

The editors of the issue consider it possible to publish this article, despite the fact that they do not share the author's views on the possibility of applicability of the approach proposed by him to describe the properties of the electroactivity of superfluid helium.

Nuclear and electronic coherence in superfluid helium

Yu. M. Poluektov

National Science Center "Kharkov Institute of Physics and Technology", Kharkiv 61108, Ukraine

E-mail: yuripoluektov@kipt.kharkov.ua

Received March 17, 2021, published online June 25, 2021

A semi-phenomenological model of a many-particle system of ^4He atoms is proposed, in which a helium atom is considered as a complex consisting of a nucleus and a bound pair of electrons in the singlet state. At zero temperature, there are two Bose–Einstein condensates of particles with opposite charges, namely, a condensate of positively charged nuclei and a condensate of negatively charged electron pairs. It is shown that in such a system there exist two excitation branches: sound and optical. On the basis of this model an interpretation of experiments on the study of the electrical activity of superfluid helium is proposed. The frequency at which the resonant absorption of a microwave radiation is observed is interpreted as a gap in the optical branch. It is shown that the distribution of the electric potential in a standing wave in a resonator is similar to that observed experimentally.

Keywords: helium atom, boson, sound and optical vibrations, superfluidity, electrical activity, Bose–Einstein condensate, coherent state.

1. Introduction

In the experiments of Rybalko [1–7] with superfluid helium, there were registered effects which demonstrate an increased electrical activity of this neutral medium. The activity manifests itself both at low frequencies in sound and torsion experiments [1, 2] and at high frequencies in the interaction with a microwave radiation [3–7]. In one group of effects, the electrical oscillations were observed under fluctuations of temperature T [1] and under oscillations of the difference of the superfluid and normal velocities $\mathbf{w} = \mathbf{v}_s - \mathbf{v}_n$ [2]. In experiments of another type, the resonant absorption of a microwave radiation was found [3–7] at a frequency close to 180 GHz. These results were mainly confirmed in later experiments [8–12].

Until now there have been carried out a significant number of theoretical works where attempts have been made to explain the observed effects. However, it seems unlikely that such effects can be explained while remaining within the framework of the traditional theory of superfluidity, where the internal structure of atoms is not taken into account. The internal structure was taken into account in theoretical works [13, 14], in which particles were considered as hydrogen-like atoms. In this work, we propose a semi-phenomenological model of a superfluid system of particles whose structure is closer to the real structure of the helium atom.

Before experiments in which the electrical activity was discovered, in the theoretical study of the superfluid properties of liquid helium atoms were usually considered as structureless particles with zero spin. At zero temperature, a system of N atoms obeying the Bose statistics is described by the wave function $\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$ being symmetric with respect to the permutations of position vectors \mathbf{r}_j . In a low-density system, when all particles are in the same state, the total wave function can be represented as a product of identical functions $\psi(\mathbf{r}_1)\psi(\mathbf{r}_2)\dots\psi(\mathbf{r}_N)$ characterizing the state of an individual particle in the condensate. The function $\psi(\mathbf{r})$ obeys the well-known Gross–Pitaevskii equation [15, 16]. Such state is coherent [13]. For dense systems the structure of the symmetric wave function proves to be more complex, and in this case an important role is also played by pair correlations and correlations of a larger number of particles [17, 18]. In this work we will not touch upon the question of the role of higher correlations. Thus, in contrast to the model of an ideal Bose gas where the condensate particles actually fall out of consideration, when taking into account the interparticle interaction the condensate particles are described by some effective complex wave function $\psi(\mathbf{r})$. We will call the condensate of interacting particles as the coherent Bose–Einstein condensate. The concept of the superfluid component of He II as a superposition of oppositely charged coherent boson

condensates (nuclear and electron) was considered in work [19], where the cause of generation of an electric field was associated with the acceleration of electrons and nuclei which have very different masses.

A helium atom consists of a nucleus (alpha particle) with zero spin and a pair of electrons. In the ground state the spins of electrons are directed oppositely, so that the total spin of a pair is zero (parahelium). A pair of electrons in such a singlet state is a very strong formation. In order to transfer a pair of electrons from the singlet state to the triplet state with the total spin equal to unity (orthohelium), an energy of 19.8 eV should be spent, and the energy of the first excited state of parahelium is 20.6 eV higher than that of the ground state. This makes it possible to consider a pair of electrons in the ground state of the helium atom as a single object resembling a Cooper pair localized near a nucleus. On this basis, in the proposed model the helium atom will be considered as a complex consisting of a spinless nucleus with charge $2|e|$ and a particle with zero spin and charge $-2|e|$.

When taking into account the internal structure of the atom, both nuclei and pairs of bound electrons pass into the condensate. Thus, this model considers a neutral system of two Bose–Einstein condensates of nuclei and electron pairs with opposite charges. The fluctuations of the densities of the number of particles in condensates are accompanied by the fluctuations of the densities of charge, current and electric potential. This article studies small oscillations of such a system of two condensates and shows that there exist two branches of elementary excitations: the sound branch and the optical branch. It is also shown that the distribution of the electric potential in a standing wave in a resonator coincides with the distribution observed in the experiment [20].

Based on the analysis of the proposed model, it was concluded that the electrical effects observed in superfluid helium are a consequence of the perturbation of its coherent system determining the value of the superfluid density. There are three parameters that lead to a change in the superfluid density: temperature, superfluid flow and pressure. Estimates show that the largest perturbation of the coherent system is induced by the temperature fluctuations. A somewhat smaller effect is caused by the fluctuations of the superfluid flow. The least influence on the coherent system is exerted by the pressure fluctuations.

2. Dynamical equations of the coherent system of nuclei and electron pairs

In the secondary quantization representation, the system of nuclei will be described by the field operator $\psi_\alpha(\mathbf{r}, t)$ and the system of pairs of bound electrons by the field operator $\psi_e(\mathbf{r}, t)$. These operators obey the usual commutation relations

$$[\psi_\alpha(\mathbf{r}, t), \psi_\alpha^\dagger(\mathbf{r}', t)] = \delta(\mathbf{r} - \mathbf{r}'), \quad [\psi_\alpha(\mathbf{r}, t), \psi_\alpha(\mathbf{r}', t)] = 0, \quad (1)$$

$$[\psi_e(\mathbf{r}, t), \psi_e^\dagger(\mathbf{r}', t)] = \delta(\mathbf{r} - \mathbf{r}'), \quad [\psi_e(\mathbf{r}, t), \psi_e(\mathbf{r}', t)] = 0$$

and commute with each other. The Hamiltonian has the form $H = H_K + H_I + H_E$, where

$$H_K = -\int d\mathbf{r} \left\{ \psi_\alpha^\dagger(\mathbf{r}) \left[\frac{\hbar^2}{2M} \Delta + \mu_\alpha \right] \psi_\alpha(\mathbf{r}) + \psi_e^\dagger(\mathbf{r}) \left[\frac{\hbar^2}{2m} \Delta + \mu_e \right] \psi_e(\mathbf{r}) \right\}, \quad (2)$$

$$H_I = \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \{ \psi_\alpha^\dagger(\mathbf{r}) \psi_\alpha^\dagger(\mathbf{r}') U_{\alpha\alpha}(|\mathbf{r} - \mathbf{r}'|) \psi_\alpha(\mathbf{r}') \psi_\alpha(\mathbf{r}) + \psi_e^\dagger(\mathbf{r}) \psi_e^\dagger(\mathbf{r}') U_{ee}(|\mathbf{r} - \mathbf{r}'|) \psi_e(\mathbf{r}') \psi_e(\mathbf{r}) + 2\psi_\alpha^\dagger(\mathbf{r}) \psi_\alpha^\dagger(\mathbf{r}') U_{\alpha e}(|\mathbf{r} - \mathbf{r}'|) \psi_e(\mathbf{r}') \psi_e(\mathbf{r}) \}, \quad (3)$$

$$H_E = \int d\mathbf{r} \left\{ e \left[\psi_\alpha^\dagger(\mathbf{r}) \psi_\alpha(\mathbf{r}) - \psi_e^\dagger(\mathbf{r}) \psi_e(\mathbf{r}) \right] \varphi(\mathbf{r}) + \frac{(\nabla\varphi(\mathbf{r}))^2}{8\pi} \right\}. \quad (4)$$

Here M , m are the effective masses of a nucleus and an electron pair, e is the electron charge. Note that the effective masses in a many-particle system of interacting particles do not have to coincide with the mass of a helium nucleus M_α and the mass of a pair of free electrons $2m_e$, but they are phenomenological parameters. For definiteness we will assume that $M > m$. The electric field is taken into account in the nonrelativistic approximation through the scalar potential $\varphi(\mathbf{r})$. For simplicity, in the following we choose the interaction potentials in the delta-like form:

$$U_{\alpha\alpha}(|\mathbf{r} - \mathbf{r}'|) \equiv g_\alpha \delta(\mathbf{r} - \mathbf{r}'), \quad U_{ee}(|\mathbf{r} - \mathbf{r}'|) \equiv g_e \delta(\mathbf{r} - \mathbf{r}'),$$

$$U_{\alpha e}(|\mathbf{r} - \mathbf{r}'|) \equiv g_{\alpha e} \delta(\mathbf{r} - \mathbf{r}').$$

We assume that $g_\alpha > 0$, $g_e > 0$, $g_{\alpha e} < 0$. The operators of the number of nuclei and the number of electron pairs, respectively, are

$$N_\alpha = \int d\mathbf{r} \psi_\alpha^\dagger(\mathbf{r}) \psi_\alpha(\mathbf{r}), \quad N_e = \int d\mathbf{r} \psi_e^\dagger(\mathbf{r}) \psi_e(\mathbf{r}). \quad (5)$$

In the Heisenberg representation, the operators depend on time and obey the equations of motion

$$i\hbar \frac{\partial \psi_\alpha(\mathbf{r}, t)}{\partial t} = [\psi_\alpha(\mathbf{r}, t), H],$$

$$i\hbar \frac{\partial \psi_e(\mathbf{r}, t)}{\partial t} = [\psi_e(\mathbf{r}, t), H]. \quad (6)$$

Using the formulas (1)–(4), we obtain an explicit form of equations for the field operators. In accordance with the fact that at temperatures close to zero most Bose particles are in a single state, by analogy to the Gross–Pitaevskii approach [15, 16] one can neglect the commutation properties of the operators and consider them as ordinary functions. As a result, we obtain the equations

$$i\hbar \frac{\partial \Psi_\alpha}{\partial t} = - \left(\frac{\hbar^2}{2M} \Delta + \mu_\alpha + |e|\varphi \right) \Psi_\alpha + g_\alpha |\Psi_\alpha|^2 \Psi_\alpha + g_{\alpha e} |\Psi_e|^2 \Psi_\alpha, \quad (7)$$

$$i\hbar \frac{\partial \Psi_e}{\partial t} = - \left(\frac{\hbar^2}{2m} \Delta + \mu_e - |e|\varphi \right) \Psi_e + g_e |\Psi_e|^2 \Psi_e + g_{\alpha e} |\Psi_\alpha|^2 \Psi_e. \quad (8)$$

Equating to zero the variation of the energy with respect to the scalar potential, we arrive at the Poisson equation

$$\Delta \varphi = -8\pi |e| (|\Psi_\alpha|^2 - |\Psi_e|^2). \quad (9)$$

The chemical potentials entering into (7), (8) can be expressed in terms of the equilibrium density of the number of nuclei and electron pairs $n_0 = |\Psi_{\alpha 0}|^2 = |\Psi_{e 0}|^2$:

$$\mu_\alpha = (g_\alpha + g_{\alpha e})n_0, \quad \mu_e = (g_e + g_{\alpha e})n_0. \quad (10)$$

The flux densities of the number of nuclei and electron pairs are given by the formulas

$$j_\alpha = \frac{i\hbar}{2M} (\Psi_\alpha \nabla \Psi_\alpha^* - \Psi_\alpha^* \nabla \Psi_\alpha),$$

$$j_e = \frac{i\hbar}{2m} (\Psi_e \nabla \Psi_e^* - \Psi_e^* \nabla \Psi_e), \quad (11)$$

and the current densities of positive and negative charges: $\mathbf{j}_{\alpha \text{ch}} = 2|e|\mathbf{j}_\alpha$, $\mathbf{j}_{e \text{ch}} = -2|e|\mathbf{j}_e$. Thus, the Eqs. (7)–(10) describe the dynamics of the coherent system of nuclei and electron pairs and the electric potential in such a system.

3. Small oscillations of the coherent system of nuclei and electron pairs

Let us consider small oscillations in the spatially homogeneous coherent system of nuclei and electron pairs in the absence of a stationary flux, writing down complex functions in the form

$$\Psi_\alpha = \sqrt{n_0} + \delta\psi_\alpha, \quad \Psi_e = \sqrt{n_0} + \delta\psi_e. \quad (12)$$

In the following, instead of the complex quantities $\delta\psi_\alpha$, $\delta\psi_e$, it will be more convenient to use the real functions

$$\begin{aligned} \delta\Psi_\alpha &= \delta\psi_\alpha + \delta\psi_\alpha^*, & \delta\Phi_\alpha &= i(\delta\psi_\alpha - \delta\psi_\alpha^*), \\ \delta\Psi_e &= \delta\psi_e + \delta\psi_e^*, & \delta\Phi_e &= i(\delta\psi_e - \delta\psi_e^*). \end{aligned} \quad (13)$$

The fluctuations of the density of the number of nuclei δn_α , the density of the number of electron pairs δn_e , the density of mass $\delta\rho_m$ and charge $\delta\rho_{\text{ch}}$, as well as the fluctuations of the flux densities in terms of the quantities (13) are given by the expressions

$$\begin{aligned} \delta n_\alpha &= \sqrt{n_0} \delta\Psi_\alpha, & \delta n_e &= \sqrt{n_0} \delta\Psi_e, \\ \rho_m &= \sqrt{n_0} (M_\alpha \delta\Psi_\alpha + 2m_e \delta\Psi_e), \\ \delta\rho_{\text{ch}} &= 2|e|\sqrt{n_0} (\delta\Psi_\alpha - \delta\Psi_e), \\ \delta j_\alpha &= -\frac{\hbar\sqrt{n_0}}{2M} \nabla \delta\Phi_\alpha, & \delta j_e &= -\frac{\hbar\sqrt{n_0}}{2m} \nabla \delta\Phi_e, \\ \delta \mathbf{j}_{\text{ch}} &= 2|e|(\delta \mathbf{j}_\alpha - \delta \mathbf{j}_e). \end{aligned} \quad (14)$$

The linearized system of Eqs. (7)–(9) for the real variables (13) has the form

$$\begin{aligned} \hbar \frac{\partial \delta\Phi_\alpha}{\partial t} &= -\frac{\hbar^2}{2M} \Delta \delta\Phi_\alpha + 2g_\alpha n_0 \delta\Psi_\alpha + \\ &+ 2g_{\alpha e} n_0 \delta\Psi_e + 2|e|\sqrt{n_0} \varphi, \end{aligned} \quad (15)$$

$$\hbar \frac{\partial \delta\Psi_\alpha}{\partial t} = \frac{\hbar^2}{2M} \Delta \delta\Phi_\alpha, \quad (16)$$

$$\begin{aligned} \hbar \frac{\partial \delta\Phi_e}{\partial t} &= -\frac{\hbar^2}{2m} \Delta \delta\Phi_e + 2g_e n_0 \delta\Psi_e + \\ &+ 2g_{\alpha e} n_0 \delta\Psi_\alpha - 2|e|\sqrt{n_0} \varphi, \end{aligned} \quad (17)$$

$$\hbar \frac{\partial \delta\Psi_e}{\partial t} = \frac{\hbar^2}{2m} \Delta \delta\Phi_e, \quad (18)$$

$$\Delta \varphi = -8\pi |e|\sqrt{n_0} (\delta\Psi_\alpha - \delta\Psi_e). \quad (19)$$

This system of five equations is equivalent to the system of two equations for the functions $\delta\Psi_\alpha$ and $\delta\Psi_e$:

$$\begin{aligned} \frac{\partial^2 \delta\Psi_\alpha}{\partial t^2} &= -\frac{\hbar^2}{4M^2} \Delta^2 \delta\Psi_\alpha + \frac{g_\alpha n_0}{M} \Delta \delta\Psi_\alpha + \\ &+ \frac{g_{\alpha e} n_0}{M} \Delta \delta\Psi_e - \omega_\alpha^2 \delta\Psi_\alpha + \omega_\alpha^2 \delta\Psi_e, \end{aligned} \quad (20)$$

$$\begin{aligned} \frac{\partial^2 \delta\Psi_e}{\partial t^2} &= -\frac{\hbar^2}{4m^2} \Delta^2 \delta\Psi_e + \frac{g_e n_0}{m} \Delta \delta\Psi_e + \\ &+ \frac{g_{\alpha e} n_0}{m} \Delta \delta\Psi_\alpha + \omega_e^2 \delta\Psi_\alpha - \omega_e^2 \delta\Psi_e. \end{aligned} \quad (21)$$

Here the plasma frequencies for nuclei ω_α and electron pairs ω_e are determined by the relations

$$\omega_\alpha^2 = \frac{8\pi e^2 n_0}{M}, \quad \omega_e^2 = \frac{8\pi e^2 n_0}{m}. \quad (22)$$

Assuming the dependencies of the quantities $\delta\Psi_\alpha$ and $\delta\Psi_e$ on coordinates and time in the form $\exp i(\omega t - \mathbf{k}\mathbf{r})$, we find from (20) and (21) the dispersion equation

$$\omega^4 - 2B\omega^2 + C = 0, \quad (23)$$

where

$$\begin{aligned}
 2B &= \omega_\alpha^2 + \omega_e^2 + \left(\frac{g_\alpha}{M} + \frac{g_e}{m}\right)n_0k^2 + \left(\frac{1}{4M^2} + \frac{1}{4m^2}\right)\hbar^2k^4, \\
 C &= \left[\omega_\alpha^2 \frac{(g_e + g_{\alpha e})}{m} + \omega_e^2 \frac{(g_\alpha + g_{\alpha e})}{M}\right]n_0k^2 + \\
 &+ \left[\hbar^2 \left(\frac{\omega_\alpha^2}{4m^2} + \frac{\omega_e^2}{4M^2}\right) + \frac{(g_\alpha g_e - g_{\alpha e}^2)n_0^2}{mM}\right]k^4 + \\
 &+ \left(\frac{g_\alpha}{4Mm^2} + \frac{g_e}{4mM^2}\right)n_0\hbar^2k^6 + \frac{\hbar^4k^8}{16M^2m^2}.
 \end{aligned}
 \tag{24}$$

Thus, there are two branches of excitations

$$\omega_\pm^2 = B \pm \sqrt{B^2 - C},
 \tag{25}$$

which are shown in Fig. 1. In the short-wavelength limit, these branches transform into the dispersion laws of free nuclei and electron pairs

$$\omega_+ = \frac{\hbar k^2}{2m}, \quad \omega_- = \frac{\hbar k^2}{2M}.
 \tag{26}$$

These limiting relations seem reasonable, but it is physically correct to consider the dispersion relations in the limit of long waves. In this case

$$\begin{aligned}
 \omega_\pm^2 &= \frac{1}{2} \left[\omega_\alpha^2 + \omega_e^2 + \left(\frac{g_\alpha}{M} + \frac{g_e}{m}\right)n_0k^2 \right] \pm \\
 &\pm \frac{1}{2} \left\{ \left[\omega_\alpha^2 + \omega_e^2 + \left(\frac{g_\alpha}{M} + \frac{g_e}{m}\right)n_0k^2 \right]^2 - \frac{4(g_\alpha g_e - g_{\alpha e}^2)n_0^2k^4}{mM} \right\}^{1/2}.
 \end{aligned}
 \tag{27}$$

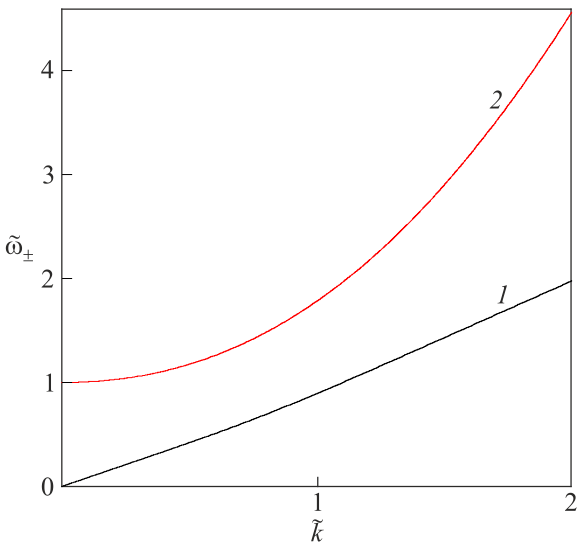


Fig. 1. The sound (1) and optical (2) branches of excitations in the system with two oppositely charged coherent Bose–Einstein condensates. Here $\tilde{\omega} \equiv \omega / \omega_0$, $\tilde{k} \equiv k / k_0$, $\omega_0^2 \equiv \omega_\alpha^2 + \omega_e^2$,

$$k_0^2 \equiv \frac{2Mm}{\sqrt{M^2 + m^2}} \frac{\omega_0}{\hbar}, \quad \gamma \equiv m / M = 0.1.$$

In the system of two neutral condensates, at $e = 0$, we have two sound branches $\omega_\pm^2 = c_{0\pm}^2 k^2$, where

$$c_{0\pm}^2 = \frac{n_0}{2} \left[\left(\frac{g_\alpha}{M} + \frac{g_e}{m}\right) \pm \sqrt{\left(\frac{g_\alpha}{M} + \frac{g_e}{m}\right)^2 - \frac{4(g_\alpha g_e - g_{\alpha e}^2)}{mM}} \right].
 \tag{28}$$

In the case of charged condensates we are interested in, there is a single sound branch $\omega_- = ck$, where the velocity is determined by the formula

$$c^2 = \frac{n_0(g_\alpha + g_e + 2g_{\alpha e})}{(m + M)}.
 \tag{29}$$

For stability of the system, the interaction constants must satisfy the condition $g_\alpha + g_e + 2g_{\alpha e} > 0$. The second branch is optical $\omega_+^2 = \omega_\alpha^2 + \omega_e^2 + \alpha k^2$, where

$$\begin{aligned}
 \alpha &= \frac{n_0}{(\omega_\alpha^2 + \omega_e^2)} \left[\frac{g_\alpha}{M} \omega_\alpha^2 + \frac{g_e}{m} \omega_e^2 - g_{\alpha e} \left(\frac{\omega_\alpha^2}{m} + \frac{\omega_e^2}{M}\right) \right] = \\
 &= \frac{n_0}{(M + m)} \left(g_\alpha \frac{m}{M} + g_e \frac{M}{m} - 2g_{\alpha e} \right).
 \end{aligned}
 \tag{30}$$

The gap in the spectrum ω_0 is determined by the relation

$$\omega_0^2 = \omega_\alpha^2 + \omega_e^2 = \frac{8\pi n_0 e^2}{M_*},
 \tag{31}$$

where $M_* = mM / (m + M)$ is the reduced mass. When deriving formulas (29), (30) from the more general formula (27), it was assumed a fulfillment of the condition

$$k^2 < \frac{(m + M)^2}{mM} \frac{e^2}{|g_i|}, \quad i = (\alpha, e, \alpha e).
 \tag{32}$$

The proposed model can pretend to provide quantitative estimates only in the case of low-density systems, whereas liquid helium is not such one. Nevertheless, it is of interest to estimate the value of the reduced mass, assuming that the formula (31) remains valid in this case and the frequency $f_0 = \omega_0 / 2\pi$ coincides with the resonant frequency of 180 GHz observed in experiments [3–7]. At the density $n_0 = 10^{22} \text{ cm}^{-3}$, it turns out that the reduced mass is four orders of magnitude greater than the mass of a helium atom: $M_* \propto 10^4 M_{\text{He}}$. Note that earlier it was drawn attention to the possibility of the existence of a gap in the energy spectrum of superfluid Bose systems due to pair correlations in works [13, 17, 18, 21]. The calculation of absorption of a microwave radiation at the resonant frequency can be performed in a similar way as in [22].

The fluctuations of the number of pairs $\delta n_e^{(-)}$ and nuclei $\delta n_\alpha^{(-)}$ in the sound wave are linked by the relation

$$\delta n_e^{(-)} = \left[1 - \frac{k^2}{8\pi e^2} \frac{(g_e M - g_\alpha m + g_{\alpha e} (M - m))}{(M + m)} \right] \delta n_\alpha^{(-)}.
 \tag{33}$$

According to (14), the fluctuations of the mass density and charge density in the sound wave are given by the formulas

$$\delta\rho_m^{(-)} = \left[M_\alpha + 2m_e - m_e \frac{k^2}{4\pi e^2} \times \frac{(g_e M - g_\alpha m + g_{\alpha e}(M - m))}{(M + m)} \right] \delta n_\alpha^{(-)}, \quad (34)$$

$$\delta\rho_{\text{ch}}^{(-)} = \frac{k^2}{4\pi |e|} \frac{(g_e M - g_\alpha m + g_{\alpha e}(M - m))}{(M + m)} \delta n_\alpha^{(-)}. \quad (35)$$

From (34) and (35) there follows the relation between the fluctuations of charge and mass densities

$$\delta\rho_{\text{ch}}^{(-)} = \frac{k^2}{4\pi |e|} \frac{(g_e M - g_\alpha m + g_{\alpha e}(M - m))}{M_{\text{He}}(M + m)} \delta\rho_m^{(-)}, \quad (36)$$

where $M_{\text{He}} \equiv M_\alpha + 2m_e$ is the mass of a helium atom ${}^4\text{He}$. As we can see, at $k \rightarrow 0$ also $\delta\rho_{\text{ch}}^{(-)} \rightarrow 0$, so that with an increase in the length of the sound wave the charge fluctuation in it decreases in comparison to the density fluctuation. It should be noted, however, that the wavelength cannot exceed the characteristic size of the system, so that always $k \geq 1/L$. This is essential, as will be seen below, when considering oscillations in a resonator. Let us also give the relation between the current density fluctuation and the mass density fluctuation:

$$\delta\mathbf{j}_{\text{ch}}^{(-)} = \frac{\hbar|e|}{M_{\text{He}}} \left(\frac{1}{m} - \frac{1}{M} \right) \nabla \delta\rho_m^{(-)}. \quad (37)$$

For the optical branch, the fluctuations of the number of pairs $\delta n_e^{(+)}$ and nuclei $\delta n_\alpha^{(+)}$ are linked by the relation

$$\delta n_e^{(+)} = -\frac{M}{m} \left[1 + \frac{k^2}{8\pi e^2} \frac{(g_e M - g_\alpha m + g_{\alpha e}(M - m))}{(M + m)} \right] \delta n_\alpha^{(+)}. \quad (38)$$

According to (14), the oscillations of the mass and charge densities in the optical branch are determined by the formulas

$$\delta\rho_m^{(+)} = \left[M_\alpha - 2\frac{m_e}{m}M - \frac{m_e M}{m} \frac{k^2}{4\pi e^2} \times \frac{(g_e M - g_\alpha m + g_{\alpha e}(M - m))}{(M + m)} \right] \delta n_\alpha^{(+)}, \quad (39)$$

$$\delta\rho_{\text{ch}}^{(+)} = 2\frac{|e|}{m} \left[(M + m) + \frac{k^2}{8\pi |e|} \times \frac{M(g_e M - g_\alpha m + g_{\alpha e}(M - m))}{(M + m)} \right] \delta n_\alpha^{(+)}. \quad (40)$$

From (39) and (40) it follows that in this case the relation between the fluctuations of charge and mass densities is given by the formula

$$\delta\rho_{\text{ch}}^{(+)} = \frac{2|e|(M + m)}{(mM_\alpha - 2m_e M)} \left[1 + \frac{k^2}{8\pi |e|} \times \frac{M(g_e M - g_\alpha m + g_{\alpha e}(M - m))}{(M + m)^2} \right] \delta\rho_m^{(+)}. \quad (41)$$

In the limit $k \rightarrow 0$, the ratio of the amplitudes of oscillations of charge and mass densities remains constant. The current density fluctuation and the mass density fluctuation are linked by the relation

$$\delta\mathbf{j}_{\text{ch}}^{(+)} = -\hbar|e| \frac{\left(\frac{M}{m} + \frac{m}{M} \right)}{(mM_\alpha - 2m_e M)} \nabla \delta\rho_m^{(+)}. \quad (42)$$

Although this work does not consider the states with stationary flows, we note that since electron pairs compensate for the charge of nuclei, stationary flux densities of the number of nuclei and pairs should be the same and the stationary electric current density should be zero in this case.

4. Low frequency oscillations in a capacitor

In the experiment [1], there were studied the standing waves of the second sound and the potential oscillations in a resonator filled with superfluid helium. Let us consider, within the framework of the proposed model, the oscillations in a capacitor the plates of which are perpendicular to the x -axis and located at the points $x = \pm L/2$. Taking into account that the flows of particles in the direction perpendicular to the plates should vanish on the plates themselves, we find that the fluctuations of the densities of nuclei and pairs are given by the formulas

$$\delta n_\alpha = \bar{n}_\alpha \sin \omega_n t \sin k_n x, \quad \delta n_e = \bar{n}_e \sin \omega_n t \sin k_n x, \quad (43)$$

where $\omega_n = ck_n$, $k_n = \pi(2n+1)/L$, and the velocity c is determined by the formula (29). Assuming that the potential on the right plate at $x = L/2$ is equal to zero and the surface charges on the capacitor plates are absent, so that the normal component of the electric field vanishes on the plates, we find from the Eq. (19) the potential distribution

$$\delta\varphi(x, t) = \frac{\Phi_m}{2} [(-1)^n - \sin k_n x] \sin \omega_n t, \quad (44)$$

where

$$\Phi_m \equiv -\frac{2(g_e M - g_\alpha m + g_{\alpha e}(M - m))}{|e|(M + m)} \bar{n}_\alpha.$$

The potential distribution for the cases when one $n = 0$ and three $n = 1$ half-waves fit along the resonator length is shown in Fig. 2.

These distributions coincide with those obtained experimentally in [20].

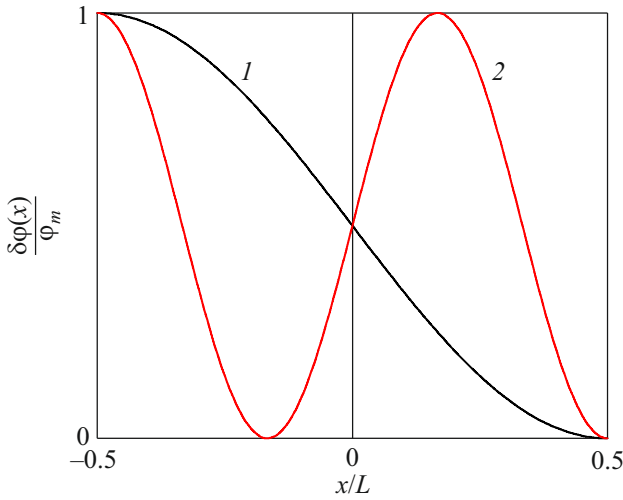


Fig. 2. The potential distribution in the capacitor at the moment $t = L / 2c$: (1) one $n = 0$, (2) three $n = 1$ half-waves fit along the resonator length.

5. Discussion

In the semi-phenomenological model of helium superfluidity proposed in this article, atoms are considered as complexes consisting of a nucleus and a bound pair of electrons in the singlet state. Since this model contains two systems of oppositely charged particles obeying the Bose statistics, there are also two coherent Bose–Einstein condensates. In dynamic processes the local fluctuations of the number of electron pairs and nuclei lead to the fluctuations of the densities of electric charge, current and potential. The model is formulated for zero temperature and under the assumption that the system is rarefied, and therefore cannot pretend to give a quantitative description of the effects observed in superfluid helium, like indeed any other model if it does not contain a set of a sufficient number of adjustable parameters. Nevertheless, it allows to qualitatively understand the cause of the observed electrical phenomena, which consists in the perturbation of the coherent system of nuclei and electron pairs. In liquid helium the coherence emerges below the lambda-transition temperature. A consequence of the emergence of the coherent Bose–Einstein condensate is the appearance of a new characteristic of the system — the superfluid density. Thus, in liquid superfluid helium the coherent subsystem of atoms forms the superfluid density, which depends on temperature T , pressure p and the difference of the superfluid and normal velocities $\mathbf{w} = \mathbf{v}_n - \mathbf{v}_s$, so that $\rho_s = \rho_s(p, T, w^2)$. As we can see, the oscillations of the superfluid density, and consequently of the coherent subsystem, in helium can arise under the influence of the fluctuations of temperature, pressure and the velocity difference. The intensity of such oscillations of the coherent subsystem can be characterized by the dimensionless parameters

$$A_p = \frac{p}{\rho_s} \left(\frac{\partial \rho_s}{\partial p} \right)_{T,w}, \quad A_T = \frac{T}{\rho_s} \left(\frac{\partial \rho_s}{\partial T} \right)_{p,w},$$

$$A_w = \frac{u_2^2}{\rho_s} \left(\frac{\partial \rho_s}{\partial w^2} \right)_{T,p}, \quad (45)$$

where u_2 is the velocity of the second sound. Let us estimate the magnitude of these coefficients. Using the data given in the appendix of the book [23], we find that at $T = 1.4$ K the coefficient $A_T \approx -0.54$. With an increase in temperature, this coefficient increases in absolute magnitude, reaching at $T = 2$ K the value $A_T \approx -7.3$. In this case, of course, ρ_s decreases.

Let us also estimate the magnitude of the coefficient A_w that determines the influence of the oscillations of the superfluid flow on the perturbation of the coherent system of helium. The derivative of the superfluid density is expressed in terms of the derivatives of the total ρ and normal ρ_n densities: $\partial \rho_s / \partial w^2 = \partial \rho / \partial w^2 - \partial \rho_n / \partial w^2$. The first term can be estimated using the thermodynamic relation [24]

$$\frac{\partial \rho}{\partial w^2} = \frac{\rho^2}{2} \frac{\partial}{\partial p} \left(\frac{\rho_n}{\rho} \right). \quad (46)$$

The dependence of the normal density on w^2 is found from formulas given in §3 of [24]. As a result, we get

$$\frac{\partial \rho_n}{\partial w^2} \approx -0.85 \cdot 10^{-9} \text{ gs}^2 / \text{cm}^5.$$

This estimate is consistent with the experimental estimate given in the appendix of the book [23] $\rho^{-1} |\partial \rho_n / \partial w^2| < 6 \cdot 10^{-8} \text{ s}^2 / \text{cm}^2$. Taking into account the value of the second-sound velocity $u_2 = 2 \cdot 10^3 \text{ cm/s}$, we get $A_w \approx -2.5 \cdot 10^{-2}$. And if we take the maximum possible value $\rho^{-1} |\partial \rho_n / \partial w^2| = 6 \cdot 10^{-8} \text{ s}^2 / \text{cm}^2$ according to the experimental data, then we get $A_w \approx -0.25$. As seen, the electrical effects caused by the fluctuations of $\mathbf{w} = \mathbf{v}_n - \mathbf{v}_s$ are close to those generated by the temperature fluctuations. The electrical effects caused by the fluctuations of the velocity difference were observed in the experiment with a torsion oscillator [2].

An estimate of the coefficient that determines the effect of pressure on the coherent subsystem at saturated vapor pressure gives $A_p \approx 10^{-5}$. Pressure has the least effect on the coherent subsystem, so that the electrical effects should be much less pronounced in experiments with the first sound. However, as the pressure increases the coefficient A_p also increases. So, at pressure of 5 atm it has an order of magnitude $A_p \approx 10^{-2}$. Note that in [25] the observation of the electric effect in the first sound wave was reported, although this effect was not observed in other works.

6. Conclusion

The article proposes a qualitative interpretation of the electrical effects observed in superfluid helium [1–12, 20, 25], based on the analysis of the model which assumes the existence of two oppositely charged coherent Bose–Einstein condensates: those of atomic nuclei and singlet electron pairs. In this approach the electron pairs are considered as delocalized, so that in nonstationary processes there exists a probability of a pair transition from atom to atom and, therefore, the possibility of the local breaking of electroneutrality, which thus leads to the appearance of the internal electric field. It is shown that there are two branches of elementary excitations: sound and optical.

The observed electrical activity in superfluid helium is explained by the disturbance due to external factors of its coherent system manifesting itself in the existence of the superfluid density. Estimates show that the strongest effect on the coherent system is exerted by the fluctuations of temperature, then follow the fluctuations of superfluid flow, and the weakest effect is due to the fluctuations of pressure. The frequency at which the resonant absorption of a microwave radiation is observed [3–7] is interpreted as a gap in the optical branch of the spectrum. The oscillations in a resonator are considered and it is shown that the distribution of the electric potential in the standing wave is consistent with experiment [20].

1. A. S. Rybalko, *Fiz. Nizk. Temp.* **30**, 1321 (2004) [*Low Temp. Phys.* **30**, 994 (2004)].
2. A. S. Rybalko and S. P. Rubets, *Fiz. Nizk. Temp.* **31**, 820 (2005) [*Low Temp. Phys.* **31**, 623 (2005)].
3. A. S. Rybalko, S. P. Rubets, E. Ya. Rudavskii, V. A. Tikhii, S. I. Tarapov, R. V. Golovashchenko, and V. N. Derkach, *Phys. Rev. B* **76**, 140503(R) (2007).
4. A. S. Rybalko, S. P. Rubets, E. Ya. Rudavskii, V. A. Tikhii, R. V. Golovashchenko, V. N. Derkach, and S. I. Tarapov, *Fiz. Nizk. Temp.* **34**, 326 (2008) [*Low Temp. Phys.* **34**, 254 (2008)].
5. A. S. Rybalko, S. P. Rubets, E. Ya. Rudavskii, V. A. Tikhii, S. I. Tarapov, R. V. Golovashchenko, and V. N. Derkach, *Fiz. Nizk. Temp.* **34**, 631 (2008) [*Low Temp. Phys.* **34**, 497 (2008)].
6. A. S. Rybalko, S. P. Rubets, E. Ya. Rudavskii, V. A. Tikhii, Yu. M. Poluektov, R. Golovashchenko, V. N. Derkach, S. I. Tarapov, and O. V. Usatenko, *Fiz. Nizk. Temp.* **35**, 1073 (2009) [*Low Temp. Phys.* **35**, 837 (2009)].
7. A. S. Rybalko, S. P. Rubets, E. Ya. Rudavskii, V. A. Tikhii, Yu. M. Poluektov, R. Golovashchenko, V. N. Derkach, S. I. Tarapov, and O. V. Usatenko, *J. Low Temp. Phys.* **158**, 244 (2010).
8. T. V. Chagovets, *Fiz. Nizk. Temp.* **42**, 230 (2016) [*Low Temp. Phys.* **42**, 176 (2016)].
9. T. V. Chagovets, *Physics B* **488**, 62 (2016).
10. A. S. Rybalko, T. V. Chagovets, and A. M. Korolev, *Fiz. Nizk. Temp.* **43**, 927 (2017) [*Low Temp. Phys.* **43**, 738 (2017)].

11. H. Yayama, Y. Nishimura, H. Ushiyama, H. Kawai, J.-P. van Woensel, and A. Hafez, *Fiz. Nizk. Temp.* **44**, 1386 (2018) [*Low Temp. Phys.* **44**, 1090 (2018)].
12. V. D. Natsik and A. S. Rybalko, *Fiz. Nizk. Temp.* **46**, 33 (2020) [*Low Temp. Phys.* **46**, 28 (2020)].
13. Yu. M. Poluektov, *Fiz. Nizk. Temp.* **37**, 1239 (2011) [*Low Temp. Phys.* **37**, 986 (2011)].
14. S. I. Shevchenko and A. S. Rukin, *Fiz. Nizk. Temp.* **38**, 1147 (2012) [*Low Temp. Phys.* **38**, 905 (2012)].
15. E. P. Gross, *Nuovo Cimento* **20**, 454 (1961).
16. L. P. Pitaevskii, *Sov. Phys. JETP* **13**, 451 (1961).
17. A. S. Peletminskii, S. V. Peletminskii, and Yu. M. Poluektov, *Condens. Matter Phys.* **16**, 13603 (2013) [arXiv:1303.5539].
18. Yu. M. Poluektov, *Fiz. Nizk. Temp.* **44**, 1326 (2018) [*Low Temp. Phys.* **44**, 1040 (2018)].
19. E. A. Pashitskii and S. M. Ryabchenko, *Fiz. Nizk. Temp.* **33**, 12 (2007) [*Low Temp. Phys.* **33**, 8 (2007)].
20. A. S. Rybalko, V. A. Tikhii, A. S. Neoneta, and K. R. Zhekov, *arXiv:1108.1272v1* [cond-mat.other] (2011).
21. A. S. Peletminskii, S. V. Peletminskii, and Yu. M. Poluektov, *Fiz. Nizk. Temp.* **40**, 645 (2014) [*Low Temp. Phys.* **40**, 500 (2014)].
22. Yu. M. Poluektov, *Fiz. Nizk. Temp.* **40**, 503 (2014) [*Low Temp. Phys.* **40**, 389 (2014)].
23. S. J. Patterman, *Hydrodynamics of Superfluid Liquid*, Mir, Moscow (1978).
24. I. M. Khalatnikov, *Theory of Superfluidity*, Nauka, Moscow (1971).
25. T. V. Chagovets, *J. Low Temp. Phys.* **187**, 383 (2017).

Ядерна та електронна когерентність у надплинному гелію

Yu. M. Poluektov

Запропоновано напівфеноменологічну модель багаточастинкової системи атомів ^4He , в якій атом гелію розглядається як комплекс, що складається з ядра та зв'язаної пари електронів у синглетному стані. При нульовій температурі існує два конденсати Бозе–Ейнштейна із протилежними зарядами, а саме, конденсат позитивно заряджених ядер та конденсат від'ємно заряджених електронних пар. Показано, що в такій системі є дві гілки збуджень: звукова та оптична. На основі цієї моделі запропоновано інтерпретацію експериментів із дослідження електричної активності надплинного гелію. Частота, на якій спостерігається резонансне поглинання НВЧ-випромінювання, трактується як щілина в оптичній гілці. Показано, що розподіл електричного потенціалу в стоячій хвилі у резонаторі аналогічний тому, що спостерігається експериментально.

Ключові слова: атом гелію, бозон, звукові та оптичні коливання, надплинність, електрична активність, конденсат Бозе–Ейнштейна, когерентний стан.