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Evolution of low-energy magnetic excitations pair spectrum in $SmMnO_{3+\delta}$

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The identification of low-energy thermal excitations in $SmMnO_{3+\delta}$ degenerate states of spin and superconducting quantum liquids in magnetic fields $H \leq 3.5$ kOe is presented. In the temperature interval 4.2–12 K, the Landau quantization of the low-energy magnetic excitations pair spectrum of Z_2 quantum spin liquid is found in the system spinon-gauge field. The formation of a broad continuum of spinon pair excitations in the "weak magnetic field" regime (H = 100 Oe, 1 kOe) in the FC regime is explained in the framework of the Landau quantization models of the compressible spinon gas with fractional values of the factor ν filling three overlapping bands. In the regime of "strong magnetic field" (H = 3.5 kOe), the quantum oscillations of temperature dependences of "supermagnetization" of the incompressible spinon liquid were observed. They have the form of three narrow steps (plateaus), corresponding to a complete filling of the non-overlapping Landau bands with integer values of the filling factor by spinons. These results are evidence for the existence of vortex gauge field fluctuations with a high density in the magnetic fields $H \geq 100$ Oe. The strong growth of vortex fluctuations can be explained by a second-kind phase transition in $SmMnO_{3+\delta}$ in the form of the vortices condensation. Growth of the external dc magnetic field strength in the $SmMnO_{3+\delta}$ samples in the interval of fields $0 < H \leq 3.5$ kOe leads to a continuous decrease in the giant magnetization jump near the temperature $T_{KT} \cong 12$ K of the topological phase transition, Kosterlitz–Thouless dissociation of 2D vortex-antivortex pairs in a local superconducting state. The suppression of the magnetization jump near the T_{KT} temperature with increasing H is explained by the polarization of vortex-antivortex pairs at temperatures below T_{KT} by an external dc magnetic field, which weakens the vortex interaction in pairs and leads to their dissociation.

Keywords: spin liquid, spinon pairs, vortex pairs, magnetization oscillations, Landau quantization.

1. Introduction

For the first time, a broad discussion of disordered states of quantum spin liquids (QSL) with the energy gap appeared during the study of unusual magnetic properties of Mott dielectrics [1]. It was found that in the case of a system of spins located at the i, j sites of a triangular crystal lattice, its ground state has a long-range antiferromagnetic (AFM) order if the exchange takes place only between the nearest neighbors. The spin system is described by the classical Heisenberg Hamiltonian $\hat{H}_{AF}$ with the spin operator $S = 1/2$ acting on electrons at i, j sites of the crystal lattice. In this ground state, there are only gapless spin-wave excitations, and each spin has an averaged polarization along some direction with small quantum fluctuations near the direction. It was also found [2] that the Heisenberg chain of spins with $S = 1$ is disordered even at zero temperature and has a correlation function $\langle S_iS_j \rangle$ exponentially decreasing at large distances. A very important consequence of the exponential decay of the correlation function is the presence of a spin gap: the system has no low-energy excitations. The ground state of the system is a spin singlet, and the first triplet excitations have a finite energy $\Delta > 0$. The presence of the gap is a consequence of the finite correlation length ξ. The spin system behaves as a finite system of size ξ, and for a finite system, the energy levels are quantized. Thus, the gap Δ is essentially the difference between the singlet ground state and the first excited triplet state of the finite system of spins. Since the magnitude of the gap is proportional to the value of the ratio of the exchange interaction J between spins to the number N of nodes in the spin clusters, then the spin gap $\Delta \propto \dfrac{J}{\left(\frac{\xi}{a}\right)^d}$, where a is the crystal lattice parameter, and d is the dimension of the spins system. Magnetic systems with a finite correlation length ξ at zero temperature and a spin gap

$\Delta \propto \frac{1}{\xi}$ are known as spin liquids by analogy with standard liquids with only short-range order.

There are a number of theoretical models used by different authors to study the unusual properties of gapped QSL in AFMs with the Heisenberg exchange interaction. Anderson first used the resonating valence bond (RVB) model [3, 4] to describe the spin system in a Mott dielectric [5]. This state is a linear superposition of a very large number of possible singlet pairings between electrons. It has been shown in [6] that the RVB state formation manifests itself as a separation between the charge and spin of the electrons. The elementary excitations of the RVB state are new quasiparticles — spinons, with spin $S = 1/2$, but without any charge. Within the mean field models, fermions in the chiral spin liquid state are spinons, neutral quasiparticles with spin $S = 1/2$. According to the mean field model, an order parameter χ_{ij} of the chiral spin liquid generates a magnetic flux. The spinons described by the Hamiltonian $\hat{H}_{\text{mean}}$ behave as if they move in a magnetic field. When the magnetic flux becomes commensurate with the spinon density and the crystal lattice, a state of the spinon gas arises in which spinons completely fill an integer number of Landau levels. In this case, the spinon gas becomes incompressible due to a finite gap of Landau levels. In the state of a chiral spin liquid with a flux commensurate with the crystal lattice, there are no density fluctuations of spinons. Kivelson described the physics of the RVB state formation in the framework of the model of "quantum dimmers" according to which the ground state of the spin system is a superposition of the dimer pairs arising due to singlet pairing of spins of nearest neighbors [7]. The stable RVB phase was first studied by Wen [8] and Reed and Sagdeev [9] in the Z_2 gauge field theory [10–13]. Examples of the existence of the Z_2 RVB phase in spin systems located in crystal lattice sites with different geometries were reviewed in [14–16]. The Kitaev spin liquid model [17, 18] is based on the assumption of the existence of spin liquid excitations in the form of exotic composite quasiparticles — anyons. According to works [19, 20], anyons have unusual statistics (neither Bose nor Fermi). The Kitaev model was solved exactly by a reduction to free fermions in the Z_2 gauge field. According to it, two types of the Z_2 spin liquid are possible: a gapped spin liquid, for which the excitations are in the form of Abelian anyons, and a gapless Z_2 spin liquid, in which non-Abelian anyons are excited. However, in the presence of an external magnetic field, the second phase acquires an energy gap. Interest in quasiparticles with intermediate statistics is primarily due to the fact, that the pairing of anions with opposite spins to form a molecular bosonic phase can lead to high-temperature superconductivity because the known high-temperature superconductors are quasi-two-dimensional systems with the antiferromagnetic ordering of magnetic moments of atoms in the layers and can be dielectrics or metals in the ground state. It is assumed that anionic superconductivity in such systems can arise at the critical temperature $T_c = N/(m^*\zeta)$, where N is the concentration of anions, m^* is their effective mass, $\zeta = 2$ is the degree of degeneration of the anyon ground state.

Earlier in the system of $La_{1-y}Sm_yMnO_{3+\delta}$ self-doped manganites ($\delta \sim 0.1$, $0 \le y \le 1$) for samples with $y = 0.85$ and 1.0, we have revealed characteristic features of nanoscale superconductivity (SC): anomalous diamagnetism, damped oscillations of the critical temperature of phase transitions and magnetic susceptibility of samples, as well as magnetic flux trapping [21]. It was assumed that the samples with $y = 0.85$ and 1.0 have properties of a multicomponent SC composite, in which at temperatures $T < 60$ K an inhomogeneous mixed state of frustrated spatially modulated AFM phases of A-, CE-type and microphases of spin, electron-hole, and superconducting quantum liquids spontaneously emerges. It has been shown that in such samples, charge correlations coexist in a wide temperature range of the short-range spin. They exhibit QSL properties and metal drops of the quantum electron-hole liquid. Moreover, in these samples, there is a small fraction of the local superconducting phase in the form of SC loops network, connected by tunneling Josephson junctions with small critical supercurrents. In [22], characteristic features of a universal Nelson–Kosterlitz jump of the quasi-two-dimensional SC liquid density in the temperature dependences of dc magnetization of superconducting samples, caused by the transition of the samples into a state with coherent superconductivity at temperatures below T_c, were found. A cusp-like feature of dc magnetization curves near the critical temperature $T_{KT} \equiv$ $\equiv T_c \cong 43$ K was observed with increasing temperature in a sample with $y = 0.85$ which is typical for the dissociation of 2D vortex pairs. A similar jump-like feature was found in the $SmMnO_{3+\delta}$ sample with samarium concentration $y = 1.0$ at much lower temperature $T_{KT} \cong 12$ K. The obtained experimental results were explained within the framework of the existing models of the topological phase transition, Kosterlitz–Thouless dissociation of 2D vortex pairs in a network of quasi-two-dimensional Josephson weak bonds. In the work [23], we found the Landau quantization of the spectrum of low-energy magnetic pair excitations of Z_2 QSL in the temperature interval 4.2–12 K in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$. The formation of a continuous excitation spectrum of the quantum spin liquid in the regime of "weak magnetic fields" ($H = 100$ Oe, 350 Oe, 1 kOe) is explained in the framework of the Landau quantization models of the spectrum of composite quasiparticles in a system spinon-gauge field with fractional values of the factor ν filling three overlapping Landau bands. In the regime of "strong external magnetic field" ($H = 3.5$ kOe) new quantum oscillations of the temperature dependences of magnetization of incompressible spinons liquid in the form of three narrow steps (plateaus), corresponding to full filling of non-overlapping Landau bands with integer values of a filling factor by spinons, are revealed.

2. Experimental technique

Self-doped manganites $SmMnO_{3+\delta}$ ($\delta \sim 0.1$) samples were grown from high-purity samarium, and electrolytic manganese oxides were taken in a stoichiometric ratio. The synthesized powder was pressed under 10 kbar into 6 mm-diameter disks 1.2 mm thick and was sintered in air at a temperature of 1170 °C for 20 h followed by the temperature decrease with a rate of 70 °C/h. The obtained pellets represented single-phase ceramics according to x-ray data. X-ray diffraction study was conducted at 300 K using a DRON-1.5 diffractometer in $NiK_{\alpha 1+\alpha 2}$ radiation. The lattice symmetry and parameters were determined by the splitting position and reflections feature of the perovskite-type pseudo-cubic lattice. The temperature dependences of the sample magnetization were measured using a VSM EGG (Princeton Applied Research) vibrating magnetometer and a non-industrial magnetometer in dc magnetic fields of 0.1, 1, and 3.5 kOe. The temperature dependences of the magnetization were obtained in the FC measurement mode in which samples were preliminarily cooled in a measuring field to 4.2 K followed by their heating to 250 K.

3. Experimental results

The magnetization-vs-temperature dependences $M(T)$ of self-doped $SmMnO_{3+\delta}$ samples obtained in the measuring magnetic fields of 100 Oe, 1 kOe, and 3.5 kOe in the FC regime are shown in Fig. 1. At $T < 20$ K, characteristic features of low-energy magnetic excitation pairs of the degenerate state of spin and superconducting quantum liquids are observed near temperatures $T_{\text{spinon}} \cong 8$ K and $T_{KT} \cong 12$ K, which shape and magnitude depend strongly on the external magnetic field strength. According to the literature data and our experimental results, we consider that in the temperature range $0\ \text{K} < T \leq 20$ K, the magnetization has three contributions, namely, the well-known dominant magnetization contribution of the RVB-phase Z_2 gapped quantum spin liquid in the form of a broad magnetization peak with a top near 20 K and weaker additional contributions to the magnetization of the degenerate thermal excitation state of spin and superconducting quantum liquids in the form of characteristic “supermagnetization” features of samples near $T_{\text{spinon}} \cong 8$ K and $T_{KT} \cong 12$ K. Here T_{spinon} is the average temperature of thermal excitations of the spinon spectrum, T_{KT} is the temperature of the topological Kosterlitz–Thouless phase transition of the dissociation of 2D vortex pairs of the superconducting quantum liquid. In the FC regime of measurement, the QSL is in the polarized state, which allows us to record weak dc magnetization changes of the samples induced by thermal excitations of the non-magnetic (singlet) ground state of the quantum spin liquid in $SmMnO_{3+\delta}$ at very low temperatures. It can be easily seen (Fig. 1) that the shape and intensity of the “supermagnetization” features at temperatures below 20 K significantly change with the increasing external magnetic field up to the critical value $H = 1$ kOe, corresponding to the beginning of the coherent state destruction of local superconductivity in $SmMnO_{3+\delta}$ samples caused by the increasing external magnetic field strength [21].

It is seen in Fig. 2 that at $H = 100$ Oe there is a threshold feature in the temperature dependence of the “supermagnetization” of $SmMnO_{3+\delta}$ near $T_d \cong 50$ K. We found a similar feature earlier in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ manganites, associated with the existence of the small pseudogap Δ_e in the electron spectrum [23]. It is the characteristic feature of a weak Mott insulator with the spinon Fermi surface. According to Figs. 2–4, in the magnetic field $H = 100$ Oe, the well-known from the literature ground state of the RVB

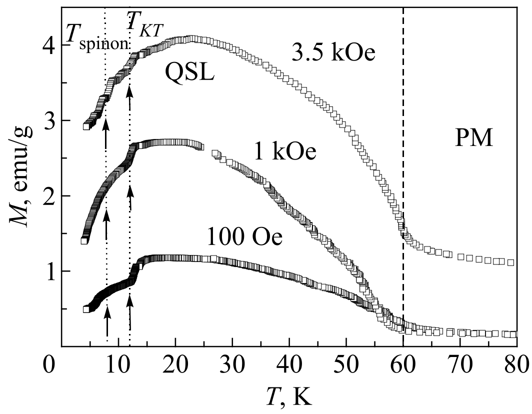

Fig. 1. Temperature dependences of the $SmMnO_{3+\delta}$ magnetization measured in the magnetic fields of 100 Oe, 1 kOe, and 3.5 kOe in the FC regime measurements in the temperature range 4.2–80 K. At temperatures $T < 20$ K, the “supermagnetization” features are observed, marked by arrows near the temperatures $T_{\text{spinon}} \cong 8$ K and $T_{KT} \cong 12$ K [quantum spin liquid (QSL), paramagnetic state (PM)].

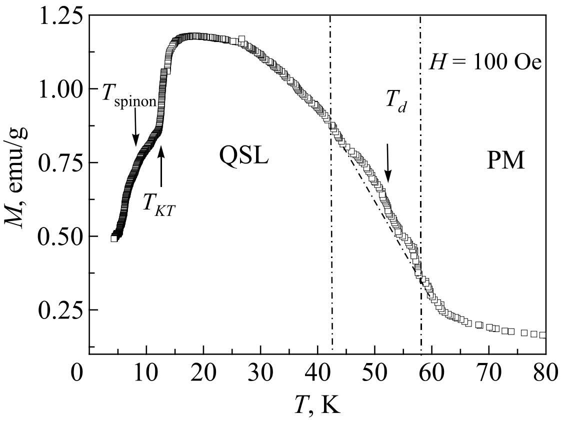

Fig. 2. Temperature dependence of the $SmMnO_{3+\delta}$ magnetization in the temperature range 4.2–80 K in the magnetic field $H = 100$ Oe in the FC regime measurements. At temperatures $T < 20$ K, the magnetization has three contributions: the dominant magnetization contribution of the RVB-phase Z_2 gapped quantum spin liquid in the form of a broad magnetization peak near 20 K and weaker additional “supermagnetization” contributions from the degenerate low-energy thermal excitation state of spin and superconducting quantum liquids near T_{spinon} and T_{KT} temperatures.

gapped spin liquid is realized in the investigated $SmMnO_{3+\delta}$ samples, with an energy gap Δ_s between ground and excited states. In the framework of widely used RVB state models of low-dimensional frustrated antiferromagnetics [3–16], the gapped phase of the spin liquid is characterized by low-energy magnetic excitations of the ground non-magnetic state of the spin system consisting of singlet pairs of spins of the nearest neighbors, as pairs of neutral quasiparticles with spin $S = 1/2$ (spinons) with different wave vectors. As shown in Figs. 2–4, thermal excitation of spinon pairs occurs over a relatively wide temperature range of 4.2–12 K. It is seen in Figs. 3, 4, the excitation of spinons with $S = 1/2$ in a magnetic field $H = 100$ Oe occurs mainly in the temperature range 6–10 K as a doublet of two broad almost overlapping peaks near the average excitation temperature $T_{\text{spinon}} \cong 8$ K, which corresponds to the thermal excitation energy of spinons $E_{\text{spinon}} \cong 0.6$ meV. At temperatures below 6 K, two weak doublets of spinon pair magnetization are observed, with the excitation energy E_{spinon} being lower than the low-energy gap $\Delta_s \approx 0.4$ meV in the spinon excitation spectrum of the 2D spin system between the non-magnetic singlet spin states in the RVB ground state and the excited magnetic spinon state with $S = 1/2$. As can be well seen in Fig. 3, the doublet excitation of spinon pairs with $S = 1/2$ takes place in the temperature interval, which overlaps with the decoupling temperatures of 2D vortex pairs of the superconducting liquid, indicating the practical coincidence of their excitation energies. As can be seen in Figs. 3 and 5,

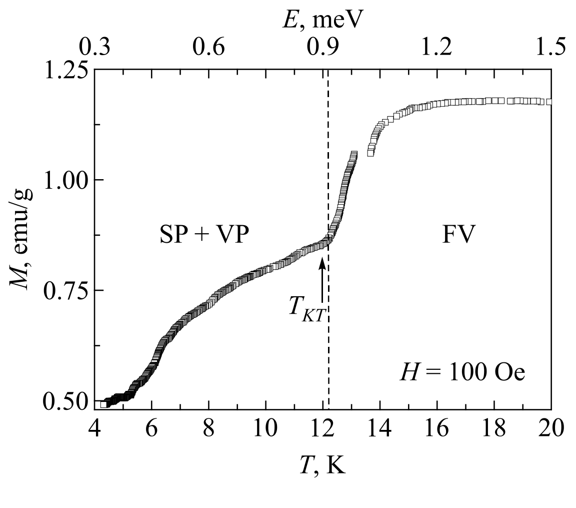

Fig. 3. The temperature dependence $M(T)$ of mixed state spin and superconducting quantum liquids in $SmMnO_{3+\delta}$, measured in the weak field $H = 100$ Oe in the temperature range 4.2–20 K. The asymmetric two-humped peak magnetization feature near $T_{\text{spinon}} \cong 8$ K corresponds to a continuous spectrum of low-energy magnetic excitations of the gapped QSL in the form of spinon pairs with $S = 1/2$. At temperatures $T \geq T_c = T_{KT} \cong 12$ K, a strong jump of the magnetization curve $M(T)$ is observed, caused by a topological phase transition of the Kosterlitz–Thouless dissociation of 2D vortex-antivortex pairs in a superconducting quantum liquid [spinon pairs (SP), vortex pairs (VP), free vortices(FV)].

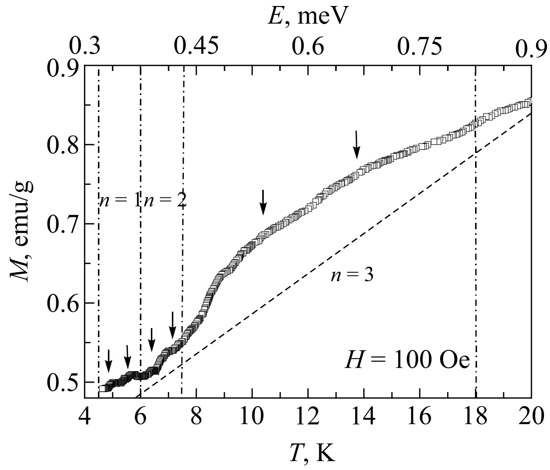

Fig. 4. Quantum oscillations of the "supermagnetization" of the $SmMnO_{3+\delta}$ samples, measured in a weak magnetic field $H = 100$ Oe in the temperature range 4.2–12 K, corresponding to the quantum continuum of spinon pair excitations with $S = 1/2$. The continuum of thermal excitations of the "supermagnetization" spinon pairs is divided into three narrow overlapping Landau bands with numbers $n = 1, 2, 3$.

the decoupling of vortex-antivortex pairs in the superconducting liquid at temperatures above T_{KT} is accompanied by a "giant" jump in the sample "supermagnetization". The magnetization jump is explained by the appearance of the free 2D vortices plasma in the SC liquid with opposite magnetic moment directions, which are easily oriented along the direction of the external magnetic field. Thus, in weak magnetic fields $H \sim 100$ Oe, thermal excitation energies of spinon pairs with $S = 1/2$ and the plasma of 2D electron vortices practically coincide, which points to the degeneracy

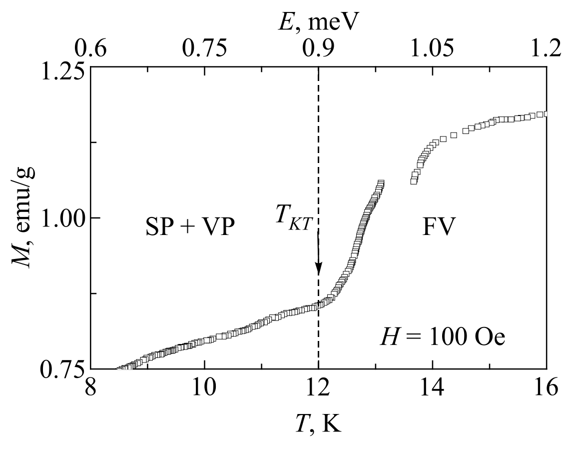

Fig. 5. Cusp-like feature of the $SmMnO_{3+\delta}$ samples magnetization in a weak magnetic field $H = 100$ Oe, measured in the temperature range 8–16 K in FC regime measurements. A giant jump in the magnetization near $T_{KT} \cong 12$ K, the temperature of the Kosterlitz–Thouless topological phase transition of the dissociation of 2D vortex-antivortex pairs in a superconducting quantum liquid is caused by a sharp increase in the number of free 2D vortices in *ab* planes with increasing temperature.

of the ground states of the spin and superconducting quantum liquids. The jump of additional magnetization near T_{KT} is almost three times higher than its increase near T_{spinon}, indicating a more significant contribution of the superconducting liquid magnetization to the total sample magnetization (Fig. 3).

According to Fig. 4, at $H = 100$ Oe, in the $SmMnO_{3+\delta}$ sample, a continuous spectrum of thermal excitations of spinon pairs is observed in the temperature range 4.2–9.5 K as a periodic double narrow peak features of the temperature dependences of "supermagnetization" near the average temperatures $T_1 \cong 4.6$ K, $T_2 \cong 5.6$ K, and $T_3 \cong 8$ K. The allowed separation of the continuum of thermal excitations of magnetization into narrow bands $n = 1$, $n = 2$, and $n = 3$, which can be explained by quantization of the spinon spectrum with fractional values of the factor ν filling three overlapping Landau bands in the regime of a weak external magnetic field [23]. Figures 6–9 clearly show that an increase in the external magnetic field strength to the value $H = 1$ kOe leads to significant changes in the temperature dependences of the $M(T)$ curves near the transition critical temperature of the samples into a coherent superconducting state. First of all, there is a complete consolidation of two weakly resolved peaks of doublet excitation of spinon pairs with $S = 1/2$ into a single symmetric peak of the magnetization temperature dependence near the average temperature $T_{\text{spinon}} \cong 8$ K and a strong broadening of the spinon excitation spectrum in the temperature range 6–10 K (Figs. 7 and 8). This result evidences the existence of "giant" fluctuations arising in the QSL ground state, which leads to smearing of the spectrum of low-energy spinon excitations.

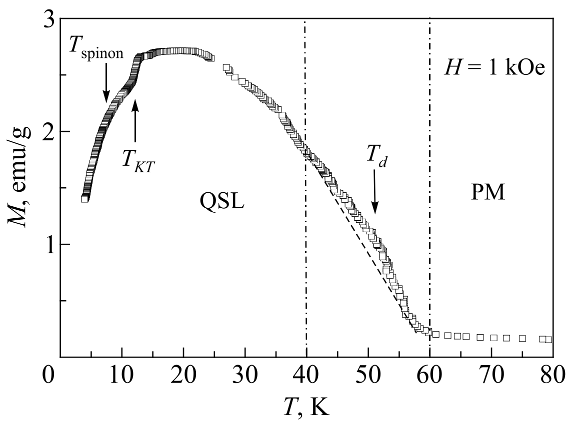

Fig. 6. The temperature dependence of the $M(T)$ magnetization of $SmMnO_{3+\delta}$ in the range 4.2–80 K in the magnetic field $H = 1$ kOe in the FC regime measurements. At temperatures $T < 20$ K, the magnetization has three components: the dominant magnetization contribution of the RVB-phase Z_2 gapped QSL in the form of a broad magnetization peak near 20 K and weaker additional "supermagnetization" contributions from the degenerate state of low-energy thermal excitations of the spin and superconducting quantum liquids near T_{spinon} and T_{KT} temperatures.

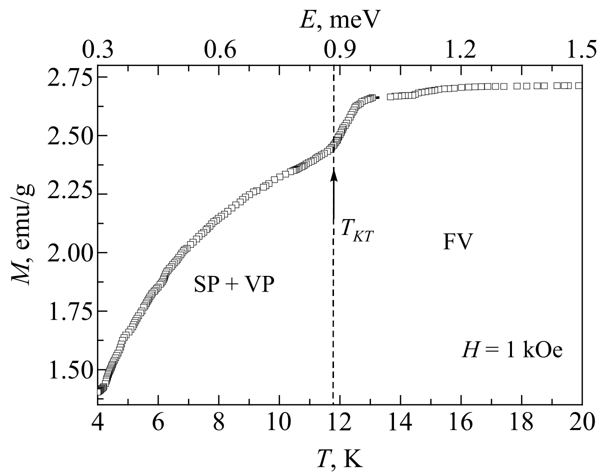

Fig. 7. The temperature dependence $M(T)$ of the mixed state spin and superconducting quantum liquids in $SmMnO_{3+\delta}$ measured in the field $H = 1$ kOe in the temperature range 4.2–20 K. The symmetric broad peak magnetization feature near $T_{\text{spinon}} \cong 8$ K corresponds to a continuum of low-energy magnetic excitations of the QSL in the form of spinon pairs with $S = 1/2$. At temperatures $T \geq T_c = T_{KT} \cong 12$ K, a weakened jump of the magnetization curve $M(T)$ caused by the topological phase transition of Kosterlitz–Thouless dissociation of 2D vortex-antivortex pairs in a superconducting quantum liquid is observed. As the magnetic field strength increases up to $H = 1$ kOe, the asymmetric two-humped peak $M(T)$ with a centre near the temperature $T_{\text{spinon}} \cong 8$ K of the spinon excitation with $S = 1/2$ transforms into a symmetric peak smeared over a wide temperature range 4.2–12 K with a top near the temperature T_{spinon}. The magnitude of the magnetization jump of a superconducting quantum liquid near $T_{KT} \cong 12$ K decreased significantly with increasing magnetic field [spinon pairs (SP), vortex pairs (VP), free vortices (FV)].

Secondly, Figs. 7 and 9 show that the growth of the external magnetic field results in a strong decrease of the "supermagnetization" jump for the samples near the critical temperature $T_{2D} \equiv T_{KT} \cong 12$ K of the 2D vortex pairs dissociation, while the intensity of the spinon pair excitation near $T_{\text{spinon}} \cong 8$ K is practically unchanged. This indicates that the QSL is much more stable to the action of an external magnetic field compared to the coherent SC state. The intensity of the spinon pair excitation near T_{spinon} is practically the same as the intensity of the vortex-antivortex pairs dissociation near T_{KT} (Fig. 7). As can be seen from Figs. 7 and 8, in a magnetic field $H = 1$ kOe, the thermal spinon excitation spectrum is dominated by a high-temperature doublet of two overlapping magnetization peaks, located in the band $n = 3$ in the temperature range ~ 6–9 K near the average excitation temperature $T_{\text{spinon}} \cong 8$ K. It strongly exceeds the intensity and width of the magnetization peaks in $n = 1$ and $n = 2$ bands. This result shows that the low-energy gap Δ_s in the spinon excitation spectrum decreases considerably with increasing H.

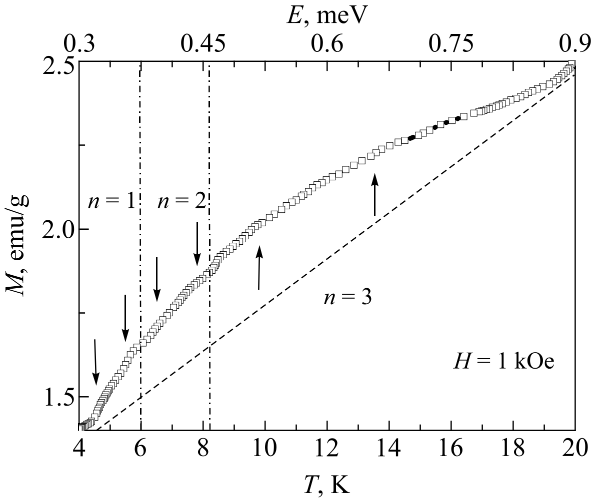

Fig. 8. Quantum oscillations of the "supermagnetization" of the $SmMnO_{3+\delta}$ samples measured in a magnetic field $H = 1$ kOe in the temperature range 4.2–12 K, corresponding to the quantum continuum of spinon pair excitations with $S = 1/2$. The continuum of thermal excitations of "supermagnetized" spinon pairs is divided into three narrow overlapping Landau bands with fractional values of the factor ν filling the bands, each with a specific magnetization feature. The quantized magnetization feature corresponds to a continuous spectrum of low-energy magnetic excitations of the QSL in the form of spinon gas smeared over the temperature by vortex fluctuations of the gauge field.

As can be seen in Figs. 10, 11, further destruction of the coherent SC state in the superconducting composite occurs at $H = 3.5$ kOe. This manifests itself in the almost complete suppression of the "giant" jump in "supermagnetization" near $T_{KT} \cong 12$ K. Only a weak jump in the temperature dependence of magnetization is observed near T_{KT}, separating the phase with new quantum spinon oscillations and the phase with a low density of vortex-antivortex pairs. A distinctive feature of the "supermagnetization" temperature dependences, obtained with increasing external magnetic field up to 3.5 kOe, is the appearance of clearly pronounced stepped oscillations of the spinon gas magnetization. As is clearly shown in Fig. 12, a characteristic feature of the "supermagnetization" oscillating in the temperature range of 4.2–9 K is the appearance of periodic threshold features of width $\Delta E \cong 0.08$–0.15 meV with characteristic narrow plateaus in the temperature dependence of the sample magnetization in Landau bands $n = 1$, $n = 2$, and $n = 3$. With increasing T, the height of thresholds and width of steps (plateaus) are growing. New quantum oscillations of "supermagnetization" temperature dependences of 2D spinon gas in the form of three narrow steps (plateaus) correspond to an integer filling of three finite gap Landau levels with spinons in a strong external magnetic field [23]. Thus, even a relatively small increase in the external magnetic field strength led to the destruction of the coherent SC state and the transition from a continuous excitation spectrum of the QSL to a discrete one.

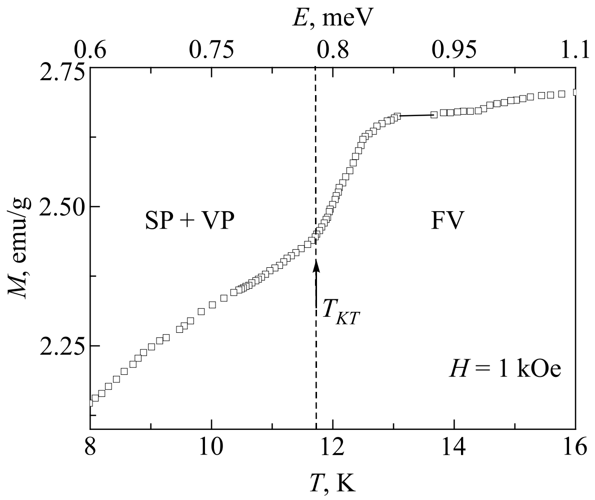

Fig. 9. Cusp-like feature of the $SmMnO_{3+\delta}$ magnetization measured in an intermediate magnetic field $H = 1$ kOe in the temperature range 8–16 K. With increasing magnetic field strength, a significant decrease of the jump in the $M(T)$ magnetization near the temperature $T_{KT} \cong 12$ K of the topological phase transition of Kosterlitz–Thouless dissociation of 2D vortex-antivortex pairs in a superconducting quantum liquid is observed. A strong decrease of magnetization jump near T_{KT} is caused by the polarization of dipole pairs by magnetic field, which leads to weakening of the bond in vortex-antivortex pairs and their dissociation.

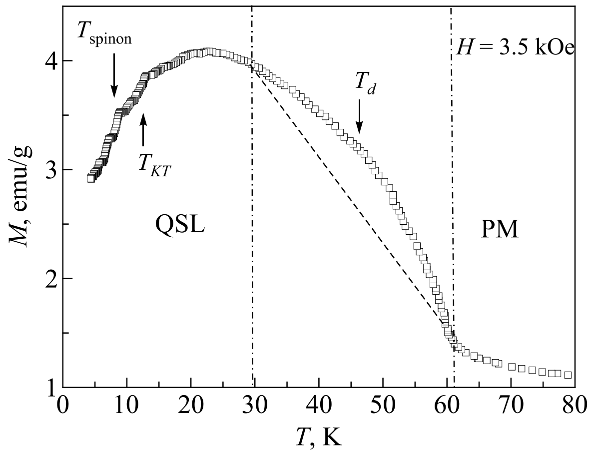

Fig. 10. Temperature dependence of the magnetization $M(T)$ of $SmMnO_{3+\delta}$ in the temperature range 4.2–80 K in the magnetic field $H = 3.5$ kOe in FC-regime measurements. At temperatures $T < 20$ K, the magnetization has three components: the dominant magnetization contribution of the RVB-phase Z_2 gapped QSL in the form of a broad magnetization peak with a top near 20 K and weaker additional "supermagnetization" contributions from the degenerate state of low-energy thermal excitations of spin and superconducting quantum liquids near T_{spinon} and T_{KT} temperatures. The magnetization jump near T_{KT} is strongly weakened.

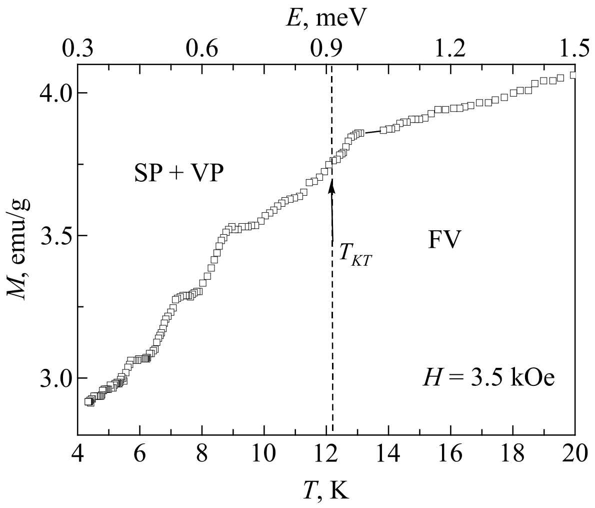

Fig. 11. The features of the $M(T)$ magnetization for a mixed state of spin and superconducting quantum liquids in $SmMnO_{3+\delta}$, measured in the field $H = 3.5$ kOe in the temperature range 4.2–20 K. With increasing strength of the magnetic field, the continuous spectrum of excitations of the QSL as a broad peak feature of "supermagnetization" with a top near $T_{\text{spinon}} \cong 8$ K is transformed into a discrete spectrum of low-energy excitations of the QSL as a series of quantum oscillations of magnetization similar to the quantum oscillations of transverse Hall resistance $\rho_{xy}(B)$ of a two-dimensional gas of electrons and composite quasiparticles in a perpendicular magnetic field. The magnetization jump near T_{KT} is strongly weakened.

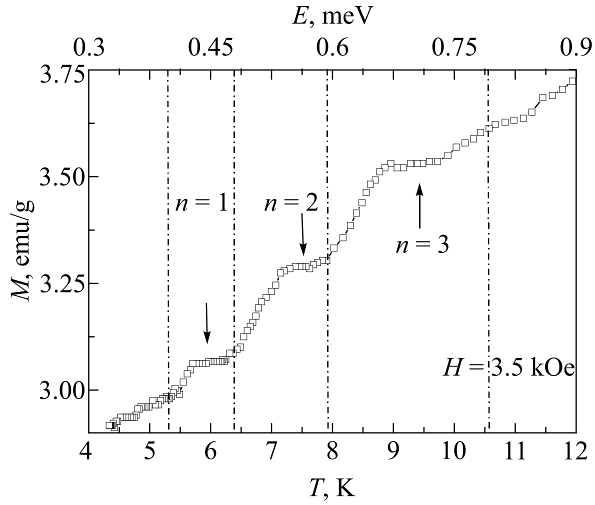

Fig. 12. Quantum oscillations of the temperature dependence of the magnetization of $SmMnO_{3+\delta}$ measured in the magnetic field $H = 3.5$ kOe in the temperature range 4.2–12 K. With an increase in the field to $H = 3.5$ kOe, a new type of quantization of the spinon spectrum appeared in the form of the formation of threshold features of magnetization in the Landau bands $n = 1$, $n = 2$, and $n = 3$ in the form of steps (plateaus) corresponding to the integer filling of three Landau bands with a finite gap by spinons. As the temperature rises, the height of the thresholds and the width of the steps increase. Emergence of new quantum oscillations of the magnetization temperature dependences in the strong field regime of measurements is caused by the Landau quantization by the gauge field of the spinon gas with a fractional spin $S = 1/2$, moving on circular orbits in the direction transverse to the direction of the "magnetic" component b of the gauge field.

4. Discussion

4.1. Landau quantization of the spinon spectrum in the chiral Z_2 state of the quantum spin liquid

In the following, we will quite often use the terminology and results of the investigation of gapped spin and superconducting quantum liquids with a topological order within the framework of the mean-field theory outlined in the review [24]. According to [24], in various mean-field models, it is customary to divide spin liquids into two main classes — gapped and gapless. The simplest example of a gapless spin liquid is a one-dimensional Heisenberg chain of spins $S = 1/2$, whereas gapped spin liquids include several disordered phases of low-dimensional spin systems with different kinds of topological order. The fermion spectrum in the spin-Peierls state has no dispersion and is described by the expression $E_k = \pm 2J_1$, where J_1 is the binding energy between nearest spins. The valence band with energy $E_k = -2J_1$ is completely filled with fermions. Spin excitations in the spin-Peierls state have a finite gap. The gapless spin liquid phase, called the flux phase, is also of interest. The ground state of the phase with flux is described by the Hamiltonian $\hat{H}_{\text{mean}}$, which is equivalent to the Hamiltonian used to describe electron jumping between sites of a crystal lattice with a "magnetic" flux π across the square. The Fermi surface for a fermion system in this state contains singular points at $(k_x, k_y) = (0, 0)$ and $(0, \pi/a)$ at half-filling of the conduction band. The spin liquid state with a flux corresponds to gapless excitations of spins through singular points of the Fermi surface. In the mean-field model, it is possible to have a gapped spin liquid phase with a topological parameter of order $\chi_{i,j}$ generating a flux. Fermions described by the Hamiltonian $\hat{H}_{\text{mean}}$ behave as when they move in a magnetic field. When the flux is strictly commensurate with the fermion density, an integer number of Landau levels will be completely filled. In this case, the fermion gas becomes incompressible, since there is a finite gap between the Landau levels. Then there are no density fluctuations of fermions. The appearance of the quantized state of fermions is accompanied by a spontaneous breaking of the time and parity inversion symmetry. Corresponding states of the spin system are called chiral spin states.

Of great interest for us is the chiral Z_2 state of the quantum spin liquid [24] that corresponds to a classical jumping motion of electrons in an external magnetic field. The relation between an ensemble of fermions and a_μ gauge field is identical to the relation between electrons and the electromagnetic field. Thus, one may expect a Hall effect-like phenomenon for a system of fermions governed by a gauge field. The "Hall effect" in this case consists in the fact that the "electric" component of the gauge field a_μ induces a current of fermions $j_x = \sigma_{xy} e_y$ in the direction transverse to the direction of the "magnetic" component b of the gauge field, where σ_{xy} is the Hall conductance, $e_i = \partial_0 a_i - \partial_i a_0$ $(i = x, y)$ is the "electric" field and the "magnetic" field $b = \partial_x a_y - \partial_y a_x$. It is believed that the Hall conductance of the filled fermion band is always quantized as an integer times 1/2π [24, 25]. It is possible to change the density of controlled fermions in a chiral spin liquid without creating fermions in the conduction band or holes in the valence band. For this purpose, a slow change of direction (rotation) of the "magnetic" flux $\Phi = \int d^2 x b$ of the gauge field a_μ induced by external influences is sufficient. The rotation of the flux induces a circular "electric" field e_θ, which in turn generates a current of driven fermions in the radial direction due to σ_{xy}. Thus, charge accumulation near the "magnetic" flux Φ occurs. It was shown that the total number of fermions induced by a change of the flux Φ is $N = -\sigma_{xy}\Phi = -\Phi/\pi$.

According to experimental results obtained in our paper, such an action changing the flux Φ of a_μ gauge field may be a change in the external magnetic field. The study of the external dc magnetic field effect on the degenerate state of spin and superconducting quantum fluids in $SmMnO_{3+\delta}$, conducted in our paper, shows a strong coupling between low energy excitations of the 2D spin and electronic subsystems in this compound in the temperature interval $4.2\ \mathrm{K} \le T \le 20\ \mathrm{K}$. They reveal themselves in coherent changes of thermal excitations caused by increasing external magnetic field. To clarify the nature of this coupling, it is of interest to consider a QSL phase with local $SU(2)$ symmetry that appears if the U_{ij} matrix is introduced into the $\hat{H}_{\mathrm{mean}}$ Hamiltonian [24]. It was found that in this case there are three possibilities. In the first case (1), the introduction of the matrix U_{ij} into the Hamiltonian breaks the translational symmetry. Then the fermion system has properties of a band dielectric and the fermion gas is incompressible. Low-energy excitations of the $SU(2)$ spin liquid phase can be studied in the gauge field theory. According to the theory, in this state, a gauge field prevents fermions with spin $S = 1/2$ from appearing in the physical spectrum. Only fermion pairs can exist as real quasiparticles with integer spin values. Spin excitations of a spin liquid have a gap. An example of this ground state of a spin liquid is the spin-Peierls state. In the second state (2), $U_{ij}^{(\mathrm{mean})}$ generates a flux. In this case, the fermions seem to move in a constant magnetic field. In the case, when the filling factor of the fermion system spectrum is expressed by a regular fraction, an integer number of Landau levels is completely filled and fluctuations of the density of states of the fermions at the Landau levels are absent. Time inversion symmetry and parity in such a state are absent. If the gauge field theory includes the Chern–Simons term, the gauge field fluctuations have an energy gap and can affect only short-range interactions. As a result, fermions cease to be confined in confined regions. Quasiparticles (spinons) in this state of the chiral spin liquid are fermions with a spin $S = 1/2$ and have fractional statistics [27–29]. In the third case (3), the local $SU(2)$ symmetry is broken. In this case, gauge field fluctuations acquire a finite energy gap due to the Anderson–Higgs mechanism. The fermion gas becomes incompressible and superconducting, as there are no gapless excitations in this phase. Thus, according to this model, three incompressible states of the QSL can arise in the fermion system: the band dielectric state (case 1); the quantum Hall liquid state (case 2); the superconducting state (case 3). It can be assumed that the quasi-two-dimensional gas of low-energy magnetic excitations studied by us in the given paper that we associate with spinons with $S = 1/2$ is due to thermal excitations of the chiral Z_2 state of the spin liquid with a local $SU(2)$ symmetry (case 2). This situation seems to be closely related to the superconducting chiral Z_2 spin liquid state without local $SU(2)$ symmetry (case 3) by strong gauge field fluctuations.

4.2. Spectrum of the low-energy thermal excitations of degenerate states of the spin and superconducting quantum liquids in weak magnetic fields

The problem of the emergence of fractional quantum states is a basic one in the modern theory of strongly correlated systems [30–37]. The appearance of particles with fractional quantum numbers occurs when quantum fluctuations of a ground state of the strongly correlated system of particles are large enough to provide a screening in which fractional components of particles are free from local integer constraints. In magnetism, a well-known example of such exemption from integer constraints is the 1D Heisenberg chain with spins $S = 1/2$, whose excitation as a semiclassical spin wave with $S = 1$ (magnon) decays into a pair of spinon excitations with $S = 1/2$, which are independent of each other [30, 31]. In the simplest case of one-dimensional antiferromagnets, spinons can arise as pair excitations of disordered 1D spin chains with the Heisenberg AFM exchange between nearest and second neighbors. In this case, the initial chain excitation as magnons with an integer spin $S = 1$ can split into two spatially separated excitations with a fractional spin $S = 1/2$, which are often considered as domain walls in a 1D antiferromagnet. Experimentally, this splitting of magnons manifests itself in the formation of a highly dispersed dynamic magnetic susceptibility continuum in neutron scattering measurements, the appearance of which is explained by the excitation of pairs of spinons with $S = 1/2$. This phenomenon has been well investigated both theoretically [30] and experimentally [31]. A single broad magnon excitation peak with spin $S = 1$ (one excitation mode) is described by the dispersion relation $\varepsilon(k) = \pi J \left|\sin(ka/2)\right|$. The two resolved peaks of spin excitations with spin $S = 1/2$ (two soft modes of spin excitations) with lower intensity are described by the dispersion relation $\varepsilon(k) = \frac{\pi J}{2}\left|\sin(ka)\right|$. Experimentally, this is well demonstrated by the study of a dynamic structural factor

$S(q,\omega)$ of spin liquids in low-dimensional AFM with geometric frustration by inelastic neutron scattering.

In [32], neutron diffraction measurements were performed on Cs_2CuCl_4 single crystals with triangular lattice spins $S = 1/2$ ions Cu^{2+} in zero magnetic field and in fields up to 70 kOe at temperatures 0.1 and 15 K. For the analysis of the experimental results, Heisenberg Hamiltonian for three possible cases was used: (i) non-interacting 1D chains ($J' = 0$); (ii) completely frustrated triangular lattice of spins ($J' = J$) and (iii) non-frustrated square lattice ($J = 0$). It was found that the exchange interaction energy inside the zigzag chains $J \cong 0.37$ meV, while the value of the inter-chain interaction $2J' \cong 0.25$ meV. This means that the value of the inter-chain coupling is of the same order as the intra-chain exchange interaction and allows us to consider Cs_2CuCl_4 as a quasi-two-dimensional frustrated antiferromagnet. The characteristic feature of dynamic spin correlations measured at $T = 0.1$ K is the broad neutron scattering peak near the energy $E \cong 0.7$ meV, which disappears at $T = 15$ K. In the magnetic excitation spectrum, there are no narrow peaks, intrinsic to neutron scattering by magnons. The presence of a broad scattering peak at $T = 0.1$ K indicates excitations that carry fractional quantum numbers. For one-dimensional Heisenberg chains not bound by interactions, such excitations are spinon pairs with $S = 1/2$ [30], which were found in neutron scattering [31]. A detailed analysis of the highly dispersive scattering spectrum found in the quasi-dimensional frustrated antiferromagnet Cs_2CuCl_4 [32] indicates that, in contrast to the non-frustrated square lattice with $J = 0$, in Cs_2CuCl_4, the excitation spectrum realizes quasi-particles with fractional spin $S = 1/2$, similar to excitations in 1D non-interacting chains with $J' = 0$. These spinons are two-dimensional, modified at all excitation energy scales.

In [33], magnetic excitations of Cs_2CuCl_4 were investigated by inelastic neutron scattering at temperatures both above and below the critical temperature $T_N = 0.62$ K of the transition to the AFM state. At temperatures above T_N in Cs_2CuCl_4, magnetic 2D layers are decoupled, whereas at $T < T_N$, due to the presence of weak interlayer coupling, mean field effects stabilize the 3D magnetic order with the disproportionate helical ordering of Cu^{2+} ion spins $S = 1/2$ in the basic *cb* planes. At temperatures above $T_N = 0.62$ K, there exists a spin liquid phase with low-energy magnetic excitations dominated by dynamic spin correlations. In the experiment, it manifested in the existence of two strongly overlapping broad neutron scattering peaks near the average energy $E \cong 0.2$ meV. In the dependence of the neutron scattering intensity $I(E)$, obtained at $T = 0.75$ K in the spin-liquid phase, there is no intense narrow peak near 0.1 meV. According to the authors, this scattering peak is caused by the excitation of magnons dominating in the spectrum of low-energy excitations at temperatures below T_N. Thus, a characteristic feature of the spin liquid phase in Cs_2CuCl_4 is the fractionation of its low-energy excitations, existing at $T < T_N$ as magnons with spin $S = 1$, into pairs of spinons with spin $S = 1/2$, which is a sign of a fractional RVB spin-liquid state. The boundaries of the continuum of the spin liquid fractional RVB state excitations are determined by dispersion relations of the magnetic excitation spectrum. With further temperature increase up to $T = 12.8$ K, the dependence of the scattering intensity on the excitation energy $I(E)$ changed dramatically. With increasing temperature, the lower scattering boundary disappears, while the upper boundary shifts up to 1 meV. The smeared neutron scattering in Cs_2CuCl_4 in a wide energy range at high temperatures is very unusual and can provide important information about the type of exciting quasiparticles. From the authors point of view, an unusual dependence $I(E)$ at 12.8 K can be explained whether by double-spinon scattering processes or by the scattering of electron-hole pairs. It should be pointed out that at temperatures below T_N magnons dominate in the spectrum of low-energy magnetic excitations, while spinons dominate in the spectrum of high-energy excitations. This result indicates the crossover of magnon and spinon excitations in Cs_2CuCl_4, associated with the proximity of AFM and RVB energy states. It is assumed that there is a short-range attraction between spinons which is sufficient to create a bound state in the form of pairs of spinons with a total spin $S = 1$ in low-energy spin liquid excitations. However, at the excitation energies higher than the attraction potential, the spinon pairs collapse. This explains the absence of bound spinon pairs at temperatures above T_N.

According to [32, 33], the appearance of particles with fractional quantum numbers occurs when quantum fluctuations of the ground state of a strongly correlated system of particles are large enough to provide screening, at which the fractional components of particles are free from local integer restrictions. A distinctive feature of such decay of magnons with $S = 1$ into states with fractional spin $S = 1/2$ is the appearance of a strongly dispersed continuum of spin excitations in experiments on inelastic neutron scattering. In most of the studies of 2D quantum magnets, the existence of spin excitations with spin $S = 1/2$ has not been found. However, in [32] in measurements of neutron scattering in the quasi-two-dimensional frustrated Heisenberg antiferromagnet Cs_2CuCl_4, the existence of a broad strongly dispersed continuum of excited states was found to be characteristic of fractional spinons with $S = 1/2$. The presence of a strongly dispersed continuum of excitations in Cs_2CuCl_4 indicates the existence of a 2D fractional RVB quantum spin liquid phase in this compound, previously predicted by Anderson in [34]. This state is characterized by the formation of singlet pairs of spins in the non-magnetic ground state and the existence of pairs of spinons with $S = 1/2$ in the excited state, which arise due to the rearrangement of valence bonds. It is assumed that the valence bonds in the RVB state fluctuate between different configurations leading to the breaking of singlet pairs and the emergence of two $S = 1/2$ spinons which propagating independently. The cause for the stability of the RVB phase with fractional excitations is frustration, which ensures the predominance of quantum fluctuations over the effects of the mean-field. A distinctive feature of the fractional RVB state, discussed in recent theoretical works [34–36], is the presence of an extended, highly dispersed continuum of spinon excitations. At present, such an excitation spectrum has not been found in any 2D magnetic systems with a rectangular lattice of spins. This is explained by the fact that in the Heisenberg square lattice of spins with $S = 1/2$ the internal fields prevent the appearance of fractional excitations of the spin system, which leads to a classic picture of fluctuations near T_N. in the form of magnons. However, geometric frustration of the antiferromagnetic order in 2D triangular and hexagonal spin lattices can neutralize these fields and enable the emergence of a dispersive spectrum of fractional excitations. In [38], the J_1–J_2 Heisenberg model of a gapped QSL existing in a system of disordered dimers on a hexagonal crystal lattice was considered in the framework of the mean-field theory. The ground state of the system is a non-magnetic singlet with a total spin $S = 0$ separated from excited states with a spin $S = 1$ by a gap. This QSL phase is characterized by short-range spin-spin correlations and the spontaneous breaking of symmetry, which leads to the degeneracy of the ground state of the spin system.

A strong change in the shape of the magnetic excitation spectrum in $SmMnO_{3+\delta}$ with a small magnetic field growth can be explained by the instability of a quasi-two-dimensional ground state concerning spin vortices and superconducting quantum liquids to the action of an external magnetic field, which leads to topological phase transitions in ensembles of vortex fluctuations existing in such liquids. For superconducting liquids, this instability leads to smearing and disappearance of the well-known in the literature cusp-like feature of dc magnetization curves near the T_{KT} temperature of the topological Kosterlitz–Thouless phase transition of vortex-antivortex pair uncoupling. At present, the nature and various manifestations of this phase transition in high-T_c (HTSC) cuprates have been studied in sufficient detail. For example, the mechanism of the Kosterlitz–Thouless phase transition instability proposed in [39] for decoupling of 2D vortex pairs in HTSC cuprates to the growth of the external magnetic field strength is simple and in good agreement with the our experimental results. The question about the influence of the magnetic field growth on the behavior of unusual quasi-two-dimensional vortices created in the QSL ground state by the gauge field is much more complicated and has been studied very little. In the Kitaev model of Z_2 quantum spin liquid, a numerical calculation of the dependence of the number of vortices, induced in the QSL ground state by the gauge field, on the strength of the external magnetic field was carried out [40]. Within the Z_2 gap gauge theory, it has been shown that the critical field strength coincides with the gap for a single vortex and thus the closing of the gap for a single vortex will lead to a field-driven phase transition in the form of the condensation of vortex fluctuations. Below the critical magnetic field, there are no vortices indicating a deconfined phase as expected in the presence of a vortex gap. At the phase transition, however, the vortices appear to condense and the number of vortices in the ground state quickly increases above the critical field strength. The nature of the phase transition might be thus framed in terms of a confinement-deconfinement transition of a non-Abelian gauge field, akin to the confinement-deconfinement transition in the Abelian discrete gauge theory. Thus, in [40] it has been found that in magnetic fields $h > h_c$ vortices condensation occurs and their number in the ground QSL state grows rapidly with increasing external magnetic field strength. In our paper, the shape and intensity of the “supermagnetization” features in $SmMnO_{3+\delta}$ at temperatures below 20 K significantly change with the increasing external magnetic field. These results evidence about the appearance of the high density vortex gauge field fluctuations in the magnetic fields $H \geq 100$ Oe. Strong growth of vortex fluctuations may be explained by a second kind of phase transition in the form of vortices condensation.

4.3. Spectrum of low-energy thermal excitations of degenerate states of spin and superconducting quantum liquids in strong magnetic fields

Quantum oscillations of the magnetization temperature dependence for the 2D spinon gas found in this paper in the temperature range $4.2\ \mathrm{K} \leq T \leq 12$ K in a magnetic field $H = 3.5$ kOe have a form similar to the field dependence of the Hall resistance $\rho_{xy}(B)$ of a $GaAs/Ga_{1-x}Al_xAs$ quasi-two-dimensional heterostructure described and explained in [41]. At present, the quantization of the spinon spectrum in a Moth dielectric with a spinon Fermi surface has only been studied theoretically. In [42], the quantization of spinons in the QSL with a spinon Fermi surface close to the Moth dielectric-metal transition was considered in the Hubbard model. The effective spin Hamiltonian in the triangular lattice for the case of the four-spin exchange interaction in the presence of an external magnetic field is constructed.

The first term of the Hamiltonian corresponds to the exchange between pairs of nearest spins with energy J_2, while the second term corresponds to the ring exchange interaction for the four nearest spins with energy J_4. In the presence of an external magnetic field, an additional term appears in the spin Hamiltonian, which is the sum of three-spin exchange interactions associated with the motion of spins in individual triangles. This term is proportional to the small flux of the external magnetic field $\Phi_{\Delta}^{\mathrm{ext}} \ll 1$ through the triangle with the coupling constant J_3. It is shown that the external magnetic field is linearly related to the chirality of spins in a triangular lattice. The chirality of the spins corresponds to the flux of the internal gauge field within the framework of the gauge field theory, which is often used to describe a spin liquid. According to the proposed model, an external magnetic field induces a static internal flux in the system of spins with four-spin exchange interaction. A quantitative evaluation of the effect has shown that the magnitude of the resulting magnetic orbital field acting on the spinons is comparable and maybe even larger than the applied field. Due to the fact, the stiffness of the internal gauge field is very small, the homogeneous state of the spinon-gauge field system is unstable in the region of low temperatures because of strong Landau quantization of the spinon energy spectrum. This instability resembles the situation in metals in the regime of a strong magnetic field but the range of temperatures and fields for the existence of the homogeneous state instability in the spinon-gauge field system is much wider. The response of the spinon-gauge field system changes significantly at low temperatures: the Landau quantization of spinons in a static internal field at temperatures below a critical value is not smeared out by temperature. It has been found that for a quantum spin liquid in organic material κ-$(ET)_2Cu_2(CN)_3$, a homogeneous state with a continuously varying internal field becomes unstable at temperatures below several Kelvins for typical laboratory fields. A similar phenomenon has been well investigated previously for magnetic oscillations in ordinary metals [43]. The mode of the homogeneous state instability in the spinon-gauge field system is much wider than in ordinary metals because the stiffness of the internal gauge magnetic field is much smaller than the external field. Hence, in the spinon-gauge field system, spinon states with integer filling of Landau levels are more stable than states with continuously varying filling. Therefore, it is energetically advantageous for the internal gauge field to adjust its discreteness. This instability allows one to study properties of the Fermi spinon surface directly using the results of experimental investigations of magnetization oscillations of the sample. It should be noted that the spinon-gauge field system has unusual properties [44–52]. In [42], the mechanism of quantization of the Landau spectrum of the spinon-gauge field system with increasing external magnetic field strength and sample temperature is discussed in detail. An inhomogeneous state of the spinon system at temperature $T = 0$ was considered. In a zero external magnetic field (flux $\Phi^{\text{ext}} = 0$) with weak ring exchanges, the lowest-energy state has $\phi_\Delta = \pi/2$ flux through each triangle, whereas in the case of strong ring exchanges the zero flux state is optimal. It was obtained enveloping function for small fluxes $|\chi_\phi| \leq |\chi_0|(1+c\phi_\Delta^2)$ with $c \approx 0.1$. When the external flux is nonzero $\Phi^{\text{ext}} \neq 0$ but small, a static internal gauge field flux $\phi^{\text{int}} = \gamma\Phi^{\text{ext}}$, where $\gamma \geq 1$. Thus, the effective orbital field is comparable in magnitude to the applied magnetic field. In the presence of the static internal gauge flux, the spinon spectrum consists of Landau bands. In the framework of the mean-field model, a numerical calculation of the dependence of the magnetic susceptibility $|\chi_\phi|$ of the spinon-gauge field system on the Landau level filling factor v was performed. The states with integer v are more stable, hence, the internal flux ϕ^{int} goes through this discrete set corresponding to the integer filling of the spinon Landau levels. It was obtained that magnetic susceptibility of the spinon-gauge field system $|\chi_\phi| = |\chi_0|(1+c\phi_\Delta^2) - \Xi_\phi^{\text{osc}}$, where $\Xi_\phi^{\text{osc}} \propto \nu^{-2}(\nu - k)(k+1-\nu)$ for the filling factor ν between integers k and k+1 (Ξ_ϕ^{osc} is an oscillatory component of the magnetic susceptibility). According to [53–55], the appearance of the magnetization plateau series of the frustrated Heisenberg AFM and chiral spin liquid by an external magnetic field is closely related to the quantization of Landau spectrum of spinons by the Chernie–Simons gauge field.

5. Conclusion

It has been found that the thermal excitation of the spinon pairs in the gapped Z_2 QSL and the excitation of the 2D vortex plasma in the SP quantum liquid occur in $SmMnO_{3+\delta}$ close temperatures $T_{\text{spinon}} \cong 8$ K and $T_{KT} \equiv T_c \cong 12$ K. This indicates that the energies of the ground state of quantum liquids in this compound are degenerate. It is shown that any asymmetric doublet of two broad almost overlapping peaks of "supermagnetization" near $T_{\text{spinon}} \cong 8$ K in a weak magnetic field $H = 100$ Oe corresponds to a continuous spectrum of low-energy magnetic excitations of gapped Z_2 QSL in the form of spinon pairs with $S = 1/2$, smeared in the temperature interval 6–10 K. As the magnetic field strength increases up to $H = 1$ kOe, the asymmetric peak feature centered near 8 K transforms into a symmetric peak smeared over a wider temperature range of 4.2–12 K with the top near the T_{spinon} temperature. The extended peak feature of the spinon magnetization in the field $H = 1$ kOe corresponds to a continuous spectrum of QSL magnetic excitations, smeared over a wider temperature interval due to the strong growth of quantum fluctuations of the gauge magnetic field. With further growth of the field up to $H = 3.5$ kOe, there is a transition from a continuous spectrum of thermal excitations of spinon pairs to a discrete one. It appears as a series of narrow steps in the temperature dependence of the $SmMnO_{3+\delta}$ magnetization near the temperature $T_{\text{spinon}} \cong 8$ K, similar to the quantum oscillations of the transverse Hall resistance $\rho_{xy}(B)$ of a 2D electron gas in a strong external magnetic field. A new type of quantization of the spinon pairs spectrum in the field $H = 3.5$ kOe is with the formation of threshold magnetization features in the Landau bands $n = 1$, $n = 2$, and $n = 3$ in the form of steps (plateaus) corresponding to an integer filling of three Landau bands with the finite gap. With increasing temperature, the height of the thresholds and the width of the steps grow. It is supposed that when the magnetic flux Φ of the "magnetic" component b of the gauge field in the Z_2 quantum spin liquid with increasing strength of the external magnetic field becomes commensurate with the density of spinons, in the sample a state like an incompressible 2D-quantum spinon gas emerges. In this state, spinons move along circular orbits in the direction transverse to the direction of the "magnetic" component *b* of the gauge field and completely fill an integer number of Landau levels. The shape and intensity of the "supermagnetization" features in $SmMnO_{3+\delta}$ at temperatures below 20 K significantly change with the increasing external magnetic field. These results evidence about the existence of the high-density vortex gauge field fluctuations in the magnetic fields $H \geq 100$ Oe. Strong growth of vortex fluctuations with the field growth is explained by a second-kind phase transition in the form of the vortices condensation.

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Еволюція спектра пар низькоенергетичних магнітних збуджень у $SmMnO_{3+\delta}$

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Представлено ідентифікацію низькоенергетичних теплових збуджень у $SmMnO_{3+\delta}$ для вироджених станів спінової та надпровідної квантової рідин у магнітних полях $H \leq 3{,}5$ кЕ. В інтервалі температур 4,2–12 К виявлено квантування Ландау низькоенергетичного парного спектра магнітних збуджень Z_2 квантової спінової рідини у спін-калібрувальному полі системи. Формування широкого континууму збуджень спінонних пар у режимі "слабкого магнітного поля" (H = 100 Е, 1 кЕ, FC режим) пояснюється у рамках моделей квантування Ландау стисливого спінонного газу з дробовими значеннями коефіцієнта заповнення ν трьох смуг, які перекриваються. В режимі «сильного магнітного поля» (H = 3,5 кЕ) спостерігалися квантові коливання температурної залежності «супернамагніченості» нестислової спінонної рідини. Вони мають вигляд трьох вузьких ступенів (плато), що відповідають повному заповненню смуг Ландау, які не перекриваються, з цілими значеннями коефіцієнта заповнення спінонами. Ці результати свідчать про існування флуктуацій вихрового каліброваного поля з високою густиною в магнітних полях $H \geq 100$ Е. Сильне зростання вихрових флуктуацій можна пояснити фазовим переходом другого роду в $SmMnO_{3+\delta}$ у вигляді конденсації вихорів. Зростання напруженості зовнішнього dc магнітного поля в зразках $SmMnO_{3+\delta}$ в інтервалі полів $0 < H \leq 3,5$ кЕ призводить до безперервного зменшення гігантського стрибка намагніченості поблизу температури $T_{KT} \cong 12$ К топологічного фазового переходу Костерліца–Таулесса двовимірних пар вихор–антивихор в локальному надпровідному стані. Зменшення стрибка намагніченості поблизу температури T_{KT} зі зростанням H пояснюється поляризацією пар вихор–антивихор при температурах нижче T_{KT} зовнішнім dc магнітним полем, яке послаблює вихрову взаємодію в парах і призводить до їх дисоціації.

Ключові слова: спінова рідина, пари спінонів, пари вихорів, коливання намагніченості, квантування Ландау.