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Transverse Anderson localization of evanescent waves propagating in randomly layered media

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We study theoretically the transverse Anderson localization of light in the simplest geometry, where the *p*-polarized wave propagates along the layers in the randomly stratified dielectric and evanesces exponentially in the direction across the layers. In this case, there exist two reasons for the localization of the wave in the direction transverse to its propagation: the usual evanescent wave confinement and the Anderson mechanism related to the randomness of the spatial distribution of permittivity. We solve the problem using the retarded-Green-function formalism in the Born approximation and show that, for fixed values of the wave frequency ω and wavenumber *q*, the random inhomogeneity results in the weakening of the wave localization. In the case of the surface plasmon-polaritons (SPPs) propagation, the Anderson mechanism changes the dispersion law for SPPs, moving the dispersion curves away from the light line. Therefore, the localization depth varies in different ways when increasing the disorder, depending on which of the values, wave vector *q* or frequency ω, is fixed. Namely, the localization depth increases for given *q*, but it decreases for given ω.

Keywords: inhomogeneous wave, depth of localization, Green's function, surface plasmon-polariton, dispersion law.

1. Introduction

In recent decades, since Anderson's seminal paper [1], the problem of transport of electromagnetic waves in one-dimensional (1D) systems with the long-range correlated disorder has attracted much attention (see, e.g., Refs. 2–9 and references therein). In this context, heterogeneous materials are of tremendous importance and interest [10]. Examples include composite materials, biological systems and others. Due to their significance, heterogeneous materials have been studied using experimental, theoretical, and different computer simulation methods. A stratified structure, the index of refraction of which depends on only one coordinate, is often used as the model medium in solving problems of wave propagation in the atmosphere, ionosphere, and ocean.

Thus, the Anderson localization is one of the fundamental phenomena in physics of disordered systems. The bibliography in this area numbers many reviews [9, 11–14], monographs [8, 15–19] and hundreds of original articles (see, e.g., Ref. 20–33). Most of these researches are related to special geometry, where a wave propagates in a direction perpendicular (or under some finite angle) to homogeneous planes with different dielectric properties that change randomly. It should be noted, however, that the transverse Anderson localization was considered in a number of papers (see, e.g., Ref. 34). Such phenomena are achievable in systems invariant along the direction of propagation and disordered in the transverse plane, e.g., in a set of randomly arranged but strictly parallel tubes. In the seminal paper [35], Schwartz and co-workers exploited a photorefractive crystal to generate complex longitudinally invariant and reconfigurable scattering patterns composed of tubes aligned along the direction of propagation.

In this paper, we study the transverse Anderson localization of light in the simplest geometry, where the wave propagates along the layers in the randomly stratified dielectric and evanesces in the direction across the layers, say, along the z axis. Thus, we investigate the situation when there are simultaneously two reasons for the localization of the wave in the direction transverse to its propagation. The first reason is the usual evanescent wave confinement due to the large value of the wave vector, $q > \omega\sqrt{\varepsilon}/c$, where ω is the wave frequency, ε_d is the average value of the dielectric permittivity, c is the speed of light in the vacuum. The second reason is the Anderson mechanism related to the randomness of the spatial distribution of $\varepsilon(z)$.

We solve the problem of p-polarized evanescent wave propagation in the randomly layered dielectric using the retarded-Green-function formalism in the Born approximation. Then we apply the obtained result to two physical realizations of the evanescent waves. The first problem is related to the so-called disturbed total internal reflection for the wave incident from an optically dense dielectric onto its boundary with the randomly inhomogeneous rare dielectric under conditions when the angle of incidence exceeds the limiting angle for the total internal reflection. It is known that, in this case, the wave penetrates into an optically rare medium in the form of the evanescent wave which can tunnel into another optically dense dielectric located nearby. We show that, contrary to expectations, the fluctuations of $\varepsilon(z)$ in the optically rare medium lead not to a weakening, but an increase in the coefficient of wave transmission through the optical tunneling barrier, in comparison with the case of a rare dielectric with constant dielectric permittivity ε_d. In other words, the Anderson mechanism does not enhance the localization of the evanescent wave with given q and ω, but weakens it.

The second solved problem is related to the propagation of surface plasmon-polaritons (SPPs). In this case, the Anderson mechanism changes the dispersion law for SPPs, moving the dispersion curves away from the light line deep into the (q,ω)-plane. It is of interest that the localization depth for SPPs varies in different ways, depending on which of the quantities, q or ω, is fixed. If one keeps the wave frequency constant, then the fluctuations of $\varepsilon(z)$ result in decrease of the localization length, whereas the disorder provides a weakening effect on the localization of SPPs with a fixed value of q.

2. Statement of the problem and main equations

Consider a monochromatic evanescent p-polarized electromagnetic wave with a single nonzero component $H_y(x,z) = H(z)\exp\left(iqx - i\omega t\right)$ of the magnetic field propagating along the x axis in a dielectric with coordinate-dependent permittivity $\varepsilon(z)$, occupying the half-space $z > 0$. From the Maxwell equations,

$$\mathrm{rot}\mathbf{E} = \frac{i\omega}{c}\mathbf{H}, \quad \mathrm{rot}\mathbf{H} = -\frac{i\varepsilon(z)\omega}{c}\mathbf{E}, \tag{1}$$

we obtain the following relation for $H(z)$,

$$\varepsilon(z)\frac{d}{dz}\left(\frac{1}{\varepsilon(z)}\frac{dH}{dz}\right) = \left(q^2 - \frac{\varepsilon(z)\omega^2}{c^2}\right)H, \tag{2}$$

If the dielectric half-space is homogeneous, $\varepsilon(z) = \mathrm{const} = \varepsilon_d$, the evanescent field decays exponentially with the spatial decrement κ_0 away from the interface,

$$H(z) = H_0\exp(-\kappa_0 z), \quad \kappa_0 = \sqrt{q^2 - \frac{\varepsilon_d\,\omega^2}{c^2}}, \quad z > 0. \tag{3}$$

Below we study the change of this decrement, $\kappa_d = \kappa_0 + \Delta\kappa$, due to a small random addition $\Delta\varepsilon(z)$ to the dielectric permittivity.

2.1. Green function for the evanescent wave

We study the role of fluctuations $\Delta\varepsilon(z)$ in the evanescent wave spectral properties in terms of the retarded Green function $G(z,z',\kappa_0)$ using the average procedure, similar to the one performed in Ref. 9 for the propagating waves. This Green function is governed by the equation,

$$\left[\frac{d^2}{dz^2} - \kappa_0^2 - V(z)\right]G(z,z',\kappa_0) = \delta(z - z'),$$

$$G(z \to \infty, z', \kappa_0) < \infty. \tag{4}$$

The weak disorder of the permittivity $\varepsilon(z)$ is incorporated into the perturbation function $V(z)$,

$$V(z) = -\frac{\Delta\varepsilon(z)\omega^2}{c^2} + \frac{\kappa_0}{\varepsilon_d}\frac{d[\Delta\varepsilon(z)]}{dz},$$

$$\Delta\varepsilon(z) = \varepsilon(z) - \langle\varepsilon(z)\rangle = \varepsilon(z) - \varepsilon_d \ll \varepsilon_d. \tag{5}$$

Without disorder, $V(z) = 0$, the unperturbed Green function $G_0(z,z',\kappa_0)$ has the form,

$$G_0(z - z', \kappa_0) = -\frac{\exp(-\kappa_0|z - z'|)}{2\kappa_0}. \tag{6}$$

The Fourier transform for the unperturbed Green function is

$$\mathcal{G}_0(k_z,\kappa_0) = \int_{-\infty}^{\infty} dz\exp(-ik_z z)G_0(z,\kappa_0)$$

$$= -\int_{-\infty}^{\infty} dz\exp(-ik_z z)\frac{\exp(-\kappa_0|z|)}{2\kappa_0} = -\frac{1}{k_z^2 + \kappa_0^2}. \tag{7}$$

In order to define the perturbation procedure over the scattering potential $V(z)$, we relate the perturbed Green function $G(z,z',\kappa_0)$ to the unperturbed one using the Green formula,

$$G(z,z',\kappa_0) = G_0(z - z',\kappa_0)$$

$$+ \int_{-\infty}^{\infty} dz'' G_0(z - z'',\kappa_0)V(z'')G(z'',z',\kappa_0). \tag{8}$$

It is easy to verify that solution (8) is satisfied to Eq. (4).

The result of the averaging procedure for the Green function is the Dyson equation,

$$\langle G(z-z',\kappa_0)\rangle = G_0(z-z',\kappa_0)$$
$$+\int_{-\infty}^{\infty} dz''dz'''G_0(z-z'',\kappa_0)M(z'',z''')\langle G(z'''-z',\kappa_0)\rangle. \quad (9)$$

The integral operator $\hat{M}$ that describes the wave interaction with the random potential, is called the self-energy or mass operator. The exact Dyson equation can be solved for a weak scattering, when the self-energy is written in the first non-vanishing (i.e., quadratic) order in the scattering potential $V(z)$. Within this approximation, known as the Born approximation, we can write,

$$M(z,z') \simeq M_B(z-z') = \langle V(z)G_0(z-z',\kappa_0)V(z')\rangle$$
$$= V_0^2 K(z-z')G_0(z-z',\kappa_0). \quad (10)$$

Here the function $K(z-z')$ is the binary/pair normalized, $K(0)=1$, correlation function of the random variable $V(z)$ with zero average value, $\langle V(z)\rangle = 0$,

$$\langle V(z)V(z')\rangle = V_0^2 K(z-z'), \quad \langle V^2(z)\rangle = V_0^2. \quad (11)$$

It is characterized by the correlation length R_c that is the scale on which the correlator effectively decreases.

Since the integral Dyson equation (9) contains the difference kernel Eq. (10), it can be readily solved by the Fourier transformation. As a result, we get a simple equation for the average Green function,

$$\langle \mathcal{G}(k_z,\kappa_0)\rangle = \mathcal{G}_0(k_z,\kappa_0) + \mathcal{G}_0(k_z,\kappa_0)\mathcal{M}_B(k_z)\langle \mathcal{G}(k_z,\kappa_0)\rangle. \quad (12)$$

Thus we have,

$$\langle \mathcal{G}(k_z,\kappa_0)\rangle = -\frac{1}{k_z^2+\kappa_0^2+\mathcal{M}_B(k_z)}. \quad (13)$$

Taking into account the explicit expression Eq. (10) for the self-energy together with the Fourier transform for the unperturbed Green function Eq. (7), one can get

$$\mathcal{M}_B(k_z) = -V_0^2 \int_{-\infty}^{\infty} \frac{dk'}{2\pi}\frac{\mathcal{K}(k'-k_z)}{k'^2+\kappa_0^2}, \quad (14)$$

where $\mathcal{K}(k)$ is the Fourier transform for the binary correlator $K(z)$.

It should be noted that one of the necessary conditions for the validity of presented here averaging procedure is the smallness of the correlation length with respect to the localization depth $1/\kappa_0$,

$$\kappa_0 R_c \ll 1. \quad (15)$$

This condition follows from the obvious physical requirement that the wave, in the region of its existence, should feel a large ensemble of realizations of the random potential. For such extremely short correlations, the function $K(z)$ can be changed by the Dirac delta-function, $K(z) \to \delta(z/R_c)$, and $\mathcal{K}(k)$ is constant. So, the integration in Eq. (14) gives the value of $\mathcal{M}_B$ independent of k_z,

$$\mathcal{M}_B = -V_0^2 \frac{R_c}{\kappa_0}, \quad (16)$$

where the correlation length is, by definition,

$$R_c = \int_0^{\infty} K(z)dz. \quad (17)$$

Now, performing the inverse Fourier transformation for Eq. (13) and using Eq. (16), we obtain the perturbed Green function $\langle G(z-z',\kappa_0)\rangle$,

$$\langle G(z-z',\kappa_0)\rangle = -\frac{\exp(-\kappa_d\,|z-z'|)}{2\kappa_d}, \quad (18)$$

with

$$\kappa_d = \kappa_0 - \frac{V_0^2 R_c}{2\kappa_0^2}. \quad (19)$$

Note that Eq. (18) differs from expression (6) for the unperturbed Green function $G_0(z-z',\kappa_0)$ by the change $\kappa_0 \to \kappa_d$ only. Thus, we have come to an important unexpected conclusion: the disorder results in the weakening of localization for the evanescent wave with given values of the frequency ω and wavenumber q. However, the change of the localization depth should be considered as a small positive correction to κ_0^{-1}, otherwise the used here Born approximation becomes inapplicable. In other words, the parameter κ_0 should not be small,

$$\kappa_0 > (V_0^2 R_c)^{1/3} \quad (20)$$

[see Eq. (19)].

It is known that the damping decrement of the averaged Green function does not define the decrement of the averaged amplitude of the *propagating waves* (see, e.g., Ref. 9). However, as was shown in Ref. 36, these damping coefficients coincide exactly for the *evanescent waves*.

2.2. Enhancement of wave transmission through an optical potential barrier

In this subsection, we apply the obtained result to the problem of disturbed total internal reflection. Consider the wave incident from an optically dense dielectric prism with permittivity $\varepsilon_p > \varepsilon_d$ onto its boundary with the randomly stratified rare dielectric under conditions when the angle θ of incidence exceeds the limiting angle θ_0 for the total internal reflection [see Fig. 1(a)]. It is known that, in this case, the electromagnetic field penetrates into an optically rare medium in the form of evanescent wave which can

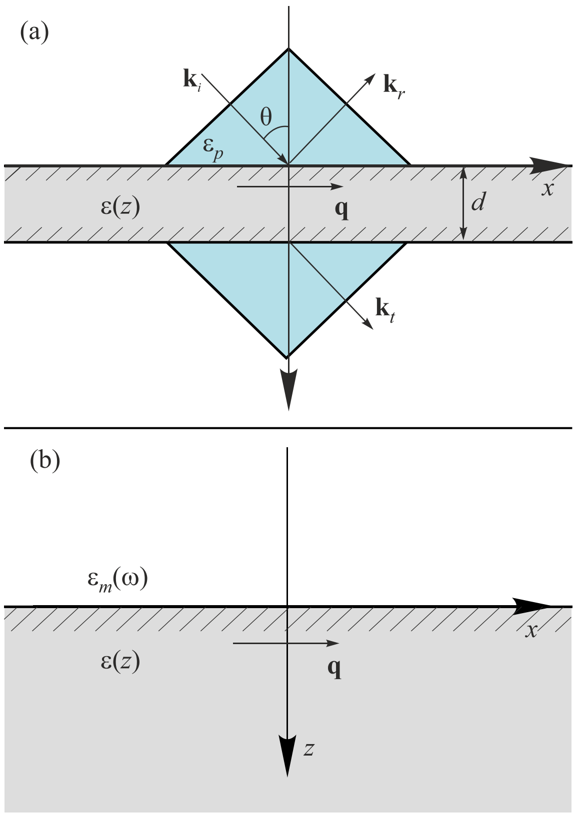

Fig. 1. Geometry of the problems: (a) For the wave transmission through an optical potential barrier. Here $\mathbf{k}_i$, $\mathbf{k}_r$, and $\mathbf{k}_t$ are the wave vectors of the incident, reflected, and transmitted waves. In this setup, the frequency and wavenumber of evanescent wave can be set independently of each other and they are independent of the distribution $\varepsilon(z)$. (b) For the surface plasmon-polariton propagation. In this case, the frequency and wavenumber of evanescent wave are related to each other via the dispersion law which, in turn, is sensitive to the distribution $\varepsilon(z)$.

tunnel into another optically dense dielectric prism located nearby. The transmission coefficient T through such an optical tunneling barrier is proportional to $\exp(-2\kappa_d d)$ where d is the distance between the prisms, and, according to Eq. (19), the fluctuations of $\varepsilon(z)$ result in its increasing,

$$T = T_0 \exp\left(\frac{V_0^2 R_c d}{\kappa_0^2}\right). \tag{21}$$

Here T_0 is the transmission coefficient through the rare dielectric gap without fluctuations. Note that, despite the smallness of the exponent in Eq. (21) with respect to $\kappa_0 d$, it can be greater than unity, and the value of T can significantly exceed T_0.

Thus, surprisingly, the Anderson mechanism does not reduce the transparency of the optical tunnel barrier but, on the contrary, can significantly increase it. It should be noted that a similar effect of enhancement of electron tunneling through a potential barrier with fluctuating amplitude was discussed in Refs. 35–37.

3. SPP propagation in disordered medium

Here we study the effect of disorder in the distribution of $\varepsilon(z)$ on the properties of the surface plasmon-polaritons. The SPP is a p-polarized electromagnetic wave propagating with the wavenumber q along the interface $z = 0$ between media with permittivities of different signs, $\varepsilon_m < 0$ for a medium occupying the half-space $z < 0$ and $\varepsilon > 0$ for a medium in half-space $z > 0$. We suppose that the dielectric medium is randomly layered and its permittivity ε is a random function of the coordinate z [see Fig. 1(b)].

The SPP is the wave exponentially evanescent in both half-spaces. The electric and magnetic fields are proportional to $\exp(-\kappa_d z)$ with κ_d given by Eq. (19) in the dielectric half-space $z > 0$ and are proportional to $\exp(\kappa_m z)$ at $z < 0$. Here

$$\kappa_m = \sqrt{q^2 + \frac{|\varepsilon_m|\omega^2}{c^2}}. \tag{22}$$

The dispersion law for the SPPs follows directly from the boundary conditions of continuity for the tangential components of the electric and magnetic fields at the interface $z = 0$. It can be written in the form,

$$\kappa_d = \kappa_m \frac{\varepsilon_d}{|\varepsilon_m|}. \tag{23}$$

Within the nondissipative Drude model, the metal permittivity ε_m is

$$\varepsilon_m = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2}, \tag{24}$$

where ω_p is the plasma frequency and ε_∞ is the dielectric constant of the ionic core. Then, using the dimensionless wave frequency Ω and wavenumber Q,

$$\Omega = \omega/\omega_p, \quad Q = q\frac{c}{\omega_p\sqrt{\varepsilon_d}}, \tag{25}$$

we can present the unperturbed dispersion law $Q_0(\Omega)$ for the surface plasmon-polaritons as

$$Q_0^2 = \Omega^2 \frac{1-\Omega^2\varepsilon_\infty}{1-\Omega^2(\varepsilon_\infty+\varepsilon_d)}. \tag{26}$$

The account of the disorder provides an additional term in the dispersion relation,

$$Q^2(\Omega) = Q_0^2(\Omega) + \gamma F(\Omega, Q), \tag{27}$$

where the function $F(\Omega, Q)$ and dimensionless constant γ are,

$$F(\Omega, Q) = \left\{\left[1 - \frac{\varepsilon_d^2\Omega^4}{(1-\varepsilon_\infty\Omega^2)^2}\right]\sqrt{Q^2-\Omega^2}\right\}^{-1}, \tag{28}$$

$$\gamma = V_0^2 R_c \frac{c^3}{\omega_p^3 \varepsilon_d^{3/2}}. \tag{29}$$

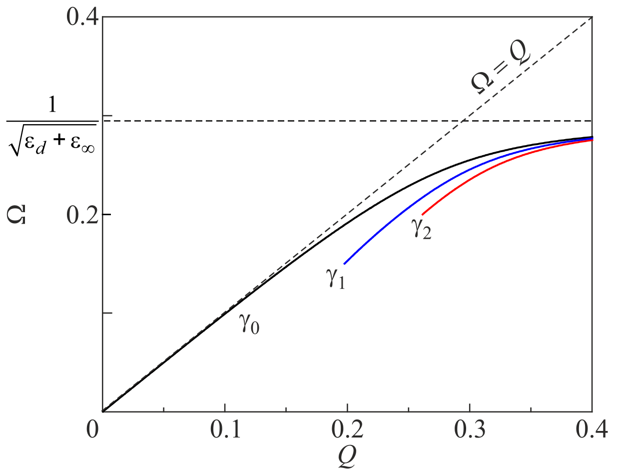

Fig. 2. Dispersion curves for SPPs plotted for $\gamma_0 = 0$, $\gamma_1 = 0.002$, and $\gamma_2 = 0.004$. The dielectric constants are $\varepsilon_d = 1.5$, $\varepsilon_\infty = 10$.

Figure 2 shows the dispersion curves Eq. (27) for the region in the (Q,Ω)-plane where the conditions Eqs. (15) and (20) are satisfied. One can see that the disorder results in the shift of the dispersion curve down from the unperturbed line $Q = Q_0(\Omega)$ for $\gamma = 0$.

Now, let us discuss the effect of disorder in the distribution of $\varepsilon(z)$ on the localization depth of the SPP. In contrast to the problem of the wave transmission through a fluctuating optical potential barrier, considered in the previous section, here the values Q and Ω are not independent parameters. They are related to each other via the dispersion law Eq. (27). Therefore, the effect of disorder turns out to be fundamentally different, depending on what value, Q or Ω, remains fixed when increasing the disorder parameter γ. To verify this, we consider Eq. (23). According to this relation and Eq. (24), the dimensionless localization depth L can be presented in the form,

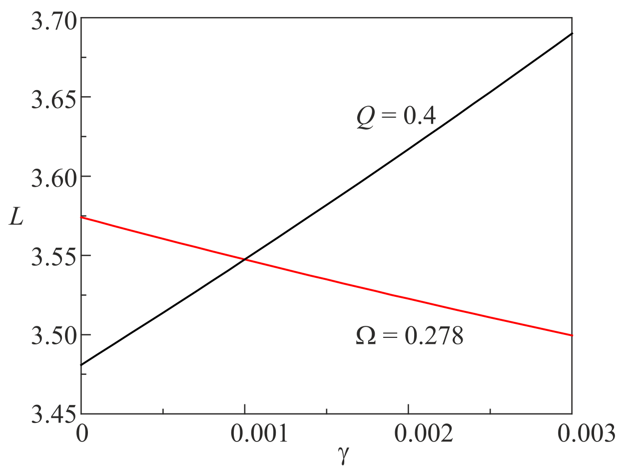

Fig. 3. Dependenses of the dimensionless localization depth $L = \omega_p\sqrt{\varepsilon_d}\,/\,c\kappa_d$ of the SPPs on the disorder parameter γ for fixed values of the wavenumber, $Q = 0.4$, or the wave frequency, $\Omega = 0.728$.

$$L = \frac{\omega_p\sqrt{\varepsilon_d}}{c\kappa_d} = \frac{1}{\Omega^2}\left[\frac{Q^2\varepsilon_d^2}{(1-\varepsilon_\infty\Omega^2)^2} + \frac{\varepsilon_d}{1-\varepsilon_\infty\Omega^2}\right]^{-1/2}. \quad (30)$$

For the case of fixed frequency Ω, the value of Q increases when increasing the parameter γ (see Fig. 2), and the localization depth Eq. (30) decreases. This means that the disorder promotes localization in this case. On the other hand, for a fixed wavenumber Q, the value of Ω decreases when increasing the parameter γ, and the localization depth Eq. (30) increases. This means that the disorder promotes delocalization of the wave with fixed Q. This fundamentally different behavior of the localization depth with an increase of disorder is demonstrated in Fig. 3, where curves $L(\gamma)$ are drown using Eqs. (27) and (30).

Note that these conclusions are valid not only within the Drude model but for any other models with decreasing dependence $|\varepsilon_m(\omega)|$.

4. Conclusion

In this paper, we have studied the effect of disorder on the properties of *p*-polarized electromagnetic waves propagating along the layers in the randomly stratified dielectric and damps in the direction across the layers. We have shown that, for fixed values of the wave frequency ω and wavenumber q, the random inhomogeneity results in the weakening of the wave localization. In the case of the surface plasmon-polaritons propagation, the Anderson mechanism changes the dispersion law for SPPs moving the dispersion curves away from the light line. Therefore, the localization depth varies in different ways when increasing the disorder, depending on what the value, wave vector q or frequency ω, is fixed. Namely, the localization depth increases for given q, but it decreases for given ω.

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Поперечна андерсонівська локалізація неоднорідних хвиль, що поширюються у випадково шаруватих середовищах

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Вивчено поперечну андерсонівську локалізацію світла в найпростішій геометрії, коли *p*-поляризована хвиля поширюється вздовж шарів у випадково стратифікованому діелектрику та згасає експоненціально в напрямку поперек шарів. У цьому випадку існує дві причини для локалізації хвилі в напрямку, поперечному її поширенню: звичайне просторове згасання неоднорідної хвилі та механізм Андерсона, який пов'язаний з випадковістю просторового розподілу діелектричної проникності. Використано формалізм запізнювальної функції Гріна у борнівському наближенні та показано, що для фіксованих значень частоти хвилі ω та хвильового числа *q* випадкова неоднорідність призводить до послаблення локалізації хвилі. У випадку поширення поверхневих плазмон-поляритонів (ППП), механізм Андерсона змінює закон дисперсії для ППП, віддаляючи дисперсійні криві від світлової лінії. Тому глибина локалізації ППП змінюється по різному при посиленні безладу, залежно від того, яке зі значень, хвильового вектора *q* чи частоти ω, фіксоване. А саме, глибина локалізації збільшується для заданого *q*, але зменшується для фіксованої ω.

Ключові слова: неоднорідна хвиля, глибина локалізації, функція Гріна, поверхневий плазмон-поляритон, закон дисперсії.