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Frustration effect on escape rate in Josephson junctions between single-band and three-band superconductors in the macroscopic quantum tunneling regime

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The escape rate of S → R switching (from superconducting S state to resistive R state) in Josephson junction formed by *s*-wave single-band (SB) and three-band (ThB) superconductors (SB/ThB junctions) in macroscopic quantum tunneling regime is investigated. We use the effective critical current approximation in SB/ThB junctions. It was shown that escape rate can exhibit qualitative features in the case of frustration effects in ThB superconductors. Inclusion of Leggett modes in ThB superconductors leads to enhancement of escape rate in three-channel Josephson junction in comparison with single-channel junctions.

Keywords: Josephson junctions, quantum tunneling, single-band and three-band superconductors.

Introduction

It is well known the Josephson effect is related with interference between the wave functions of weekly linked two superconductors. The supercurrent carries the information of gap structures in electrodes of Josephson junction (JJ). Therefore, the phase-sensitive experiments were used as a tool to detect the symmetry of the order parameter in unconventional superconductors. It is useful to note that *d*-wave pairing symmetry in cuprate compounds was investigated by Josephson interferometry in tricrystal junctions [1]. In [2, 3], the question of pairing mechanism of electrons in oxypnictides and other classes of superconductors is discussed. There is growing interest in these multi-band superconductors in which superconductivity in different bands couples through the interband coupling. Detail investigation of dynamics of $S \to R$ switching of JJ, based in new superconductors seems also interesting for the study of order parameter structure in these compounds. The influence of multiple tunneling channels on JJ dynamics becomes also actual due to the fabrication of JJ, based on many band compounds such as iron-based [4, 5] or MgB_2 [6, 7]. In Ref. 8, the escape rate of JJ based on single-band/two-band superconductors is calculated in terms of effective critical current in thermal activation and macroscopic quantum tunneling (MQT) regimes. It was shown that in all cases, increasing of critical current in a two-channel case with respect to the single-channel case leads to a decreasing in the escape rate. Radical change in the case of the s^+-wave symmetry in thermal and the MQT regimes [8]. The inclusion of Leggett modes in two-band superconductors leads to enhancement of escape rate in the MQT regime.

In the case, of JJ formed by single-band (SB) and three-band (ThB) superconductors, the phase dynamics and the influence of fluctuations effects can exhibit new features. In iron-based superconductors, the angle-resolved photoemission spectroscopy experiments show three and four different gaps at Fermi surfaces [3]. Sign reversals between Cooper pairing of different parts of the Fermi surface were discussed [3]. Details of investigation of the phase difference in the ground state of three-band superconductors presented in Refs. 9, 10. Therefore, it is of interest to investigate the consequences of repulsive interband-coupling from a general point of view. Especially, the presence of a frustrating ground state in many band superconductors leads to the Josephson system with φ-junction peculiarity [11–13]. Overall, in this study, we analyze the escape rate in SB/ThB JJs in MQT regimes.

For the study of the influence of the fluctuations on the $S \to R$ switching of JJ based on two *s*-wave superconductors, we consider that the current growth rate across junction [14, 15] is small. It is well known that the phase dy-

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namics of JJ are equivalent to the behavior of a particle in "washboard" potential

$$U = \frac{\hbar I_c}{2e}\{1-\cos\varphi - i_e\varphi\},$$

where I_c and φ are the critical current and phase of JJ, correspondingly [3, 15]. In the approach of a small growth rate of current, at high temperatures thermal fluctuations can initiate switching from S state to resistive R state in the vicinity to critical current I_c. At low temperatures, the phase dynamics are influenced by the quantum fluctuations. For detail calculations of these effects, we use expressions for the potential barrier height [15, 16]:

$$\Delta U = \frac{4\sqrt{2}}{3}E_J(1-i_e)^{3/2}, \quad E_J = \frac{\hbar I_c}{2e}, \tag{1}$$

where $i_e = I_e / I_c$ is the normalized external current. For the probability of such $\mathrm{S} \to \mathrm{R}$ switching is true the expression [15, 17]:

$$p(t) = 1 - \exp\left(1 - \int_{-\infty}^{t} \Gamma(I(t'))dt'\right), \tag{2}$$

where $\Gamma(I)$ is the tunneling rate of junctions in general case. In the MQT regime in unshunted tunnel JJ (McCumber parameter $\beta = \dfrac{2\pi I_c R_N^2 C}{\Phi_0} >> 1$), $\Gamma(I)$ is calculated as [17, 18]

$$\Gamma(I) = \sqrt{\frac{864\pi\Delta U}{\hbar\Omega_p}}\frac{\Omega_p}{2\pi}\exp\left[-\frac{\Delta U(I)}{\hbar\Omega_p}\left(1+\frac{0{,}87}{Q}\right)\right], \tag{3}$$

where the quality factor of JJ is determined by the expression $Q = \Omega_p R_N C$, R_N and C are the resistance and capacitance of JJ, respectively. In Eq. (3), the plasma frequency Ω_p of JJ calculated as [15, 16]:

$$\Omega_p = \left(\frac{2eI_c}{\hbar C}\right)^{1/2}(1-i_e)^{1/4}. \tag{4}$$

The crossover between two regimes (thermal and MQT regimes) takes place at the temperature T_{cr} [19]

$$T_{\text{cr}} = \frac{\hbar\Omega_p}{2\pi k}\sqrt{1+\frac{4}{Q^2}-\frac{1}{2Q}}, \tag{5}$$

MQT phenomena in JJ based on low-temperature superconductors were experimentally observed in Refs. 20, 21 many years ago.

In the derivation of the above presented Eqs. (1)–(5) supercurrent between single-band *s*-wave superconductors considered as $I = I_c \sin\chi$. For the JJs on SB/ThB structures (Fig. 1) supercurrent is the sum of currents in three tunneling channels [11–13],

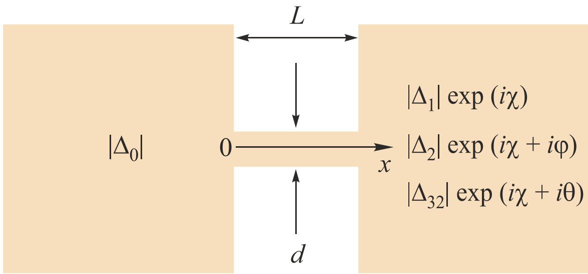

Fig. 1. Schematic diagram of JJ based between SB/ThB superconductors.

$$I = I_{c1}\sin\chi + I_{c2}\sin(\chi+\phi) + I_{c3}\sin(\chi+\theta), \tag{6}$$

where $I_{c1,2,3}$ are the critical currents in three different channels. In the three-band Ginzburg–Landau free energy functional [22] (see also [23])

$$F = \int d^3r \sum_{ij}\left(F_{ii} - F_{ij} + \frac{H^2}{8\pi}\right), \tag{7a}$$

where

$$F_{ii} = \frac{\hbar^2}{4m_i}\left|\left(\nabla - \frac{2\pi i\mathbf{A}}{\Phi_0}\right)\Psi_i\right|^2 + \alpha_i(T)|\Psi_i|^2 + \beta_i|\Psi_i|^4/2, \tag{7b}$$

$$F_{ij} = \varepsilon_{ij}(\Psi_i^*\Psi_j + \text{c.c.}) +$$
$$+\varepsilon_1^{ij}\left\{\left(\nabla + \frac{2\pi i\mathbf{A}}{\Phi_0}\right)\Psi_i^*\left(\nabla - \frac{2\pi i\mathbf{A}}{\Phi_0}\right)\Psi_j + \text{c.c.}\right\}, \tag{7c}$$

m_i is the masses of electrons belonging to different bands (i = 1–3); $\alpha_i = \gamma_i(T - T_{ci})$ is the quantity linearly dependent on the temperature T; β_i and γ_i are constant coefficients; $\varepsilon_{ij} = \varepsilon_{ji}$ and $\varepsilon_1^{ij} = \varepsilon_1^{ji}$ describe the interaction between order parameters and their gradients of different bands, respectively; H is the external magnetic field and Φ_0 is the quantum of magnetic flux.

As shown in [24, 25], in the case of identical and positive interband interaction term $\varepsilon_{ij} = \varepsilon_{ji} = \varepsilon > 0$, one of thephase difference is zero and another phase differences $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}2\pi/3\\ -2\pi/3\end{pmatrix}$ and $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}-2\pi/3\\ 2\pi/3\end{pmatrix}$ is in frustration states [Fig. 2(a)], Refs. 25, 26. Another frustration state corresponds to phase differences $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}0\\ \pi\end{pmatrix}; \begin{pmatrix}\pi\\ 0\end{pmatrix}$ and $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}\pi\\ \pi\end{pmatrix}$ [Fig. 2(b)]. The potential energy of single-band/ThB JJ under external current I_e in the general case can be written as (Fig. 1)

$$U(\chi,\phi,\theta) = -\frac{\hbar I_{c1}}{2e}\cos\chi - \frac{\hbar I_{c2}}{2e}\cos(\chi+\phi) -$$
$$-\frac{\hbar I_{c3}}{2e}\cos(\chi+\theta) - \frac{\hbar I_e}{2e}f(\chi,\phi,\theta). \tag{8}$$

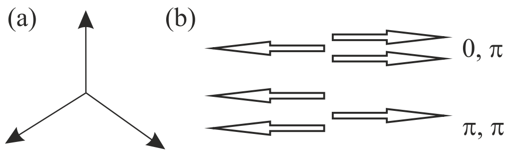

Fig. 2. Frustrated state of three-band superconductors.

This expression for potential energy can be simplified in frustration state [Fig. 2(a)] $\begin{pmatrix}\phi\\ \theta\end{pmatrix}=\begin{pmatrix}2\pi/3\\ -2\pi/3\end{pmatrix}$:

$$U(\chi)=-\frac{\hbar}{2e}\left(I_{c1}-\frac{I_{c2}}{2}-\frac{I_{c3}}{2}\right)\cos\chi-$$
$$-\frac{\hbar}{2e}\frac{\sqrt{3}}{2}(I_{c3}-I_{c2})\sin\chi-\frac{\hbar I_e}{2e}\chi\,. \quad (9)$$

In the case of state $\begin{pmatrix}\phi\\ \theta\end{pmatrix}=\begin{pmatrix}-2\pi/3\\ 2\pi/3\end{pmatrix}$, the terms I_{c2} and I_{c3} in Eq. (6) changed by the place. In the frustration case $\begin{pmatrix}\phi\\ \theta\end{pmatrix}=\begin{pmatrix}0\\ \pi\end{pmatrix}$ the potential energy has a form

$$U(\chi)=-\frac{\hbar}{2e}\cos\chi(I_{c1}+I_{c2}-I_{c3})-\frac{\hbar I_e}{2e}\chi. \quad (10)$$

$\begin{pmatrix}\phi\\ \theta\end{pmatrix}=\begin{pmatrix}\pi\\ \pi\end{pmatrix}$ frustration state corresponds to the potential energy:

$$U(\chi)=-\frac{\hbar}{2e}\cos\chi(I_{c1}-I_{c2}-I_{c3})-\frac{\hbar I_e}{2e}\chi. \quad (11)$$

For the calculation of potential energy height ΔU and plasma frequency Ω_{pl} in Eq. (3), we will use general relations presented in Ref. 27:

$$\Delta U=\frac{4\sqrt{2}}{3}\left(\frac{\gamma_c^3}{U'''(\chi_c)}\right)^{1/2}\left(\frac{\delta I_e}{I_c}\right)^{3/2}, \quad (12)$$

$$\Omega_{\mathrm{pl}}=\left(2\gamma_c U'''(\chi_c)\right)^{1/4}\left(\frac{\delta I_e}{I_c}\right)^{1/4}. \quad (13)$$

Here χ_c is determined from the Eqs. $U''(\chi_c)=0$ and $(\delta I_e/I_c)_c=U'(\chi_c)$ [27]. U', U'', U''' are correspondingly derivatives of potential energy functions, presented above.

Result and discussion

Result of calculations using Eqs. (6)–(10) presented below: in frustrated case $\begin{pmatrix}\phi\\ \theta\end{pmatrix}=\begin{pmatrix}-2\pi/3\\ 2\pi/3\end{pmatrix};\begin{pmatrix}2\pi/3\\ -2\pi/3\end{pmatrix}$ we have expressions

$$\frac{\Delta U}{\hbar\Omega_{\mathrm{pl}}}=\frac{\hbar}{2e}\left[\left(I_{c1}-\frac{I_{c2}}{2}-\frac{I_{c3}}{2}\right)^2+\frac{3}{4}(I_{c3}-I_{c2})^2\right]^{1/4}, \quad (14a)$$

$$\Omega_{\mathrm{pl}}=\left(\frac{2e}{\hbar C}\right)^{1/2}\left[\left(I_{c1}-\frac{I_{c2}}{2}-\frac{I_{c3}}{2}\right)^2+\frac{3}{4}(I_{c3}-I_{c2})^2\right]^{1/4}, \quad (14b)$$

$$\tan\chi_c=\frac{I_{c1}-\frac{I_{c2}}{2}-\frac{I_{c3}}{2}}{\frac{\sqrt{3}}{2}\left|I_{c3}-I_{c2}\right|}\,. \quad (14c)$$

In state $\begin{pmatrix}\phi\\ \theta\end{pmatrix}=\begin{pmatrix}0\\ \pi\end{pmatrix};\begin{pmatrix}\pi\\ 0\end{pmatrix}$ calculations give results

$$\frac{\Delta U}{\hbar\Omega_{\mathrm{pl}}}=\frac{\hbar}{2e}\left[I_{c1}+I_{c2}-I_{c3}\right]^{1/2}, \quad (15a)$$

$$\Omega_{\mathrm{pl}}=\left(\frac{2e}{\hbar C}\right)^{1/2}\left[I_{c1}+I_{c2}-I_{c3}\right]^{1/2}. \quad (15b)$$

Analogically, in the case $\begin{pmatrix}\phi\\ \theta\end{pmatrix}=\begin{pmatrix}\pi\\ \pi\end{pmatrix}$, the expressions are true

$$\frac{\Delta U}{\hbar\Omega_{\mathrm{pl}}}=\frac{\hbar}{2e}\left[I_{c1}-I_{c2}-I_{c3}\right]^{1/2}, \quad (16a)$$

$$\Omega_{\mathrm{pl}}=\left(\frac{2e}{\hbar C}\right)^{1/2}\left[I_{c1}-I_{c2}-I_{c3}\right]^{1/2}, \quad (16b)$$

Calculation of the ratio of escape rate in a three-channel case Γ to single-channel case Γ_0 in MQT regime for frustrated case $\begin{pmatrix}\phi\\ \theta\end{pmatrix}=\begin{pmatrix}2\pi/3\\ -2\pi/3\end{pmatrix}$ using Eqs. (14a), (14b), and (14c) leads to expression

$$\frac{\Gamma}{\Gamma_0}=\left[\left(1-\frac{I_{c2}}{2I_{c1}}-\frac{x}{2}\right)^2+\frac{3}{4}\left(x-\frac{I_{c2}}{I_{c1}}\right)^2\right]^{1/4}\times$$
$$\times\exp\left\{B\left(1-\left[\left(1-\frac{I_{c2}}{2I_{c1}}-\frac{x}{2}\right)^2+\frac{3}{4}\left(x-\frac{I_{c2}}{I_{c1}}\right)^2\right]^{1/4}\right)\right\}, \quad (17)$$

here $x=I_{c3}/I_{c1}$, $B=(\hbar/2e)^{3/2}\sqrt{I_{c1}C}$. As followed from Eq. (17) in the limit $I_{c3}\to I_{c2}$ we have $\chi_c\to\pi/2$, which is consistent with the classical results for SB/SB junctions [15]. Results of calculations using Eq. (17) are presented in Fig. 3. It is clear that the case of the $I_{c3}=I_{c2}$ is identical to SB/TB junctions in the $\phi=\pi$ state [8]. For the value of $I_{c2}=I_{c1}=0$ escape rate remains practically invariable. With the variation of the ratio $I_{c2}/I_{c1}=0.4$; 0.8, the changing escape rate becomes important, revealing minimum at the intermediate values of $x=I_{c3}/I_{c1}$.

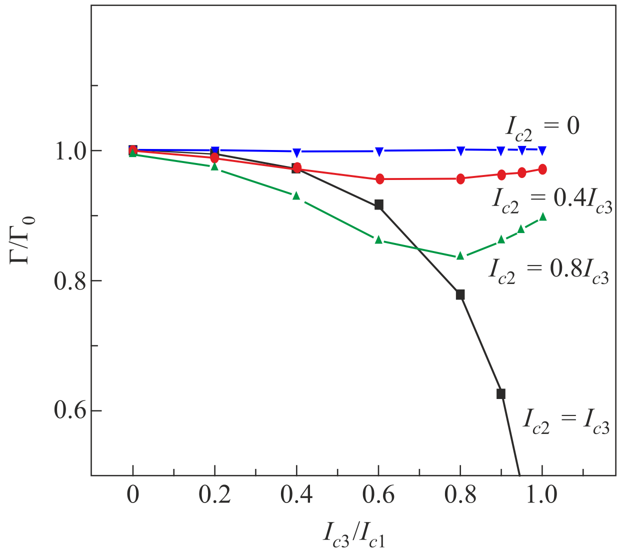

Fig. 3. Normalized escape rate of SB/ThB JJ in MQT regime in frustrated state $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}2\pi/3\\ -2\pi/3\end{pmatrix}$.

For the calculation of the ratio of escape rate in the three-channel case Γ to single-channel case Γ_0 in MQT regime, for frustrated case $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}0\\ \pi\end{pmatrix};\begin{pmatrix}\pi\\ 0\end{pmatrix}$ and $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}\pi\\ \pi\end{pmatrix}$ were taken into account Eqs. (15a), (15b), (16a), and (16b). In this case, results of calculations are presented in Fig. 4. In both frustrating case $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}0\\ \pi\end{pmatrix};\begin{pmatrix}\pi\\ 0\end{pmatrix}$ and $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}\pi\\ \pi\end{pmatrix}$ for the values $I_{c2} = I_{c1} = 0$ expressions for escape rate are similar to the case of SB/TB junction in $\phi = \pi$. In this state, escape

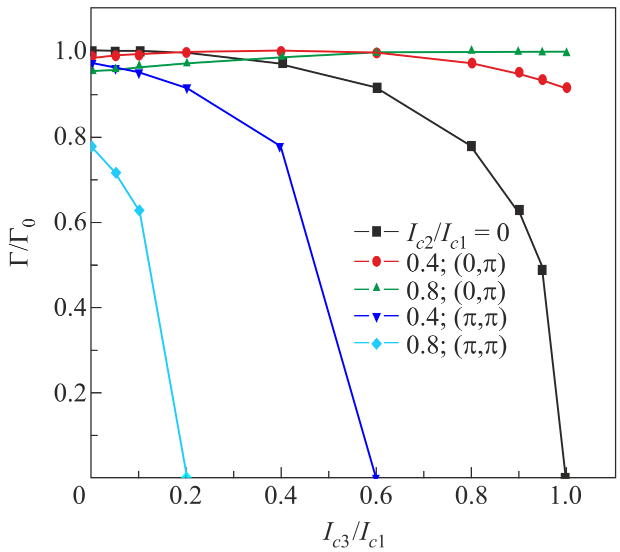

Fig. 4. Normalized escape rate of SB/ThB JJ in MQT regime in frustrated state $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}0\\ \pi\end{pmatrix};\begin{pmatrix}\pi\\ 0\end{pmatrix}$ and $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}\pi\\ \pi\end{pmatrix}$.

rate decreases monotonically to zero. In a frustrated state $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}0\\ \pi\end{pmatrix};\begin{pmatrix}\pi\\ 0\end{pmatrix}$ and for values $I_{c2}/I_{c1} = 0.4;\ 0.8$ escape rate changes insignificant. In the frustrated state $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}\pi\\ \pi\end{pmatrix}$ and for $I_{c2}/I_{c1} = 0.4;\ 0.8$ that escape change is considerable. changing of escape. In this case, the whole region of changing of I_{c3}/I_{c1} becomes correspondingly 0.2; 0.6.

Above presented result means that plasma frequency in JJ between single-band and three-band superconductors are affected by the number of channels in such junctions. In the literature no direct experimental confirmation of the reducing of plasma frequency and escape rate in SB/ThB junctions in the frustrated regime. However, there is an experimental indications of reducing of critical current in the case of the ThB based in JJ. In Ref. 28 it was considered SB/ThB JJ and predicted effects such as of asymmetric critical current, subharmonic Shapiro steps. The effect of asymmetrical critical current has been observed in the edge-type hybrid junction between PbIn and two-band Co-doped $BaFe_2As_2$ thin-film presented in Ref. 29. In this junctions I_cR_N-product of about 12 μV. In Ref. 30, another similar experiment with JJ between Pb and the $Ba_{1-x}K_x(FeAs)_2$ $x = 0.29$ and 0.49 were conducted. In Ref. 31, it was investigated experimentally $PbIn/BaK(FeAs)_2$ point-contact junction. In this study, authors used the theoretical approach of three bands superconducting state scenarios for the treatment of experimental data. Very recently in Ref. 32 was reported about $Nb/BaNa(FeAs)_2$ based JJ with very small I_cR_N-product approximately 3 μV. Authors explain this fact with the cancellation of opposite supercurrents in frustrated state three-band FeAs-based superconductors. Considerable reducing of Josephson plasma frequency in such three-band structures also obtained by the calculations [28].

Finally it is useful to discuss the influence of the Leggett mode on the above-presented results in the MQT regime. Leggett modes related with coupling of density oscillations in many band SC with oscillations of phase difference [34, 35]. Calculations based on three-band GL equations [22] confirms that the Leggett oscillations leads to renormalization of interband interaction parameter ε

$$\varepsilon_{12} \to \varepsilon\left(\frac{1}{2} - \frac{\langle\delta\phi^2\rangle}{4}\right);\ \varepsilon_{13} \to \varepsilon\left(\frac{1}{2} - \frac{\langle\delta\theta^2\rangle}{4}\right);$$
$$\varepsilon_{23} \to \varepsilon\left(\frac{1}{2} - \frac{\langle\delta(\phi-\theta)^2\rangle}{4}\right), \tag{18}$$

where (see [8, 36])

$$\langle\delta\phi^2\rangle = \langle\delta\theta^2\rangle = \langle\delta(\phi-\theta)^2\rangle = \frac{1}{2}\sqrt{\frac{E_C}{\varepsilon}};\ E_C = \frac{e^2}{2C}. \tag{19}$$

As followed from the Eqs. (11) and (12), the suppression of critical temperature δT_c due to Leggett oscillations in frustration state is proportional

$$\delta T_c \propto \frac{3}{4}\sqrt{\varepsilon E_C}. \quad (20)$$

As shown in Refs. 11–13, in multichannel JJs the amplitude of the critical current in different channels is proportional to the multiplication of order parameters in ThB superconductors $\Psi_0\Psi_{1,2,3}$. It means that, taking into account Leggett oscillations into three-band superconductors, leads to suppression of critical currents in three-channel junctions, and as result the ratio I_{c3}/I_{c1} in Eq. (15) also decreases. The Leggett mode for a three-band superconductor was also considered in the paper [37]. Unlocking of phase difference in multiband superconductors at critical temperature T_c was analyzed using Bardeen–Cooper–Schrieffer theory. It was demonstrated that interband phase difference fluctuation has a large effect on gap evolution similar to our result. In our approach, we take into account renormalization by the Leggett mode at zero temperature, i. e., quantum fluctuations of phase difference [33, 36] [see also above Eqs. (18)–(20)].

As clear from Fig. 3, in frustration state $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}2\pi/3\\ -2\pi/3\end{pmatrix}$ in the limit, $I_{c2} \to 0$ the influence of Leggett oscillations on escape rate in the MQT regime can be ignored. With increasing amplitude of I_{c2} the influence of Leggett oscillations on escape rate becomes important and under $I_{c2} \to I_{c3}$ such effect reach maximum level. In frustration state, $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}0\\ \pi\end{pmatrix}$ the influence of Leggett mode in the limit $I_{c2} \to 0$ is small for all values of I_{c3}. Under $I_{c2} = 0$ for high values of I_{c3} Leggett modes strongly influence escape rate. In a frustration state, $\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}\pi\\ \pi\end{pmatrix}$ the influence of this modes becomes important under $I_{c3} \to I_{c1} - I_{c2}$. It means enhancement of escape rate in SB/ThB JJs under Leggett oscillations (see Figs. 3 and 4). We expect that coexistence of Josephson plasma modes and Leggett's mode can be detected in a junction system and obtained above theoretical results will be verified experimentally.

Thus, in this study it was calculated the escape rate in Josephson junction formed by *s*-wave single-band and three-band superconductors (SB/ThB junctions). It was shown that in all cases of the presence of additional channels leads to a decreasing in escape rate in the MQT regime. Coupling between order parameters in many-band superconductors cause Leggett modes of density oscillations, which causes enhancement of escape rate in MQT regime.

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Вплив фрустрації на частоту переходів у джозефсонівських контактах між односмуговими та трисмуговими надпровідниками у режимі макроскопічного квантового тунелювання

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Досліджено частоту переходів S → R (з надпровідного S-стану в резистивний R-стан) у джозефсонівському контакті, який утворений *s*-хвильовими однозонними (SB) та трьохзонними (ThB) надпровідниками (переходи SB/ThB) у режимі макроскопічного квантового тунелювання. Використано наближення ефективного критичного струму в SB/ThB контактах. Показано, що частота переходів має особливість у разі фрустраційних ефектів у ThB надпровідниках. Включення режимів Леггетта у ThB надпровідниках призводить до збільшення частоти переходів у трьохканальних джозефсонівських контактах в порівнянні з одноканальними.

Ключові слова: джозефсонівські контакти, квантове тунелювання, односмугові та трисмугові надпровідники.