Low Temperature Physics/Fizika Nizkikh Temperatur, 2021, vol. 47, No. 8, pp. 703–709

Phonon model of heat radiation into superfluid helium by a solid with a flat surface

(Review Article)

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Received March 2, 2021, published online June 25, 2021

This paper reviews the quasiparticle model of superfluid helium and its application to describe heat transfer between a heated solid and superfluid helium. In this case, a problem is considered in which the surface of the heater is flat and the helium is at practically zero temperature. Under these conditions, heat transfer between solid and superfluid helium is determined by the transformation of the phonons of the solid into helium phonons. The work considers certain types of such transformation — elastic processes of phonon transformation, in which the number of phonons is conserved, and inelastic processes, in which the number of phonons changes. The main attention in this work is paid to the development of a quantum-mechanical approach for calculating the contribution of various polarizations phonons of solid to the heat flux formation, its magnitude, and angular distribution. The results of the work are used to explain the experimentally observed features of heat transfer from a heated solid to superfluid helium.

Keywords: Kapitsa jump, phonons, heat flux, inelastic processes.

Contents

1. Quasiparticle model of heat transfer in superfluid helium

Superfluid ^{4}He and solutions of its isotopes are systems in which quantum laws are most clearly manifested on a macroscopic scale. The theory of superfluidity of He II was developed by L. D. Landau [1]. Landau's theory is based on the quasiparticle method, according to which certain properties of superfluid are completely determined by the corresponding properties of a gas of quasiparticles. This quasiparticle approach has proven to be very productive for the description of condensed matter. Landau's theory of superfluidity, in particular, allows one to go from a system of strongly interacting particles to a system of weakly interacting quasiparticles, the thermodynamic parameters of which can be calculated in the ideal gas model, and the kinetic properties — using the perturbation theory. Moreover, the quasiparticle approach to the description of superfluid helium turned out to be one of the examples of a successful theoretical model, which not only described the phenomena for the explanation of which it was created [1]. The application of this model made it possible to explain unexpected phenomena observed in subsequent experiments, as well as to predict new properties of superfluid helium and its solutions.

In pure superfluid helium, quasiparticles are phonons and rotons — thermal excitations of a quantum liquid with definite dispersion laws. Taking these quasiparticles into account made it possible to explain the thermodynamic properties of He II, in the region up to 2 K, when the gases of quasiparticles can be considered practically ideal. Also, the quasiparticle model explains the physical nature of such phenomena as second sound, thermal expansion of helium, fountain effect, etc. The use of physical kinetic methods for a gas mixture of quasiparticles made it possible to calculate the main dissipative parameters of He II — the coefficients of the first and second viscosities, the coefficient of thermal conductivity, the absorption coefficients of the first, second, and other sounds. In the model of quasiparticles, it was possible to construct a kinetic theory of superfluid solutions of ^{3}He–^{4}He. The theoretical description of these solutions was made in a three-component system of quasiparticles — phonons and rotons (thermal excitations of He II), as well as impuritons (quasiparticles of ^{3}He). This theory has successfully explained the unusual properties of superfluid ^{3}He–^{4}He solutions and provided explanations for a large number of thermodynamic, hydrodynamic, and kinetic properties of these quantum fluids.

One example of the application of the quasiparticle model is the description of heat transfer processes between solids and superfluid helium, in particular, the so-called Kapitsa jump. P. L. Kapitsa [2] found that when heat is radiated by a solid that is in contact with liquid He II, a constant temperature difference arises between the solid and liquid helium (Kapitsa jump). This difference turned out to be proportional to the radiated heat flux, and the value of the thermal resistance, equal to the ratio of the temperature difference to the heat flux, in Kapitsa's experiments increased with decreasing temperature according to the cubic law.

Since then, the phenomenon of heat transfer between superfluid helium and a solid has been intensively studied both experimentally and theoretically. There are at least three reasons that stimulate this research. At the first, the unusual phenomenon is observed not only at the superfluid helium–solid interface, but also at the interfaces of another quantum continuous media. At the second, the need to take into account the Kapitsa jump in all low-temperature experiments, since the presence of a Kapitsa jump at ultralow temperatures significantly reduces the efficiency of heat exchangers in superfluid helium. And, at the third, despite the progress achieved over the years in understanding the physics of heat transfer between two quantum continuous media, several issues have remained unresolved to this day.

The first theoretical explanation of the Kapitsa jump was proposed by Khalatnikov in the quasiparticle model [3]. According to this theory, heat transfer between two quantum continuous media is due to the transition of phonons from one medium to another. Heat transfer, in this case, is strongly limited due to the inconsistency of the acoustic impedance of the media and the smallness of the critical angle of incidence for phonons in liquid helium, above which total internal reflection occurs.

Further experiments, carried out in different years for various solids, gave results that sometimes differed significantly for different authors, even for the boundaries of the same solids with superfluid helium. In this case, both the temperature dependence and the numerical value of the heat transfer coefficient were observed to differ from the results of the Khalatnikov theory by one or two orders of magnitude.

A significant contribution to the study of heat transfer was made by direct experiments by Adrian Wyatt, in which the energy and angle distribution of phonons that were emitted by a heated solid into cold ($T < 100$ mK) superfluid helium were measured. These direct experiments, carried out on various ideal crystal surfaces, showed the presence of two separated channels of phonon emission.

The first channel formed a sharp peak of phonons, which radiated into a narrow cone of angles, the axis of which was normal to the surface of the solid. The solid angle of this cone for various solids coincided with the results of the classical acoustic theory, on which Khalatnikov's theory was based [3–5]. This radiation channel was called acoustic.

Along with phonons, emitted into a narrow cone, they observed the phonons emitted in all directions with a cosine-like angular distribution. This channel of phonon emission is called background radiation. In this case, the total phonon energy contained in the background radiation was an order of magnitude higher than the phonon energy that was emitted into the acoustic channel.

According to the acoustic theory [3–5], the phonons of solid falling on the interface between a solid and superfluid helium are transformed with a certain probability into phonons of a liquid. In this case, the energy of the incident solid-state phonon is equal to the energy of the liquid phonon. This process of phonon transition from one medium to another can be called elastic because energy and number of phonons are conserved.

The existence of background radiation was tried to be explained by the inelastic processes when the number of phonons in the initial and final states is different. Experimental work [6] shows a diagram of an inelastic process in which one solid phonon transforms into two liquid phonons, which can move at any angle to the interface. In this case, the average phonon energy of the liquid turns out to be less than the energy of the solid-state phonon. As noted in [6], the possibility of inelastic processes contributing to the background radiation was indicated by the fact that the phonons generating the background radiation have lower energy than phonons and are concentrated in a narrow acoustic peak.

In this regard, theoretical studies of inelastic processes were carried out, and their contribution to the energy flux through the solid-superfluid helium interface. One of the possible inelastic processes, which differed from the process depicted in [6], was considered by Khalatnikov in [3], who showed that the contribution of this process to the Kapitsa jump is relatively small.

In [7], an attempt was made for the first time to create a microscopic theory that would describe both elastic and inelastic processes in a unified manner. Work [8] developped the approach proposed in work [7]. However, in [7, 8], it was not possible to create a self-consistent approach that would allow obtaining results that coincide with the results of Khalatnikov for the elastic process. This circumstance is related to the fact that in [7, 8] the calculations were not brought to the final analytical formulas and specific numerical values.

In [9–11], the results of constructing a unified self-consistent theory describing both elastic and inelastic processes at the superfluid helium–solid interface were presented for the first time. These results were reduced to obtaining the Hamiltonian of the superfluid helium–solid interface and considering an elastic process, as well as some inelastic processes that contribute to the energy flux from a heated solid to cold superfluid helium.

In this work, the technique developed in [9–11] is used to study the contributions of different types of phonons of a solid to the formation of heat flux from a solid to superfluid helium.

2. Energy of excitations and quantization of hydrodynamic variables

To describe the processes of heat transfer from a solid to superfluid helium, we use the theory of elastic and inelastic interaction of helium and solid phonons at the interface between these media [9]. This theory is based on the use of the hydrodynamic Hamiltonian of thermal excitations of helium in an external field created by the oscillating surface of a solid. To obtain an explicit form of this Hamiltonian, we represent the contribution of thermal excitations to the energy density of a liquid in the following form:

$$E_t = \frac{1}{2}\rho_t \mathbf{v}_t^2 + E_\rho(\rho_t) - E_\rho(\rho_L), \tag{1}$$

here ρ_t and $\mathbf{v}_t$ are total density and velocity of helium, receptively; ρ_L is the equilibrium value of the density, and E_ρ is the density of the potential energy of the liquid. The helium density can be presented in the following form: $\rho_t = \rho_L + \rho + \rho_i$, where ρ is the density deviation, caused by the own helium excitations, and ρ_i is the density deviation, caused by the helium excitations created by wall oscillations. In the same way, we present the total fluid velocity as the sum of two terms $\mathbf{v}_t = \mathbf{v} + \mathbf{v}_i$. Further, assuming the perturbations to be small, we restrict ourselves to considering the quadratic and cubic terms in the expansion of energy (1) in terms of these perturbations:

$$E = \frac{\rho_L}{2}(\mathbf{v}+\mathbf{v}_i)^2 + \frac{c_L^2}{2\rho_L}(\rho+\rho_i)^2 + \frac{1}{2}(\rho+\rho_i)(\mathbf{v}+\mathbf{v}_i)^2 + \frac{(2u-1)c_L^2}{6\rho_L^2}(\rho+\rho_i)^3. \tag{2}$$

Here

$$c_L^2 = \rho_t \left(\frac{\partial^2 (E_\rho(\rho_t))}{\partial \rho_t^2} \right)_{\rho_t=\rho_L} \tag{3}$$

is the sound velocity in helium

$$u = \frac{\partial \ln c_L}{\partial \ln \rho_L} \tag{4}$$

is Gruneisen constant.

To obtain the Hamiltonian of the interaction between the phonons of helium and a solid, we will leave in Eq. (2) only the terms that contain both internal perturbations of helium and perturbations caused by vibrations of the walls. Then we represent the final expression for the interaction energy as the sum of two terms:

$$E_{\text{int}} = E_2 + E_3. \tag{5}$$

The first of the terms is quadratic in perturbations:

$$E_2 = \rho_L \mathbf{v}\mathbf{v}_i + \frac{c_L^2}{\rho_L}\rho\rho_i, \tag{6}$$

and the second term contains cubic terms:

$$E_3 = (\rho+\rho_i)\mathbf{v}\mathbf{v}_i + \frac{1}{2}(\rho\mathbf{v}_i^2 + \rho_i\mathbf{v}^2) + \frac{(2u-1)c_L^2}{2\rho_L^2}\rho\rho_i(\rho+\rho_i). \tag{7}$$

Let us proceed to the definition of the explicit form of perturbations of the hydrodynamic variables of superfluid helium — density and velocity — and their subsequent quantization. We start with quantizing the perturbations of the hydrodynamic variables of superfluid helium density and velocity ρ_i and $\mathbf{v}_i$, which are caused by the presence of an oscillating flat wall — the surface of a solid body. These perturbations are determined from the system of hydrodynamic equations:

$$\begin{cases} \dfrac{\partial \rho_i}{\partial t} + \rho_L \operatorname{div} \mathbf{v}_i = 0, \\ \dfrac{\partial \mathbf{v}_i}{\partial t} = -\dfrac{c_L^2}{\rho_L}\nabla\rho_i \end{cases} \tag{8}$$

in the presence of the boundary condition at the helium–solid interface for the normal component of the fluid velocity:

$$v_{iz}(x,y,z=0,t) \equiv V_{Bz}(x,y,t). \tag{9}$$

Here the z-axis is chosen perpendicular to the wall, and V_{Bz} is the amplitude of the wall vibration velocity. Wall vibrations, in turn, can be caused by mechanical action (membrane or tuning fork), or heating of a solid. In the latter case, the phonons of the solid, falling on the boundary and reflecting from it, lead to its oscillations [5]. In this case, the expression for V_{Bz} can be represented as

$$V_{Bz}(x,y,z=0,t) = \sum_{\mathbf{q}_\parallel} \beta_{\mathbf{q}} A_{\mathbf{q}} e^{i\mathbf{q}_\parallel \boldsymbol{\tau}} e^{-i\Omega t}. \tag{10}$$

Here $\beta_q = \sqrt{\hbar\Omega/\rho_S V_S}$, V_S is the volume of the solid, ρ_S is the solid density, $\mathbf{q}$ and Ω is the wave vector and frequency of the incident phonon of solid, respectively. The $\mathbf{q}_\parallel$ is the component of the phonon wave vector that is parallel to the wall. The normalization factor β_q is chosen so that the energy in the incident wave equals to $\hbar\Omega$. The vibration amplitudes of wall A take into account the reflection laws in solid of phonons with different polarizations [5] and will be discussed below. The solution to system (8) under boundary condition (9) has the following form:

$$v_{iz}(\mathbf{r},t)=\sum_{\mathbf{q}_{\|}}\beta_{\mathbf{q}}A_{\mathbf{q}}\,\mathrm{e}^{i\mathbf{q}_{\|}\boldsymbol{\tau}}\mathrm{e}^{-i\Omega t}\mathrm{e}^{iK_z z} \tag{11}$$

and presents plane waves running from the wall. The value $K_z=K_z(\Omega,\mathbf{q}_{\|})$ is the zth component of the wave vector of the wave radiated into helium:

$$K_z=K_z(\Omega,\mathbf{q}_{\|})=\sqrt{\Omega^2/c_L^2-q_{\|}^2}\,. \tag{12}$$

The transverse components of this vector and the frequency are equal to the corresponding values of the incident phonon: $\mathbf{K}_{\|}=\mathbf{q}_{\|}$ и $\Omega=\omega$. The density and other (parallel to the wall) components of the velocity are determined from the solution of the system (8)

$$v_{ix,y}(\mathbf{r},t)=\sum_{\mathbf{q}_{\|}}\beta_{\mathbf{q}}A_{\mathbf{q}}\frac{K_{x,y}}{K_z}\mathrm{e}^{i\mathbf{q}_{\|}\boldsymbol{\tau}}\mathrm{e}^{-i\Omega t}\mathrm{e}^{iK_z z}, \tag{13}$$

$$\rho_i(\mathbf{r},t)=\frac{\rho_L}{c_L}\sum_{\mathbf{q}_{\|}}\beta_{\mathbf{q}}A_{\mathbf{q}}\frac{K}{K_z}\mathrm{e}^{i\mathbf{q}_{\|}\boldsymbol{\tau}}\mathrm{e}^{-i\Omega t}\mathrm{e}^{iK_z z}, \tag{14}$$

where $K=(\mathbf{K}_{\|}^2+K_z^2)^{1/2}$ is absolute value of the wave number the of the wave radiated from solid.

After carrying out the second quantization procedure in a solid [5, 9], we obtain that the operator $\hat{V}_{BZ}$ is defined by the operators of creation and annihilation of phonons in solid $\hat{b}_{\mathbf{q}}^{+}$ и $\hat{b}_{\mathbf{q}}$.

$$\hat{V}_{Bz}=\sum_{\mathbf{q}_{\|}}\beta_{\mathbf{q}}\left[\hat{b}_{\mathbf{q}}-\hat{b}_{-\mathbf{q}}^{+}\right]A\,\mathrm{e}^{i\mathbf{q}_{\|}\boldsymbol{\tau}}. \tag{15}$$

The quantization procedure used to obtain relation (15) takes into account, in particular, the fact that a solid occupies half of the space, as well as the extremely small impedance of the solid–superfluid helium interface. Relation (15) was first obtained in [9] and takes into account the presence of only one type of phonons in a solid.

Operator (15) was used in [9] to describe all types of interactions containing two or three types of phonons. In [10, 11], it was shown that the main contribution to the heat transfer is made by the process in which one phonon incident on the boundary of a solid is converted into one phonon (elastic process) or two phonons (the first inelastic process) of helium. For this reason, in the article, we will restrict ourselves to taking into account only these two processes. In this regard, in the operator of the velocity of oscillations of the boundary (15), we restrict ourselves to taking into account one operator of annihilation of the phonon of a rigid body incident on the boundary:

$$\hat{V}_{Bz}=i\sum_{\mathbf{q}_{\|}}\beta_{\mathbf{q}}A_{\mathbf{q}}\hat{b}\mathrm{e}^{i\mathbf{q}_{\|}\boldsymbol{\tau}}. \tag{16}$$

To take into account different polarizations of the phonons of a solid, it is necessary to use the expressions for the vibration amplitudes $A_{\mathbf{q}}^{(l)}$ and $A_{\mathbf{q}}^{(t)}$ of the wall surface velocity when longitudinal and transverse phonons incident on it:

$$\hat{V}_{Bz}^{(l.t)}=i\sum_{\mathbf{q}_{\|}}\beta_{\mathbf{q}}A_{\mathbf{q}}^{(l,t)}\hat{b}\mathrm{e}^{i\mathbf{q}_{\|}\boldsymbol{\tau}}, \tag{17}$$

where the creation and annihilation operators refer to phonons of the corresponding polarization. Amplitudes $A_{\mathbf{q}}^{(l)}$ are $A_{\mathbf{q}}^{(t)}$ presented in [5] and correspond to the process of longitudinal phonon incidence with subsequent reflection of the longitudinal and transverse phonons

$$A_{\mathbf{q}}^{(l)}=\frac{c_l^2\cos\theta\cos 2\theta_t}{c_t^2\sin 2\theta_t\sin 2\theta+c_l^2\cos^2 2\theta_t}, \tag{18}$$

as well as to the process of transverse phonon incidence with the subsequent reflection of the longitudinal and transverse phonons

$$A_{\mathbf{q}}^{(t)}=\frac{c_t^2\cos\theta\sin 2\theta_l}{c_t^2\sin 2\theta_l\sin 2\theta+c_l^2\cos^2 2\theta}. \tag{19}$$

As a result, for velocity $\hat{\mathbf{v}}_i$ and density $\hat{\rho}_i$ operators of forced vibrations inside helium, we obtain the following expressions:

$$\hat{\mathbf{v}}_i(\mathbf{r},t)=\sum_{\mathbf{q}_{\|}}\beta_{\mathbf{q}}A_{\mathbf{q}}^{(l,t)}\hat{b}_{\mathbf{q}}\frac{\mathbf{K}}{K_z}\mathrm{e}^{i\mathbf{q}_{\|}\boldsymbol{\tau}}\mathrm{e}^{iK_z z}, \tag{20}$$

$$\hat{\rho}_i(\mathbf{r},t)=\frac{\rho_L}{c_L}\sum_{\mathbf{q}_{\|}}\beta_{\mathbf{q}}A_{\mathbf{q}}^{(l,t)}\hat{b}_{\mathbf{q}}\frac{K}{K_z}\mathrm{e}^{i\mathbf{q}_{\|}\boldsymbol{\tau}}\mathrm{e}^{iK_z z} \tag{21}$$

which we will be used to determine the interaction Hamiltonian.

At the next stage, we quantize the phonon field of internal perturbations of superfluid helium. The procedure for such a quantization was carried out in [9] and takes into account the fact that helium occupies a half-space, as well as the presence of a fixed impenetrable boundary.

$$\hat{\rho}=\frac{\rho_L}{c_L}\sum_{\mathbf{k}}\alpha_k\left(\hat{a}_{\mathbf{k}}+\hat{a}_{-\mathbf{k}}^{+}\right)\left(\frac{\mathrm{e}^{ik_z z}+\mathrm{e}^{-ik_z z}}{2}\right)\mathrm{e}^{i\mathbf{k}_{\|}\boldsymbol{\tau}}, \tag{22}$$

$$\hat{v}_z=\sum_{\mathbf{k}}\alpha_k\frac{k_z}{k}\left(\hat{a}_{\mathbf{k}}-\hat{a}_{-\mathbf{k}}^{+}\right)\left(\frac{\mathrm{e}^{ik_z z}-\mathrm{e}^{-ik_z z}}{2}\right)\mathrm{e}^{i\mathbf{k}_{\|}\boldsymbol{\tau}}, \tag{23}$$

$$\hat{\mathbf{v}}_{\|}=\sum_{\mathbf{k}}\alpha_k\frac{\mathbf{k}_{\|}}{k}\left(\hat{a}_{\mathbf{k}}-\hat{a}_{-\mathbf{k}}^{+}\right)\left(\frac{\mathrm{e}^{ik_z z}+\mathrm{e}^{-ik_z z}}{2}\right)\mathrm{e}^{i\mathbf{k}_{\|}\boldsymbol{\tau}}, \tag{24}$$

where the normalization constant $\alpha_{\mathbf{k}}=\sqrt{\hbar\omega/\rho_L V_L}$.

3. Hamiltonian of interaction and heat flux in elastic processes

To determine the Hamiltonian that describes the elastic process of phonon transformation at the boundary, we use expression (6) for the energy density, which contains the quadratic terms in the deviation of the velocity and density from the equilibrium values:

$$\hat{H}_2 = \rho_L \hat{\mathbf{v}}\hat{\mathbf{v}}_i + \frac{c_L^2}{\rho_L}\hat{\rho}\hat{\rho}_i. \tag{25}$$

Using relations (20)–(24) from Eq. (25), after integration over the volume of the liquid, we obtain the Hamiltonian, which describes the transformation of a solid body phonon into a helium phonon:

$$\hat{H}_{\text{el}} = \sqrt{\frac{\rho_L V_L}{\rho_S V_S}} \sum_{\mathbf{q}_{\|}} \sqrt{\hbar\Omega\hbar\omega}\,\hat{a}_{\mathbf{k}}^{+}\hat{b}_{\mathbf{q}} A_{\mathbf{q}} \frac{K}{K_z}. \tag{26}$$

Such a Hamiltonian differs from the Hamiltonian used in [7, 8] and, at the same time, gives the correct result for the elastic process obtained in the Khalatnikov theory [5]. The heat flux in the proposed approach is determined by the general quantum-mechanical formula:

$$W = \int w \sum_f E_f \cos\theta_f \prod_f \left(1 + n(E_f)\right) d\Gamma_f \prod_i n(E_i) d\Gamma_i. \tag{27}$$

Here w is the probability of the transition of phonons from the initial state to the final state per unit time through the unit of surface area, E_f and E_i are the total phonon energies in the final and initial states, respectively. The value of the probability w of the transition from the initial state to the final state is determined by the relation

$$w_k = \frac{2\pi}{\hbar S}\left|M_{fi}\right|^2 \delta\left(E_f - E_i\right), \tag{28}$$

where the matrix element of the Hamiltonian (26) of the transition from the initial to the final state is used:

$$M_{fi}^{(\text{el})} = \left\langle f \left| \hat{H}_{\text{el}} \right| i \right\rangle. \tag{29}$$

When calculating (29), the final state is normalized to the value LK_z, where L is the longitudinal size of helium, and K_z is the longitudinal component of the wave vector of the radiation field of an oscillating surface in helium (12). The final result for the heat flux (27) for the process under consideration has the form:

$$W_{\text{el}} = c_L \rho_L \int \left|A_{\mathbf{q}}\right|^2 \frac{\hbar\Omega}{2\rho_S} n_S d\Gamma_S, \tag{30}$$

which coincides with the result of the acoustic theory [5].

4. Hamiltonian of interaction and heat flux in an inelastic process

To determine the Hamiltonian, which describes the inelastic process of transformation of one phonon of a solid into two phonons of a liquid at the interface, we use expression (7) for the energy density. This expression contains cubic terms in the deviation of velocity and density from equilibrium values, and those in which one of the factors corresponds to a solid, and two to helium:

$$\hat{H}_3 = \frac{1}{2}\left(\hat{\rho}\hat{\mathbf{v}} + \hat{\mathbf{v}}\hat{\rho}\right)\hat{\mathbf{v}}_i + \frac{1}{2}\left(\hat{\rho}_i\hat{\mathbf{v}}_i + \hat{\mathbf{v}}_i\hat{\rho}_i\right)\hat{\mathbf{v}} + $$
$$+ \frac{1}{2}\left(\hat{\rho}\hat{\mathbf{v}}_i^2 + \hat{\rho}_i\hat{\mathbf{v}}^2\right) + \frac{(2u-1)c_L^2}{2\rho_L^2}\hat{\rho}\hat{\rho}_i\left(\hat{\rho} + \hat{\rho}_i\right). \tag{31}$$

In this expression, a symmetrization procedure is carried out to take into account the non-commutativity of operators.

Using relations (20)–(24) and Eq. (30), after integration over the volume of the liquid, we obtain the Hamiltonian, which describes the transformation of a phonon of a solid into two phonons of helium:

$$\hat{H}_{\text{el}} = \frac{\hbar^{3/2}}{c_L\sqrt{\rho_S V_S}} \sum_{\mathbf{q},\mathbf{k}1,\mathbf{k}2} \sqrt{\Omega\omega_1\omega_2}\,\hat{a}_{\mathbf{k}1}^{+}\hat{a}_{\mathbf{k}2}^{+}\hat{b}_{\mathbf{q}} A(\mathbf{q},\mathbf{k})\delta(k_{1z} + k_{2z} + K_z)\delta(\mathbf{k}_{1\|} + \mathbf{k}_{2\|} + \mathbf{q}_{\|}), \tag{32}$$

where $\omega_{1,2}$ and Ω are respectively, the phonon frequencies of helium and solid, $\mathbf{k}_{1,2}$ and $\mathbf{q}$ are the phonon wave vectors of the helium and the solid, respectively; amplitudes $A(\mathbf{q},\mathbf{k})$ are equal to

$$A(\mathbf{q},\mathbf{k}) = A_{\mathbf{q}}\left\{2u - 1 + \frac{k_{1z}}{k_1} + \frac{k_{2z}}{k_2} + \frac{\mathbf{k}_1}{k_1}\frac{\mathbf{k}_2}{k_2}\right\}, \tag{33}$$

and it is used the abbreviation

$$\delta(n) = \delta_{n,0}. \tag{34}$$

The calculation of the heat flux from a solid with a temperature T_S to liquid helium with a zero temperature is carried out based on relations (27)–(29) and gives the following expression:

$$W_{\text{inel}} = \int \frac{\hbar k_1 k_{2z}}{k_{1z} + k_{2z}} \left|A(\mathbf{q},\mathbf{k})\right|^2 \frac{\hbar\Omega}{2\rho_S} n_S d\Gamma_S d\Gamma_L. \tag{35}$$

The boundaries of integration in the phase space $d\Gamma_L$ of the phonon momentum in helium are determined from the laws of conservation of energy and the tangential component of the phonon momentum. The features of the angular distributions of the interacted phonons are considered in more detail in [11].

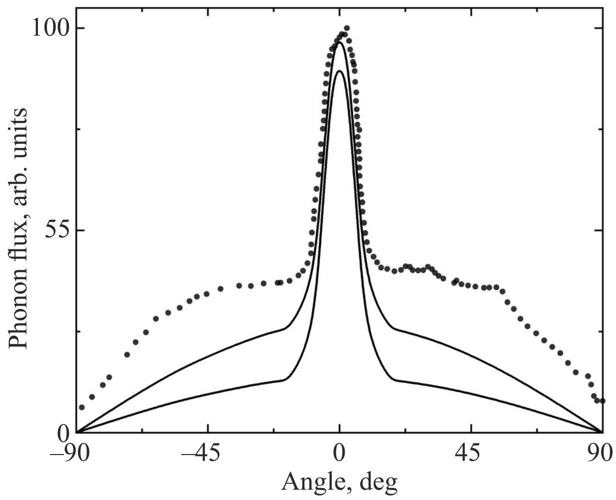

Fig. 1. Angular distribution of phonons emitted into superfluid helium by a heated solid. Points represent the experimental data, the upper solid line corresponds to the temperature $T_S = 4$ K, the lower one — $T_S = 3$ K.

In contrast to the elastic process in the considered inelastic process, the phonons that have radiated in helium will move in all directions to the surface. In this case, in the angular distribution of phonons, along with a sharp acoustic peak due to the elastic process, phonons emitted in all directions should also be observed. Such an angular distribution of phonons was detected in experiments [6, 12–15], where two separated channels of phonon emission by a heated solid into cold (T < 100 mK) superfluid helium were observed.

The result for the heat flux was first obtained in [11] for the case of incidence of longitudinal phonons of solid helium on the boundary. Relationship (35) allows one to take into account the contribution of other types of excitations of solids, in particular, transverse phonons or surface waves.

Figure 1 shows the angular distribution of phonons emitted into helium. The solid line corresponds to theoretical calculations using formulas (30) and (35) for certain heater temperatures. Such a high heater temperature, which exceeds the superfluid transition temperature, is explained by the fact that it is not the equilibrium temperature of the solid, but is the instantaneous effective temperature of the metal film heater through which a short electric current pulse passes.

Conclusion

The paper reviewed the quasiparticle model of heat transfer from a solid to superfluid helium and considered inelastic phonon interactions at the interface between these two media. In the frame of this model, the interaction Hamiltonian for elastic (26) and inelastic (32) processes that give the main contribution for heat flow are derived. The obtained expressions take into account the contribution of different types of thermal excitations in solid — longitudinal and transverse phonons and surface waves. From the derived Hamiltonian, the respective heat flows were calculated. The result for the elastic process (30) coincides with the result of the acoustic theory and the result for the heat flow caused by the inelastic processes (35) allows to describe the wide angular distribution of emitted phonons, which is observed in experiments.

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Фононна модель теплового випромінювання твердим тілом з плоскою поверхнею в надплинний гелій

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Проведено огляд квазічастинкової моделі надплинного гелію та її застосування до опису теплообміну між нагрітим твердим тілом та надплинним гелієм. При цьому розглядається задача, в якій поверхня нагрівача є абсолютно плоскою, а гелій знаходиться при практично нульовій температурі. У цих умовах теплообмін між твердим тілом та надплинним гелієм визначається трансформацією фононів твердого тіла в фонони гелію. Розглядено певні види такої трансформації — пружні процеси перетворення фононів, в яких кількість фононів зберігається, та непружні процеси, в яких кількість фононів змінюється. Основну увагу приділено розробці квантово-механічного підходу для обчислення внеску фононів твердого тіла різних поляризацій у формування потоку тепла, його величини та кутового розподілу. Результати роботи застосовуються для пояснення спостережених в експерименті особливостей перенесення тепла від нагрітого твердого тіла в надплинний гелій.

Ключові слова: стрибок Капіци, тепловий потік, нееластичні процеси.