l}$, Eq. (13) with $\Phi_{n,l+1}$ and integrating the resulting equations by term over r und using the below identies of associated Laguerre polynomials [16]:

$$\int_0^\infty x^{\alpha-1}e^{-cx}L_m^\gamma(bx)L_n^\lambda(cx)\,dx = \frac{(1+\gamma)_m(1-\alpha+\lambda)_n\Gamma(\alpha)}{m!n!}\times$$
$$\times {}_3F_2\left(-m,\alpha,\alpha-\lambda,1+\gamma,\alpha-\lambda-n;\ \frac{b}{c}\right) \quad (14)$$

and also carrying out the normalization,we find that the coefficients A_J and B_J satisfy eigenvalue equation:

$$\begin{pmatrix} E_{n,l,0} + \frac{1}{2}g\mu_B B - E & T1+T2+T3 \\ Y1+Y2+Y3 & E_{n,l+1,0} - \frac{1}{2}g\mu_B B - E \end{pmatrix}\begin{pmatrix} A_J \\ B_J \end{pmatrix} = 0, \quad (15)$$

where $(\gamma)_m$ is a Pochhammer symbol, ${}_3F_2\left(-m,\alpha,\alpha-\lambda,1+\gamma,\alpha-\lambda-n;\ \frac{b}{c}\right)$ the generalized hypergeometric function.

To determine of the solutions of eigenvalue Eq. (15), we set the determinant of Eq. (15) to zero. We obtain energy spectrum of electrons:

$$E = \frac{1}{2}\left(E_{n,l,0} + E_{n,l+1,0} \pm \sqrt{\left(E_{n,l,0} + E_{n,l+1,0}\right)^2 - 4C}\right), \quad (16)$$

where

$$C = \left(E_{n,l,0} + \frac{1}{2}g\mu_B H\right)\left(E_{n,l+1,0} - \frac{1}{2}g\mu_B H\right) - (T1+T2+T3)(Y1+Y2+Y3), \quad (17)$$

where

$$T1 = 2^{k1}C1C2\pi\lambda\alpha(1+l+M2)\times \frac{\Gamma[1+M1+n]}{\Gamma[1+M1]}\frac{\Gamma[1-k1+M2+n]}{\Gamma[1-k1+M2]}\frac{\Gamma[k1]}{n!(n)!}\times {}_3F_2(-n,k1,k1-M2,1+M1,k1-M2-n;\ 1), \quad (18)$$

$$T2 = 2^{k2}C1C2\pi\lambda\alpha\left(-\frac{1}{2} + \frac{\pi H\lambda^2 e}{\hbar}\right)\times \frac{\Gamma[1+M1+n]}{\Gamma[1+M1]}\frac{\Gamma[1-k2+M2+n]}{\Gamma[1-k2+M2]}\frac{\Gamma[k2]}{n!(n)!}\times {}_3F_2(-n,k2,k2-M2,1+M1,k2-M2-n;\ 1), \quad (19)$$

$$T3 = 2^{k2}\pi\lambda\alpha C1C2\times \frac{\Gamma[1+M1+n]}{\Gamma[1+M1]}\frac{\Gamma[1-k2+1+M2+1-n]}{\Gamma[1-k2+1+M2]}\frac{\Gamma[k2]}{n!(n-1)!}\times {}_3F_2(-n,k2,k2-M2-1,1+M1,k2-M2-n;\ 1), \quad (20)$$

$$k2 = \frac{3+M1+M2}{2}, \quad k1 = \frac{1+M2+M1}{2}, \quad (21)$$

$$Y1 = 2^{k2}\pi\lambda\alpha C1C2\times \frac{\Gamma[1+M2+n]}{\Gamma[1+M2]}\frac{\Gamma[1-k2+1+M1+n-1]}{\Gamma[1-k2+M1]}\frac{\Gamma[k2]}{n!(n-1)!}\times {}_3F_2(-n,k2,k2-M1-1,1+M2,k2-M1-n;\ 1), \quad (22)$$

$$Y2 = 2^{k2}C1C2\pi\lambda\alpha\times \left(1 + \frac{\pi H\lambda^2 e}{\hbar}\right)\frac{\Gamma[1+M1+n]}{\Gamma[1+M1]}\frac{\Gamma[1-k2+M2+n]}{\Gamma[1-k2+M2]}\frac{\Gamma[k2]}{n!(n)!}\times {}_3F_2(-n,k2,k2-M2,1+M1,k2-M2-n;\ 1), \quad (23)$$

$$Y3 = 2^{k1}C1C\pi\lambda\alpha(l-M1)\times \frac{\Gamma[1+M1+n]}{\Gamma[1+M1]}\frac{\Gamma[1-k1+M2+n]}{\Gamma[1-k1+M2]}\frac{\Gamma[k1]}{n!(n)!}\times {}_3F_2(-n,k1,k1-M2,1+M1,k1-M2-n;\ 1), \quad (24)$$

$$C2 = \frac{1}{\lambda}\sqrt{\frac{n!}{2^{M2+1}\Gamma(n+M2+1)\pi}}. \quad (25)$$

For obtaining the magnetoresistance of the electrons in semiconductor rings it is necessary to find the ballistic conductance of the electron gas in the quantum ring. The ballistic conductance of the electron gas can be determined of the Landauer formula [9]:

$$G(H) = \frac{e^2}{h} \sum_{n,\sigma} T_n^{\sigma\sigma}(H, E_F), \tag{26}$$

where $T_n^{\sigma\sigma}(H, E_F)$ is the magnetic-field-dependent transmission coefficient of the nth channel in the leads at the Fermi energy E_F. If we assume that the two leads are weakly coupled to the ring, the electron in one lead can reach the other one only by tunneling through the quasibound circular states in the ring. In such a case, the conductance can be approximately expressed in the form of [9]:

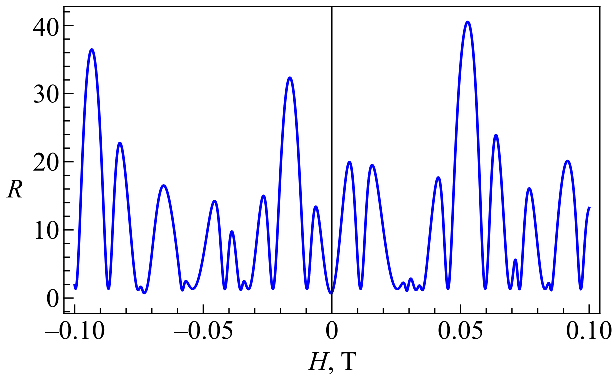

Fig. 3. The dependence of the dimensionless resistance $R\frac{h}{e^2}$ as a function of magnetic field for value of Rashba coupling $\alpha = 3$ meV·nm. Fermi energy is kept fixed at $E_F = 2$ meV.

$$G(H) = \frac{e^2}{h} \sum_{n,m,\sigma} \frac{\Gamma^e_{n,m,\sigma}\Gamma^c_{n,m,\sigma}}{\left[E_F - E_{n,m,\sigma}(H)\right]^2 + \frac{1}{4}\left(\Gamma^e_{n,m,\sigma} + \Gamma^c_{n,m,\sigma} + \Gamma^i_{n,m,\sigma}\right)^2} \frac{\Gamma^e_{n,m,\sigma} + \Gamma^c_{n,m,\sigma} + \Gamma^i_{n,m,\sigma}}{\Gamma^e_{n,m,\sigma} + \Gamma^c_{n,m,\sigma}}, \tag{27}$$

$E_{n,m,\sigma}(H)$ is the energy of the (n, m, σ) the quasibound ring states. Furthermore, we can approximate the energies of these quasibound states with those of the isolated ring given in Eq. (16). $\Gamma^e_{n,m,\sigma}, \Gamma^c_{n,m,\sigma}$ and $\Gamma^i_{n,m,\sigma}$ are the broadening of the (n, m, σ)th ring state caused by leaking into the emitter (collector) and inelastic scattering, respectively. In order to determine the qualitative dependence of the conductance on the magnetic field, we assume that $\Gamma^e_{n,m,\sigma}$ and $\Gamma^i_{n,m,\sigma}$ are constant, $\Gamma^e_{n,m,\sigma} = \Gamma^c_{n,m,\sigma} = \Gamma^{e,c} = 0.005$ meV, $\Gamma^i_{n,m,\sigma} = \Gamma^i = 0.004$ meV [9].

3. Results

For our calculation we consider the parameter corresponding to GaAs materials: $m_n = 0.067\, m_0$, where m_0 is the free electron mass, and $g_e = -0.5$, ring radius $r_0 = 800$ nm and a ring width $\Delta r = 300$ nm at $E_F = 2$ meV are taken from the literature [9].

In Fig. 2 we plot the dimensionless electron energy spectrum vs magnetic field at fixed Rashba spin-orbit parameter $\alpha = 3$ meV·nm for quantum number $n = 0$, $|m| \le 5$, $\sigma = \pm 1$.

The dimensionless resistance $R(h/e^2)$ vs magnetic field with Rashba spin-orbit interaction at fixed Fermi energy $E_F = 2$ meV in quantum ring is shown in Fig. 3. In Fig. 3 calculated with $n = 0$, $|m| \le 500$, $\sigma = \pm 1$. As seen from Fig. 3 the influence Rashba spin-orbit interaction destroys beating in the magnetic field dependence of magnetoresistance. It follows from the general considerations that the destructive interference of the contributions made by different electron trajectories to the wave function phase

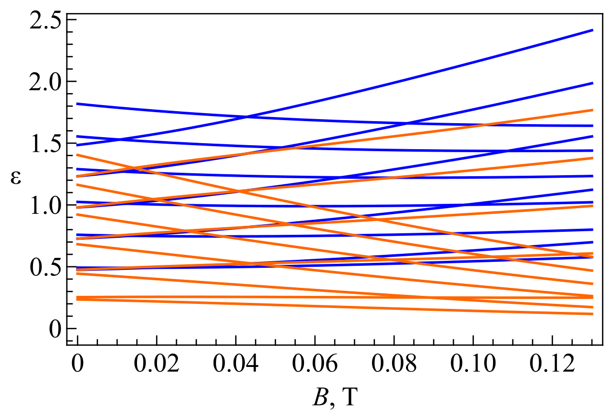

Fig. 2. The dependence dimensionless energy levels $(\varepsilon = E / \frac{\hbar^2}{2m_n r_0^2})$ as a function magnetic field at fixed Rashba spin-orbit parameter $\alpha = 3$ meV·nm for quantum number $n = 0$, $|m| \le 5$, $\sigma = \pm 1$.

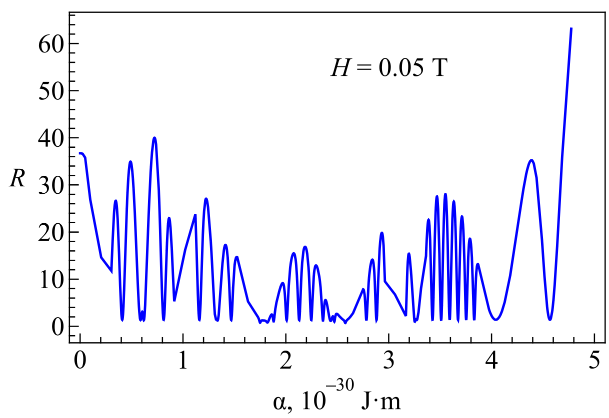

Fig. 4. The dependence of the dimensionless resistance $R\frac{h}{e^2}$ as a function of Rashba spin-orbit interaction parameter for magnetic field value $H = 0.05$ T in quantum ring. Fermi energy is kept fixed at $E_F = 2$ meV.

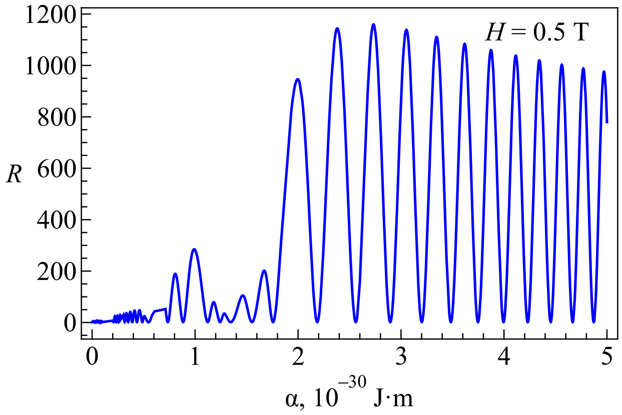

Fig. 5. The dependence of the dimensionless resistance $R\frac{h}{e^2}$ as a function of Rashba spin-orbit interaction parameter for magnetic field value $H = 0.5$ T in quantum ring. Fermi energy is kept fixed at $E_F = 2$ meV.

should distort the strict periodicity of the Aharonov–Bohm oscillations. The computed resistance versus Rashba spin-orbit parameter α shows in Fig. 4 at magnetic field $H = 0.05$ T and in Fig. 5 at magnetic field $H = 0.5$ T. The Fermi energy fixed at $E_F = 2$ meV. As see from Fig. 4 and Fig. 5 the magnetoresistance undergoes oscillation as function of Rashba spin-orbit interaction parameter α.

4. Conclusions

To summarize, in the paper we consider 2D electron gas magnetoresistance in semiconductor quantum ring with pseudoharmonic potential profile. The energy spectrum and wave functions of electrons are calculated for a semiconductor quantum ring of finite width under the uniform perpendicular magnetic field and the Rashba spin-orbit interactions. We show that 2D electron gas magnetoresistance depending on Rashba spin-orbit interaction constant oscillates with random amplitudes.

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Магнітоопір електронів у квантовому кільці зі спін-орбітальною взаємодією Рашби

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Досліджено вплив спін-орбітальної взаємодії Рашби на транспортні властивості двовимірного квантового кільця з кінцевою шириною за наявності рівномірного перпендикулярного магнітного поля. Досліджено залежність магнітоопору від магнітного поля та параметра спінової орбіти Рашби у квантовому кільці з кінцевою шириною. Було показано, що за наявності спін-орбітальної взаємодії Рашби картина биття руйнується.

Ключові слова: магнітоопір, балістична провідність.