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Spin wave propagation through the interface between two ferromagnets without/with Dzyaloshinskii–Moriya interaction

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The analytical model is constructed for the description of the spin wave propagation through a system consisting of two ferromagnets without and with the Dzyaloshinskii–Moriya interaction, separated by a flat interface. The dependences of transmission and reflection coefficients of spin wave are found as a function of Dzyaloshinskii–Moriya constant which is known to be strongly temperature dependent, tending to a significant increase at low temperature.

Keywords: ferromagnet, spin waves, Dzyaloshinskii–Moriya interaction.

Introduction

The Dzyaloshinskii–Moriya interaction (DMI) [1, 2], also known as the antisymmetric exchange coupling, allows to describe the numerous magnetic properties of compounds with broken symmetry. Chiral, topological, and non-reciprocal features of magnetic structures caused by DMI are the subject of theoretical and experimental studies. Studies of systems with non-collinear spin textures, such as chiral domain walls [3, 4], skyrmions [5–7], skyrmion lattices [8–10], magnetic spirals [11] in ferromagnetic materials have a wide range of applications, for example, in devices of spin-wave logic [12, 13], coding bits for racetrack memory [14–16].

The contribution of DMI into the total energy of a ferromagnet increases significantly at low temperatures compared to room temperature [15].

In this paper, the propagation of a spin wave is considered from ferromagnetic plate without DMI into the ferromagnetic plate with DMI, separated by a flat interface. The dependences of transmission and reflection coefficients of spin wave in this system are found as a function of DMI constant and spin wave frequency.

2. Theory and calculation

The system consists of two semi-infinitely long uniaxial ferromagnets, which are characterized by saturation magnetizations M_{01} and M_{02}, the exchange stiffness constant $A_1^{\text{ex}}(x)$, $A_2^{\text{ex}}(x)$ and the uniaxial magnetic anisotropy constants of ferromagnets and interface $K_1(x)$, $K_2(x)$, and $K'(x)$, correspondingly. Those ferromagnets contact along the plane *YOZ*. The external homogeneous constant magnetic field $\mathbf{H}_0$ is directed along the direction of the easy axis *OZ* (Fig. 1).

The Landau–Lifshitz equations without dissipation in first and second ferromagnet have the form

$$\begin{cases} \dfrac{\partial \mathbf{M}_1}{\partial t} = -|\gamma|\mu_0\left[\mathbf{M}_1 \times \mathbf{H}_{\text{eff}}^{(1)}\right] \\ \dfrac{\partial \mathbf{M}_2}{\partial t} = -|\gamma|\mu_0\left[\mathbf{M}_2 \times \mathbf{H}_{\text{eff}}^{(2)}\right] \end{cases}, \tag{1}$$

where $\mathbf{M}_1$ and $\mathbf{M}_2$ are the magnetization vectors in the first and second ferromagnets, respectively, γ is the gyromagnetic ratio, μ_0 is the vacuum permeability, $\mathbf{H}_{\text{eff}}^{(1)}$ and $\mathbf{H}_{\text{eff}}^{(2)}$ are the effective magnetic field strength in the first and second ferromagnet, respectively:

$$\mathbf{H}_{\text{eff}}^{(1)} = -\frac{\delta W}{\delta \mathbf{M}_1}, \qquad \mathbf{H}_{\text{eff}}^{(2)} = -\frac{\delta W}{\delta \mathbf{M}_2}. \tag{2}$$

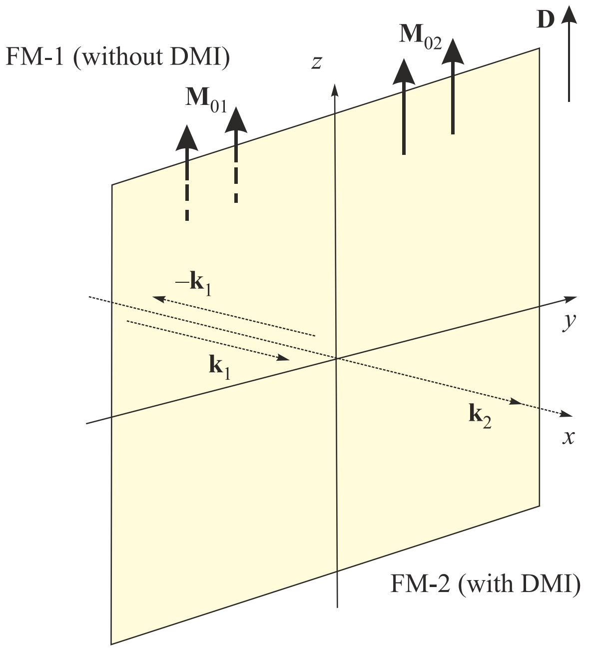

Fig. 1. A schematic image of the system of two semi-infinite ferromagnetic media FM-1 and FM-2 separated by the interface of thickness δ which is much less than the spin wave length. The SW normally incidents on the interface with the wave vector $\mathbf{k}_1$ and reflects with the wave vector $-\mathbf{k}_1$. Transmitted SW has the wave vector $\mathbf{k}_2$. $\mathbf{D}$ represents DMI vector.

The total energy of the system is as follows:

$$W = \int_V d\mathbf{r} \left\{ -\frac{A(x)}{M_{01}M_{02}}\mathbf{M}_1\mathbf{M}_2 + \frac{1}{2}\frac{A_1^{\text{ex}}(x)}{M_{01}^2}\left(\frac{\partial \mathbf{M}_1}{\partial x}\right)^2 + \right.$$
$$+\frac{1}{2}\frac{A_2^{\text{ex}}(x)}{M_{02}^2}\left(\frac{\partial \mathbf{M}_2}{\partial x}\right)^2 - \frac{1}{2}\frac{K_1(x)}{M_{01}^2}(\mathbf{M}_1\mathbf{n}_1)^2 -$$
$$-\frac{1}{2}\frac{K_2(x)}{M_{02}^2}(\mathbf{M}_2\mathbf{n}_2)^2 - \frac{K'(x)}{M_{01}M_{02}}(\mathbf{M}_1\mathbf{n}_2)(\mathbf{M}_2\mathbf{n}_2) - \quad (3)$$
$$-(\mathbf{M}_1 + \mathbf{M}_2)\mu_0\mathbf{H}_0^{(i)} -$$
$$\left. -\frac{D}{M_{02}^2}\left[\mathbf{e}_z\left(\mathbf{M}_2 \times \frac{\partial \mathbf{M}_2}{\partial x}\right) - \mathbf{e}_x\left(\mathbf{M}_2 \times \frac{\partial \mathbf{M}_2}{\partial z}\right)\right]\right\}.$$

$A(x)$ is the parameter of uniform exchange interaction at the interface between the ferromagnets. It is a function of the x coordinate, which is zero inside the both ferromagnets and essentially nonzero inside the interface, $A(x) = A\delta(x)$, where $\delta(x)$ is the Dirac delta function. Vectors $\mathbf{n}_1$, $\mathbf{n}_2$ are the unit vectors directed along the axes of uniaxial magnetic anisotropy, $\mathbf{e}_x$, $\mathbf{e}_z$ are orts of the OX and OZ axes. $A_1^{\text{ex}} = \lim_{x\to-\infty} A_1^{\text{ex}}(x)$, $A_2^{\text{ex}} = \lim_{x\to+\infty} A_2^{\text{ex}}(x)$ are the exchange stiffness constants in first and second ferromagnet at distances much greater than the interface thickness δ. D is Dzyaloshinskii–Moriya constant.

We consider the same directions of anisotropy for both ferromagnets and the interface $\mathbf{n}_1 = \mathbf{n}_2 = \mathbf{n}$, here OZ is chosen parallel to the vectors $\mathbf{n}_1$, $\mathbf{n}_2$. The normal to the interface between two ferromagnets is parallel to the OX axis. $\mathbf{H}_0^{(i)}$ is internal homogeneous constant magnetic field directed along the direction of the easy axis.

The magnetization vector can be represented as $\mathbf{M}_1 = \mathbf{M}_{01} + \mathbf{m}_1$, where $\mathbf{M}_{01}$ is the saturation magnetization of the first ferromagnet, $\mathbf{m}_1$ is the small deviation magnetization from the ground state. Similar considerations are valid for the magnetization vector $\mathbf{M}_2$ in the second ferromagnet: $\mathbf{M}_2 = \mathbf{M}_{02} + \mathbf{m}_2$, where $\mathbf{M}_{02}$ is the saturation magnetization of the second ferromagnet, $\mathbf{m}_2$ is the small deviation magnetization from the ground state.

Two ferromagnets with the same parameters $M_{01} = M_{02} = M_0$, $A_1^{\text{ex}} = A_2^{\text{ex}} = A_{\text{ex}}$, $K_1 = K_2 = K$ are considered except that there is no DMI in the first of them, and there is nonzero DMI in the second one.

The flat spin wave (SW) is considered further in both ferromagnets. The magnetization vectors of the incident and reflected waves can be represented as the following solution of the linearized Landau–Lifshitz Eqs. (1) and (2) in the first ferromagnet [18]:

$$m_{1x} = A_0\cos(k_1x - \omega_1t + \varphi_{01}) + R\cos(-k_1x - \omega_1t + \tilde{\varphi}_{01}),$$
$$m_{1y} = A_0\sin(k_1x - \omega_1t + \varphi_{01}) + R\sin(-k_1x - \omega_1t + \tilde{\varphi}_{01}), \quad (4)$$

where A_0, R are the amplitudes of the incident and reflected SW, respectively, k_1 is the wave number of the incident SW, $\varphi_{01} = \text{const}$, $\tilde{\varphi}_{01} = \text{const}$, ω_1 is the frequency of the SW in the first ferromagnet.

The magnetization components for SW transmitted into the second ferromagnet can be represented as the following solution of the linearized Landau–Lifshitz Eqs. (1), (2) in the second ferromagnet [16]

$$m_{2x} = A_2\cos(k_2x - \omega_2t),$$
$$m_{2y} = A_2\sin(k_2x - \omega_2t), \quad (5)$$

where A_2 is the amplitude of the SW transmitted into the second medium, k_2 is the wave number of a spin wave in the second ferromagnet, ω_2 is the frequency of the SW in the second ferromagnet.

The dispersion relation for SW in the first ferromagnet without the DMI has the form [18]

$$\omega_1(k_1) = \gamma\sqrt{\frac{1}{M_0^2}\left(A_{\text{ex}}k_1^2 + \mu_0H_0^{(i)}M_0 + K\right)^2 + \mu_0\left(A_{\text{ex}}k_1^2 + \mu_0H_0^{(i)}M_0 + K\right)}. \quad (6)$$

The dispersion relation for SW in the second ferromagnet with the DMI has the form [19]

$$\omega_2(k_2) = -\frac{2\gamma D k_2}{M_0} + \gamma\sqrt{\frac{1}{M_0^2}\left(A_{ex}k_2^2 + \mu_0 H_0^{(i)} M_0 + K\right)^2 + \mu_0\left(A_{ex}k_2^2 + \mu_0 H_0^{(i)} M_0 + K\right)}. \quad (7)$$

The boundary conditions are used to find the coefficients of transmission and reflection of SW at the interface

$$\begin{cases} \left[\mathbf{M}_1 \times \left(\dfrac{A_1^{ex}}{M_{01}^2}\dfrac{\partial \mathbf{M}_1}{\partial x} - \dfrac{A}{M_{01}M_{02}}\mathbf{M}_2\right)\Bigg|_{x=0}\right] = 0, \\ \left[\mathbf{M}_2 \times \left(\dfrac{A_2^{ex}}{M_{02}^2}\dfrac{\partial \mathbf{M}_2}{\partial x} - \dfrac{D}{M_{02}^2}(\mathbf{e}_z \times \mathbf{M}_{02}) + \dfrac{A}{M_{01}M_{02}}\mathbf{M}_1\right)\Bigg|_{x=0}\right] = 0. \end{cases} \quad (8)$$

Substituting small deviations of the magnetization from the ground state, we can write the boundary conditions in the form:

$$\begin{cases} \left(A_{ex}\dfrac{\partial m_{1x}}{\partial x} - Am_{2x} + Am_{1x}\right)\Bigg|_{x=0} = 0, \\ \left(A_{ex}\dfrac{\partial m_{1y}}{\partial x} - Am_{2y} + Am_{1y}\right)\Bigg|_{x=0} = 0, \\ \left(A_{ex}\dfrac{\partial m_{2x}}{\partial x} + Am_{1x} - Am_{2x} + Dm_{2y}\right)\Bigg|_{x=0} = 0, \\ \left(A_{ex}\dfrac{\partial m_{2y}}{\partial x} + Am_{1y} - Am_{2y} - Dm_{2x}\right)\Bigg|_{x=0} = 0. \end{cases} \quad (9)$$

After substituting the components of the magnetization (4), (5) in the boundary conditions (9), the following equations are obtained:

$$\begin{cases} -A_{ex}k_1\left(A_0\sin\varphi_{01} - R\sin\tilde{\varphi}_{01}\right) - AA_2 + A\left(A_0\cos\varphi_{01} + R\cos\tilde{\varphi}_{01}\right) = 0, \\ A_{ex}k_1\left(A_0\cos\varphi_{01} - R\cos\tilde{\varphi}_{01}\right) + A\left(A_0\sin\varphi_{01} + R\sin\tilde{\varphi}_{01}\right) = 0, \\ A\left(A_0\cos\varphi_{01} + R\cos\tilde{\varphi}_{01}\right) - AA_2 = 0, \\ A_{ex}k_2A_2 + A\left(A_0\sin\varphi_{01} + R\sin\tilde{\varphi}_{01}\right) - DA_2 = 0. \end{cases} \quad (10)$$

It follows from the boundary conditions (10) that $\omega_1 = \omega_2 = \omega$.

Dependence of k_1 on ω has the form:

$$k_1 = \sqrt{\frac{-\left(K + \mu_0 H_0^{(i)} M_0 + \frac{1}{2}\mu_0 M_0^2\right) + M_0\sqrt{\left(\left(\frac{1}{2}\mu_0 M_0\right)^2 + \left(\frac{\omega}{\gamma}\right)^2\right)}}{A_{ex}}}, \quad (11)$$

where the condition should be satisfied

$$\omega > \frac{\gamma}{M_0}\sqrt{\left(K + \mu_0 H_0^{(i)} M_0 + \frac{1}{2}\mu_0 M_0^2\right)^2 - \left(\frac{1}{2}\mu_0 M_0^2\right)^2}.$$

Dependence of k_2 on ω was obtained from (7).

The expression for the square of the reflection coefficient $\tilde{R} = R/A_0$ has the form as follows from the system (10)

$$\tilde{R}^2 = \frac{A^2\left(A_{ex}k_1 - A_{ex}k_2 + D\right)^2 + \left(A_{ex}k_1\right)^2\left(A_{ex}k_2 - D\right)^2}{A^2\left(A_{ex}k_1 + A_{ex}k_2 - D\right)^2 + \left(A_{ex}k_1\right)^2\left(A_{ex}k_2 - D\right)^2}. \quad (12)$$

The expression for the square of the transmission coefficient $\tilde{A}_2 = A_2/A_0$ has the form as follows from the system (10)

$$\tilde{A}_2^2 = \frac{\left(2A_{ex}k_1A\right)^2}{A^2\left(A_{ex}k_1 + A_{ex}k_2 - D\right)^2 + \left(A_{ex}k_1\right)^2\left(A_{ex}k_2 - D\right)^2}. \quad (13)$$

3. Results and Discussion

The material parameters are represented in Table 1 for analysing the Ex. (13) for SW transmission from ferromagnet without DMI into the ferromagnet with DMI.

Table 1. The material parameters of thulium iron garnet ($Tm_3Fe_5O_{12}$, TmIG) for analysing the SW transmission from ferromagnet without DMI into the ferromagnet with DMI

Values of material parameters
$M_0 = 10^5$, A/m [20]
$H_0^{(i)} = 795.8$, A/m
$K = 11.88$, kJ/m^3 [21]
$A_{ex} = 2.3$, pJ/m [22]
$A = 0.1$, J/m^2

The dependences of the SW transmission coefficient $\tilde{A}_2^2$ on the Dzyaloshinskii–Moriya constant D are represented in Fig. 2.

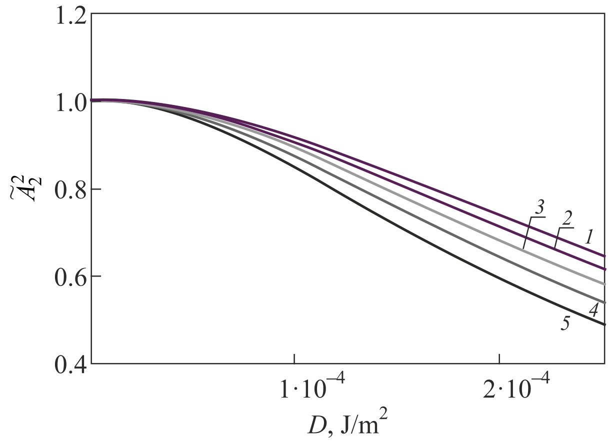

Fig. 2. Dependence of the SW transmission coefficient $\tilde{A}_2^2$ on the Dzyaloshinskii–Moriya constant D; curves are plotted for different values of frequency ω, GHz: 70 (*1*), 65 (*2*), 60 (*3*), 55 (*4*), 50 (*5*).

The intensity of transmitted SW $\tilde{A}_2^2$ decreases with increasing D. The results of this paper illustrate that it is possible to control SW transmission coefficient from ferromagnet without DMI into ferromagnet with DMI by means of changing the temperature as the DMI constant D depends significantly on temperature trending to significant increase at low temperature [15].

Conclusions

Boundary conditions (8) for the Landau–Lifshitz equation are obtained at the interface between a ferromagnet without DMI and a ferromagnet with DMI. The reflection and transmission coefficients are calculated for the SW passing through the interface between a ferromagnet without DMI and a ferromagnet with DMI taking into account these boundary conditions. The dependences of transmission and reflection coefficients of spin wave on Dzyaloshinskii–Moriya constant confirms that SW control in magnonic devices can be achieved by changing temperature because the DMI constant is known to be strongly temperature dependent.

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Поширення спінової хвилі через інтерфейс двох феромагнетиків без / із взаємодією Дзялошинського–Морія

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Побудовано аналітичну модель для опису поширення спінової хвилі через систему, що складається з двох феромагнетиків без взаємодії та із взаємодією Дзялошинського-Морія, які розділені інтерфейсом. Знайдено залежності коефіцієнтів пропускання та відбиття спінової хвилі як функції від константи Дзялошинського–Морія, яка сильно залежить від температури та має тенденцію до значного збільшення при низькій температурі.

Ключові слова: феромагнетик, спінові хвилі, взаємодія Дзялошинського–Морія.